Computational Performance of the Current-voltage (IV) Linearization of the ACOPF: Before Extensions

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Views expressed are not necessarily those of the Commission.

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Overview

- Motivations and Challenges
- Successive Linear Program (SLP)
- Preliminary Results
- Current Work
- Discussion

Motivations and Challenges

- **Changing Energy Landscape**
- **Operational and Planning Challenges**
- **Co-Optimize for Market Efficiency and Security**

ACOPF Feasible Region for a Three-Bus Network

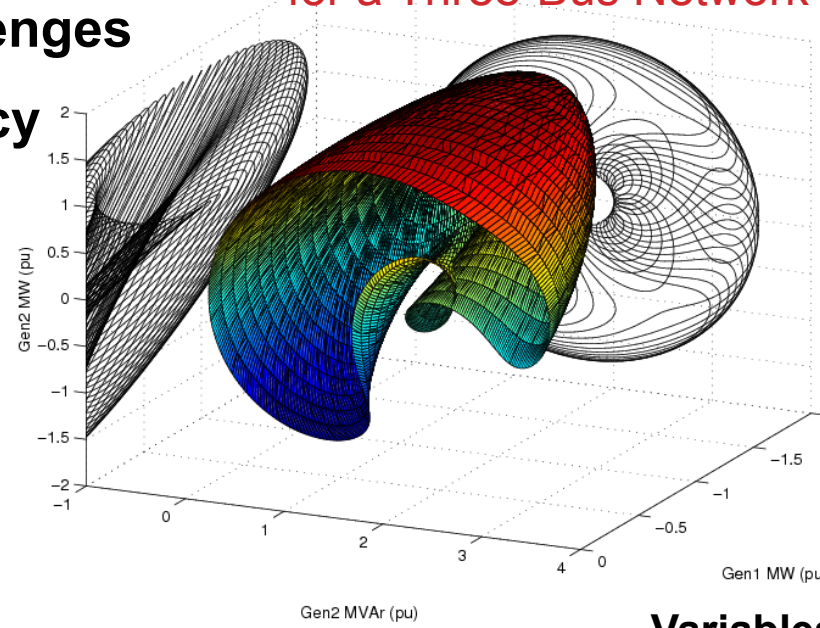


Image Source: Hiskens, 2001.

ACOPF
Co-optimizes Real and Reactive Power

- **Carpentier (1962)**
- **Nonconvex, NP-Hard**
- **Multiple Local Optima**

Variables:

P (Real Power)

Q (Reactive Power)

I (Current Phasor)

V (Voltage Phasor)

ACOPF Formulation in Rectangular Coordinates

$$\text{Min} \sum_n c(P_n)$$ — Cost function

s.t.

$$P_n^{\min} \le P_n = V_n^r I_n^r + V_n^j I_n^j \le P_n^{\max}, P_n = P_n^G - P_n^L, \forall n$$

$$Q_n^{\min} \le Q_n = V_n^j I_n^r - V_n^r I_n^j \le Q_n^{\max}, Q_n = Q_n^G - Q_n^L, \forall n$$

— Nodal n Power Limits

$$I = YV$$ — Network Balancing

$$\left(V_n^{\min}\right)^2 \le \left|V_n\right|^2 = V_n^r V_n^r + V_n^j V_n^j \le \left(V_n^{\max}\right)^2, \forall n$$ — Nodal n Voltage Limits

$$\left|I_k\right|^2 = I_k^r I_k^r + I_k^j I_k^j \le \left(I_k^{\max}\right)^2, \forall k$$ — Line k Current Limits

$$\left|\theta_n - \theta_m\right| \le \theta_{nm}^{\max}, \forall nm$$ — Voltage Angle Differences

where

$$V_n = \left|V_n\right| \angle \theta_n = V_n^r + \mathbf{j} V_n^j \qquad I_n = I_n^r + \mathbf{j} I_n^j$$

$$\theta_n = \tan^{-1}\left(V_n^j / V_n^r\right) \qquad I_k = I_k^r + \mathbf{j} I_k^j$$

SLP IV-ACOPF Linear Approximation

$$\text{Min } c(P) + penalty(P, Q, I, V)$$

s.t. $P^{\min} \le P \approx \underline{V}^r I^r + \underline{V}^j I^j + V^r \underline{I}^r + V^j \underline{I}^j - \underline{V}^r \underline{I}^r - \underline{V}^j \underline{I}^j \le P^{\max}$

$Q^{\min} \le Q \approx \underline{V}^j I^r - \underline{V}^r I^j + V^j \underline{I}^r - V^r \underline{I}^j - \underline{V}^j \underline{I}^r + \underline{V}^r \underline{I}^j \le Q^{\max}$

$I = YV, \quad P = P^G - P^L, \quad Q = Q^G - Q^L$

Voltage Pre-processed Polygon Cuts & Infeasibility Cuts

Current Pre-processed Polygon Cuts & Infeasibility Cuts

Step-Size Limit§ $\left| V^r - \underline{V}^r \right| \le aV^{\max} / k^b$

$\left| V^j - \underline{V}^j \right| \le aV^{\max} / k^b$

Iteration $k = k + 1$, do a fixed point update:

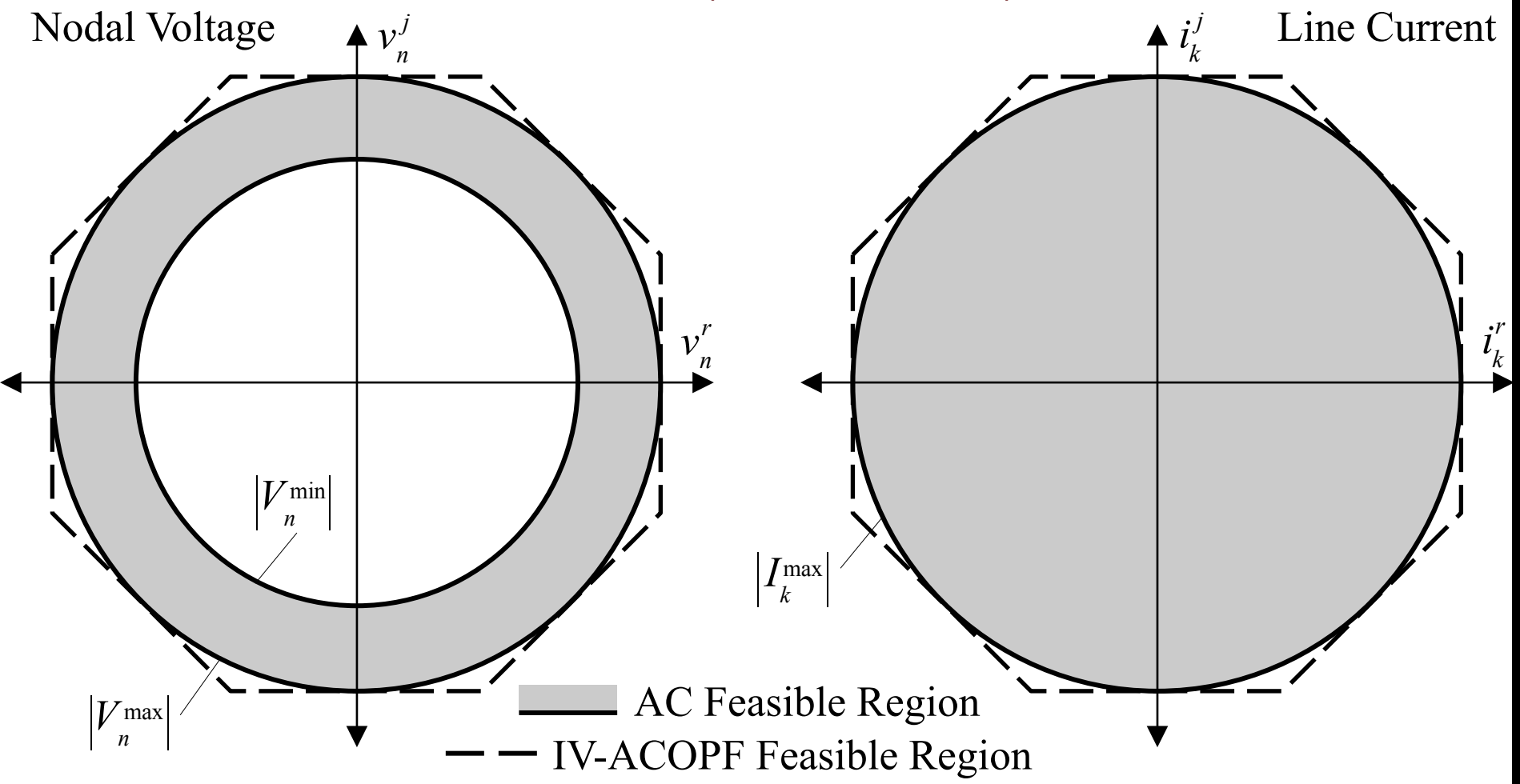
$$\underline{P}^{(k+1)} = P^*, \underline{Q}^{(k+1)} = Q^*, \underline{I}^{(k+1)} = I^*, \underline{V}^{(k+1)} = V^*$$

Source: O'Neill, Castillo & Cain (2013) FERC

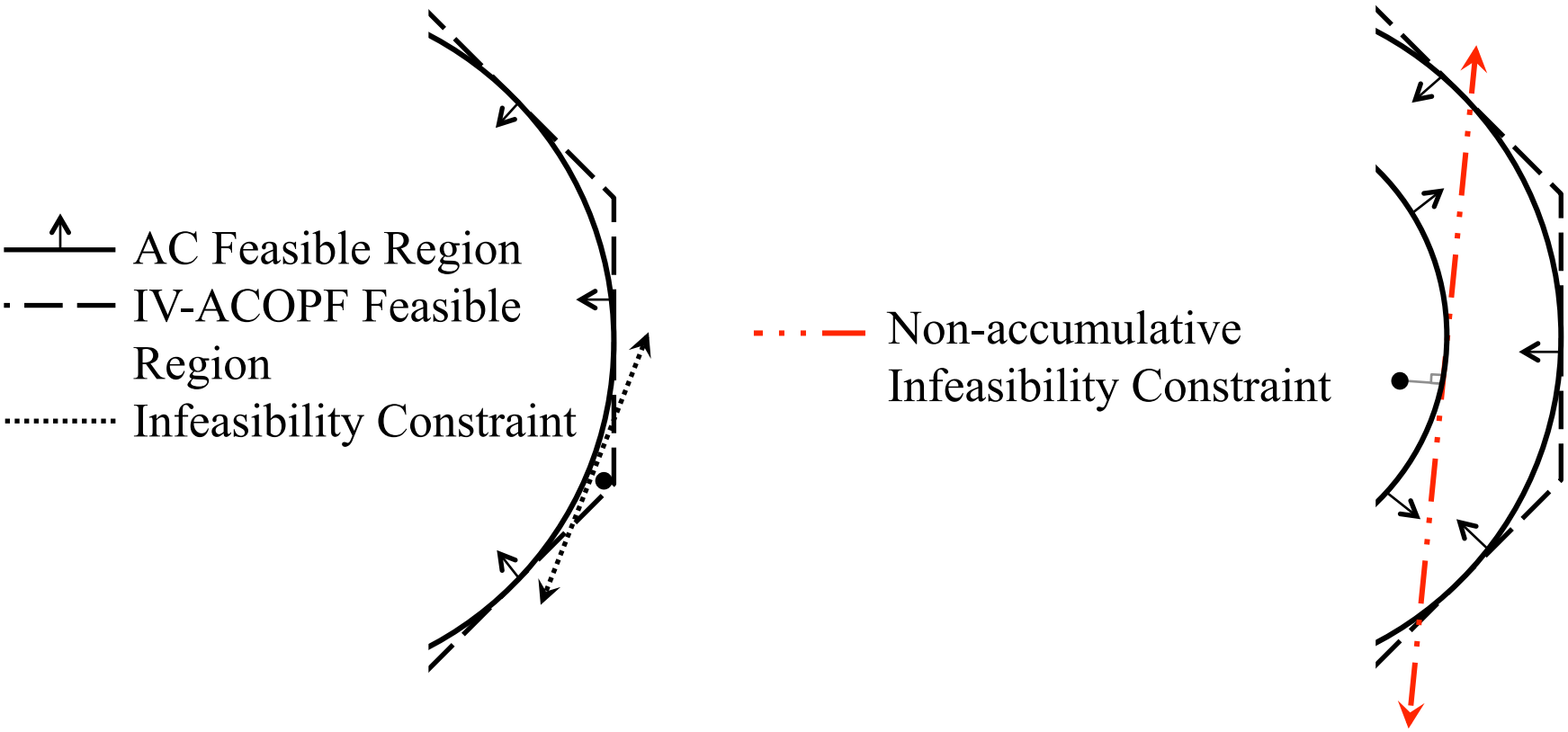
§ Source: Campaigne et al. (2013) FERC

SLP IV-ACOPF
Pre-processed Polygon Cuts

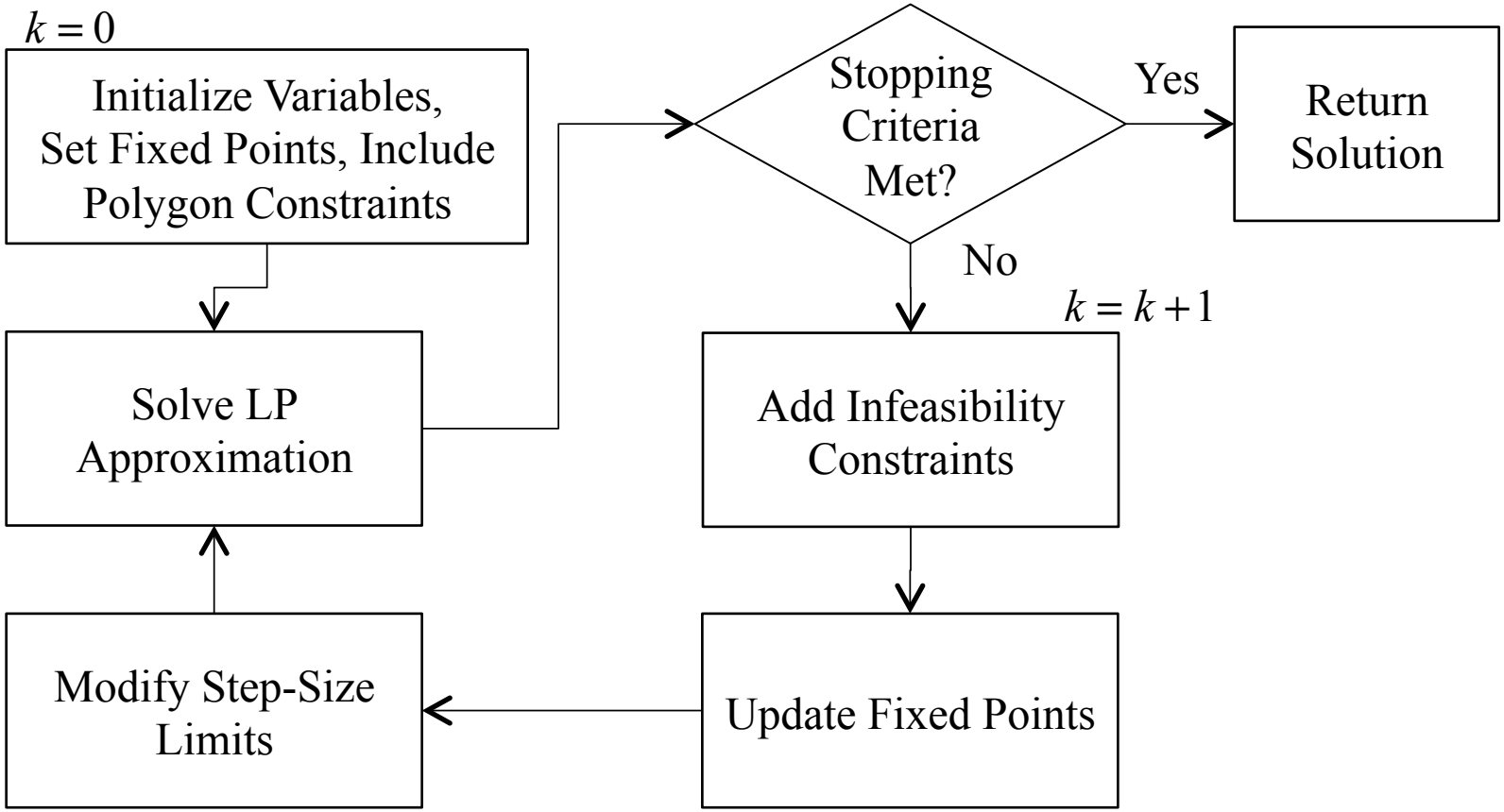
N-SIDED (N=8 Shown here)



SLP IV-ACOPF Infeasibility Cuts



SLP IV-ACOPF Solution Technique



Step-size Limit

$$P^{\text{actual}} = V^r I^r + V^j I^j$$ ACOPF

$$P^{\text{approx}} = \underline{V}^r I^r + \underline{V}^j I^j + V^r \underline{I}^r + V^j \underline{I}^j - \underline{V}^r \underline{I}^r - \underline{V}^j \underline{I}^j$$ SLP

- **At each iteration *k*, solve the SLP to determine:**

$$ratio^{(k)} = 1 - \frac{c\left(P^{\text{approx}}\right)}{c\left(P^{\text{actual}}\right) + penalty\left(P^{\text{actual}}\right)}$$

$$\boxed{\begin{array}{l} \text{if } ratio^{(k)} \le 0.01: \\ \quad \beta = 3,\ \alpha = 1/\beta \\ \text{else if } ratio^{(k)} \le 0.1: \\ \quad \beta = 2,\ \alpha = 1/\beta \\ \text{else if } ratio^{(k)} \ge 0.75: \\ \quad \beta = 1,\ \alpha = 3\beta/2 \\ \text{else}: \\ \quad \beta = 1.5,\ \alpha = 2\beta/3 \end{array}}$$

- **For iteration $k = k + 1$, update constraints:**

$$\left| V^r - \underline{V}^r \right| \le \alpha V^{\max} / k^{\beta}$$

$$\left| V^j - \underline{V}^j \right| \le \alpha V^{\max} / k^{\beta}$$

Note: Convergence extremely sensitive to α and β.

Stopping ('Convergence') Criteria

$$P^{\text{actual}} = V^r I^r + V^j I^j$$ ACOPF

$$P^{\text{approx}} = \underline{V}^r I^r + \underline{V}^j I^j + V^r \underline{I}^r + V^j \underline{I}^j - \underline{V}^r \underline{I}^r - \underline{V}^j \underline{I}^j$$ SLP

- **Optimality:**
  - Mismatch error (feasibility tolerance) for each bus is:

$$e_n^P = \left| P_n^{\text{actual}} - P_n^{\text{approx}} \right| \le 10e-4$$

$$e_n^Q = \left| Q_n^{\text{actual}} - Q_n^{\text{approx}} \right| \le 10e-4$$

  - OR mismatch error for the network is:

$$\sum_n e_n^P + e_n^Q \le 50e-4$$

(Model normalized to p.u. by 100 MVA)

- **Termination:**
  - Iteration limit:

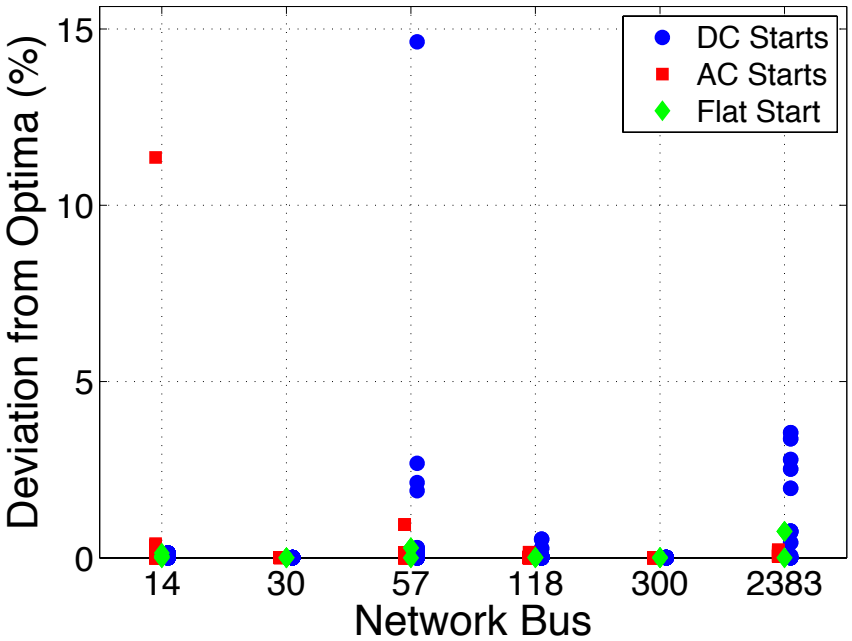
$$k \ge MAX_ITERATION_LIMIT = 100$$

Test-bed Specs

- **Networks:**
  - 14, 30, 57, 118, 300, and 2383-bus networks
- **Initialization Technique:**
  - Flat Start
  - DC-OPF Hot Start (10 Samples)
  - AC-OPF Hot Start (10 Samples)
- **Transmission Constraints:**
  - Tight $\left|I_{ni\ell}\right| \le S_{ni\ell}^{\max} / V_{ni\ell}^{\max}$, Loose $\left|I_{ni\ell}\right| \le S_{ni\ell}^{\max} / V_{ni\ell}^{\min}$
- **Performance results for 16-sided polygon constraints and 10-segment piecewise linear cost function§**
- **Solvers: Gurobi 5.5, Cplex 12.5, Ipopt 3.11.4**
- **Software Platform: Python 2.7, Coopr 3.2 (Sandia)**

Optimal Solution of the SLP

Quality Index (QI)			
Network Buses	Gurobi SLP	Cplex SLP	IPOPT NLP
14	1.03	1.03	1.02
30	1.03	1.03	1.03
57	1.03	1.02	1.02
118	1.01	1.01	1.01
300	1.02	1.02	1.24
2383	1.05	1.05	1.04



$$QI = \sum_{n \in N} \left[\frac{\text{cost}_n - \text{penalty}_n}{\text{cost for SDP}^{\S}} \right] \Big/ N$$

for N simulations

Stable convergence from various types of starting points

§ Globally Optimal Solution with Zero-Duality Gap

Hardware Matters§

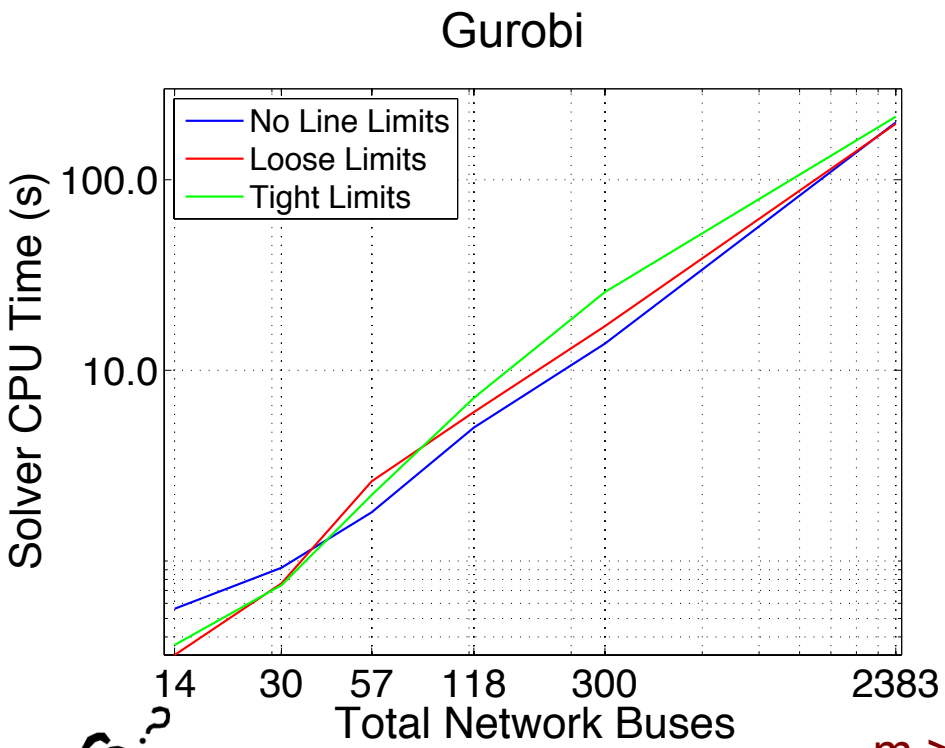
§ software implementation matters also, but comparisons not implemented

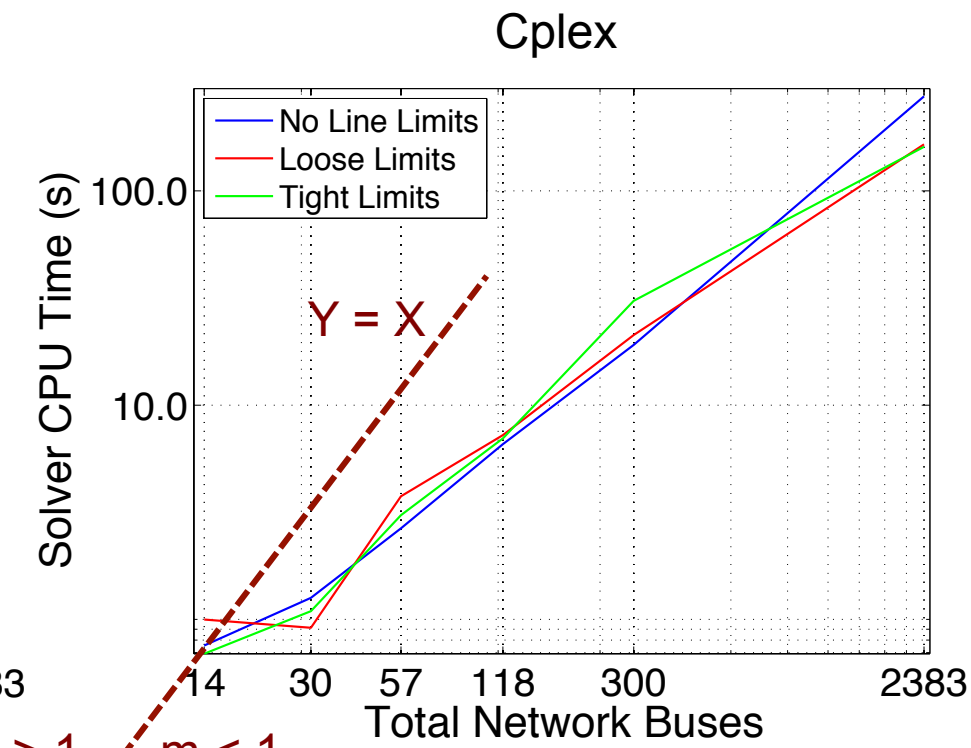
	32-BIT WORKSTATION*			64-BIT WORKSTATION**		
Network Buses	Gurobi 4.0	Cplex 12.2	Ipopt 3.8 (w/ MUMPS)	Gurobi 4.0	Cplex 12.2	Ipopt 3.8 (w/ MUMPS)
14	0.02 sec	0.06 sec	0.45 sec	0.02 sec	0.06 sec	0.30 sec
30	0.17	0.30	0.76	0.05	0.10	0.63
57	0.28	0.15	2.72	0.08	0.11	0.83
118	1.89	1.89	9.08	0.49	0.57	1.54
300	16.2	9.9	16	1.59	1.19	3.42

* Intel CoreTM i5-540M (2.53GHz), 4GB RAM

** Intel CoreTM i7-3520M (2.90GHz), 8GB RAM

Stable Performance of the SLP In Congestion Scenarios





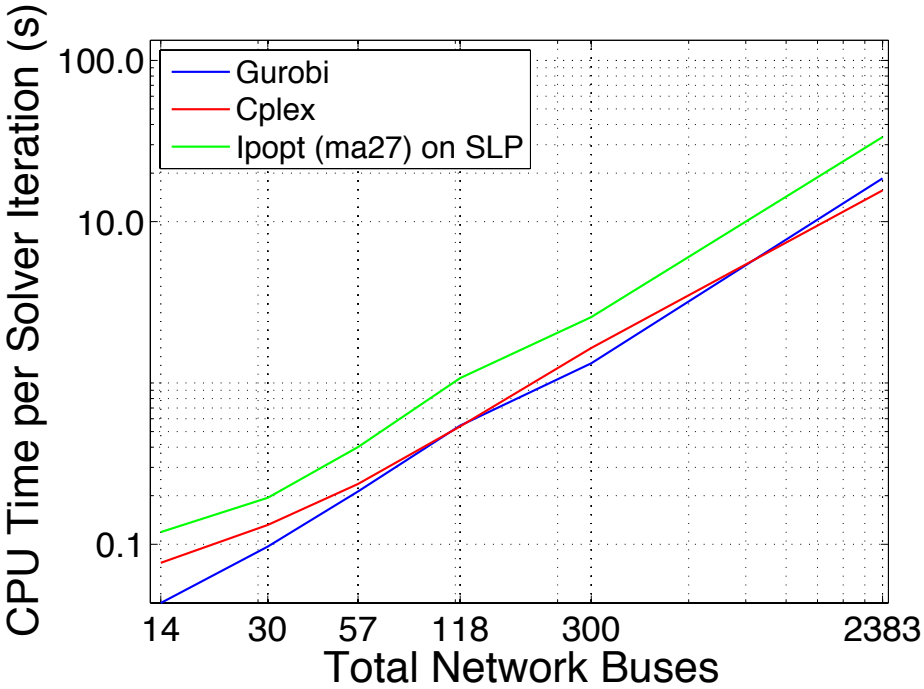
m > 1 m < 1

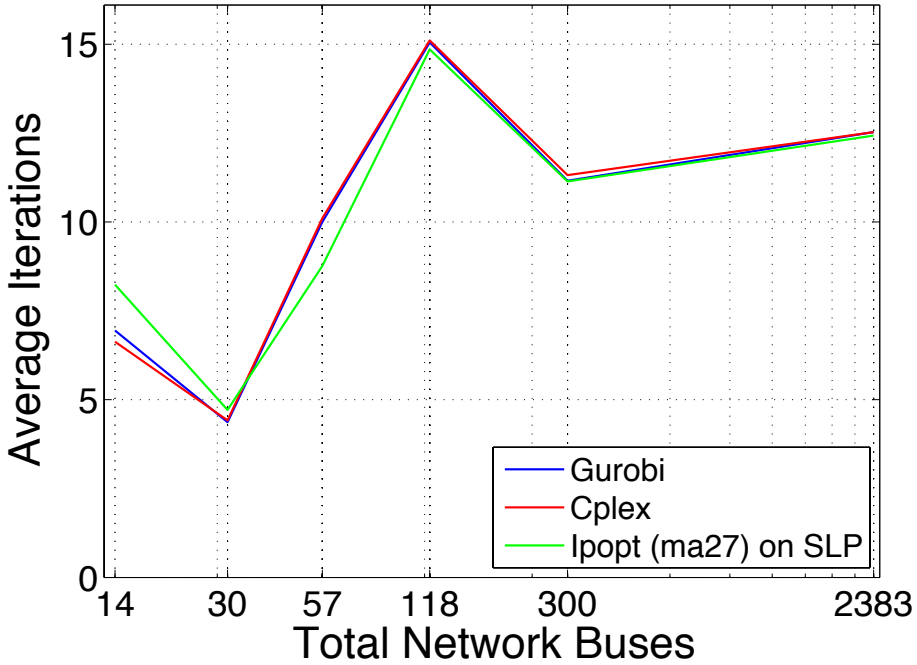
Log-Log Plot Refresher:

- Power Function $Y = bX^m$
- Linear Relationship → slope m = 1
- Higher Order Relationship → m > 1

Workstation: 1.8 GHz Intel Core i7, 4 GB RAM

SLP Iterative Performance





- **Performance per iteration is less than linear**
- **Number of iterations increase dramatically with larger networks**

Workstation: quad-core Xeon 2.7 GHz processors, 512GB RAM

Room for Improvement?

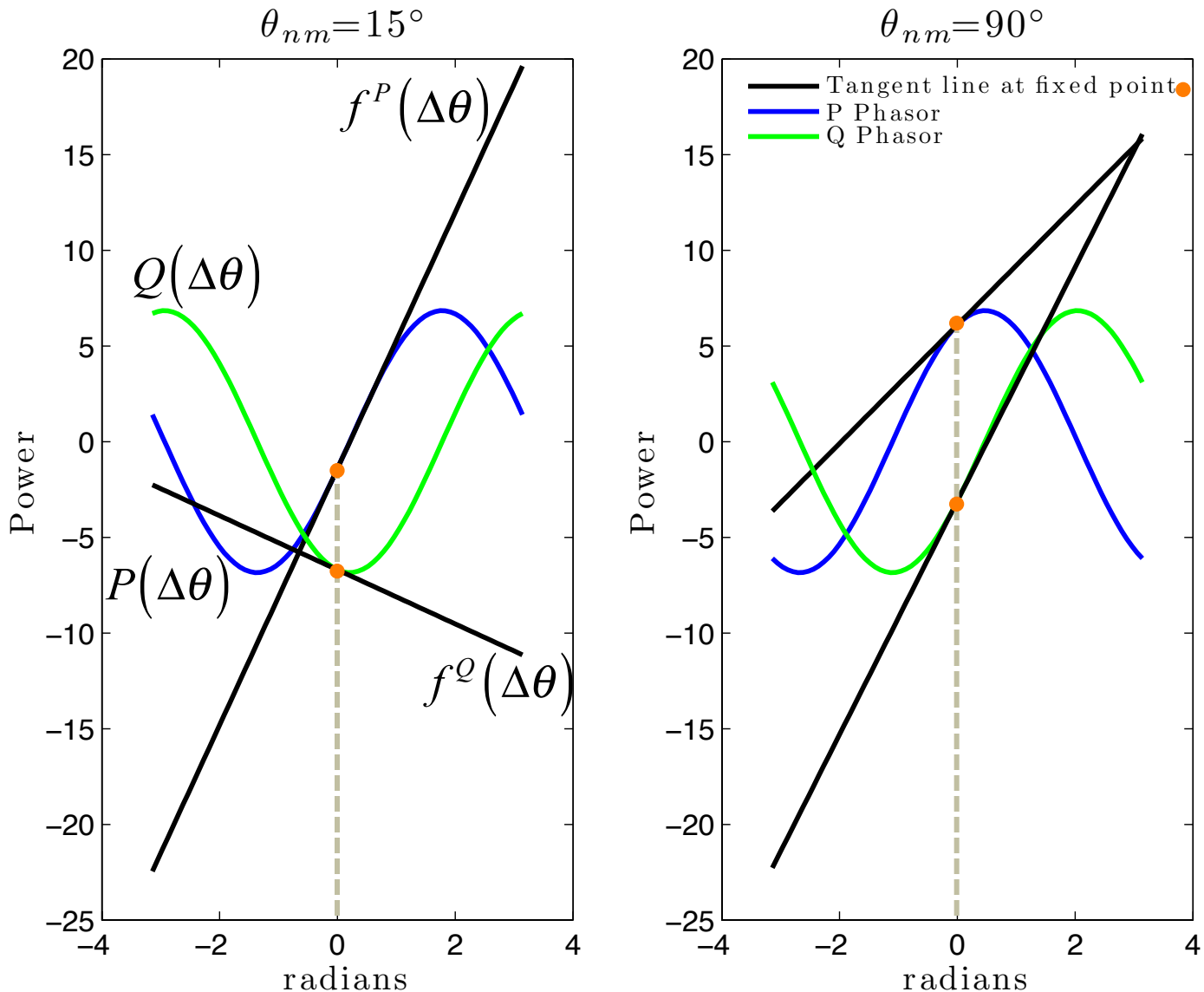
- **Parameterization tuning**
  - Step-size limits and stopping criteria
  - Number of polygon constraints
  - Solver parameterization
- **Hardware and software improvements**
  - 32-bit v. 64-bit, memory, cache, processor etc.
  - GAMS, Pyomo (Python), AIMMS or another application platform

Current Work…

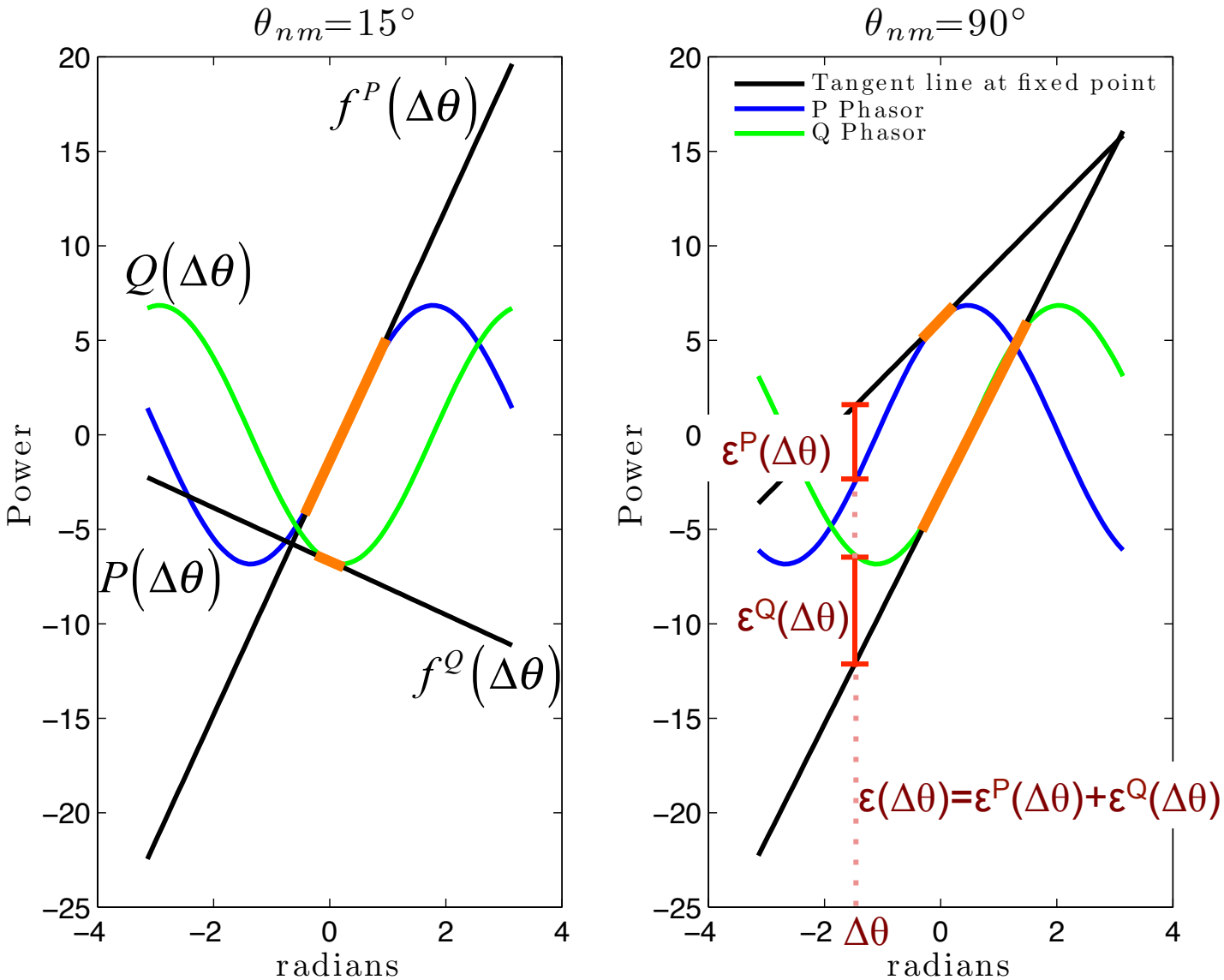
Adaptive Step-size Limits

We want to be able to guarantee that we will achieve some lower bound on the pre-determined accuracy in the linearization.

Example: First-order approximation of power line flows between n and m



Example: Error in power line flow as a function of change in angle difference $\Delta\theta$



Limiting the Approximation Error: Adaptive Step-size Limits

For any $\Delta\theta$ from the fixed point, we *could* calculate the total distance in error(s), i.e. the Euclidean norm:

$$\varepsilon(\Delta\theta) = \varepsilon^P(\Delta\theta) + \varepsilon^Q(\Delta\theta) = \sqrt{\left(P(\Delta\theta) - f^P(\Delta\theta)\right)^2 + \left(Q(\Delta\theta) - f^Q(\Delta\theta)\right)^2}$$

But we just want to know our error term in relation to the power flow.

Relative Error Factor:

For iteration k, we need a SLP IV-ACOPF constraint of the form

$$\left|\theta_{nm} - \underline{\theta}_{nm}\right| \le \Delta\theta^{(k)} \quad \text{where } \theta_{nm} = \tan^{-1}\left(v_n^j / v_n^r\right) - \tan^{-1}\left(v_m^j / v_m^r\right)$$

(linearize)

for the power flow from bus n to m. Given an error factor of $0 \le \eta < 1$, we solve

$$\eta = \min\left(\left|\frac{P(\Delta\theta) - f^P(\Delta\theta)}{P(\Delta\theta)}\right|, \left|\frac{Q(\Delta\theta) - f^Q(\Delta\theta)}{Q(\Delta\theta)}\right|\right)$$

(N-R method)

to find the smallest absolute root $\Delta\theta^{(k)} = \Delta\theta > 0$ using the fixed point (optimal solution from iteration $k - 1$).

Advantages of the Adaptive Step-size Approach

Properties:

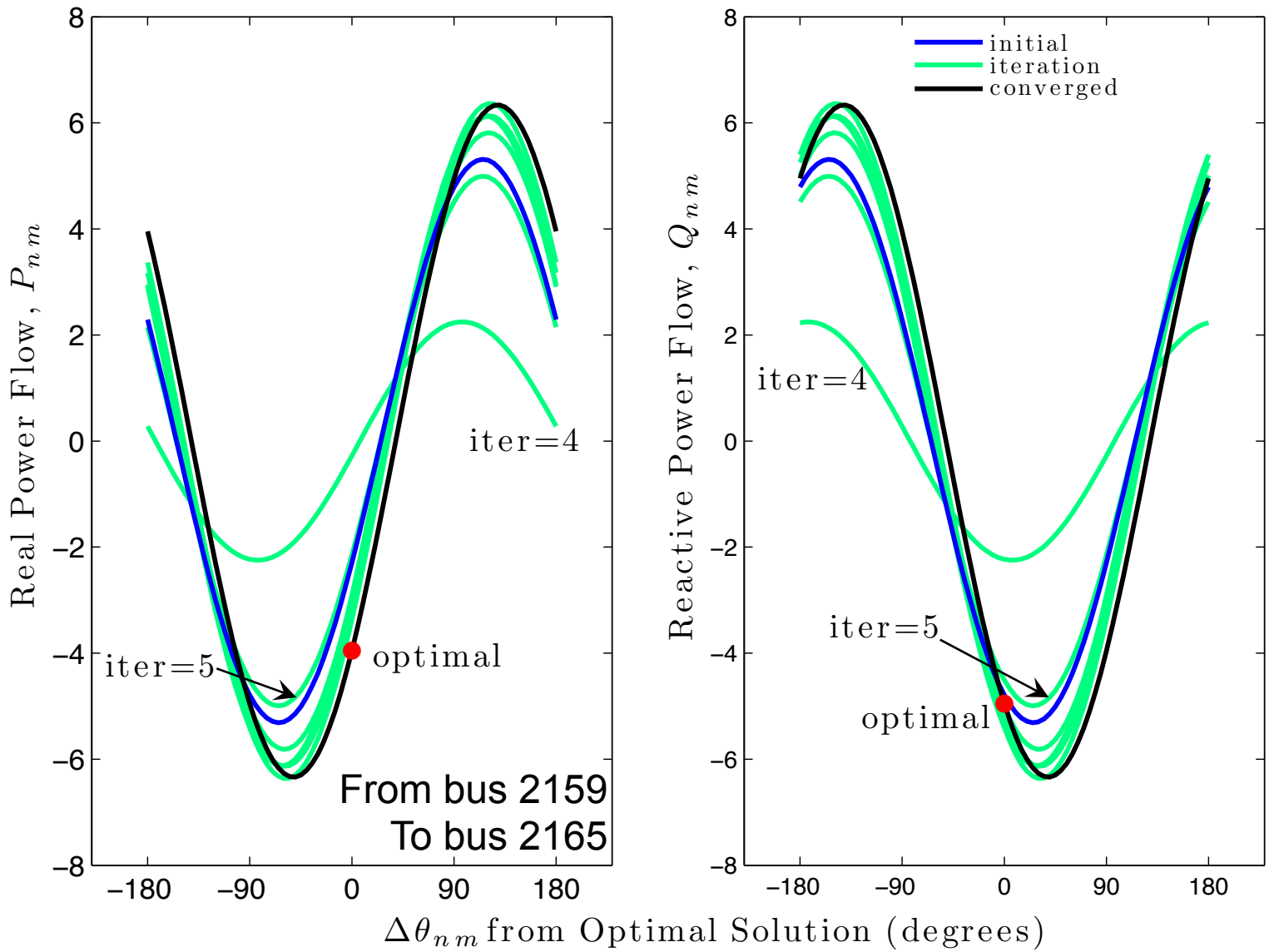
- $\Delta\theta$ **(between buses n and m) is invariant to the fixed points for the voltage magnitudes $|v_n|$ and $|v_m|$**
- $\Delta\theta$ **(between buses n and m) is dependent on line parameters (G_{nm}, B_{nm}) and the fixed point $\underline{\theta}_{nm}$**

Solution Technique:

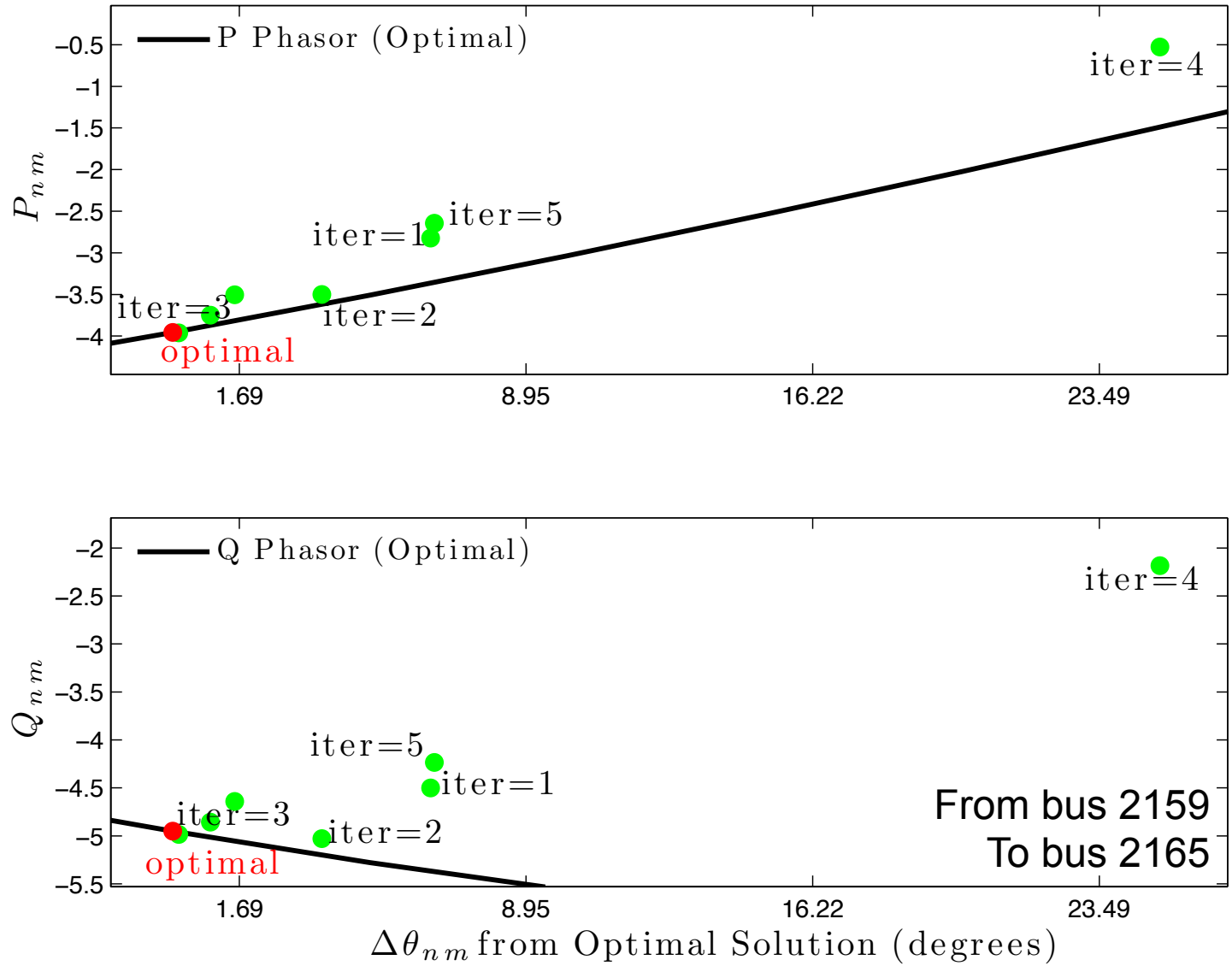
- **Pre-process the adaptive step-size limits (offline):**
  - Solve the smallest root $\Delta\theta$ for discrete combinations of $\underline{\theta}_{nm}$ and error η, where $-\pi \leq \underline{\theta}_{nm} \leq \pi$ and preferably $\eta < 0.1$ (e.g. no more than 10% error)
- **Look-up prior to solving each iteration k (online):**
  - Returns $\Delta\theta^{(k)}$ for given $\underline{\theta}_{nm}$ and η, i.e.

$$f^{look-up}\left(\underline{\theta}_{nm}^{(k)}, \eta\right) = \Delta\theta^{(k)}$$

Part I. Analyzing the Iterative F.P. for the 2383-bus



Part II. Analyzing the Iterative F.P. for the 2383-bus



Applying the Adaptive Step-size where η = 0.1 (10% Error)

- **2383-Network results for power flow between buses 2159 and 2165:**

Iteration k	$\lvert\Delta\theta^{(k)}\rvert$ LIMIT	$\lvert\theta_{nm} - \underline{\theta}_{nm}\rvert$	Valid Fixed Point?
1	N/A	0°	N/A
2	14.75°	2.76°	Yes
3	14.75°	2.82°	Yes
4	15.25°	24.07°	No
5	10.25°	18.39°	No
6	14.75°	5.06°	Yes
7 (Optimal)	15.25°	1.42°	Yes

- **The adaptive step-size limits remove 2 of the 7 SLP iterations**
- **Depending on how the error factor η is parameterized, the adaptive step-size can effectively address some of the solution zig-zagging**

Preliminary Results

- **The SLP converges to an optimal cost that is on average within 5% error from the globally optimal solution**
- **We note, *a less than linear* relationship between CPU time (*overall and per iteration*) and network size for the proposed SLP, even with thermal line limits**
- **Adaptive step-size can decrease the number of iterations**

Future Extensions

- **Comprehensive speed-up with the adaptive step-size limit**
- **Reliability Unit Commitment (RUC) with SLP IV-ACOPF**
  - Extend to realistic power systems
  - Applications in current market design
  - Incorporate decision-making processes (MILP formulation)
- **Corrective Switching with SLP IV-ACOPF (Paula Lipka)**
  - Stay tuned for next presentation

Thank You!

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Richard O'Neill, FERC

Jean-Paul Watson, Sandia National Laboratories

REFERENCE:

Castillo, A. et al. "The Current-Voltage (IV) Formulation of the AC Optimal Power Flow Problem" (*manuscript in preparation*, 2014)