Decomposable Formulation of Security Constraints for Power Systems Optimization and Optimal Energy Exchange

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Decentralized Optimization: Motivation

- **Power system optimization is still performed centrally**:
  - Economic Dispatch
  - Day-Ahead Unit Commitment
  - Transmission Expansion Planning
- **Limitations of centralized approach**:
  - Scalability and performance issues
  - Privacy and cybersecurity issues
  - Unsuitable for decentralized studies (e.g. energy exchange, coordinated congestion relief)
- **Decentralized approach**:
  - Problem is subdivided into smaller subproblems, solved independently
  - Adjustments are made until local solutions become compatible

Decentralized Optimization: Challenges

- **Challenge**: Power flow equations
  - Non-linear and non-convex; based on physical laws
  - Global effect: injection at any location may affect entire network
- **Commonly used DC power flow formulations**:
  - Phase-Angle Formulation
  - Injection Shift Factors Formulation
- Neither formulation is well suited for large-scale decentralized optimization

Phase-Angle Formulation

- n_b: net injection at bus $b \in B$
- θ_b: phase-angle at bus $b \in B$
- f_{uv}: flow in transmission line $(u, v) \in L$

$$
\begin{aligned}
& n_b + \sum_{u:(u,b)\in L} f_{ub} = \sum_{u:(b,u)\in L} f_{bu} && \forall b \in B \\
& f_{uv} = B_{uv} (\theta_u - \theta_v) && \forall (u, v) \in L \\
& -F_{uv} \le f_{uv} \le F_{uv} && \forall (u, v) \in L
\end{aligned}
$$

- Advantages: Constraints are local; easily decomposable
- Drawbacks:
  - Flow must be computed for the entire network
  - Does not scale well for large-scale systems or multiple topologies (N-1 security)

Injection Shift Factors (ISF) Formulation

- n_b: net injection at bus $b \in B$

$$\sum_{b \in B} n_b = 0$$

$$-F_l \leq \sum_{b \in B} \delta_{lb} n_b \leq F_l \qquad \forall l \in L$$

- Advantages:
  - Allows enforcing thermal limits on subset of transmission lines
  - Scales very well for large systems and multiple topologies (N-1 security)
- Drawbacks: Very dense constraints, not decomposable

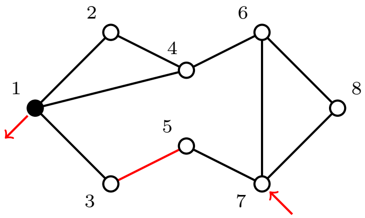
Our Contribution

- **New decomposable formulation of transmission and security constraints**:
  - Less dense, more decomposable than ISF
  - Allows selective enforcement of transmission limits
  - Scalable to large networks and multiple topologies (N-1 security)
  - Enables large-scale decentralized studies (e.g. optimal energy exchange)
- **Computational experiments**:
  - Multi-Zonal Security-Constrained Unit Commitment
  - Realistic, large-size test systems with up to 6,515 buses
  - All test cases solved reliably and efficiently
  - Previous methods fail to handle even smallest test cases

Injection Shift Factors: Intuition

- Suppose 1 MW of power is injected at bus b and withdrawn from slack bus
- δ_{lb} is the fraction of that power that flows through line l

	1	2	3	4	5	6	7	8
(1, 2)	0	-0.65	-0.06	-0.29	-0.12	-0.23	-0.19	-0.21
(1, 3)	0	-0.06	-0.81	-0.12	-0.62	-0.31	-0.44	-0.38
(1, 4)	0	-0.29	-0.12	-0.58	-0.25	-0.46	-0.38	-0.42
(2, 4)	0	0.35	-0.06	-0.29	-0.12	-0.23	-0.19	-0.21
(3, 5)	0	-0.06	0.19	-0.12	-0.62	-0.31	-0.44	-0.38
(4, 6)	0	0.06	-0.19	0.12	-0.38	-0.69	-0.56	-0.62
(5, 7)	0	-0.06	0.19	-0.12	0.38	-0.31	-0.44	-0.38
(6, 7)	0	0.04	-0.12	0.08	-0.25	0.21	-0.38	-0.08
(6, 8)	0	0.02	-0.06	0.04	-0.12	0.10	-0.19	-0.54
(7, 8)	0	-0.02	0.06	-0.04	0.12	-0.10	0.19	-0.46



- Other injections: linearity + superposition

Decomposed ISF: Intuition

- Let (L_1, L_2) be a partition of L and $(B_1, B_\cap, B_2)$ a suitable partition of B
- **Observation 1**: Each line may use a different slack bus.
- **Observation 2**: When computing flows in L_1, any net injection in buses B_2 can be simulated by an equivalent net injection in buses $B_\cap$.

	1	2	3	4	5	6	7	8
(1, 2)	0	-0.65	-0.06	-0.29	-0.12	-0.23	-0.19	-0.21
(1, 3)	0	-0.06	-0.81	-0.12	-0.62	-0.31	-0.44	-0.38
(1, 4)	0	-0.29	-0.12	-0.58	-0.25	-0.46	-0.38	-0.42
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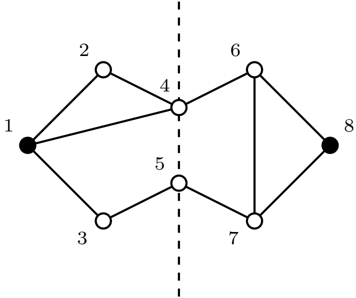
(3, 5)	0	-0.06	0.19	-0.12	-0.62	-0.31	-0.44	-0.38
(4, 6)	0.62	0.69	0.44	0.75	0.25	-0.06	0.06	0
(5, 7)	0.38	0.31	0.56	0.25	0.75	0.06	-0.06	0
(6, 7)	0.08	0.12	-0.04	0.17	-0.17	0.29	-0.29	0
(6, 8)	0.54	0.56	0.48	0.58	0.42	0.65	0.35	0
(7, 8)	0.46	0.44	0.52	0.42	0.58	0.35	0.65	0



- **Example**: 1 MW in bus 7 $\equiv$ 0.38 MW in bus 4 + 0.62 MW in bus 5

Decomposed ISF: Theorem

- Let (L_1, L_2) be a partition of L and $(B_1, B_\cap, B_2)$ a suitable partition of B
- Let Δ be the matrix of Injection Shift Factors, partitioned as follows:

	B_1	$B_\cap$	B_2
L_1	Δ_{11}	$\Delta_{1\cap}$	Δ_{12}
L_2	Δ_{21}	$\Delta_{2\cap}$	Δ_{22}

- **Theorem**: The columns of Δ_{12} are convex combinations of the columns of $\Delta_{1\cap}$. Similarly, the columns of Δ_{21} are convex combinations of the columns of $\Delta_{2\cap}$.

Decomposed ISF: Formulation

- n_b: original net injection at bus $b \in B$
- w_b^1, w_b^2: artificial net injection at bus $b \in B_\cap$

$$w_b^1 = \sum_{c \in B_2} \gamma_{bc}^2 n_c^2 \qquad \forall b \in B_\cap, \tag{1a}$$

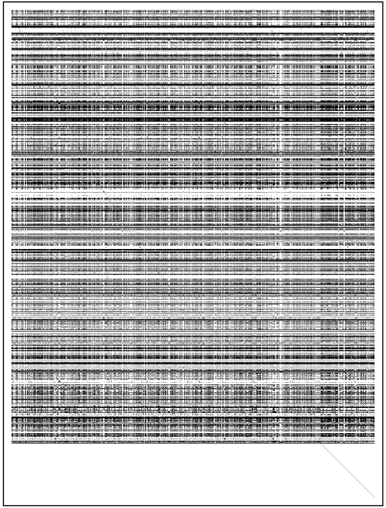
$$w_b^2 = \sum_{c \in B_1} \gamma_{bc}^1 n_c^1 \qquad \forall b \in B_\cap, \tag{1b}$$

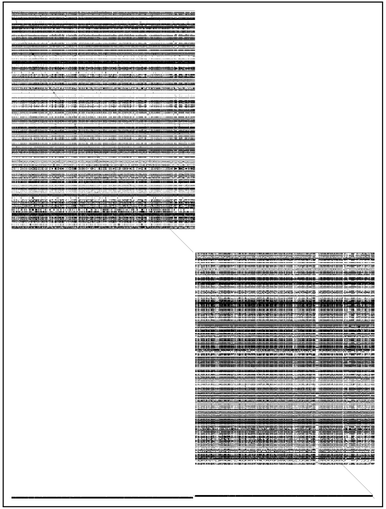
$$\sum_{b \in B_k} n_b^k + \sum_{b \in B_\cap} w_b^k = 0 \qquad \forall k \in \{1, 2\}, \tag{1c}$$

$$f_l^k = \sum_{b \in B_k} \delta_{lb} n_b^k + \sum_{b \in B_\cap} \delta_{lb} w_b^k \qquad \forall k \in \{1, 2\}, l \in L_k \tag{1d}$$

$$-F_l \leq f_l^k \leq F_l \qquad \forall k \in \{1, 2\}, l \in L_k, \tag{1e}$$

Decomposed ISF: Heat Maps





`case1888rte` (Franch VHV System, 2013)

Decentralized Optimization via ADMM

- **Problem**: $\min_x f_1(x) + f_2(x) + \ldots + f_n(x)$
- **Goal**: Minimize each $f_i(x)$ in parallel
- **Alternating Direction Method of Multipliers** (ADMM):
  1. Initialize $\bar{x}, \lambda_1, \ldots, \lambda_n, \rho \in \mathbb{R}$
  2. $x_i \leftarrow \arg\min_x \left[f_i(x) + \lambda_i(x - \bar{x}) + \rho(x - \bar{x})^2\right]$
  3. $\bar{x} \leftarrow \frac{x_1 + x_2 + \ldots + x_n}{n}$
  4. $\lambda_i \leftarrow \lambda_i + \rho(x_i - \bar{x})$
  5. Goto step 2
- Phase-Angle Formulation:
  - Duplicate θ_b and f_l for buses and lines in the boundary
  - Feizollahi, Costley, Ahmed, Grijalva (2015)
- Decomposed ISF:
  - Duplicate artificial net injections w_b^1, w_b^2

Enforcement of N-1 Security Constraints

- Consider a monitored line m and an outaged line q
- **Internal outages**: Easily handled via Line Outage Distribution Factors (LODF)

$$\tilde{f}_{mq}^k = f_m^k + \phi_{mq} f_q^k$$
$$-F_m \le \tilde{f}_{mq}^k \le F_m$$

- **External outages**: LODF cannot be used; ignored in existing literature
  - **Our approach**: Update transmission limits after each ADMM iteration
  - All transmission line contingencies are fully considered
  - No additional variables or constraints required; good performance
  - Little information sharing across zones

$$-F_m - \underbrace{\left(\min_{\tau \in T} \tilde{g}_{m\tau}^k\right)}_{\tilde{g}_{m,\min}^k} \le f_m^k \le F_m - \underbrace{\left(\max_{\tau \in T} \tilde{g}_{m\tau}^k\right)}_{\tilde{g}_{m,\max}^k}$$

Computational Experiments: Instances

- **Problem**: Multi-Zonal Security-Constrained Unit Commitment
  - Decision: Optimal generation schedule; optimal energy exchange
  - Constraints: Supply equals demand; ramping; minimum-up and down; others
- **Instances**: Realistic and large-scale instances from MATPOWER:

	Total			Zone 1			Zone 2		
Instance	Buses	Units	Lines	Buses	Units	Lines	Buses	Units	Lines
case1888rte	1,888	297	2,531	1,113	211	1,498	784	86	1,033
case1951rte	1,951	391	2,596	1,037	119	1,415	923	272	1,181
case2848rte	2,848	547	3,776	1,481	226	1,957	1375	321	1,819
case3012wp	3,012	502	3,572	1,637	322	1,938	1,388	180	1,634
case3375wp	3,374	596	4,161	1,649	334	2,007	1,696	262	2,154
case6468rte	6,468	1,295	9,000	2,896	544	4,049	3,588	751	4,951
case6515rte	6,515	1,388	9,037	3,536	800	4,831	2,994	588	4,206

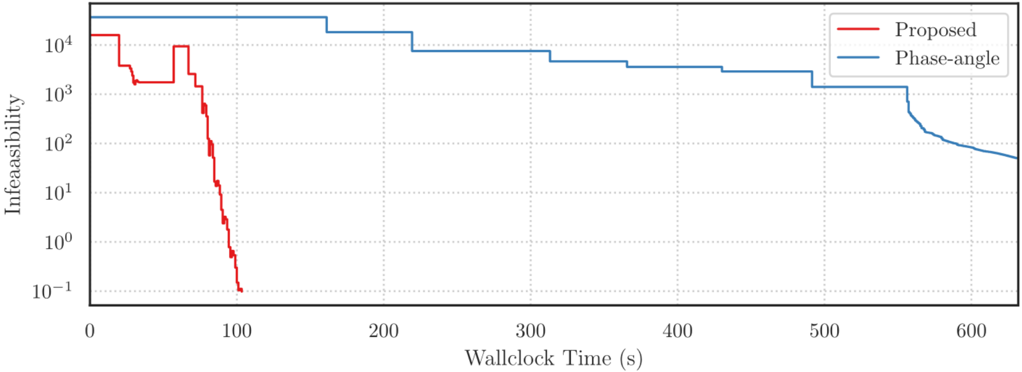
Computational Experiments: Implementation

- **Tools & Libraries**:
  - Implemented in Julia 1.1, JuMP 0.19 and MPI
  - IBM ILOG CPLEX 12.8.0 used as QP and MIQP solver
- **Decentralized solution method**:
  - One optimization subproblem for each geographical zone
  - Subproblems optimized independently and simultaneously
  - Transmission constraints added lazily to the formulation
- **Revised Fix-and-Release Procedure**:
  - **Challenge**: ADMM does not guarantee convergence for MIQPs
  - **Solution**: Dynamically alternate between QPs and MIQPs
  - Improved version of method by Feizollahi, Costley, Ahmed, Grijalva (2015)

Transmission-Constrained Unit Commitment: Results

	Proposed				Phase-Angle			
Instance	Time (s)	Infeas.	Iter.	Gap (%)	Time (s)	Infeas.	Iter.	Gap (%)
`case1888rte-2z`	83.7	9.826e-02	45.0	0.02	631.9	4.996e+01	102.0	nan
`case1951rte-2z`	185.3	7.128e-02	51.0	0.09	nan	nan	nan	nan
`case2848rte-2z`	169.2	6.573e-02	42.0	0.27	nan	nan	nan	nan
`case3012wp-2z`	216.3	8.209e-02	37.0	0.10	3872.4	3.027e+03	11.0	nan
`case3375wp-2z`	286.1	6.781e-02	42.0	0.07	6045.5	1.281e+04	1.0	nan
`case6468rte-2z`	696.0	5.850e-02	37.0	0.00	3600.3	2.991e+02	66.0	nan
`case6515rte-2z`	850.2	9.058e-02	31.0	0.08	3607.6	1.064e+02	120.0	nan
Average	**355.3**	**7.632e-02**	**40.7**	**0.09**	**3551.5**	**3.258e+03**	**60.0**	**nan**

Transmission-Constrained Unit Commitment: Results



Security-Constrained Unit Commitment: Results

	Proposed			
Instance	Time (s)	Infeas.	Iter.	Gap (%)
`case1888rte-2z`	129.4	8.316e-02	47.0	0.03
`case1951rte-2z`	206.6	6.245e-02	47.0	0.09
`case2848rte-2z`	243.8	3.704e-02	42.0	0.21
`case3012wp-2z`	755.9	9.917e-02	42.0	0.04
`case3375wp-2z`	1183.4	6.869e-02	56.0	0.07
`case6468rte-2z`	1240.2	8.808e-02	36.0	0.00
`case6515rte-2z`	3146.9	6.668e-02	77.0	0.00
Average	**986.6**	**7.218e-02**	**49.6**	**0.06**

Conclusion

- **Our contribution**:
  - Less dense, more decomposable formulation of DC Power Flows
  - Scalable to large networks and multiple topologies (N-1 security)
  - Enables large-scale decentralized studies (e.g. optimal energy exchange)
- **Future work**:
  - Validation with real-world datasets
  - Application to coordinated congestion relief
  - Impact to other problems (Transmission Switching, Expansion Planning)

Thank You