Optimal Convex Hull Pricing and MISO Case Studies

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Joint work with Yanan Yu and Yonghong Chen

Overview

Overview

Unit Commitment (UC)

- Two types of decisions to make:
  - Which generators to turn on/off and when.
  - Generation amount of those "on" generators.
- Constraints to respect:
  - Generator characteristics (e.g., min-up/-down time, ramping rate capacity).
  - System-wise restrictions (e.g., transmission line capacity, system spinning reserve).

Notation

- Parameters
  - $\overline{C}(\underline{C})$: capacity upper(lower) bound.
  - $L(\ell)$: minimum-up(-down) time limit.
  - V : stable ramp-up/-down rate limit.
  - $\overline{V}$: start-up/shut-down ramp rate limit.
- Decision Variables
  - u : on/off status (binary).
  - v : start-up operation (binary).
  - x : generation amount (continuous).

Dynamic Programming Formulation - Single Generator

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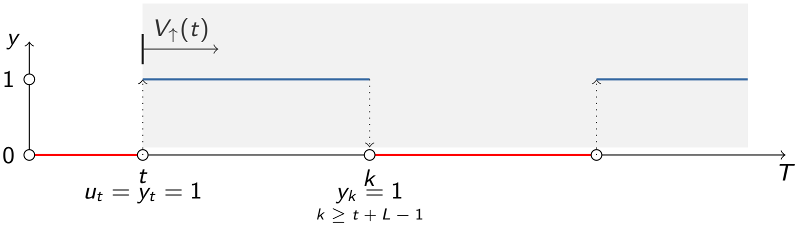

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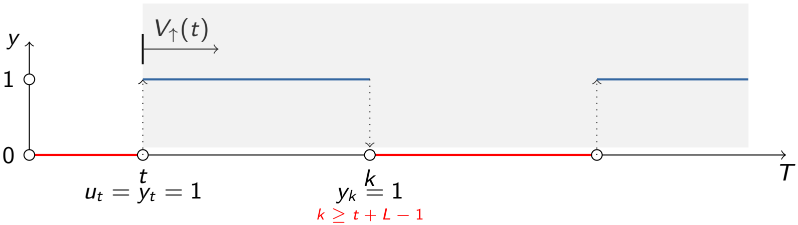

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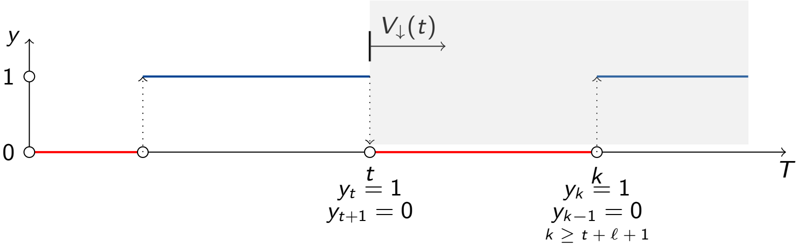
$$V_{\downarrow}(t) = \min_{k \in [t+\ell+1, T]_{\mathbb{Z}}} \left\{ g^{+}(k-t-1) + V_{\uparrow}(k), 0 \right\}, \ \forall t \in [L, T-\ell-1]_{\mathbb{Z}}.$$



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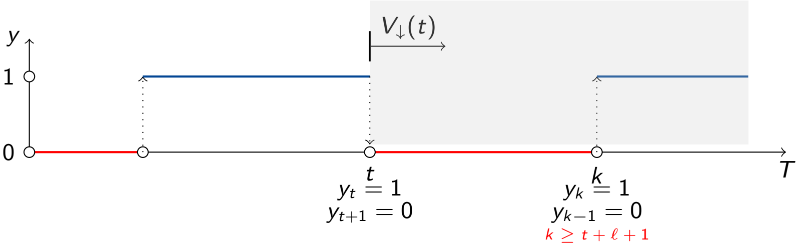

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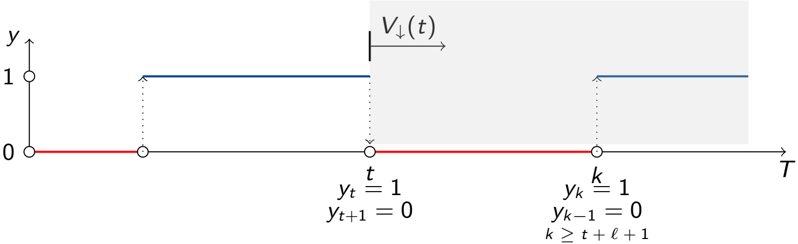

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$$V_{\downarrow}(t) = \min_{k \in [t+\ell+1, T]_{\mathbb{Z}}} \left\{ g^{+}(k-t-1) + V_{\uparrow}(k), 0 \right\}, \ \forall t \in [L, T-\ell-1]_{\mathbb{Z}}.$$

Proposition: This dynamic program can be solved in $\mathcal{O}(T^2)$ time if $C(t,k)$ is pre-calculated.

An Integral Formulation (NUC Formulation)

$$\min \sum_{t=1}^{T} g^+(s_0 + t - 1) w_t + \sum_{t=1}^{T} \sum_{k=t+L-1}^{T-1} g^-(k - t + 1) y_{tk} +$$

$$\sum_{t=L}^{T-\ell-1} \sum_{k=t+\ell+1}^{T} g^+(k - t - 1) z_{tk} + \sum_{tk \in \mathcal{TK}} \sum_{s=t}^{k} \phi_{tk}^s$$

$$\text{s.t.} \quad \sum_{t=1}^{T} w_t \le 1, \quad \sum_{k=t+L-1}^{T} y_{tk} - \sum_{k=L}^{t-\ell-1} z_{kt} = w_t, \ \forall t \in [1, T]_{\mathbb{Z}},$$

$$\sum_{k=1}^{t-L+1} y_{kt} - \sum_{k=t+\ell+1}^{T} z_{tk} = \theta_t, \ \forall t \in [L, T - \ell - 1]_{\mathbb{Z}},$$

$$\underline{C} y_{tk} \le q_{tk}^s \le \overline{C} y_{tk}, \ \forall s \in [t, k]_{\mathbb{Z}}, \forall tk \in \mathcal{TK},$$

$$q_{tk}^t \le \overline{V} y_{tk}, \quad q_{tk}^k \le \overline{V} y_{tk}, \ \forall tk \in \mathcal{TK}, k \le T - 1,$$

$$q_{tk}^{s-1} - q_{tk}^s \le V y_{tk}, q_{tk}^s - q_{tk}^{s-1} \le V y_{tk}, \ \forall s \in [t+1, k]_{\mathbb{Z}}, \forall tk \in \mathcal{TK},$$

$$\phi_{tk}^s - m_j q_{tk}^s \ge n_j y_{tk} \ \forall s \in [t, k]_{\mathbb{Z}}, \forall j \in [1, N]_{\mathbb{Z}}, \forall tk,$$

$$w, \theta, y, z \ge 0.$$

Remark: $\mathcal{O}(T^2)$ binary decision variables and $\mathcal{O}(T)$ constraints for the network flow formulation part.

Overview

Convex Hull Pricing

System Optimization Model

$$Z^*_{\text{QIP}} = \min_{w^j,\phi^j,\theta^j,y^j,z^j,q^j} \sum_{j\in\Lambda} g_j(w^j,\phi^j,\theta^j,y^j,z^j,q^j) \tag{2}$$

$$\text{s.t.} \quad \sum_{j\in\Lambda} \sum_{tk\in\mathcal{TK}} q^j_{tk} = d \tag{3}$$

$$(w^j,\phi^j,\theta^j,y^j,z^j,q^j) \in \mathcal{X}^j_I, \ \forall j \in \Lambda \tag{4}$$

$$y^j \text{ and } z^j \text{ are binary}, \ \forall j \in \Lambda$$

Remark:

1. Transmission constraints can be incorporated similarly.
2. $\mathcal{X}^j_I$ represents the feasible region for the integral formulation of generator j.

Convex Hull Pricing

Profit Maximization for Each Participant

$$
\begin{aligned}
v_j(\pi) = \max_{w^j,\phi^j,\theta^j,y^j,z^j,q^j} \ & \pi^T \sum_{tk \in \mathcal{TK}} q^j_{tk} - g_j(w^j,\phi^j,\theta^j,y^j,z^j,q^j) \\
\text{s.t.} \quad & (w^j,\phi^j,\theta^j,y^j,z^j,q^j) \in \mathcal{X}^j_I \\
& y^j \text{ and } z^j \text{ are binary}
\end{aligned}
$$

Uplift Payment

$$
\begin{aligned}
U_j(\pi,\bar{w}^j,\bar{\phi}^j,\bar{\theta}^j,\bar{y}^j,\bar{z}^j,\bar{q}^j) &= \\
& v_j(\pi) - (\pi^T \sum_{tk \in \mathcal{TK}} \bar{q}^j_{tk} - g_j(\bar{w}^j,\bar{\phi}^j,\bar{\theta}^j,\bar{y}^j,\bar{z}^j,\bar{q}^j))
\end{aligned}
$$

Convex Hull Pricing

Lagrangian Relaxation and Convex Hull Pricing

$$\begin{aligned}
\max_{\pi} \quad & \sum_{j\in\Lambda}\Big(\min_{(w^j,\phi^j,\theta^j,y^j,z^j,q^j)} g_j(w^j,\phi^j,\theta^j,y^j,z^j,q^j) \\
& \qquad -\pi^T \sum_{tk\in\mathcal{TK}} q^j_{tk}\Big) + \pi^T d \\
\text{s.t.} \quad & (w^j,\phi^j,\theta^j,y^j,z^j,q^j) \in \mathcal{X}^j_I \\
& y^j \text{ and } z^j \text{ are binary}
\end{aligned}$$

Remark: An optimal Lagrangian multiplier is an optimal convex hull price.

Convex Hull Pricing

Theorem: If the general cost function $f^s_{tk}(q^s_{tk})$ is piecewise linear, then the optimal convex hull price can be obtained by solving the linear program (2)-(4), and the optimal convex hull price is equal to the dual values corresponding to the load balance constraints (3).

An Example

$T = 3$, $d_1 = 70$MW, $d_2 = 80$MW, and $d_3 = 90$MW.

$G1$: $\underline{C}_1 = 0$ and $\overline{C}_1 = 40$MW; The unit generation cost: $c_1 = \$4/\text{MWh}$, $c_2 = \$5/\text{MWh}$, and $c_3 = \$5/\text{MWh}$.

$G2$: $\underline{C}_2 = 20$MW, $\overline{C}_2 = 100$MW, $\overline{V}_2 = 55$MW/h, $V_2 = 5$MW/h and $L_2 = \ell_2 = 2$. The start-up cost for $G2$ is \$100. The convex generation cost for $G2$ is approximated by a two-piece piecewise linear function (e.g., $\phi^s_{tk} \geq 100y_{tk} + q^s_{tk}$ and $\phi^s_{tk} \geq 370y_{tk} - q^s_{tk}$).

An Example

1. $Z^*_{\text{QIP}} = \$1315$ with the optimal solution $\bar{x}^1_1 = 15, \bar{x}^1_2 = 20, \bar{x}^1_3 = 25, \bar{x}^2_1 = 55, \bar{x}^2_2 = 60, \bar{x}^2_3 = 65$ for both the 2-Bin and NUC MILP formulations. The corresponding LMPs are $\pi^1_1 = 4$, $\pi^1_2 = 5$, and $\pi^1_3 = 5$.
2. $Z^{\text{2B}}_{\text{QP}} = \$940$ with the corresponding dual values $\pi^2_1 = 5.459$, $\pi^2_2 = 5$, and $\pi^2_3 = 5$.
3. $Z^*_{\text{QP}} = \$1267.27$ with the corresponding dual values $\pi^3_1 = 4.41$, $\pi^3_2 = 5$, and $\pi^3_3 = 7.5$ (the corresponding solution is $x^1_1 = 40, x^1_2 = 34, x^1_3 = 40, q^1_{13} = 30, q^2_{13} = 32.7, q^3_{13} = 35.5, q^2_{23} = 13.3, q^3_{23} = 14.5, y_{13} = 0.545, y_{23} = 0.242$).
4. Using π^1, π^2, and π^3 as the input, the uplift payments are \$185, \$141.23, and \$47.75, respectively.

Overview

Special Cases

There are different types of generators in practice within MISO. We first present a traditional 3-bin MILP UC formulation as follows as a base for building our model:

$$
\begin{aligned}
Z^*_{\text{QIP}} \quad &= \min_{f,x,u,v,e,\delta} \sum_{i \in \mathcal{G}} \sum_{t \in T} \left(\sum_{s \in S} d_{is}\delta_{ist} + S_i e_{it} + a_i u_{it} + f_{it} \right) && (5) \\
\text{s.t.} \quad & \sum_{i \in \mathcal{G}} x_{it} = D_t, \forall t \in \mathcal{T}, && (6) \\
& (f_i, x_i, \delta_i, u_i, v_i, e_i) \in \mathcal{X}_i, \forall i \in \mathcal{G}. && (7)
\end{aligned}
$$

Special Cases

$$
\begin{aligned}
\mathcal{X}_i = \{ & f_i, x_i, \delta_i, u_i, v_i, e_i \in \mathbb{R}^T \times \mathbb{R}^T \times \mathbb{R}^T \times \mathbb{B}^T \times \mathbb{B}^{T-1} \times \mathbb{B}^{T-1} | \\
& f_{it} \geq h_{it}^k x_{it} - p_{it}^k u_{it}, \forall k \in N, \forall t \in \mathcal{T}, && (8)\\
& \delta_{its} \leq \sum_{j=\underline{T}_i^s}^{\overline{T}_i^s} e_{i(t-j)}, \forall s \in S/\{3\}, t \in \mathcal{T}, && (9)\\
& \sum_{s \in \mathcal{S}} \delta_{its} = v_{it}, t \in \mathcal{T}, u_{it} - u_{i(t-1)} = v_{it} - e_{it}, \forall t \in [2, T]_{\mathbb{Z}}, && (10)\\
& \sum_{j=t-L_i+1}^{T} v_{ij} \leq u_{it}, \forall t \in [L_i + 1, T]_{\mathbb{Z}}, && (11)\\
& \sum_{j=t-\ell_i+1}^{T} v_{ij} \leq 1 - u_{i(t-\ell)}, \forall t \in [\ell_i + 1, T]_{\mathbb{Z}}, && (12)\\
& x_{it} \geq \underline{C}_i u_{it}, x_{it} \leq \overline{C}_i u_{it}, \forall t \in \mathcal{T}, && (13)\\
& x_{it} - x_{i(t-1)} \leq V_i u_{i(t-1)} + \overline{V}_i v_{it}, \forall t \in \mathcal{T}, && (14)\\
& x_{i(t-1)} - x_{it} \leq V_i u_{it} + \overline{V}_i e_{it}, \forall t \in \mathcal{T} \}. && (15)
\end{aligned}
$$

Special Cases

The set of generators with constant start-up costs and without ramping constraints (labelled as set $\mathcal{G}_1$):

$$
\begin{aligned}
Z^*_{\text{QIP}} &= \min_{x,u,v,e} \sum_{i \in \mathcal{G}} \sum_{t \in T} (S'_i v_{it} + S_i e_{it} + a_i u_{it} + f_{it}) && (16)\\
\text{s.t.} &\quad (6), (f_i, x_i, u_i, v_i, e_i) \in \mathcal{X}'_i, \forall i \in \mathcal{G}.\\
\mathcal{X}'_i &= \{f_i, x_i, u_i, v_i, e_i \in \mathbb{R}^T \times \mathbb{R}^T \times \mathbb{B}^T \times \mathbb{B}^{T-1} \times \mathbb{B}^{T-1} |\\
&\quad (8),\ (10)-(13)\}. && (17)
\end{aligned}
$$

The convex hull for such a single generator i, $\mathcal{X}'_i$, is [Hua and Baldick, 2017]:

$$
\begin{aligned}
\bar{\mathcal{D}}'_i &= \{f_i, x_i, u_i, v_i, e_i \in \mathbb{R}^T \times \mathbb{R}^T \times \mathbb{R}^T \times \mathbb{R}^{T-1} \times \mathbb{R}^{T-1} |\\
&\quad (8),\ (10)-(13),\\
&\quad v_{it} \geq 0, e_{it} \geq 0\}. && (18)
\end{aligned}
$$

Special Cases

The set of generators with constant start-up costs and only start-up ramping constraints (labelled as set $\mathcal{G}_2$ [Gentile et al., 2017]):

$$\begin{aligned}\hat{\mathcal{D}}_i' \quad = \{ & f_i, x_i, u_i, v_i, e_i \in \mathbb{R}^T \times \mathbb{R}^T \times \mathbb{R}^T \times \mathbb{R}^{T-1} \times \mathbb{R}^{T-1} | \\ & (8),\ (10) - (13),\ (18), \\ & x_{it} \leq \overline{C}_i u_{it} + (\overline{V}_i - \overline{C}_i) v_{it}, \forall t \in [2, T]_{\mathbb{Z}} \}. \end{aligned} \tag{19}$$

General Cases: Flexible Min-up/-down Time Restrictions

Redefine the range of y_{tk} and z_{tk} to accommodate this.

- If $\kappa_t = 1$, we define $y_{tk}, \forall k \in [t + L - 1, T]_{\mathbb{Z}}$, which indicates the generator must stay online for L time periods after its start-up time t based on the min-up time constraint.
- If $\kappa_t = 0$, we define $y_{tk}, \forall k \in [t, T]_{\mathbb{Z}}$, which indicates the min-up time is relaxed to be 1 and the generator can shut down any time after its start-up time t.

General Cases: Maximum-on Time Restrictions

For some generators in the MISO market (labelled as $\mathcal{G}_3$), there are restrictions on maximum time periods that the generator can stay online because of machine deterioration.

$$
\begin{aligned}
\tilde{\mathcal{D}}_i' \quad = \{ & f_i, x_i, u_i, v_i, e_i \in \mathbb{R}^T \times \mathbb{R}^T \times \mathbb{R}^T \times \mathbb{R}^{T-1} \times \mathbb{R}^{T-1} | \\
& (8),\ (10) - (13),\ (18),\ (19), \\
& \sum_{j=t+1}^{t+\overline{L}_i} v_{ij} \geq u_{i(t+\overline{L}_i)}, \forall t \in \mathcal{T}. \}
\end{aligned} \tag{20}
$$

General Cases: Dynamic Parameter Settings

The capacity and ramping rates for certain generators are varying in different time periods.

General Formulation for MISO

After considering all the general aspects described above, we refine the integral formulation as follows (named group $\mathcal{G}_4$ [Yu et al., 2019]):

$$\min \sum_{tk \in \overline{\mathcal{TK}^1}, k<T} S'(t) w_k + \sum_{tk \in \overline{\mathcal{TK}^2}, k<T} S'(k-t+1) y_{tk}$$

$$+ \sum_{kt \in \overline{\mathcal{KT}}} S z_{kt}(t-k-1) + \sum_{tk \in \overline{\mathcal{TK}}} \sum_{s=t}^{k} \phi_{tk}^s \tag{21}$$

$$\text{s.t.} \quad \sum_{t=t_0}^{T} w_t = 1, \tag{22}$$

$$-w_{t'} + \sum_{tk \in \overline{\mathcal{KT}}, t=t'} z_{t'k} - \sum_{kt \in \overline{\mathcal{TK}^2}, t=t'} y_{kt'} + \theta_{t'} = 0, \forall t' \in [t_0, T-1]_{\mathbb{Z}}, \tag{23}$$

$$\sum_{tk \in \overline{\mathcal{TK}^2}, t=t'} y_{t'k} - \sum_{kt \in \overline{\mathcal{KT}}, t=t'} z_{kt'} = 0, \forall t' \in [t_0 + \ell_t + 1, T]_{\mathbb{Z}}, \tag{24}$$

General Formulation for MISO

$$\underline{C}_s w_k \leq q^s_{tk} \leq \overline{C}_s w_k, \ \forall s \in [t,k]_{\mathbb{Z}}, \forall tk \in \overline{\mathcal{TK}^1}, \tag{25}$$

$$\underline{C}_s y_{tk} \leq q^s_{tk} \leq \overline{C}_s y_{tk}, \ \forall s \in [t,k]_{\mathbb{Z}}, \forall tk \in \overline{\mathcal{TK}^2}, \tag{26}$$

$$q^t_{tk} \leq \overline{V}^u_t y_{tk}, \ \forall tk \in \overline{\mathcal{TK}^2}, \tag{27}$$

$$q^k_{tk} \leq \overline{V}^e_k w_k, \ \forall tk \in \overline{\mathcal{TK}^1}, k \leq T-1, \tag{28}$$

$$q^k_{tk} \leq \overline{V}^e_k y_{tk}, \ \forall tk \in \overline{\mathcal{TK}^2}, k \leq T-1, \tag{29}$$

$$q^{s-1}_{tk} - q^s_{tk} \leq V^e_s w_k, \ q^s_{tk} - q^{s-1}_{tk} \leq V^u_s w_k,$$

$$\forall s \in [t+1,k]_{\mathbb{Z}}, \forall tk \in \overline{\mathcal{TK}^1}, \tag{30}$$

$$q^{s-1}_{tk} - q^s_{tk} \leq V^e_s y_{tk}, \ q^s_{tk} - q^{s-1}_{tk} \leq V^u_s y_{tk},$$

$$\forall s \in [t+1,k]_{\mathbb{Z}}, \forall tk \in \overline{\mathcal{TK}^2}, \tag{31}$$

$$\phi^s_{tk} - a^s_j q^s_{tk} \geq b^s_j w_k, \forall s \in [t,k]_{\mathbb{Z}}, j \in [1,N]_{\mathbb{Z}}, \forall tk \in \overline{\mathcal{TK}^1}, \tag{32}$$

$$\phi^s_{tk} - a^s_j q^s_{tk} \geq b^s_j y_{tk}, \forall s \in [t,k]_{\mathbb{Z}}, j \in [1,N]_{\mathbb{Z}}, \forall tk \in \overline{\mathcal{TK}^2}, \tag{33}$$

$$w, z, y \geq 0, \theta_t \geq 0, \forall t \in \mathcal{L}. \tag{34}$$

General System Optimization Model for MISO

$$(P) : Z^*_{\text{QIP}} = \min_{f,x,u,v,e,w,z,y,\theta,q,\phi} \sum_{i \in \mathcal{G}_1 \cup \mathcal{G}_2 \cup \mathcal{G}_3} \sum_{t \in T} g_{it} + \sum_{i \in \mathcal{G}_4} g'_{it} \tag{35}$$

$$\begin{aligned} \text{s.t.} \quad & (6), (f_i, x_i, u_i, v_i, e_i) \in \bar{\mathcal{D}}'_i, \forall i \in \mathcal{G}_1, \\ & (f_i, x_i, u_i, v_i, e_i) \in \hat{\mathcal{D}}'_i, \forall i \in \mathcal{G}_2, \\ & (f_i, x_i, u_i, v_i, e_i) \in \tilde{\mathcal{D}}'_i, \forall i \in \mathcal{G}_3, \\ & (w_i, z_i, y_i, \theta_i, q_i, \phi_i) \in \dot{\mathcal{D}}'_i, \forall i \in \mathcal{G}_4, \end{aligned}$$

where the cost function g_{it} of each generator i in $\mathcal{G}_1 \cup \mathcal{G}_2 \cup \mathcal{G}_3$ can be expressed as

$$g_{it} = S'_i v_{it} + S_i e_{it} + a_i u_{it} + f_{it} \tag{36}$$

and the cost function g'_{it} of each generator i in $\mathcal{G}_4$ can be expressed as in the integral formulation.

Methodology Background

$$
\begin{aligned}
Z^*_{\text{QIP}} &= \min\{c^T x | Dx = d, x \in \mathcal{X}\}, \\
\mathcal{X} &= \{x_1 \in \mathbb{B}^{n_1}, x_2 \in \mathbb{R}^{n_2} | Ax \leq b\}.
\end{aligned}
\tag{37}
$$

General Integral Formulation for MISO

Lemma

For $A \in \mathbb{R}^{m \times n}$, $A' \in \mathbb{R}^{m \times n}$, $D \in \mathbb{R}^{p \times n}$, $b \in \mathbb{R}^m$, $b' \in \mathbb{R}^m$, $d \in \mathbb{R}^p$ and $c \in \mathbb{R}^n$, we consider the integer optimization problem (37), its tightened linear programming relaxation problem (P_C) in which $A'x \leq b'$ dominates $Ax \leq b$, and the Lagrangian relaxation $\mathcal{D}_C$ corresponding to a dual value γ as follows:

$$(P_C): \; z_C = \min\{c^T x | A'x \leq b', Dx = d, x \in \mathbb{R}^n\}, \tag{38}$$

$$(\mathcal{D}_C): z_C(\gamma) = \min\{c^T x + \gamma^T (d - Dx) | A'x \leq b', x \in \mathbb{R}^n\}. \tag{39}$$

Given an optimal dual value $\bar{\gamma}$ for constraints $Dx = d$ in (38), if

1. $x^* = \{x_1^*, x_2^*\}$ *is an optimal solution to problem* $z_C(\bar{\gamma})$,
2. x_1^* *are all binaries,*

then the uplift payment given price $\bar{\gamma}$ can be calculated as $U = Z_{QIP}^* - z_C(\bar{\gamma})$.

General Integral Formulation for MISO

Theorem

For $A_1 \in \mathbb{R}^{m \times n}$, $A_2 \in \mathbb{R}^{m \times n}$, $D \in \mathbb{R}^{p \times n}$, $b_1 \in \mathbb{R}^m$, $b_2 \in \mathbb{R}^m$, $d \in \mathbb{R}^p$, and $c \in \mathbb{R}^n$, considering the integer optimization problem (37)*, its linear programming relaxation (for notation brevity, we define $\mathcal{X}_L = \{x \in \mathbb{R}^n | Ax \leq b\}$), and two tightened linear programming relaxation problems $(P_{C1}), (P_{C2})$ shown below,*

$$
\begin{aligned}
(P_{C1}): \quad & z_{C1} = \min\{c^T x | Dx = d, x \in \mathcal{X}_1\}, && (40) \\
(P_{C2}): \quad & z_{C2} = \min\{c^T x | Dx = d, x \in \mathcal{X}_2\}, && (41)
\end{aligned}
$$

in which $\mathcal{X}_2 \subseteq \mathcal{X}_1 \subseteq \mathcal{X}_L$ with $\mathcal{X}_1 = \{x \in \mathbb{R}^n | A_1 x \leq b_1\}$ and $\mathcal{X}_2 = \{x \in \mathbb{R}^n | A_2 x \leq b_2\}$. Under these two formulations, we use the dual value for constraints $Dx = d$ as the market clearing price, written as $\bar{\gamma}_1$ and $\bar{\gamma}_2$. Then the uplift payment under $\bar{\gamma}_2$ will be no larger than that under $\bar{\gamma}_1$ if conditions (i) *and* (ii) *in the previous Lemma hold.*

The Iterative Algorithm

Master ISO minimization problem to be solved:

$$
\begin{aligned}
(P1): Z^*_{\text{QIP}_1} \quad &= \min_{f,x,u,v,e} \sum_{i \in \mathcal{G}} \sum_{t \in T} g_{it} \qquad (42) \\
\text{s.t.} \quad & (6), (f_i, x_i, u_i, v_i, e_i) \in \bar{\mathcal{D}}'_i, \forall i \in \mathcal{G}_1, \\
& (f_i, x_i, u_i, v_i, e_i) \in \hat{\mathcal{D}}'_i, \forall i \in \mathcal{G}_2, \\
& (f_i, x_i, u_i, v_i, e_i) \in \tilde{\mathcal{D}}'_i, \forall i \in \mathcal{G}_3 \cup \mathcal{G}_4.
\end{aligned}
$$

The Iterative Algorithm

Profit maximization problem to be solved:

$$
\begin{aligned}
(P2): Z(\bar{\gamma}) = \bar{\gamma}^T d + \min_{f,x,u,v,e} \sum_{i \in \mathcal{G}} \sum_{t \in T} (g_{it} - \bar{\gamma} x_{it}) \quad (43) \\
\text{s.t.} \qquad (f_i, x_i, u_i, v_i, e_i) \in \bar{\mathcal{D}}'_i, \forall i \in \mathcal{G}_1, \\
(f_i, x_i, u_i, v_i, e_i) \in \hat{\mathcal{D}}'_i, \forall i \in \mathcal{G}_2, \\
(f_i, x_i, u_i, v_i, e_i) \in \tilde{\mathcal{D}}'_i, \forall i \in \mathcal{G}_3 \cup \mathcal{G}_4.
\end{aligned}
$$

The Iterative Algorithm

Feasibility check problem:

$$(P3)\ \dot{Z}_i = \min_{w_i, z_i, y_i, \theta_i, q_i, \phi_i} 1 \tag{44}$$

$$\text{s.t.} \quad (w_i, z_i, y_i, \theta_i, q_i, \phi_i) \in \dot{\mathcal{D}}'_i,$$

$$x^*_{is} = \sum_{tk \in \overline{\mathcal{TK}}, t \le s \le k} q^s_{tk},\ f^*_{is} = \sum_{tk \in \overline{\mathcal{TK}}, t \le s \le k} \phi^s_{tk}, \forall s \in [1, T]_{\mathbb{Z}}, \tag{45}$$

$$v^*_{is} = \sum_{kt \in \overline{\mathcal{KT}}, t = s} z_{kt},\ e^*_{is} = \sum_{kt \in \overline{\mathcal{KT}}, k = s} z_{kt} + \theta_s, \forall s \in [1, T]_{\mathbb{Z}}, \tag{46}$$

$$u^*_{is} = \sum_{tk \in \overline{\mathcal{TK}}^1, k \ge s} w_k + \sum_{tk \in \overline{\mathcal{TK}}^2, t \le s \le k} y_{tk}, \forall s \in [1, T]_{\mathbb{Z}}. \tag{47}$$

The Iterative Algorithm

The basic steps:

- First solve (P1) and get the dual value $\bar{\gamma}$ of equality (6).
- Then we solve (P2) which is the profit maximization problem for generators based on the given price $\bar{\gamma}$. For each generator $i \in \mathcal{G}_4$, we may get fractional values in u_i, v_i, e_i, since $\tilde{\mathcal{D}}'_i$ is a relaxation of $\mathcal{D}'_i$. Usually, strong valid inequalities are added to cut off fractional solutions and ultimately only convex hull description with convex envelop of the objective function can ensure to get an integer solution.
- For the given (P1) solution, we check (P3) feasibility. If not, then we add the integral formulation constraints.
- Repeat above steps until no further improvement is found.

MISO Case Studies

- More than 1000 generators,
- Operation periods: 36 hours,
- Run in 32-processor Intel(R) Xeon(R) CPU E5-2667 v4 @ 3.20GHz 528GB,
- Optimizer: Gurobi 8.0.1,
- The required relative MIP gap is set to be 1e-6.

MISO Case Studies

Table: Convex hull pricing test results on the MISO system

Case	Model	Solution ($)	Uplift Payment ($)	Time (s)	Save ($)	Optimal
C1	MIP	39986855	-	-	-	-
	LMP	-	3521	36	-	N
	TLP	39978837	471	24	3049	N
	IA1	39986726	129	226	+342	Y
	IA2	39986726	129	291	+342	Y
	IAC1	39986726	129	$\diamond(+0)$	+342	Y
	IAC2	39986726	129	$\diamond(+0)$	+342	Y
	OPT	39986726	129		+342	$\star$
C2	MIP	55311277	-	-	-	-
	LMP	-	13242	35	-	N
	TLP	55291978	3187	26	10056	N
	IA1	55309361	1916	201	+1270	Y
	IA2	55309361	1916	215	+1270	Y
	IAC1	55309361	1916	$\diamond(+0)$	+1270	Y
	IAC2	55309361	1916	$\diamond(+0)$	+1270	Y
	OPT	55309361	1916		+1270	$\star$

MISO Case Studies

Table: Convex hull pricing test results on the MISO system

Case	Model	Solution ($)	Uplift Payment ($)	Time (s)	Save ($)	Optimal
C3	MIP	69299295	-	-	-	-
	LMP	-	18114	35	-	N
	TLP	69290109	2890	18	15224	N
	IA1	69297643	1652	399	+1237	Y
	IA2	69297643	1652	301	+1237	Y
	IAC1	69297643	1652	$\diamond(+0)$	+1237	Y
	IAC2	69297643	1652	$\diamond(+0)$	+1237	Y
	OPT	69297643	1652		+1237	$\star$
C4	MIP	50763423	-	-	-	-
	LMP	-	8948	34	-	N
	TLP	50754612	2033	23	6915	N
	IA1	50762469	953	541	+1080	Y
	IA2	50762469	953	333	+1080	Y
	IAC1	50762469	953	$\diamond(+0)$	+1080	Y
	IAC2	50762469	953	$\diamond(+0)$	+1080	Y
	OPT	50762469	953		+1080	$\star$

MISO Case Studies

Table: Convex hull pricing test results on the MISO system

Case	Model	Solution ($)	Uplift Payment ($)	Time (s)	Save ($)	Optimal
C5	MIP	49946355	-	-	-	-
	LMP	-	7637	35	-	N
	TLP	49861587	6046	24	1592	N
	IA1	49945716	639	670	+5407	Y
	IA2	49945004	1351	406	+4695	N
	IAC1	49945716	639	$\diamond(+0)$	+5407	Y
	IAC2	49945716	639	$\diamond(+165)$	+5407	Y
	OPT	49945716	639		+5407	$\star$
C6	MIP	58880777	-	-	-	-
	LMP	-	36630	38	-	N
	TLP	58861963	3889	48	32741	N
	IA1	58879927	850	733	+3039	N
	IA2	58880049	727	592	+3162	Y
	IAC1	58880049	727	$\diamond(+106)$	+3162	Y
	IAC2	58880049	727	$\diamond(+0)$	+3162	Y
	OPT	58880049	727		+3162	$\star$

MISO Case Studies

Table: Convex hull pricing test results on the MISO system

Case	Model	Solution ($)	Uplift Payment ($)	Time (s)	Save ($)	Optimal
C7	MIP	59195530	-	-	-	-
	LMP	-	11613	36	-	N
	TLP	59193235	1899	17	9715	N
	IA1	59194229	1302	116	+597	Y
	IA2	59194229	1302	115	+597	Y
	IAC1	59194229	1302	$\diamond(+0)$	+597	Y
	IAC2	59194229	1302	$\diamond(+0)$	+597	Y
	OPT	59194229	1302		+597	$\star$
C8	MIP	57307363	-	-	-	-
	LMP	-	51424	37	-	N
	TLP	57288519	1832	19	49593	N
	IA1	57306079	1284	559	+547	N
	IA2	57306120	1243	479	+589	Y
	IAC1	57306120	1243	$\diamond(+38)$	+589	Y
	IAC2	57306120	1243	$\diamond(+0)$	+589	Y
	OPT	57306120	1243		+589	$\star$

MISO Case Studies

Table: Convex hull pricing test results on the MISO system

Case	Model	Solution ($)	Uplift Payment ($)	Time (s)	Save ($)	Optimal
C9	MIP	69977540	-	-	-	-
	LMP	-	11472	35	-	N
	TLP	69950201	5230	19	6242	N
	IA1	69976587	954	630	+4276	N
	IA2	69977111	429	556	+4801	Y
	IAC1	69977111	429	$\diamond(+55)$	+4801	Y
	IAC2	69977111	429	$\diamond(+0)$	+4801	Y
	OPT	69977111	429		+4801	$\star$
C10	MIP	47889206	-	-	-	-
	LMP	-	8875	38	-	N
	TLP	47860497	4088	20	4787	N
	IA1	47887537	1669	213	+2419	Y
	IA2	47887537	1669	235	+2419	Y
	IAC1	47887537	1669	$\diamond(+0)$	+2419	Y
	IAC2	47887537	1669	$\diamond(+0)$	+2419	Y
	OPT	47887537	1669		+2419	$\star$

Reference I

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