A Scalable Mixed-Integer Decomposition Approach for Optimal Black-Start and Restoration of Power Systems

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Optimal Power System Restoration (OPSR)

- Human error, attacks, and extreme weather events can result in blackouts → **power systems must be resilient**
- OPSR lies at the heart of **evaluation** and **improvement** of power system resiliency:
  - Interruption time, interrupted load
  - Cranking paths, energization priorities, black-start allocation
- OPSR corresponds to a large-scale mixed-integer nonlinear program
  - Large-scale due to **size of realistic networks** (thousands of buses)
  - Nonlinear due to **power flow equations**
  - Mixed-integer due to **binary energization decisions**
- Simultaneous attack or damage to communication systems may affect optimal restoration decisions

OPSR without Communications Considerations

- **Objective: Restore normal system operation as fast as possible**
- **Decision variables:**
  - Startup sequence of generators
  - Energization of buses, lines, transformers and other components
  - Pickup of loads
- **Restrictions/Constraints:**
  - Energization consistency (integer, linear)
  - Sequential energization, maximum number of energizations per step
  - Optimal power flow constraints
    - AC power flow equations (continuous, nonlinear)
    - Thermal and voltage limits
- **Challenges:**
  - **Problem size:** 1500-bus grid problem leads to ~**1M decision variables**
  - **Problem complexity:** mixed integer linear optimization problem (piecewise linear AC approximation)

I. Aravena, D. Rajan, G. Patsakis, "Mixed-integer linear approximations of AC power flow equations for systems under abnormal operating conditions," *IEEE PES General Meeting, 2018* .

Previous Algorithmic Developments

- **Specialized integer L-shaped** method for power system restoration exploiting time and spatial decomposition

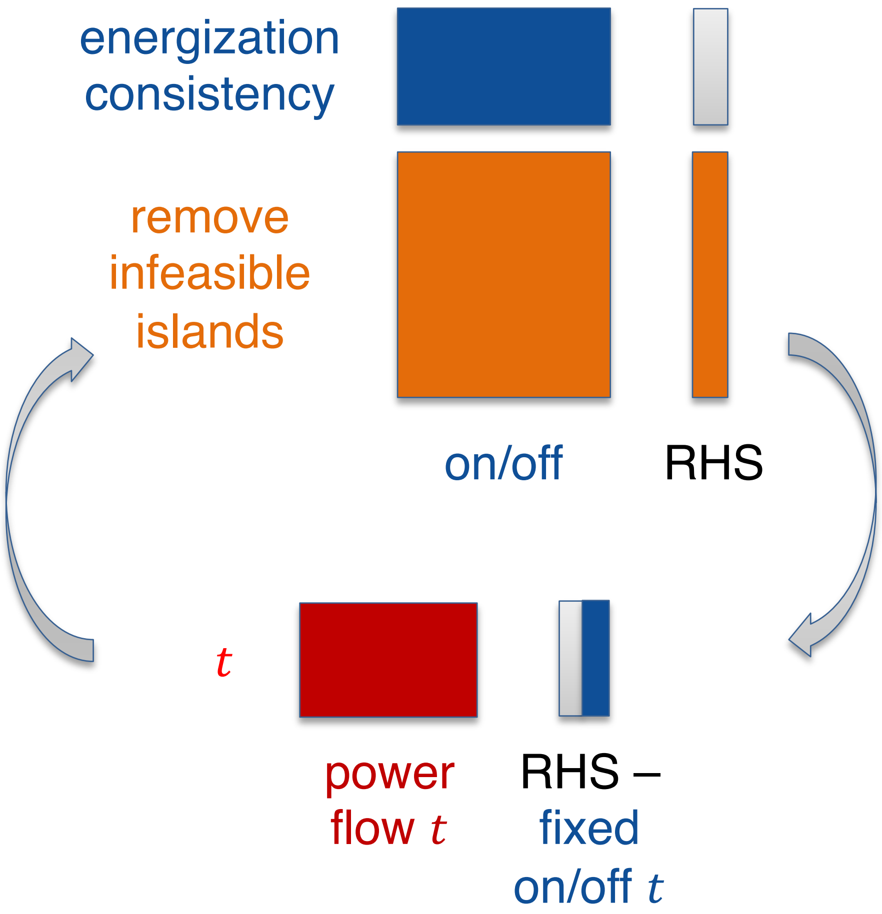
- Add the following novel cuts:
  - **Energization cuts**: cuts and strengthening inequalities for islands without online generators
  - **Hybrid Benders-binary island-based cuts**: remove islands with no power flow solution to linear relaxation
  - **Binary island-based cuts**: remove islands for which linear power flow is feasible but MIP power flow is not

- **Heuristics** to:
  - Greedily generate an initial solution
  - Round relaxed energization solutions to consistent plans
  - Generate improved energization plans out of known feasible configurations

G. Patsakis, D. Rajan, I. Aravena, S. Oren, "Strong Mixed-Integer Formulations for Power System Islanding and Restoration," *IEEE Trans. Power Systems*, 2019.

I. Aravena, D. Rajan, G. Patsakis, S. Oren, J. Rios, "A scalable mixed-integer decomposition approach for optimal power system restoration," *Optimization Online*, 2019.

Specialized integer L-shaped for OPSR



1. Solve problem considering only on/off variables and consistency
2. Check power flow feasibility feasibility of all islands
3. Add infeasibility cuts, gradually ban infeasible islands

Numerical performance: settings

- Implementation using Julia/JuMP/LightGraphs, Ipopt, Gurobi

- Solving restoration using
  - *DC* power flow
  - *LP AC* linear approximation
  - *MILP AC* linear approximation

- Each experiment running on a single node of Quartz with a time limit of 48 hours or 1% optimality

- Multi-threaded (24 threads) evaluation of island feasibility with static balancing based on island size

Numerical performance: black start instances

System	buses	branches	gens	Power flow	Value [-]	Gap [%]	Solution time [s]	Time composition [%] B&B	Cuts	Heur.
IEEE39	39	46	10	DC	26.41	0.01	13.0	14.6	0.8	84.6
				LRAC	26.41	0.01	17.5	10.3	1.1	88.6
				MIAC	26.41	0.01	22.2	8.1	1.8	90.1
IEEE118	118	186	54	DC	18.73	0.03	22.6	13.3	2.7	84.1
				LRAC	18.73	0.03	49.6	5.8	2.6	91.5
				MIAC	18.58	0.84	19 901.1	0.1	99.0	0.9
Chile	1 548	1 954	297	DC	32.52	0.77	33 123.7	90.2	4.6	5.2
				LRAC	30.65	*6.88	84 044.5	69.7	23.9	6.4
				MIAC	30.64	*7.26	172 188.3	5.4	14.3	80.2

- Performance highly dependent on power flow model used: convex models (DC, LRAC) much easier than non-convex (MIAC) due to over-voltages in early stages of black start
- **Algorithm *works!*** (without algorithm: problem cannot be even loaded in memory for large instances, large constants in MILP make relaxations very lose)

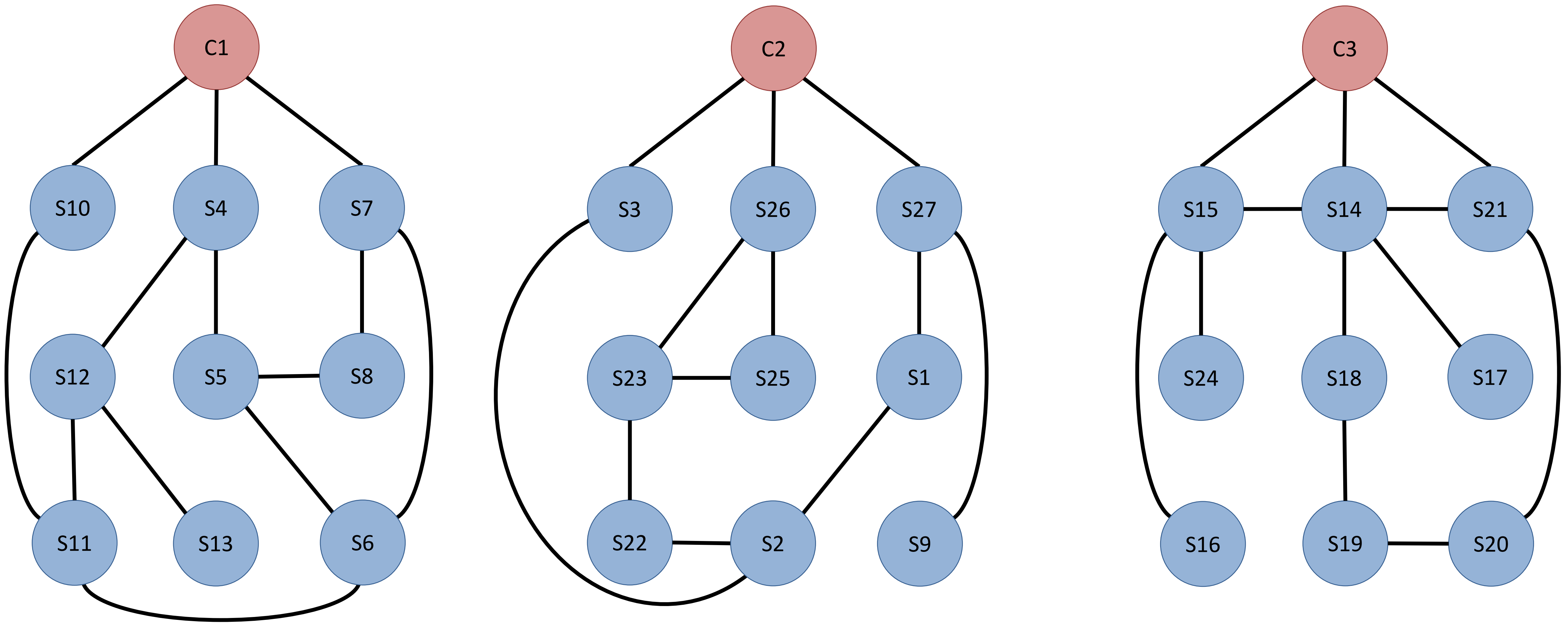
Control System Restoration

- Extreme events and cyber-attacks may damage or compromise communications systems used to monitor and control power systems
- Equipment cannot be energized if a control signal cannot be sent to the substation where the bus is located
- Restoration with compromised communications workflow:

Asses damage → **Repair/replace** → **Energize**

- Problem: limited trained human resources to repair/replace damaged/compromised equipment
- Questions:
  - How are energization decisions affected if the control network is damaged along with the power system?
  - What is the optimal restoration sequence for damaged communication systems?

Communications Network Structure – IEEE 39-Bus Example



Other Communications Model Assumptions

1. A communication channel is operational if and only if the communications equipment at both terminals (substations or control centers) of the channel is functioning

2. If the communications equipment at a substation is not functioning, the substation will be de-energized for operational security

3. Control centers are always energized, though the communications equipment at control centers may be damaged or compromised

Communications in Power System Model

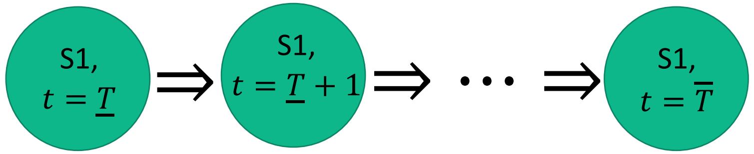
- Overlay communications model on power system model developed in [1]
- Only consider energization decisions in conjunction with communication equipment repairs to start with
- Objective unchanged
- Added decision variables:
  - $w_{a,t}$: indicator of the operational status of communications equipment at substation, control center, or communications channel a at time t
  - $w_{a,t}^{rep}$: binary variable that's 1 if repair begins on communications equipment at substation or control center a at time t, 0 otherwise

[1] I. Aravena, D. Rajan, G. Patsakis, S. Oren, J. Rios, "A scalable mixed-integer decomposition approach for optimal power system restoration," *Optimization Online*, 2019.

Communications System Constraints

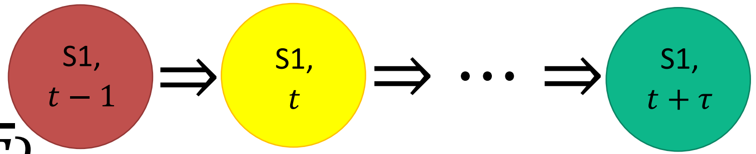
- Communications equipment that is operational at the start of the time horizon will remain operational,

$$w_{a,t} = 1 \; \forall \; a \in C_0 \cup S_0, t \in T$$



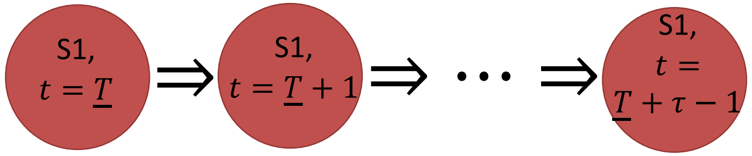
- All other communications equipment will become operational τ_a periods after repairs begin,

$$w_{a,t} - w_{a,t-1} = w^{\text{rep}}_{a,t-\tau_a} \; \forall \; a \in \{C \backslash C_0\} \cup \{S \backslash S_0\}, t \in \{\underline{T} + \tau_a, \ldots, \overline{T}\}$$



- The operational status of damaged systems must be 0 until enough time has passed for repairs to be made,

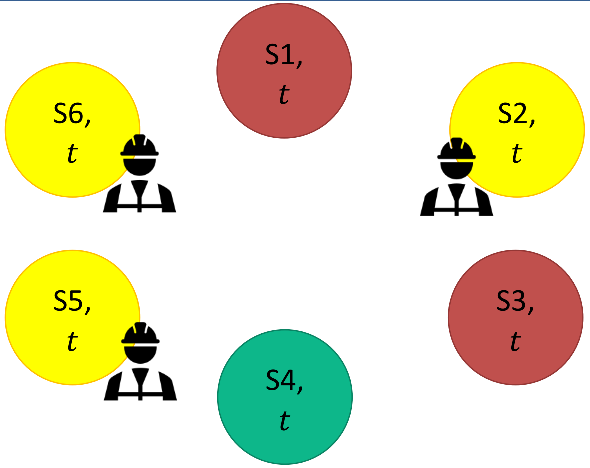
$$w_{a,t} = 0 \; \forall \; a \in \{C \backslash C_0\} \cup \{S \backslash S_0\}, t \in \{\underline{T}, \ldots, \underline{T} + \tau_a - 1\}$$



Communications System Constraints (cont.)

- The amount of equipment simultaneously undergoing repair is limited by the amount of workers available,

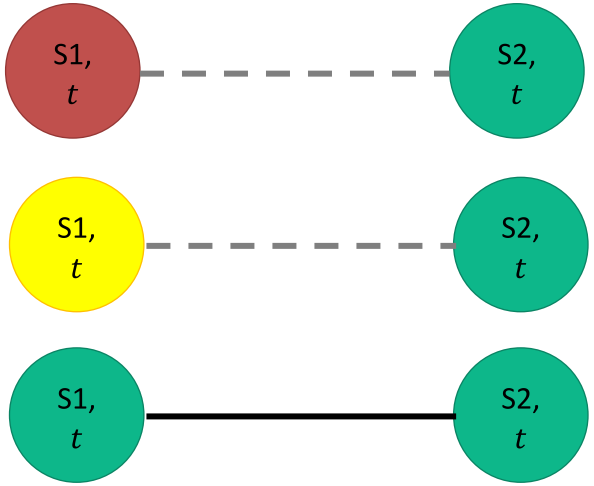
$$\sum_{a\in\{C\setminus C_0\}\cup\{S\setminus S_0\}} \sum_{t'=\max\{\underline{T},t-\tau_a+1\}}^{t} w_{a,t'}^{\mathrm{rep}} \leq K^{\mathrm{comm}} \;\forall\, t \in T$$



- Communications lines are only operational if the communications equipment at both terminals is operational,

$$w_{l,t} \leq w_{a,t} \;\forall\, l \in L^{\mathrm{comm}}, a \in I(l), t \in T$$

$$w_{l,t} \geq \sum_{a\in I(l)} w_{a,t} - 1 \;\forall\, l \in L^{\mathrm{comm}}, t \in T$$



Communications System Constraints (cont.)

- A bus can only be energized if there is an operational communications path from the control center to its substation,

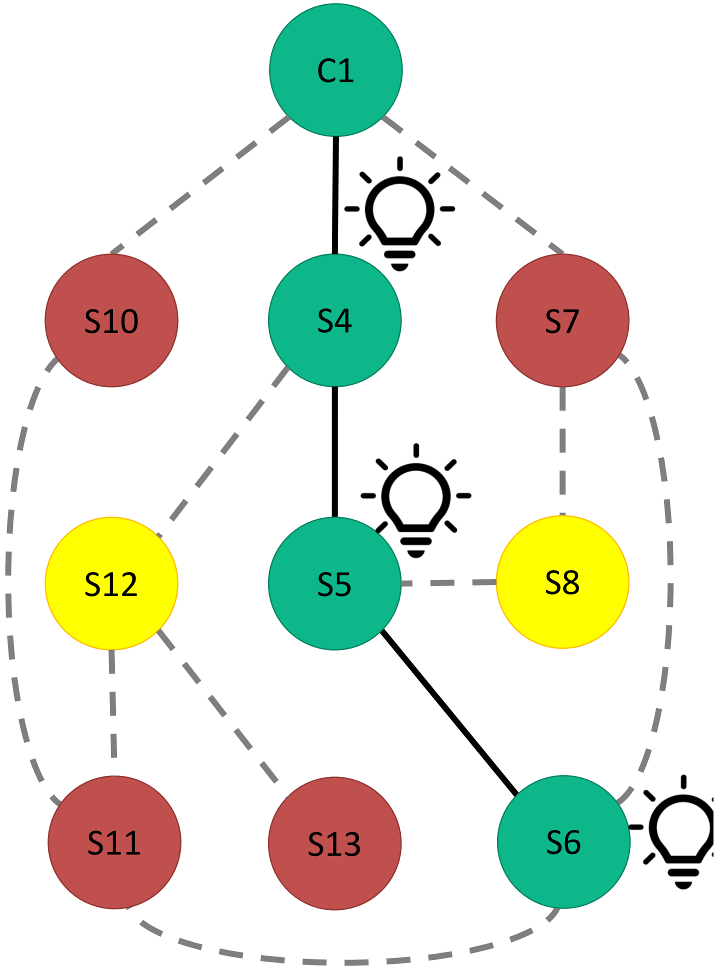
$$u_{n,t} \leq w_{c,t} \forall\, c \in C, s(n) \in S(c)$$

$$u_{n,t} \leq \sum_{l \in L^{\text{comm}}(\delta(M))} w_{l,t} \forall\, c \in C, M \subseteq S(c) : s(n) \in M, t \in T$$

  — Exponentially many inequalities, but separable as min-cut/max-flow

- For a bus to be energized, the communications equipment at its substation must be operational,

$$u_{n,t} \leq w_{s(n),t} \forall\, n \in N, t \in T$$



Exponential constraints separation

- Incumbent separation (at integer candidates) much faster (linear time) than general separation (quadratic time)
- For each time t:

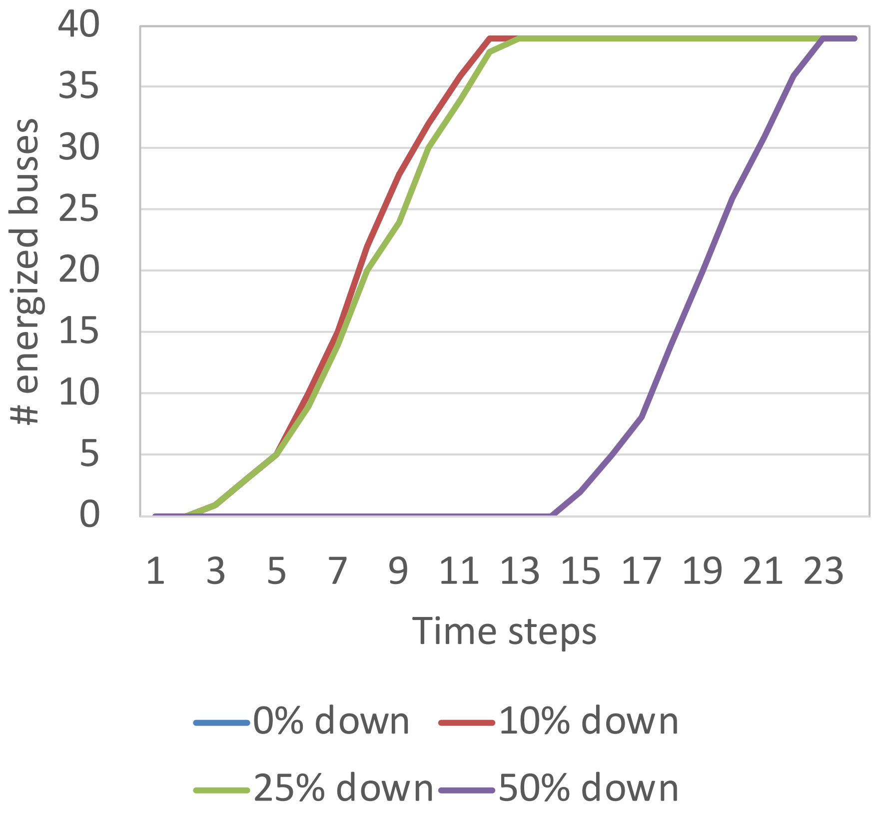
1. Construct graph with available communications channels ($w_{l,t} = 1$)
2. Split graph into connected components
3. Find connected components without control centers
4. Find energized buses ($u_{n,t} = 1$) within connected components without control centers → **violated communication constraints**

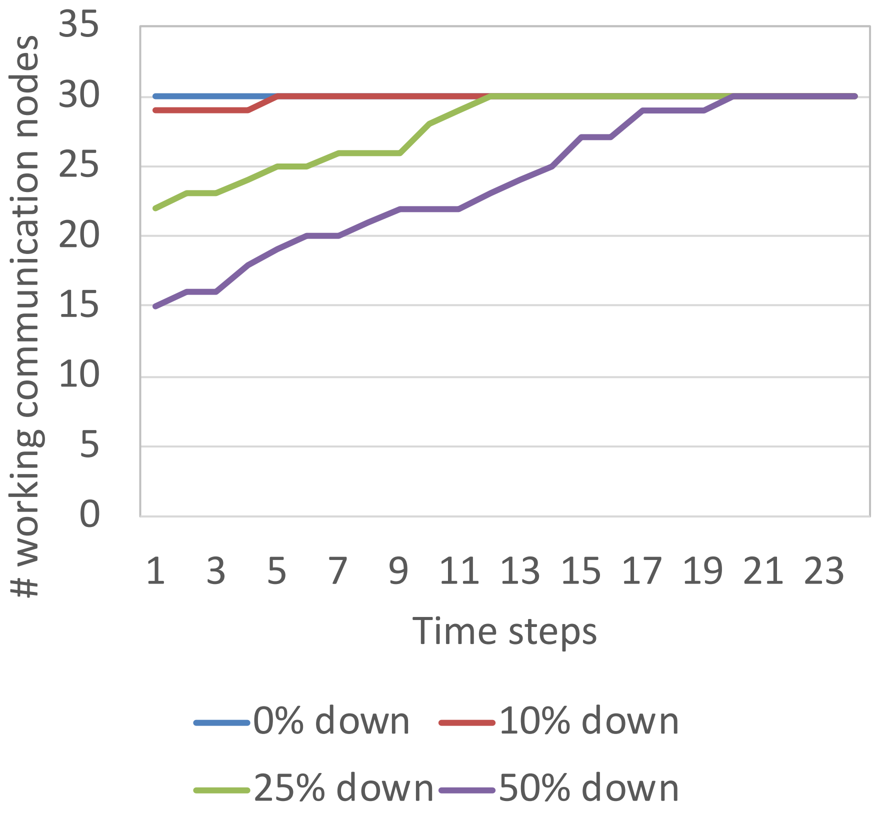
Numerical experiments

- Minimal power flow representation: existence of at least one generator providing voltage to each electrical island (relaxation of DC and AC power flows)
- Running experiments on IEEE39 and IEEE118 tests systems with communications networks generated following procedure of [2]
- Performance observations:
  - All experiments finished in less than 15 seconds
  - At most 40 constraints from the exponential class generated on the fly
  - Further study with power flow constraints and larger instances in progress

[2] M. Korkali et al., "Reducing Cascading Failure Risk by Increasing Infrastructure Network Interdependence," Nature Scientific Reports 7, 2017.

IEEE039: Energizations decisions for different levels of damage in communications network





(similar, but less pronounced effects, observed for IEEE118)

Conclusions

- Specialized integer L-shaped method in conjunction with initialization heuristics allows us to solve
  - Instances up to **10x the size of the state-of-the-art** when using an LP approximation of the power flow equations
  - Instances of the size of the state-of-the-art using an MILP approximation **(taking over-voltages into consideration)**
- Adding control network to the model allows us to analyze the impact of a cyber-attack or damaged communication systems on restoration planning
- Damaged/weak communications network can significantly delay restoration of power to critical services and end costumers
- Perform grid energization actions (starting up generators, switching lines) in parallel to repairs to the communication network results critical in attaining good restoration performance

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Generic integer L-shaped method

- Assume you want to solve

$$\min_{x\in\{0,1\}^n} \quad f(x)$$
$$s.t. \quad Ax \geq b$$
$$g(x) \geq 0$$

where $g(x)$ is a **complicated function**.

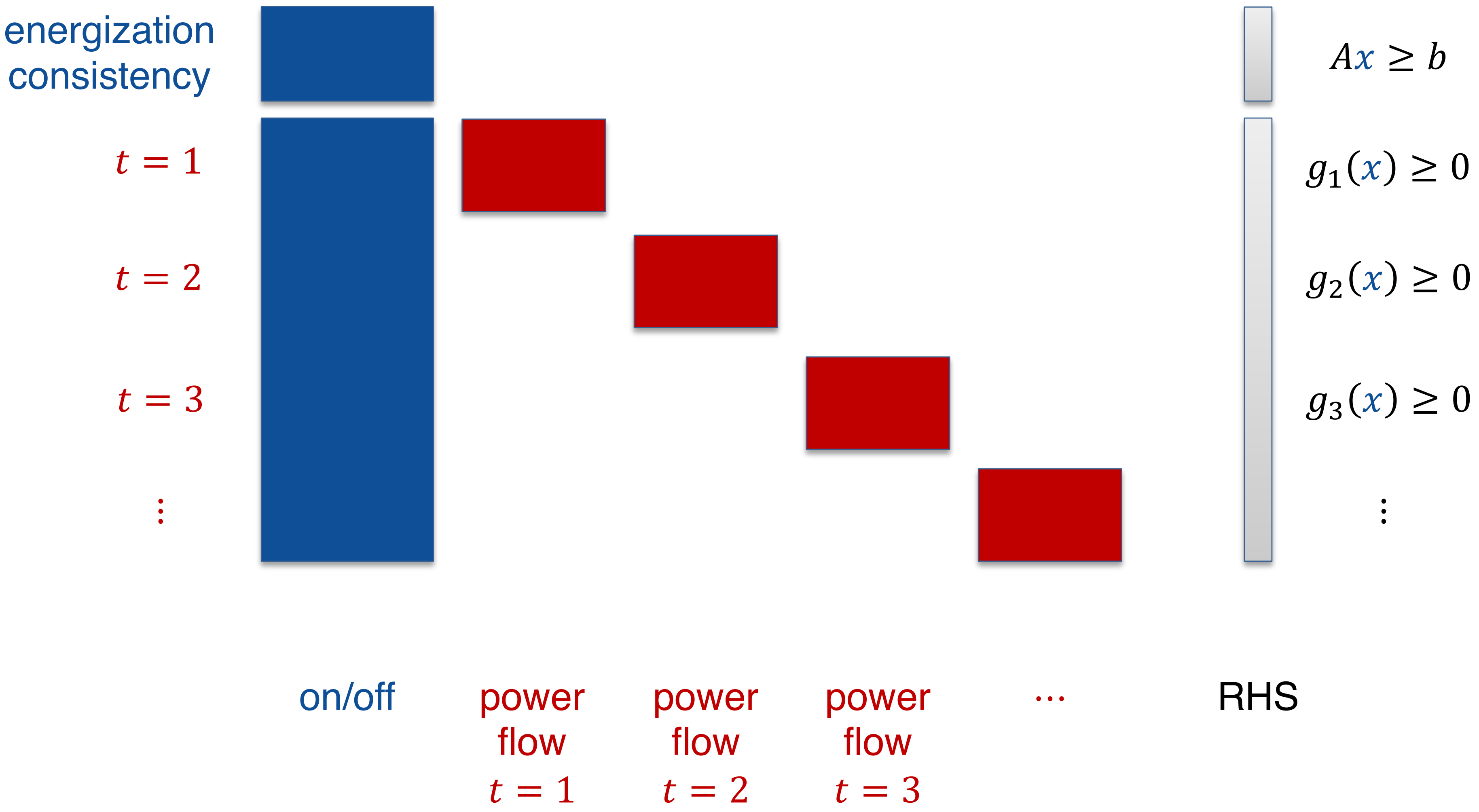
- Note: for a given $\tilde{x}$, if $g(\tilde{x}) \leq 0$ we can separate $\tilde{x}$ by the following cut

$$\sum_{i:\tilde{x}_i=0} x_i + \sum_{i:\tilde{x}_i=1} (1 - x_i) \geq 1$$

Integer L-shaped method

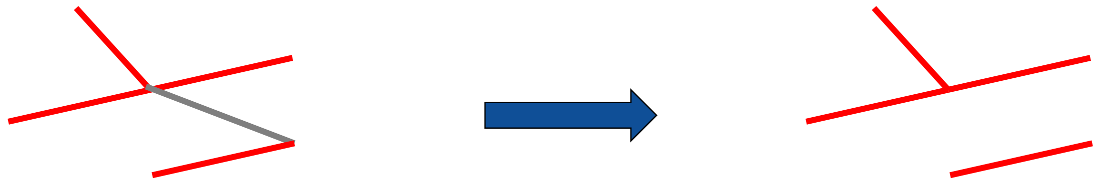
1. Let $C = [], d = []$
2. Solve
$$\min_{x\in\{0,1\}^n} \quad f(x)$$
$$s.t. \quad Ax \geq b, Cx \geq d$$
3. Evaluate $g(x^*)$
4. If $g(x^*) \geq 0$, then $\boldsymbol{x}^*$ **is optimal for the original problem.**
Else, append cut parameters to C, d and return to 2.

OPSR: Structure of the constraint matrix



OPSR: Further structure for given energization decisions

- For any time t, a $\{0,1\}$ energization status for all components allows to decompose the system in islands



- Power flow feasibility can be determined at the island level → **large potential for parallelism** in large scale systems
- An energization plan $\tilde{x}$ (respecting consistency) is **feasible iff all of its islands are feasible**

Infeasibility cuts

- Power flow approximation is LP feasible, but MILP infeasible: **integer L-shaped cut for island $\mathcal{J}$ at time t**

$$\sum_{l:l\in L(\mathcal{J}),\,\tilde{x}_l=0} x_{l,t} + \sum_{l:l\in L(\mathcal{J}),\,\tilde{x}_l=1} (1-x_{l,t}) + \sum_{l:l\in L(\partial\mathcal{J})} x_{l,t} + \cdots \geq 1$$

- Power flow approximation is LP infeasible: **hybrid Benders-integer cut for island $\mathcal{J}$at time t**

$$v_{\mathcal{J}} + \sum_{g\in G(\mathcal{J})} w_{\mathcal{J},g} x_{g,t} \leq M \cdot \left(\sum_{l:l\in L(\mathcal{J}),\,\tilde{x}_l=0} x_{l,t} + \sum_{l:l\in L(\mathcal{J}),\,\tilde{x}_l=1} (1-x_{l,t}) + \sum_{l:l\in L(\partial\mathcal{J})} x_{l,t} \right)$$

  where v, w describe an unbounded ray of the dual of the LP relaxation

- Islands tend to reappear at different times → **MAINTAIN hash table with all evaluated islands**

Creating an initial solution through a greedy heuristic

- Bare branch-and-bound will terminate, however, **practical performance also relies on heuristics**
- An initial solution provides with a starting point for solver heuristics (e.g. local search) and helps in fathom B&B nodes. We create an initial restoration plan by executing the following steps for each t

1. Clone $t - 1$ solution
2. Update online generator status (**cranking → on**, if possible)
3. Greedily **energize branches** in border of each island (decreasing capacity), up to energization budget
4. Greedily **startup more generators**

Rolling-horizon heuristics for rounding at B&B nodes

- At certain nodes of the B&B tree, we can round the node solution to a consistent or, even, feasible restoration plan

- Generating a consistent restoration plan (respecting $Ax \geq b, Cx \geq d$):

  For each time t, obtain consistent energization decisions by **minimizing L1 rounding error subject to $\boldsymbol{Ax \geq b}$, $\boldsymbol{Cx \geq d}$ (existing cuts) and $\boldsymbol{x \in \{0, 1\}^n}$**

- Generating a feasible restoration plan:

  For each time t, **construct a configuration using feasible islands** from the **hash table**, such that it respects sequential energization and the maximum number of energizations