Period Optimal Power Flow Model in Power Systems with High Penetrations of Intermittent Power Sources

Names : Zongjie Wang, C. Lindsay Anderson

CONTENT

Background

Generation Scheduling Plan

- Executed based on net load forecasts;
- Actual power curve: unknown, continuous;
- Forecasted power output: known, predicted at discrete timepoints, piece-wise linear function.

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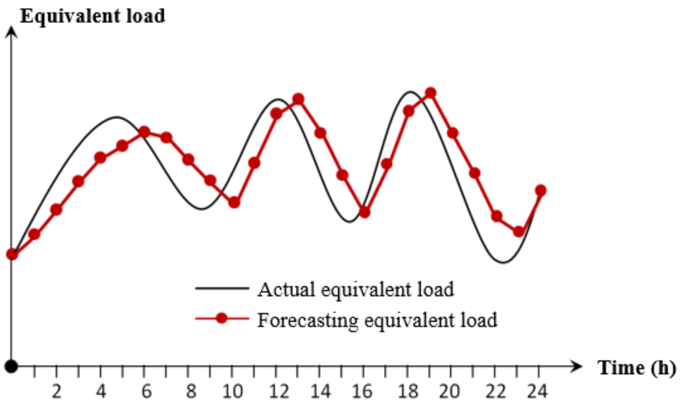

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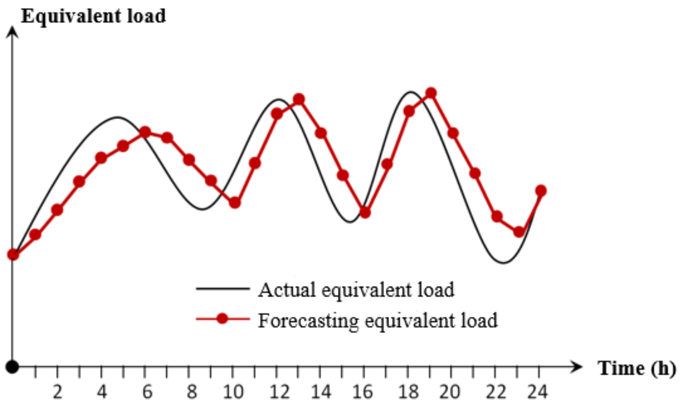

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- A single specified timepoint;
- Applies the decision to the time period.

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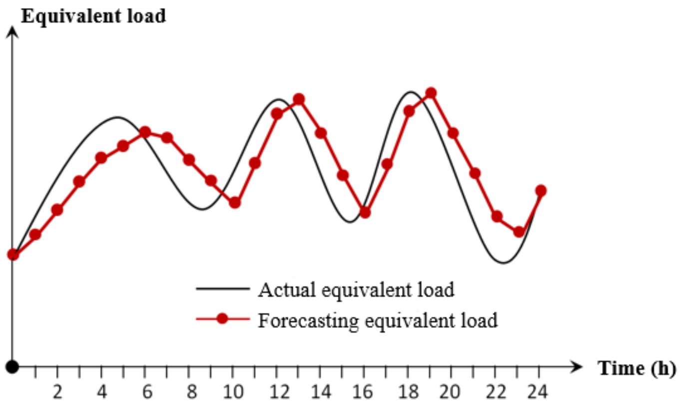

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- Constraints satisfaction are not guaranteed.

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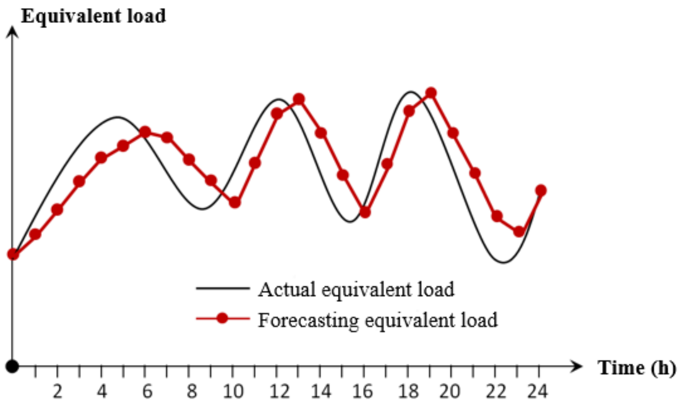

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Questions

How to **select** the specified timepoints to better represent the entire time period?

Desirable Characteristics and Approaches

Desirable Characteristics

- Present a fast and accurate OPF model over a time period
- Reduce the complexity of finding solutions under non-linear time-varying equations
- Theoretically guarantee system's economy and security over a time period
- Coherent scheme between short-term and real-time dispatch

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3. **Combined time-varying node voltage function:** the order of maximum voltage error is restricted within 10^{-4}

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2. Under LI, each node voltage is **approximate linear function** of time.
3. **Combined time-varying node voltage function:** the order of maximum voltage error is restricted within 10^{-4}
4. **Period optimal power flow (POPF) :** objective function—median timepoint; constraints--two terminal timepoints

Linear-time Interval (LI)

Norm of Jacobian matrix

Proposition 1 Under a LI, in the rectangular coordinate, the norm of *k*-th derivative of Jacobian satisfies:

$$\left\|\boldsymbol{J}^{(k)}(t)\right\| \leq \left\|\boldsymbol{J}(t)\right\| \left\|\boldsymbol{x}^{(k)}(t)\right\|, (k \geq 0)$$

where *x* is node voltage vector.

Norm of node voltage derivatives

Proposition 2 Under a LI, for a *d*-th order derivative of node voltage vectors, it follows:

$$\left\|\boldsymbol{x}^{(d)}(t)\right\| \leq (2d-3)!!\rho_t^{d-1} \left\|\boldsymbol{x}^{(1)}(t)\right\|^d, \ (d \geq 2)$$

where $(\bullet)!!$ is double factorial, ρ_t is the condition number: $\rho_t = \left\|\boldsymbol{J}^{-1}(t)\right\|_p \left\|\boldsymbol{J}(t)\right\|_p, (p = 1, 2, \infty)$

By selecting a proper coefficient $\alpha_t \in (0,1)$, ρ_t is corrected as follows:

$$\left\|\boldsymbol{x}^{(d)}(t)\right\| \approx (2d-3)!!\tilde{\rho}_t^{d-1} \left\|\boldsymbol{x}^{(1)}(t)\right\|^d, \ (d \geq 2) \quad \tilde{\rho}_t = \alpha_t \rho_t \leq \rho_t$$

Time-varying node voltage properties

Property 1 Under a LI, the norms of node voltage derivatives present a “U” curve when the order of derivatives increases.

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Combined Time-varying Node Voltage Function

Taylor series for the beginning timepoint t_0 and the ending timepoint t_e are shown as

$$\begin{cases} \boldsymbol{x}(t,t_0) = \boxed{\boldsymbol{x}(t_0)} + \Delta t_0 \boldsymbol{x}^{(1)}(t_0) + \frac{1}{2}\Delta^2 t_0 \overline{\boldsymbol{x}}^{(2)} \\ \boldsymbol{x}(t,t_e) = \boxed{\boldsymbol{x}(t_e)} + \Delta t_e \boldsymbol{x}^{(1)}(t_e) + \frac{1}{2}\Delta^2 t_e \overline{\boldsymbol{x}}^{(2)} \end{cases} \quad \text{where} \begin{cases} \Delta t_0 = t - t_0 \\ \Delta t_e = t - t_e \end{cases}$$

$\overline{\boldsymbol{x}}^{(2)}$ is given by the following numerical differentiation:

$$\overline{\boldsymbol{x}}^{(2)} = \frac{1}{\Delta T}(\boldsymbol{x}^{(1)}(t_e) - \boldsymbol{x}^{(1)}(t_0))$$

Combined node voltage function:

$$\boldsymbol{x}(t) = \boldsymbol{x}(t,t_0) + \alpha(t)(\boldsymbol{x}(t,t_e) - \boldsymbol{x}(t,t_0)), \ \alpha(t) = \frac{1}{\Delta T}\Delta t_0$$

$$\boldsymbol{x}(t) = \boldsymbol{x}(t_0) + \Delta t \boldsymbol{x}^{(1)}(t_0)$$

Property: Under a LI, for the combined node voltage function, the order of maximum node voltage error is restricted within 10^{-4}.

LI Algorithm for Time-varying Power Flow

1) LI partition

M+1 combined discrete timepoints $t_l, (l = 0,1,...,m)$ partitions the specified time period into *m* combined LIs $T_l, (l = 1,2,...,m)$.

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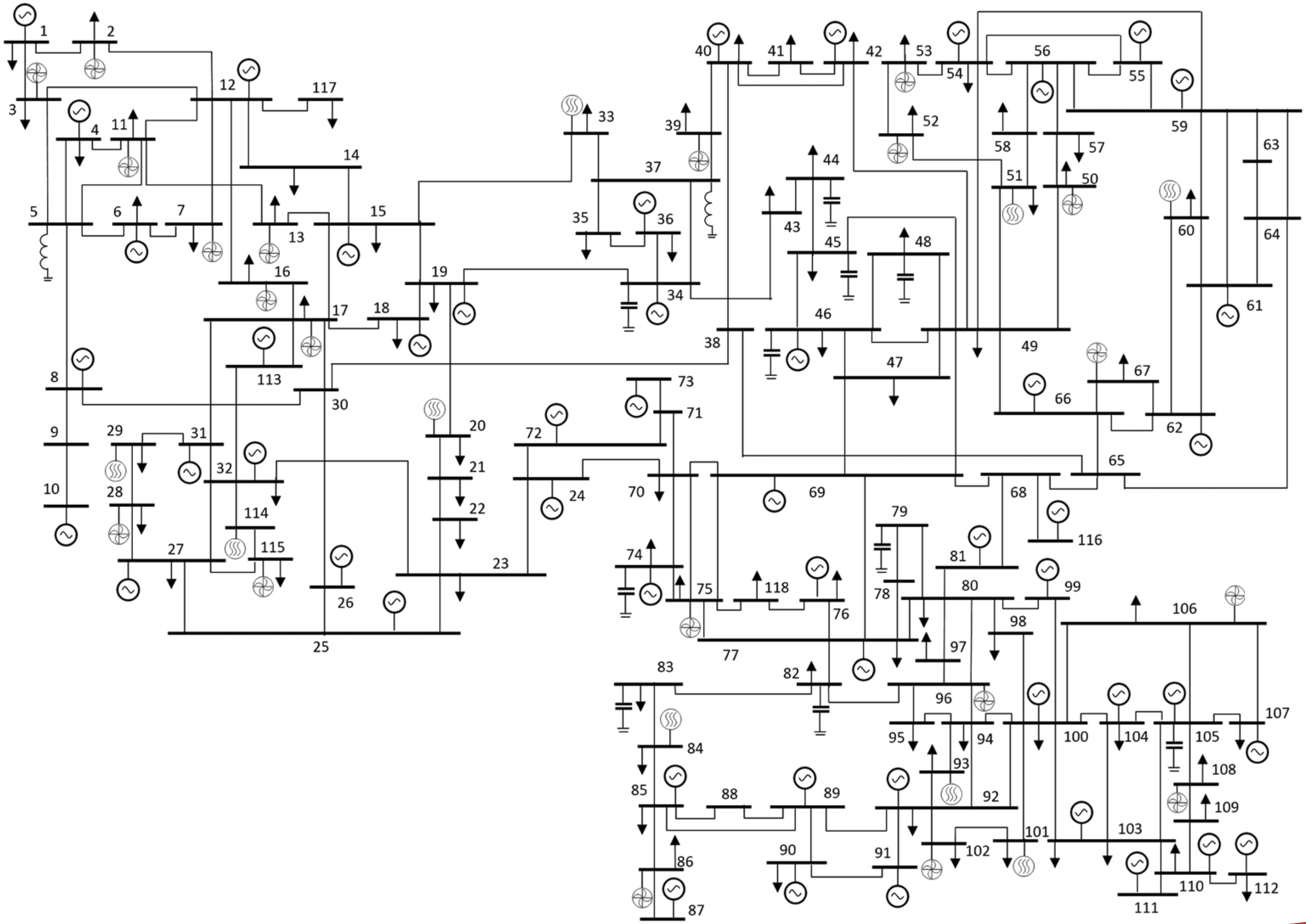
4) Calculations for other parameters

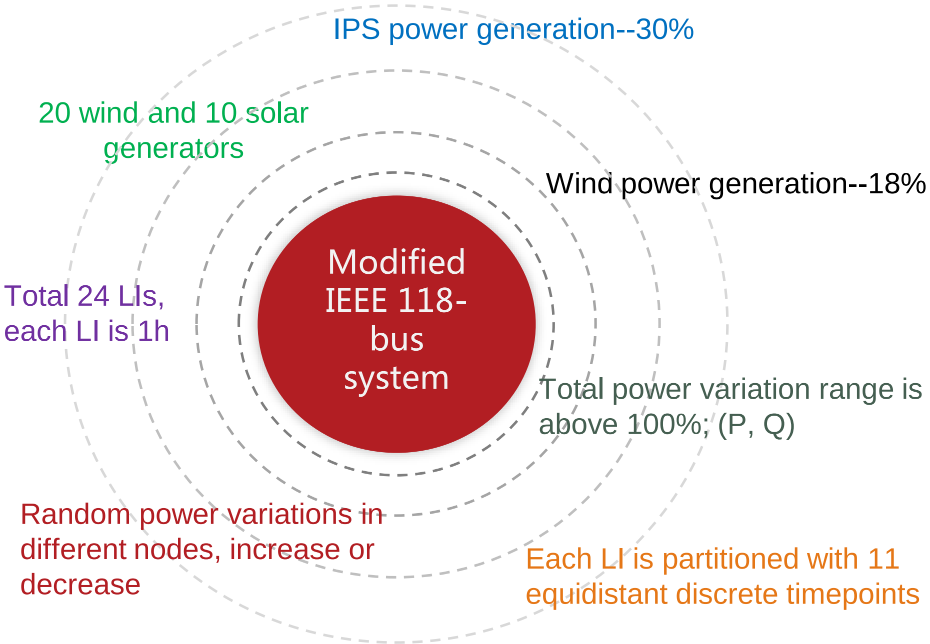
Distributions of other parameters at any timepoint of an interval can be also calculated.

Case Studies

Modified IEEE 118-bus system.



Case Studies



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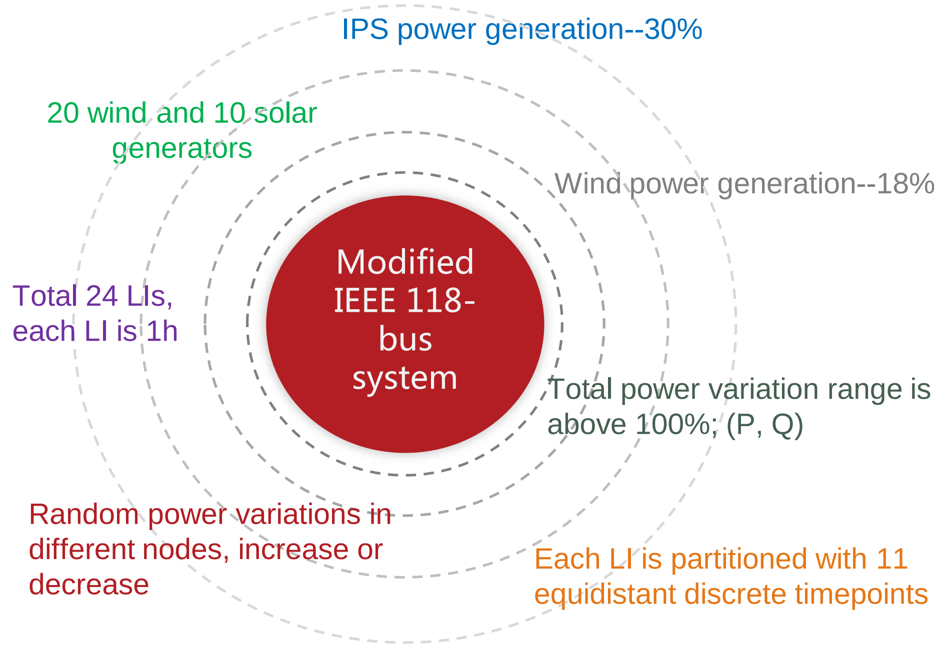
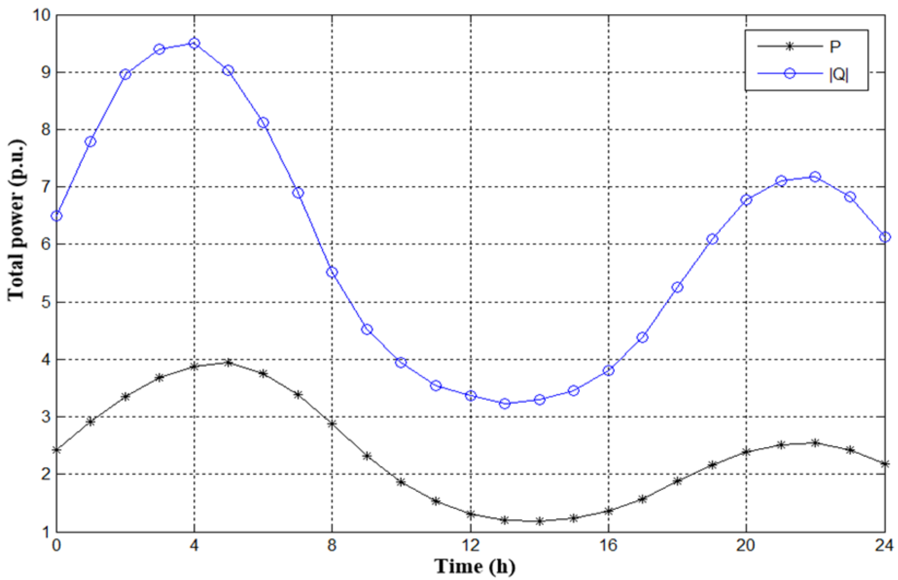
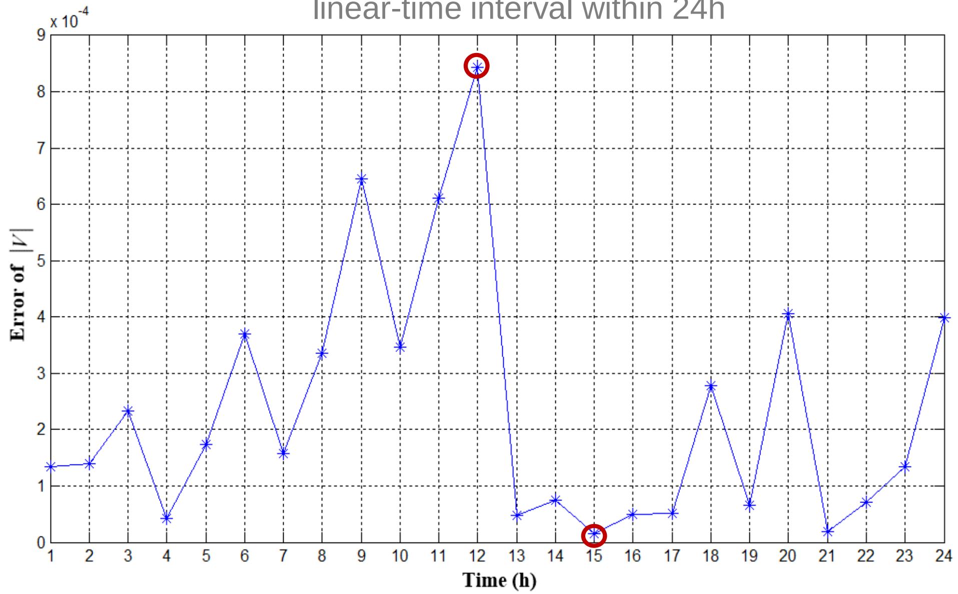




Total power curves within 24h

Case Studies

Maximum voltage errors for each linear-time interval within 24h



- Maximum voltage error---12th time interval: 8.4×10^{-4}
- Minimum voltage error---15th time interval: 1.4×10^{-5}
- For any timepoint---the order of each node voltage error is within 10^{-4}. **High accuracy**

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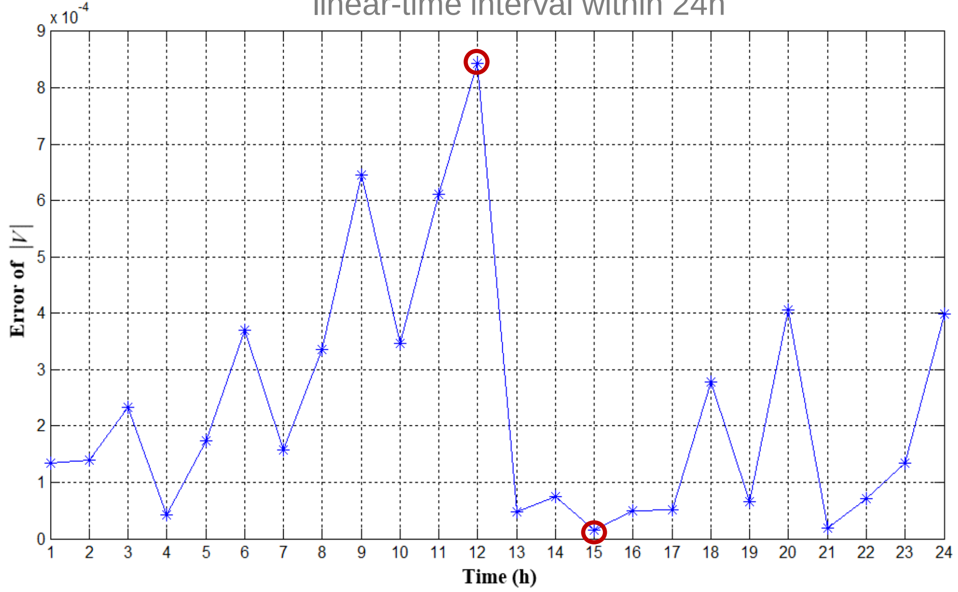
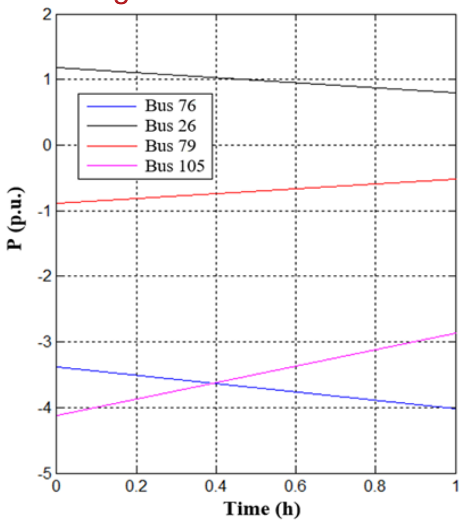
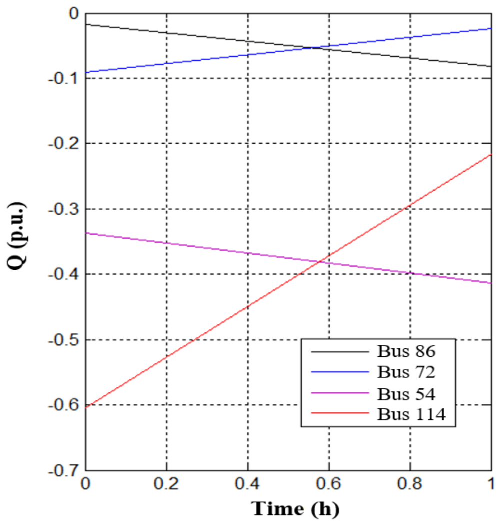


First four nodes with the largest power variation ranges in the 21st LI:





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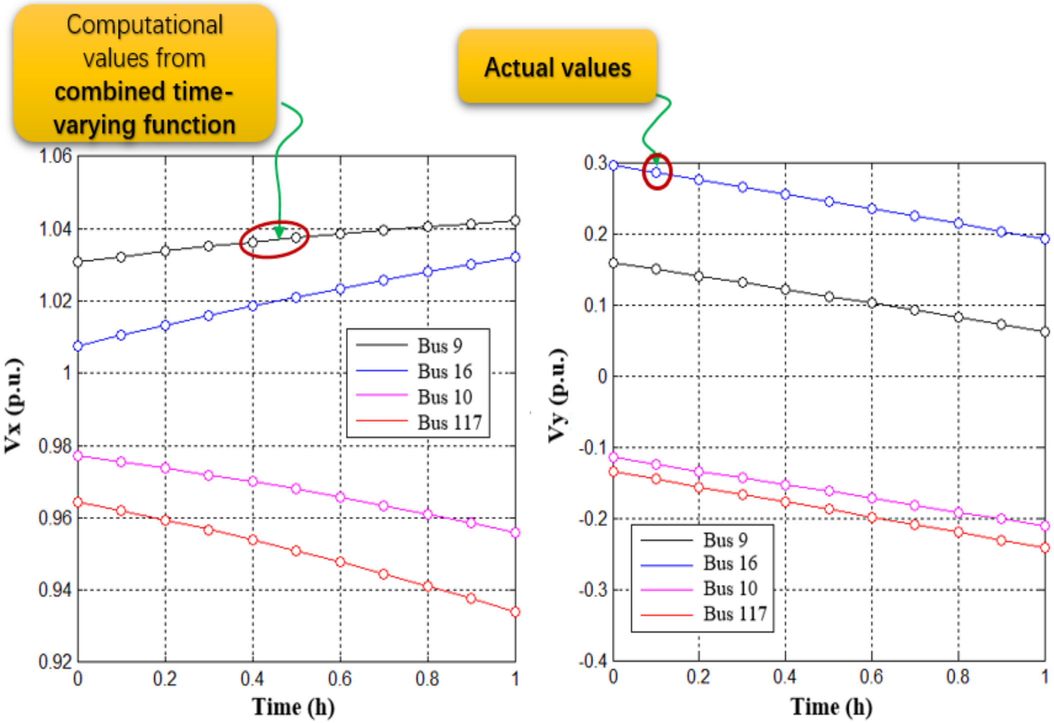
Case Studies (the 21st LI)

Voltage computational results from combined node voltage functions in the 21st LI.



Observation 1: Under a LI, real and imaginary parts of node voltage curves are approximate linearity with respect to time.

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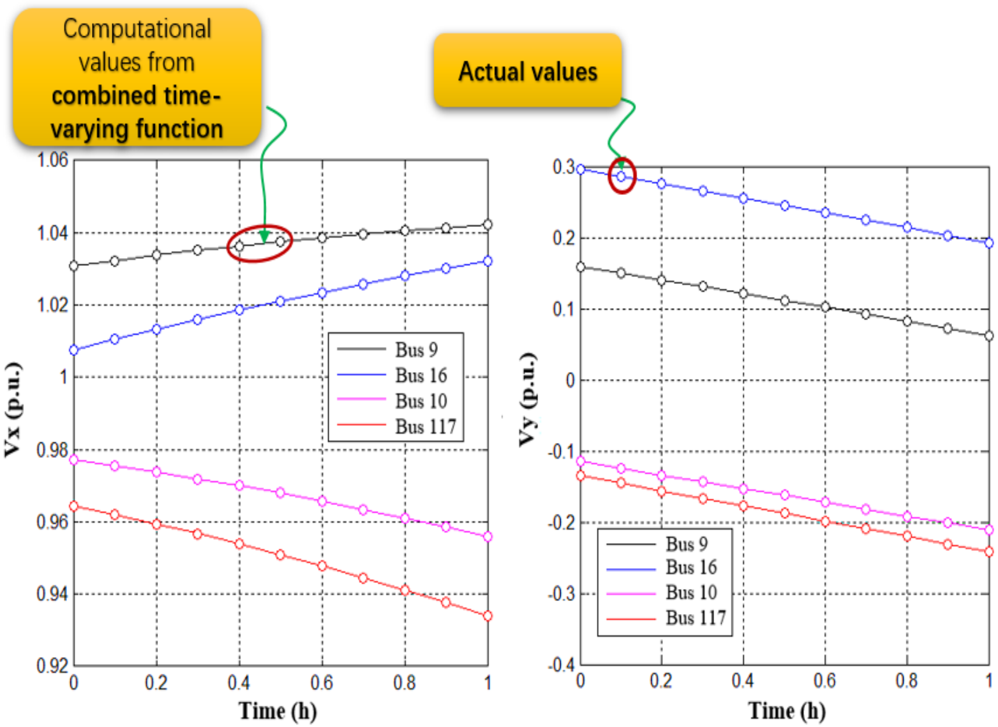

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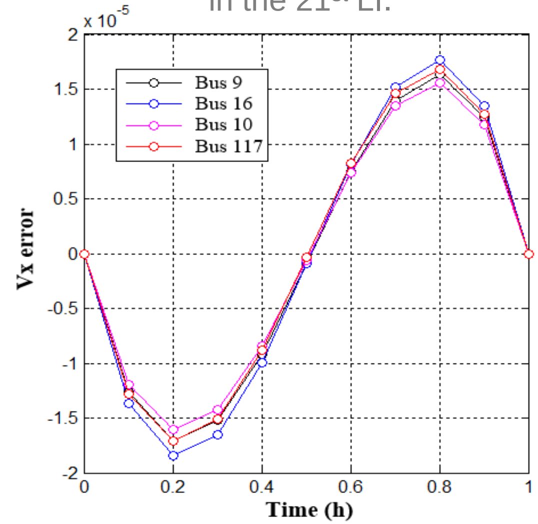

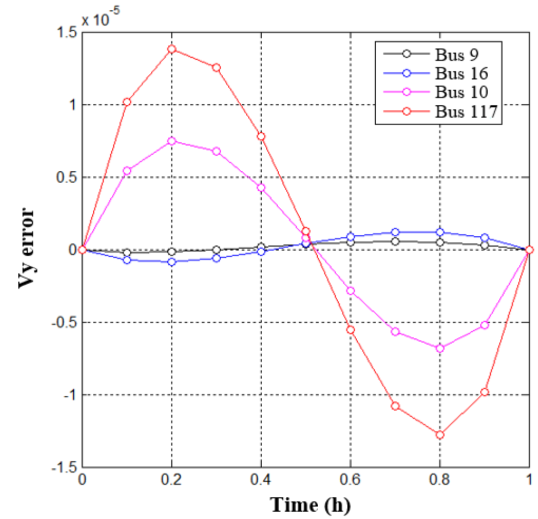

Observation 2: Orders of the first four maximum voltage errors are all within 10^{-5}.

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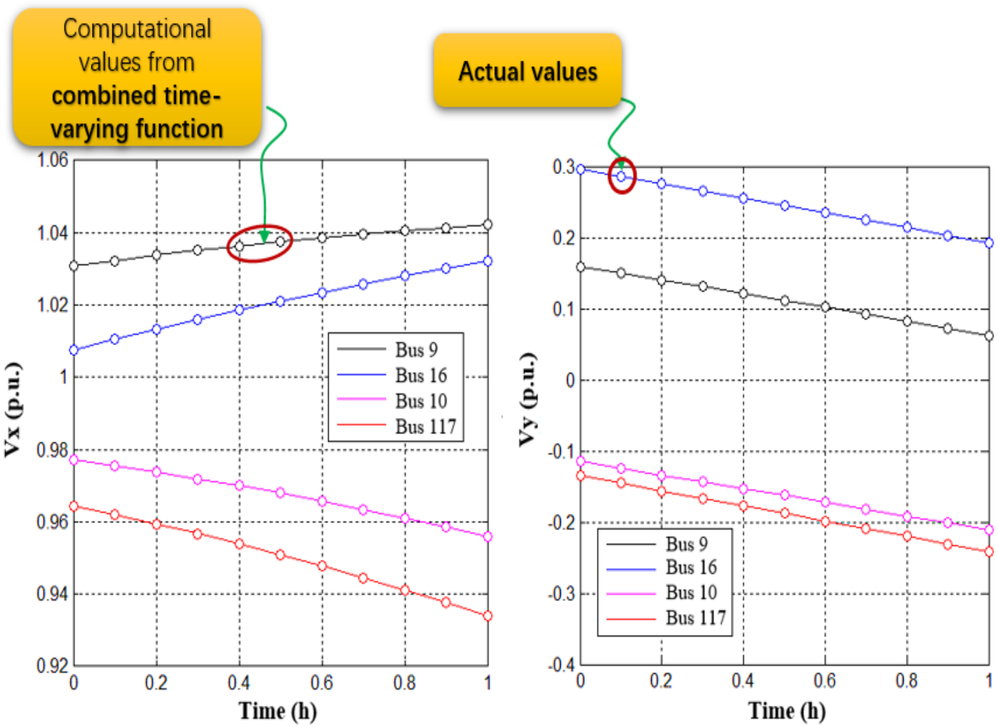

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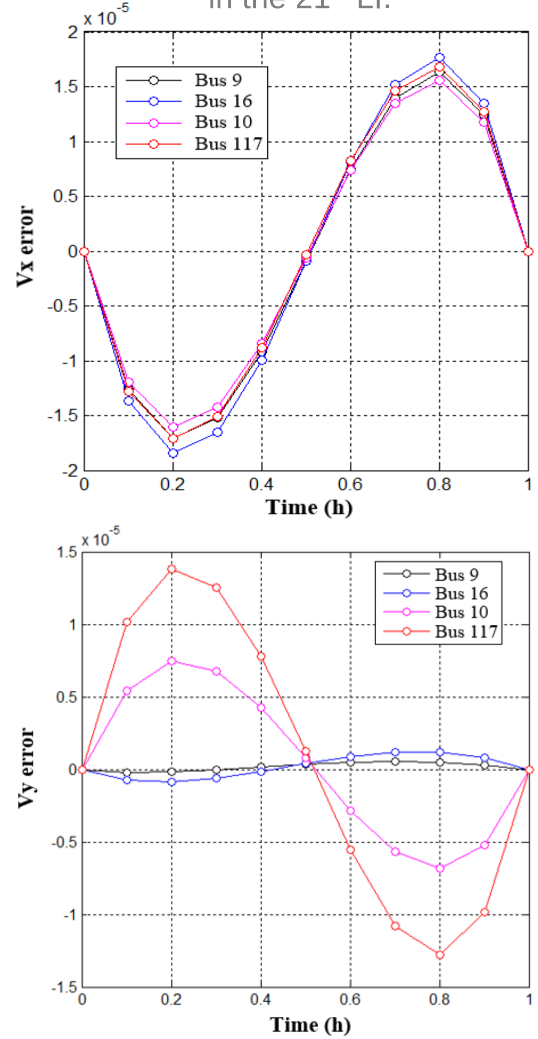
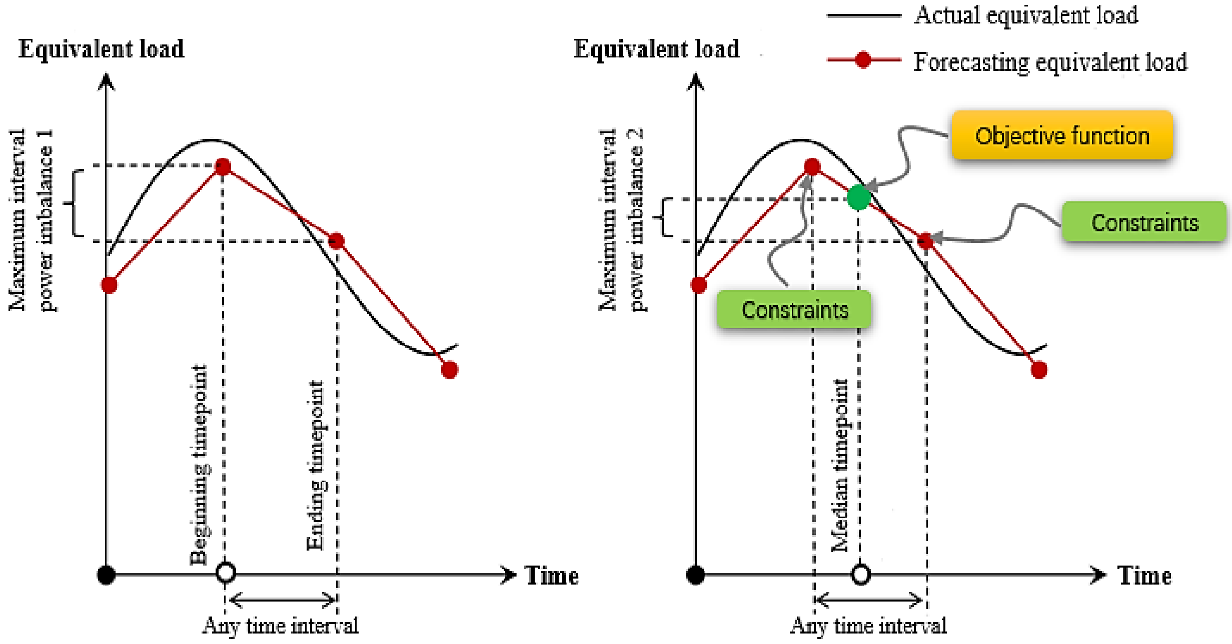


Observation 1: Under a LI, real and imaginary parts of node voltage curves are approximate linearity with respect to time.

Observation 2: Orders of the first four maximum voltage errors are all within 10^{-5}.

Observation 3: The median and two terminal timepoints have the smallest voltage errors---close to 0.

Period Optimal Power Flow (POPF)



TOPF

POPF

- Energy integral property
- Smallest voltage error
- Terminal constraint satisfaction

Period Optimal Power Flow (POPF)

- Objective Function (median timepoint) $\forall T_k : \min f(\boldsymbol{U}, \boldsymbol{X}, t_k^m)$
- Constraints (median timepoint) $\forall T_k : \boldsymbol{h}(\boldsymbol{U}, \boldsymbol{X}, t_k^m) = 0$

- Constraints (Two terminal timepoints)

Power flow equality constraints

$$\forall T_k : \begin{cases} \boldsymbol{h}(\boldsymbol{U}, \boldsymbol{X}, t_{k-1}) = 0 \\ \boldsymbol{h}(\boldsymbol{U}, \boldsymbol{X}, t_k) = 0 \end{cases}$$

Voltage constraints

$$\forall T_k : \begin{cases} \boldsymbol{V}_{\min} \le \boldsymbol{V}(t_{k-1}) \le \boldsymbol{V}_{\max} \\ \boldsymbol{V}_{\min} \le \boldsymbol{V}(t_k) \le \boldsymbol{V}_{\max} \end{cases}$$

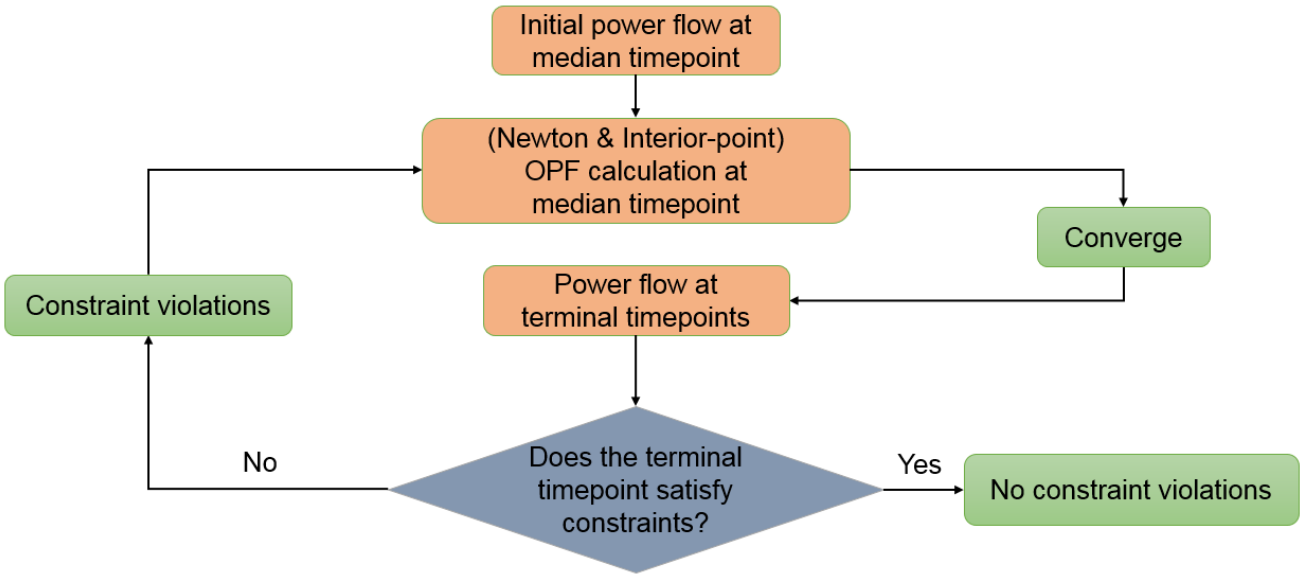
Generator constraints

$$\forall T_k : \begin{cases} \boldsymbol{P}^c_{\min} \le \boldsymbol{P}^c(t_{k-1}) \le \boldsymbol{P}^c_{\max} \\ \boldsymbol{Q}^c_{\min} \le \boldsymbol{Q}^c(t_{k-1}) \le \boldsymbol{Q}^c_{\max} \\ \boldsymbol{P}^c_{\min} \le \boldsymbol{P}^c(t_k) \le \boldsymbol{P}^c_{\max} \\ \boldsymbol{Q}^c_{\min} \le \boldsymbol{Q}^c(t_k) \le \boldsymbol{Q}^c_{\max} \end{cases}$$

Line power constraints

$$\forall T_k : \begin{cases} \boldsymbol{P}^{line}(t_{k-1}) \le \boldsymbol{P}^{line}_{\max} \\ \boldsymbol{P}^{line}(t_k) \le \boldsymbol{P}^{line}_{\max} \end{cases}$$

POPF Algorithm



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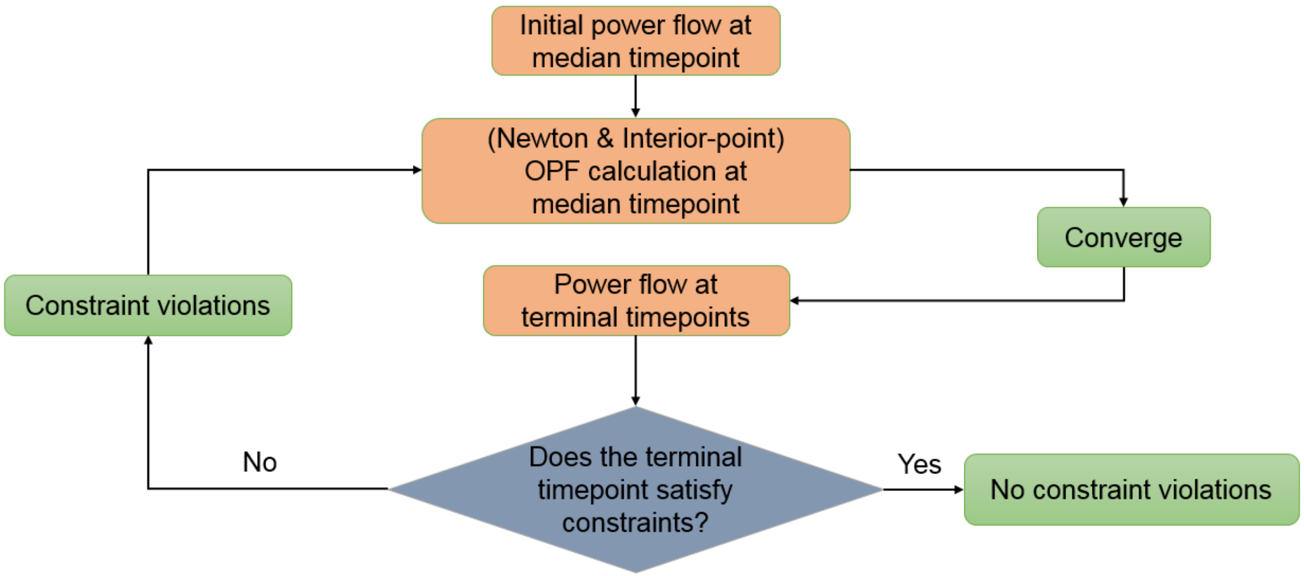


POPF in adjacent LIs:

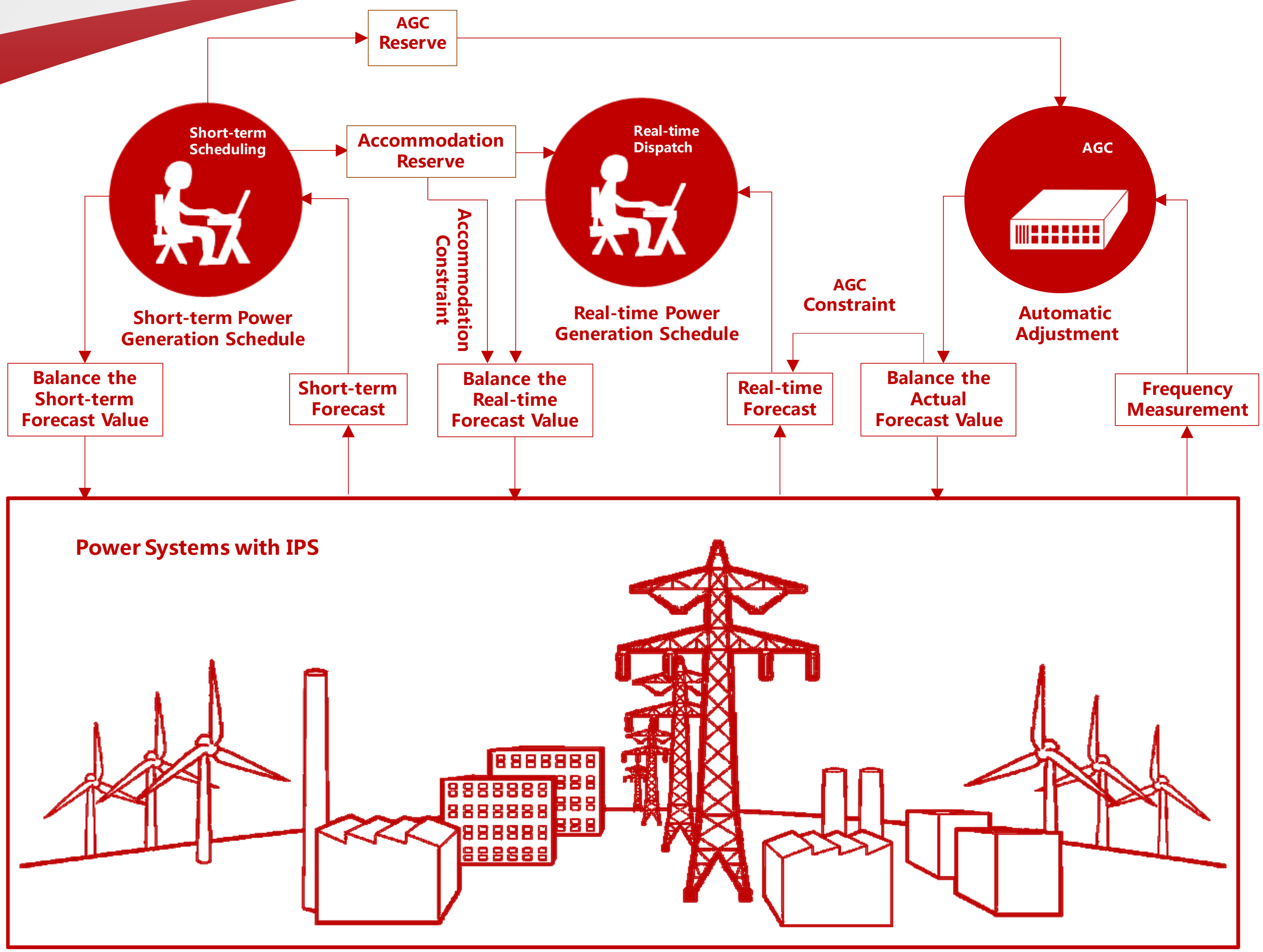
$$\begin{cases} \boldsymbol{P}_0^c(t_k^m) = \boldsymbol{P}^c(t_{k-1}^m) \\ \boldsymbol{V}_0^c(t_k^m) = \boldsymbol{V}^c(t_{k-1}^m) \end{cases}, (k = 1, 2, \ldots, N)$$

where $\boldsymbol{P}_0^c$ is initial values of controllable active power;

$\boldsymbol{V}_0^c$ is initial values of node voltage magnitudes;

N is total number of terminal timepoints.

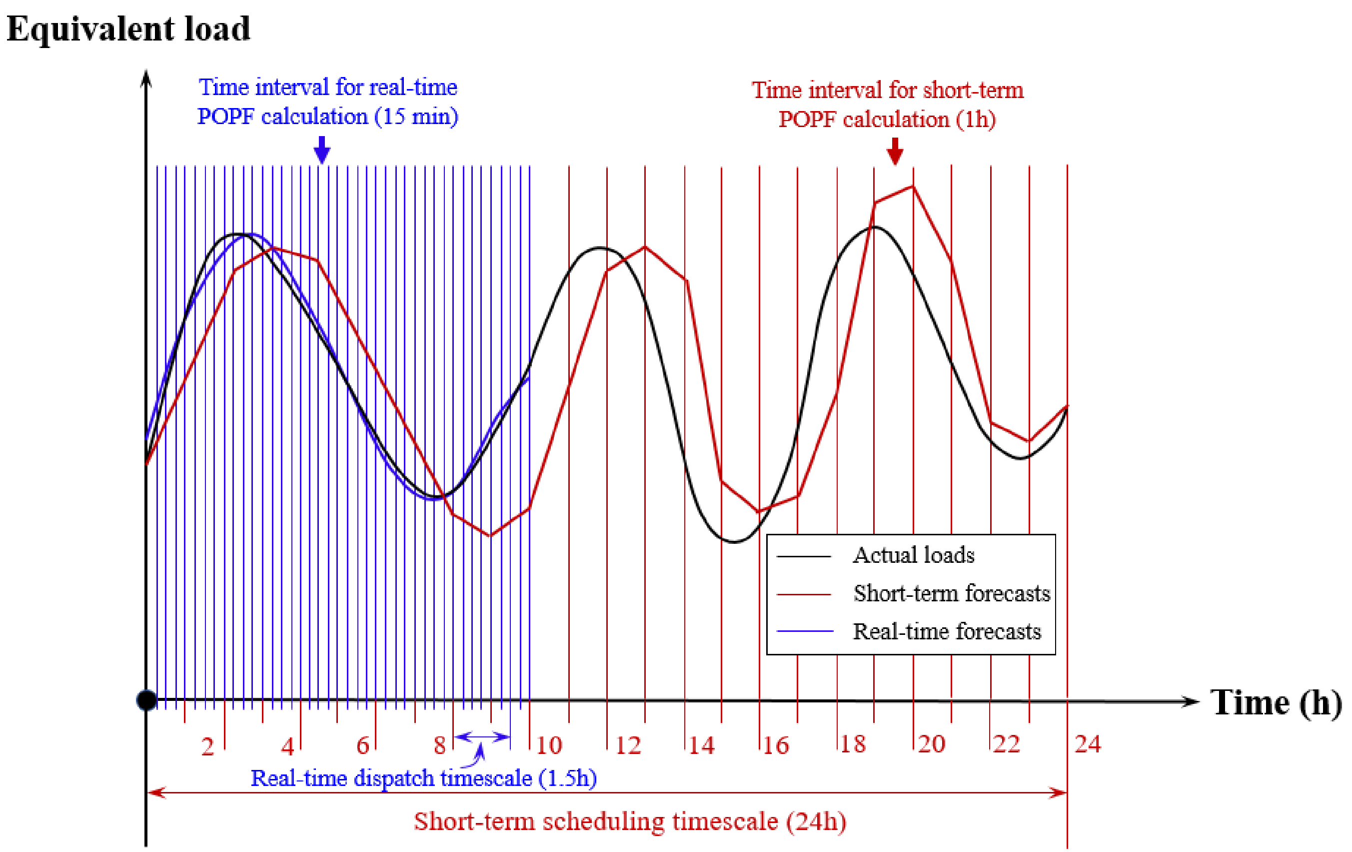
Dispatch Control Hierarchy



Logic relation diagram for the power balancing of dispatch control system

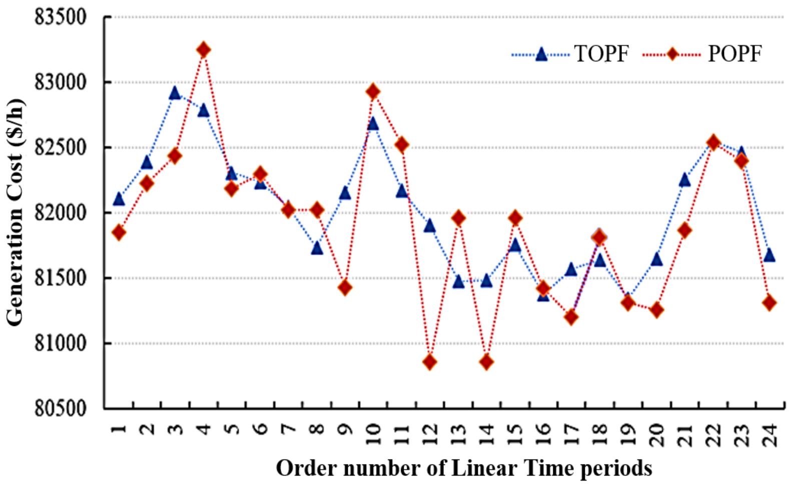
- Dispatch system
- Resources
- Constraints

Coherent Scheme

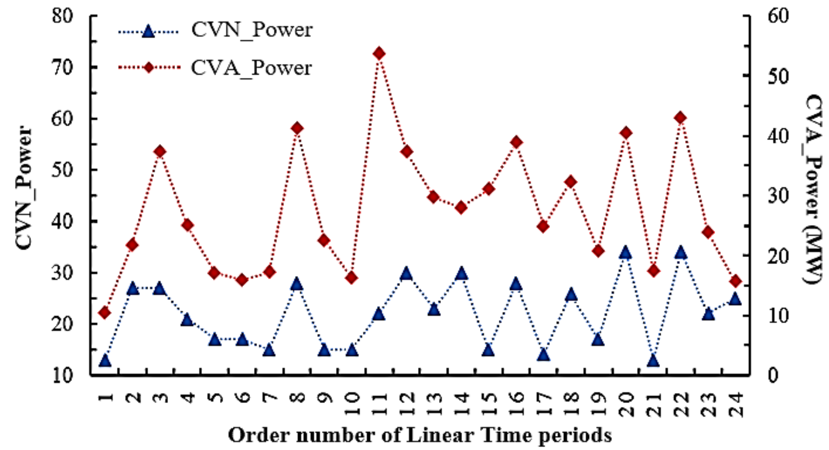


1. **Zongjie Wang**, Zhizhong Guo. On Critical Timescale of Real-time Power Balancing in Power Systems with Intermittent Power Sources [J]. *Electrical Power Systems Research*, 2018, 155:246-253.
2. **Zongjie Wang**, Zhizhong Guo. Quantitative Characterization of Uncertainty Levels of Intermittent Power Sources [J]. *Journal of Renewable and Sustainable Energy*, 2018, 10(4): 043304.

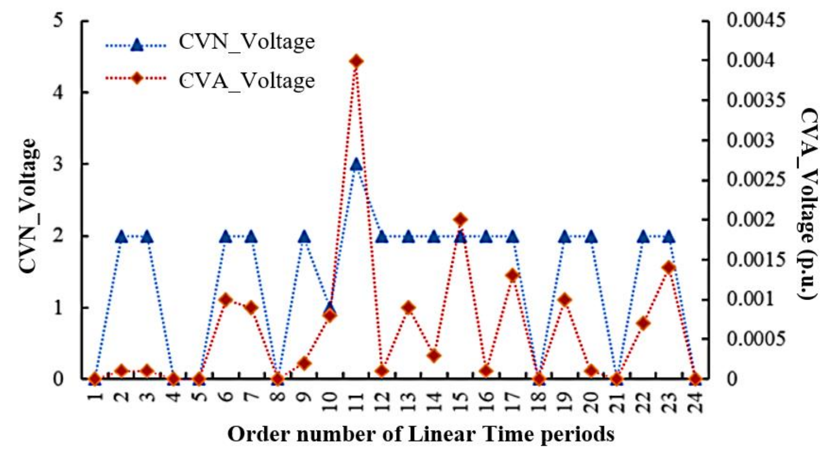
POPF results in short-term scheduling (118-bus)



Generation cost of TOPF and POPF under short-term scheduling



CVN and CVA on branch power of the TOPF under short-term scheduling



CVN and CVA on node voltage of the TOPF under short-term scheduling

- All the constraints are satisfied in the POPF;
- TOPF results in power constraint violations in all the 24 LIs and a number of voltage constraint violations;
- In general, objective function values of POPF are close to TOPF; some cases: POPF is smaller than TOPF;

POPF results in Real-time Dispatch (118-bus)

Simulation results of POPF and TOPF under real-time dispatch.

Linear-time Interval	Parameters	TOPF	POPF
0h-0.25h	CVN-Power	9	0
	CVA-Power (p.u.)	2.3479	0
	CVN-Voltage	2	0
	CVA-Voltage (p.u.)	0.0001	0
	Generation cost ($/h)	92030.4	92429.1
0.25h-0.5h	CVN-Power	14	0
	CVA-Power (p.u.)	4.0001	0
	CVN-Voltage	4	0
	CVA-Voltage (p.u.)	0.0004	0
	Generation cost ($/h)	91823.3	92304.6
0.5h-0.75h	CVN-Power	14	0
	CVA-Power (p.u.)	6.4342	0
	CVN-Voltage	2	0
	CVA-Voltage (p.u.)	0.0001	0
	Generation cost ($/h)	92384.1	92738.9
0.75h-1h	CVN-Power	15	0
	CVA-Power (p.u.)	4.3408	0
	CVN-Voltage	4	0
	CVA-Voltage (p.u.)	0.0001	0
	Generation cost ($/h)	91859.3	92381.0
1h-1.25h	CVN-Power	7	0
	CVA-Power (p.u.)	1.756	0
	CVN-Voltage	0	0
	CVA-Voltage (p.u.)	0	0
	Generation cost ($/h)	92247.2	92849.4
1.25h-1.5h	CVN-Power	15	0
	CVA-Power (p.u.)	2.8245	0
	CVN-Voltage	4	0
	CVA-Voltage (p.u.)	0.0001	0
	Generation cost ($/h)	92192.0	92696.3

- Real-time dispatch timescale: 1.5h;
- Divided into six LIs of 15 minutes;

- POPF satisfies all the constraints;
- TOPF produces over-limits in both branch power and voltage magnitudes;
- Compared with short-term scheduling, TOPF violation levels are generally lower;
- For TOPF and POPF, objective function values in real-time are larger than short-term.

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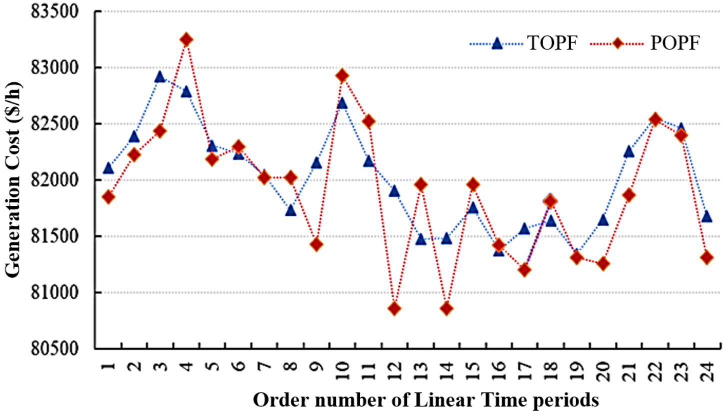
	CVA-Power (p.u.)	6.4342	0
	CVN-Voltage	2	0
	CVA-Voltage (p.u.)	0.0001	0
	Generation cost ($/h)	92384.1	92738.9
0.75h-1h	CVN-Power	15	0
	CVA-Power (p.u.)	4.3408	0
	CVN-Voltage	4	0
	CVA-Voltage (p.u.)	0.0001	0
	Generation cost ($/h)	91859.3	92381.0
1h-1.25h	CVN-Power	7	0
	CVA-Power (p.u.)	1.756	0
	CVN-Voltage	0	0
	CVA-Voltage (p.u.)	0	0
	Generation cost ($/h)	92247.2	92849.4
1.25h-1.5h	CVN-Power	15	0
	CVA-Power (p.u.)	2.8245	0
	CVN-Voltage	4	0
	CVA-Voltage (p.u.)	0.0001	0
	Generation cost ($/h)	92192.0	92696.3



Conclusions

Period Optimal Power Flow Model in Power Systems with High Penetration of Intermittent Power Sources

Conclusions

Linear-time Interval (LI)

- Node voltage is approximately linear function of time;
- Key to discretize and linearize the time-varying problems.

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Simulation case studies on a modified IEEE 118-bus system demonstrate the effectiveness of the proposed POPF model.

POPF applications in dispatch system

Presenter：Zongjie Wang

Institute：Cornell University

Thanks

For Listening