Power System Optimization with an Inertia Study on the IEEE 30-Bus Test System

Sandeep Sadanandan

Introduction

- Inertia on the power system is an important issue for study and analysis. With the national movement to low inertia green energies, the lack of inertia on the power system could be a significant issue.

- The purpose of this paper is to include inertia in a power system optimization of the IEEE 30-bus system.

Inertia on the Power System

- $KE = \frac{1}{2} J\omega^2$
  - KE is the Kinetic Energy or Stored Energy of our Rotating Mass
  - KE is in MW-s or MJoules for transmission power system analysis
  - J is the Moment of Inertia of the Mass
  - ω is the Speed of the Rotor
- Here we can see the rotor and stator of a synchronous generator:

Inertia on the Power System

- Define an inertia constant (H) for generators
- H is proportional to the stored energy in the rotor of the generator
- Stored Energy (E) is proportional to the Moment of Inertia (J) and velocity of our rotating mass
- As the mass of the rotor increases, Moment of Inertia increases (J)
- Thus, Kinetic or Stored Energy (E) of the unit increases as the Moment of Inertia increases
- H increases as Stored Energy Increases

Inertia on the Power System

- As a Unit's Inertia Constant (H) increases, there is more stored energy (or kinetic energy) on a per MVA Basis

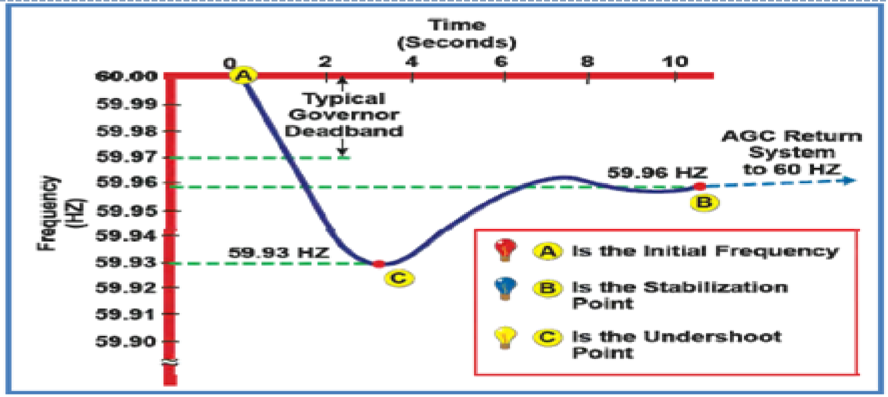
- In order to calculate Kinetic Energy on the system, the sum of the H*MVArating for each online unit must be obtained

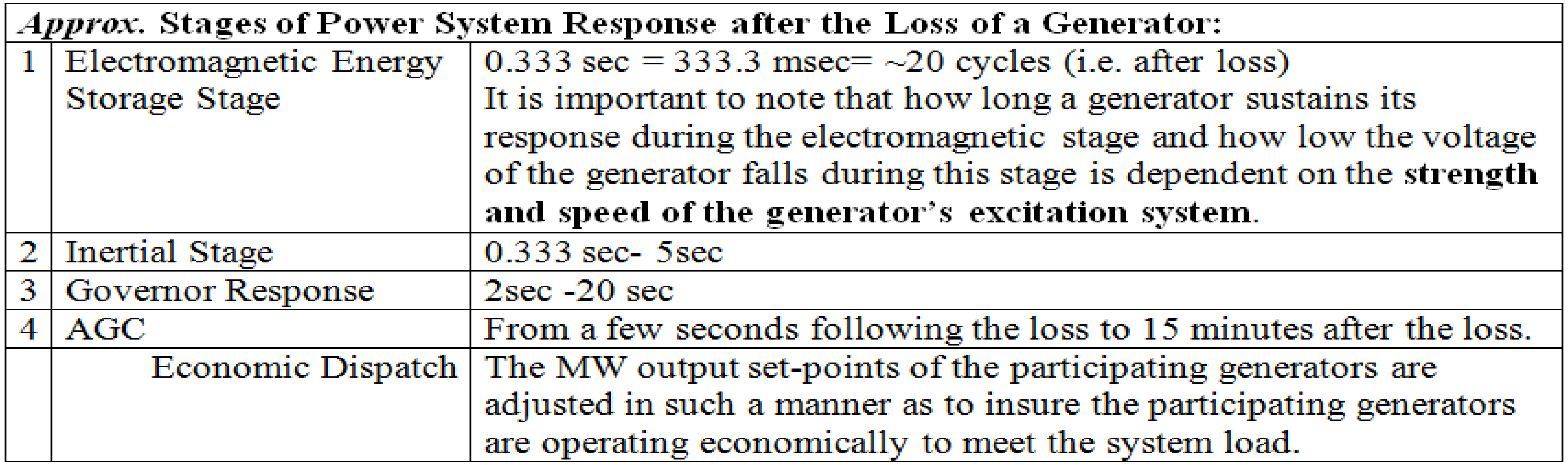
$$KE(t) = SUM\ (H_i * MVA_i)$$

- How much kinetic energy and inertia do we need?
  - We want our system to arrest frequency drops before there is too steep a decline in frequency
  - As more non-synchronous generators (wind and solar) with lower inertia constants enter our system, the inertia may not be adequate to arrest frequency before a steep decline occurs

Inertia on the Power System

- Arrest Frequency Decline due to a Large Generator Trip
- From the EPRI Power Systems Dynamics Tutorial. EPRI, Palo Alto, CA, 2009:





Approx. **Stages of Power System Response after the Loss of a Generator:**		
1	Electromagnetic Energy Storage Stage	0.333 sec = 333.3 msec= ~20 cycles (i.e. after loss) It is important to note that how long a generator sustains its response during the electromagnetic stage and how low the voltage of the generator falls during this stage is dependent on the **strength and speed of the generator's excitation system.**
2	Inertial Stage	0.333 sec- 5sec
3	Governor Response	2sec -20 sec
4	AGC	From a few seconds following the loss to 15 minutes after the loss.
	Economic Dispatch	The MW output set-points of the participating generators are adjusted in such a manner as to insure the participating generators are operating economically to meet the system load.

Objective Function

- The objective function of generation cost is minimized subject to constraints of
  - active power generator limits,
  - active power reserves, and
  - system inertia.
- As generation is lost on the system, frequency drops. The early response of the system comes from the inertia on the system. With the replacement of large synchronous machines by renewable resources, which are often lower inertia units, the need to maintain a system inertia-constant (Hsys) becomes a necessary goal of power system planners and operators.

Minimizing Generation Cost without Reserves

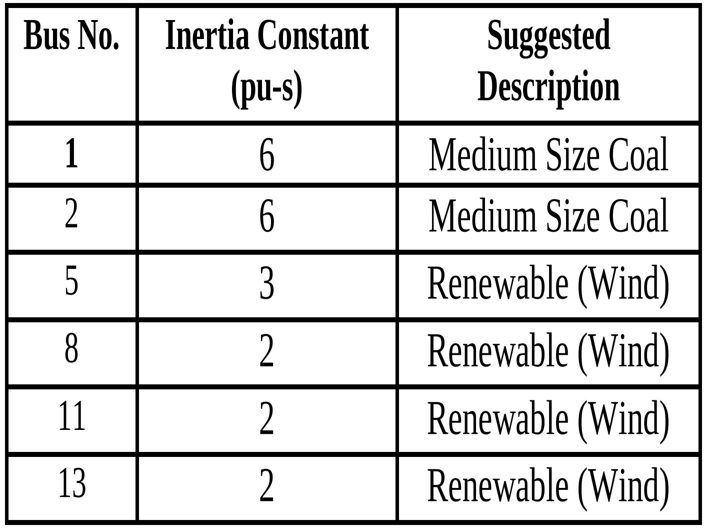
- Obj.Fn.:$min\ Cost(P_{gk}) = \ a\ +\ bP_{gk}\ + dP_{gk}^2$ (7)
- subject to
- C1: $P_{ld}\ \ <\ P_{gk}$
- C2: $P_{gk}min\ <\ P_{gk}\ <\ P_{gk}max$

Minimizing Generation Cost with Reserves & Inertia Constraints

- Obj1: $min\ Cost(P_{gk}) = \ a \ + \ bP_{gk} \ + dP_{gk}^2 + \ a \ + \ bRsv_{gk} \ + dRsv_{gk}^2$
- C1: $P_{ld} \ + \ Rsvreq \ < \ P_{gk} \ + \ Rsv_{gk}$
- C2: $P_{ld} \ < \ P_{gk}$
- C3: $P_{gk}min \ < \ P_{gk} \ < \ P_{gk}max$
- C4: $Rsvreq \ < \ Rsv_{gk}$
- C5: $P_{gk} \ + \ Rsv_{gk} \ < \ P_{gk}max$
- C6: $0 \leq \ Rsv_{gk}$

Designated Inertia Constants

- Assume that the system inertia constant is constrained to $Hsys = 5$ pu-s. The higher the inertia constant, the better the system can reduce the rate of change of frequency after a significant drop in generation. The inertia constant for each unit is designated in Table 2.



Bus No.	**Inertia Constant (pu-s)**	**Suggested Description**
1	6	Medium Size Coal
2	6	Medium Size Coal
5	3	Renewable (Wind)
8	2	Renewable (Wind)
11	2	Renewable (Wind)
13	2	Renewable (Wind)

Adding Kinetic Energy Constraint

- $Kinetic\ Energy\ =\ \sum(H_i MVA_i)\ ,$
- $where\ i = Individual\ Units$
-
- $H_{sys}MVA_{sys} = H_1MVA_1 + H_2MVA_2 + H_5MVA_5 + H_8MVA_8 + H_{11}MVA_{11} + H_{13}MVA_{13}$
- Thus, the following Kinetic Energy constraint is added to the analysis:
- $H_{sys}P_{load} - H_1Pgen_1 - H_2Pgen_2 - H_5Pgen_5 - H_8Pgen_8 - H_{11}Pgen_{11} - H_{13}Pgen_{13} \leq\ 0$

Evaluation of Frequency and Inertia Constant

- Starting with Energy and Real Power,
- $E = ½\,J\omega_m^2 = ½\,J(2\pi f_m)^2$ (13)
-
- $P\ [MW] = \frac{dE}{dt} = J * 2f_m * \frac{d(2\pi f)}{dt}$ (14)
- Using the swing equation,
- $\mathrm{Pacc} = \mathrm{Pmech} - \mathrm{P_{elec}} = (2½\mathrm{J})\omega_m \frac{d\omega_m}{dt}$ (15)
-
- With,

$$\mathrm{H} = \frac{Stored\ Energy\ at\ synch\ speed}{Machine\ MVA\ rating} = \frac{½\ J\omega_s^2}{MVA_{rating}}$$

Evaluation of Frequency and Inertia Constant

- so,
- $\text{Pacc} = \ 2HMVA_{rating}\frac{1}{\omega_s^2}\omega_m\frac{d(\omega_m)}{dt}$
- $\frac{Pacc}{MVArat} = \frac{Pmech-Pelect}{MVA_{rating}} = \frac{2H}{\omega_s}\frac{\omega_m}{\omega_s}\frac{d\omega_m}{dt}$
- $\frac{\Delta P_{pu}}{2Hsys} = \frac{1}{\omega_s}\omega_{m_{pu}}\frac{d\omega_m}{dt} = \frac{1}{2\pi f_s}\omega_{m_{pu}}\frac{d(2\pi f_m)}{dt}$
-
- Assume $\omega_{m_{pu}}$=1,
- $\frac{\Delta P_{pu}}{2Hsys} = \frac{d(f_m/f_s)}{dt}$
- Thus,
- $$\frac{d(f_{pu})}{dt} = \frac{\Delta P_{pu}}{2Hsys}$$

Final Constraint

- Using this final equation with frequency drop constrained to 0.3Hz (or 0.3/60 pu) in 1-second and Hsys=5 pu-s, the following constraint can be added to the cost optimization problem formulation:
-

(22)

- $\Delta P_{pu} - \frac{d(f_{pu})}{dt} 2Hsys \leq 0 \; or \; \Delta P_{pu} - \left(\frac{\frac{0.3}{60}}{1}\right) * 2 * 5 \leq 0$
- $or \; \Delta P_{pu} - 0.05 \leq 0$
- THIS IS NO LONGER A CONVEX PROBLEM. THE INTERIOR POINT METHOD GIVES US A LOCAL MINIMUM AND NOT A TRUE OPTIMIZATION.

Results

- Table 3- Power Generation Results of Minimizing Cost

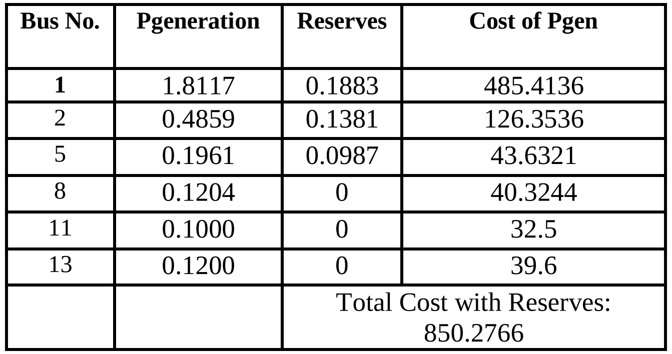
Bus No.	**Pgeneration**	**Cost**
1	1.8540	499.7117
2	0.4687	120.4739
5	0.1912	41.9824
8	0.1	33.3340
11	0.1	32.5000
13	0.12	39.6000
		Total Cost: 767.6021

- Table 5- Power Flow Results of the 30-bus system

Bus No.	**Pgeneration**	**Cost**
1	1.9741	540.9602
2	0.4687	120.4739
5	0.1912	41.9824
8	0.1	33.3340
11	0.1	32.5000
13	0.12	39.6000
		Total Cost: 808.8506

Results

- Table 5- Minimizing Cost with Reserves

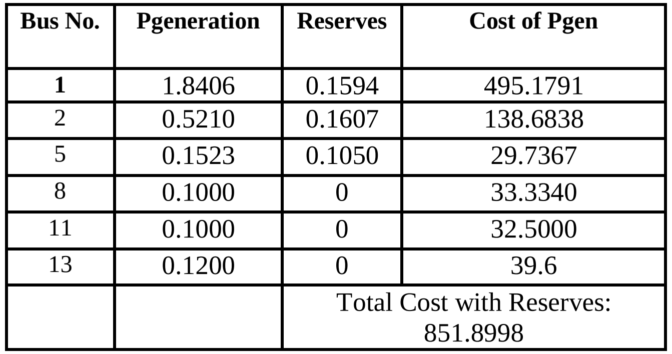


Bus No.	Pgeneration	Reserves	Cost of Pgen
1	1.8117	0.1883	485.4136
2	0.4859	0.1381	126.3536
5	0.1961	0.0987	43.6321
8	0.1204	0	40.3244
11	0.1000	0	32.5
13	0.1200	0	39.6
		Total Cost with Reserves: 850.2766	

- Comparing Tables 3 and 5, it is evident that as reserve requirements are added to the system, generation gets reallocated. Some generators lose the opportunity cost of generating MWs and must contribute to the reserve requirements.

Results

- Table 6- Minimizing Cost with Hsys=5 pu-s



Bus No.	**Pgeneration**	**Reserves**	**Cost of Pgen**
1	1.8406	0.1594	495.1791
2	0.5210	0.1607	138.6838
5	0.1523	0.1050	29.7367
8	0.1000	0	33.3340
11	0.1000	0	32.5000
13	0.1200	0	39.6
		Total Cost with Reserves: 851.8998	

- As illustrated by the values in Table 6, the generation shifts to the higher inertia units (at Bus 1 and 2). These higher inertia units then have less headroom to help meet reserve requirements.

Results

- The last constraint we add is the frequency drop constraint.
- The final result of the optimization with the inertia constraints was that the system could accommodate a 0.0239pu loss of generation and still keep frequency above 59.7Hz or higher during the 1 second.

Conclusions

- The IEEE 30-bus system was analyzed to minimize the cost function subject to several constraints. It is good to note that Matlab fmincon gave the local minimum and not a true optimization. One of these constraints was inertia. The system inertia constant was kept to 5pu-s.
- The result was that the 30-bus system, under the inertia constants designated, could keep frequency at 59.7 Hz or higher for generation trip of 0.0239 pu.