Data-Driven Stochastic Optimization for Power Grids Scheduling under High Wind Penetration

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OUTLINE

Two-Stage Stochastic Unit Commitment (SUC)

- Suppose that F^c is the underlying "correct" stochastic model characterizing the uncertainty of wind power generation ξ.
- We consider the two-stage stochastic unit commitment problem

$$
\begin{aligned}
\min_{\mathbf{u}} G(\mathbf{u}) \quad &\equiv \quad C_1\mathbf{u} + \mathrm{E}_{\boldsymbol{\xi}\sim F^c}\left[\min_{\mathbf{y}} C_2\mathbf{y}(\mathbf{u},\xi)\right] \\
\text{s.t.} \quad & \quad A\mathbf{u} \le B \\
& \quad H\mathbf{u} + Q\mathbf{y}(\mathbf{u},\xi) \le M(\xi)
\end{aligned}
$$

Current Practice

- However, F^c is unknown and estimated by finite real-world data. Denote the input model estimate as $\widehat{F}$.
- The empirical SUC considers

$$\begin{aligned} \min_{\mathbf{u}} \widehat{G}(\mathbf{u}) \quad &\equiv \quad C_1\mathbf{u} + \mathrm{E}_{\boldsymbol{\xi}\sim\widehat{F}}\left[\min_{\mathbf{y}} C_2\mathbf{y}(\mathbf{u},\boldsymbol{\xi})\right] \\ \text{s.t.} \quad & \quad A\mathbf{u} \le B \\ & \quad H\mathbf{u} + Q\mathbf{y}(\mathbf{u},\boldsymbol{\xi}) \le M(\boldsymbol{\xi}) \end{aligned}$$

- Since $\mathrm{E}\left[\min_{\mathbf{y}} C_2\mathbf{y}(\mathbf{u},\boldsymbol{\xi})\right]$ has no closed-form, the SAA is often used

$$\min_{\mathbf{u}} \bar{G}(\mathbf{u}) = C_1\mathbf{u} + \frac{1}{S}\sum_{s=1}^{S}\left[\min_{\mathbf{y}} C_2\mathbf{y}(\mathbf{u},\boldsymbol{\xi}_s)\right]$$

It introduces the finite sampling error.

- There are three sources of uncertainties, including
  - Stochastic Uncertainty: characterized by F^c
  - Model Estimation Uncertainty: unknown F^c is estimated by finite real-world data
  - Finite Sampling Error: induced by using SAA
- The current practice on SUC ignores the input model estimation uncertainty and finite sampling error.

Data-Driven Stochastic Unit Commitment

- Given the valid historical data, denoted by $\mathcal{D}$, the posterior predictive distribution

$$f^p(\xi) \equiv p(\xi|\mathcal{D}) = \int p(\xi|F)p(F|\mathcal{D})dF$$

can quantify the forecasting uncertainty, accounting for inherent wind power stochastic uncertainty and model estimation error.

- We propose the data-driven SUC,

$$\begin{aligned} \min_{\mathbf{u}} G^p(\mathbf{u}) \quad &\equiv \quad C_1\mathbf{u} + \mathrm{E}_{\boldsymbol{\xi}\sim F^p}\left[\min_{\mathbf{y}} C_2\mathbf{y}(\mathbf{u},\xi)\right] \\ \text{s.t.} \quad & \quad A\mathbf{u} \leq B \\ & \quad H\mathbf{u} + Q\mathbf{y}(\mathbf{u},\xi) \leq M(\xi) \end{aligned}$$

Our data-driven SUC can be applied to both parametric and nonparametric situations.

- If the parametric family of F^c is known, the posterior of model parameters $p(\boldsymbol{\theta}|\mathcal{D})$ can characterize the model estimation error.
- It can be combined with nonparametric probabilistic forecast; for example the infinite state Markov-switching autoregressive (IMSAR)

$$
\begin{aligned}
& f(\boldsymbol{\xi}_t|\boldsymbol{\xi}_{[1:t-1]}, F) \\
&= \sum_{i=1}^{+\infty} p(s_t = i|\boldsymbol{\xi}_{[1:t-1]}) h(\boldsymbol{\xi}_t|\boldsymbol{\theta}_{s_t}, \boldsymbol{\xi}_{[1:t-1]}, s_t = i)
\end{aligned}
$$

with

$$
p(s_t = i|\boldsymbol{\xi}_{[1:t-1]}) = \sum_{j=1}^{+\infty} p(s_t = i|s_{t-1} = j) p(s_{t-1} = j|\boldsymbol{\xi}_{[1:t-1]})
$$

Two-Phase Data-Driven SUC Optimization Procedure

Step (0) Specify the total budget T (number of second-stage economic dispatch problems) for second phase selection.

Step (1) Use L CPUs to solve the SAA approximated SUC problems in parallel, and obtain the optimal candidate decisions $\widehat{\mathbf{u}}_1^\star, \ldots, \widehat{\mathbf{u}}_L^\star$.

Step (2) Add the additional ΔT resource to $\widehat{\mathbf{u}}_1^\star, \ldots, \widehat{\mathbf{u}}_L^\star$

$$\begin{aligned} \frac{N_i}{N_j} &= \left(\frac{\delta_j}{\delta_i}\right)^2, i, j \neq b \\ N_b &= \sigma_b \sqrt{\sum_{\ell=1, \ell \neq b}^{L} \frac{N_\ell^2}{\sigma_\ell^2}}, \end{aligned}$$

where $\sigma_\ell^2 = \text{Var}\left[\min_{\mathbf{y}} C_2 \mathbf{y}(\widehat{\mathbf{u}}_\ell^\star, \xi)\right]$ and $\delta_\ell \equiv \frac{\bar{G}^p(\widehat{\mathbf{u}}_\ell^\star) - \bar{G}^p(\widehat{\mathbf{u}}_b^\star)}{\sigma_\ell}$.

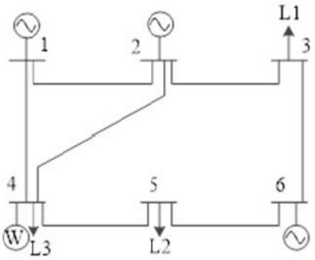
Step (3) Update the best candidate $\widehat{\mathbf{u}}_b^\star \equiv \text{argmin}_{\ell=1,\ldots,L} \bar{G}^p(\widehat{\mathbf{u}}_\ell^\star)$.

Step (4) Repeat Steps (2) and (3) until reaches to the budget. Return $\widehat{\mathbf{u}}_b^\star$.

Case Studies

The six bus system is used to study the performance of our approach.

1. Case Study of SUC with Parametric Input Model
2. Case Study of SUC with Nonparametric Input Model
3. Case Study of Two-Phase Optimization Procedure



Case Study of SUC with Parametric Input Model

- Suppose the distribution of wind power ξ_t for t-th hour in the past r days is the same, $\xi_t \sim N(\mu_t, \phi_t^2)$ with ϕ_t proportional to μ_t.
- Suppose the true mean μ_t is unknown and estimated by the data from past r days.
- We compare the performance of proposed data-driven SUC with the empirical SUC.
  - Data-driven SUC: $\sum G^p = \sum_{d=1}^{n_d} G(\mathbf{u}_d^{\star p})$ based on F^p
  - Empirical SUC: $\sum G^e = \sum_{d=1}^{n_d} G(\mathbf{u}_d^{\star e})$ based on $\widehat{F}$

- We set the scenarios size $S = 50$ and let $n_d = 20$ days, $r = 1$.
- The proposed data-driven SUC has better performance than the empirical SUC.
- The advantage tends to be larger as the wind power penetration becomes higher.

	$\sum G^p$	$\sum G^e$
$\phi_t = 5\%\mu_t$	2,002,300	2,320,620
$\phi_t = 10\%\mu_t$	2,175,780	2,445,560
$\phi_t = 20\%\mu_t$	2,151,380	2,710,900

Case Study of SUC with Nonparametric Input Model

- We use the real-world wind power data to compare data-driven SUC having IMSAR model with empirical SUC having probabilistic persistent model.
- We consider the intra-day market with the planning horizon length $n_h = 4$ hours. All three generators are fast start generators.
- Since F^c is unknown, the real dispatch cost is used for evaluation,

$$G^r(\mathbf{u}_{dh_t}) \equiv C_1\mathbf{u}_{dh_t} + \min_{\mathbf{y}} C_2\mathbf{y}(\mathbf{u}_{dh_t}, \boldsymbol{\xi}^r_{dh_t^+})$$

  where $\boldsymbol{\xi}^r_{dh_t^+} \equiv (\xi_{d(h_t+1)}, \ldots, \xi_{d(h_t+n_h)})$ is the wind power realizations.
- We compare the accumulated costs obtained by data-driven SUC and empirical SUC, $\sum G^p_r \equiv \sum_{d=1}^{n_d} \sum_{h_t=1}^{6} G^r(\mathbf{u}^{\star p}_{dh_t})$ and $\sum G^e_r \equiv \sum_{d=1}^{n_d} \sum_{h_t=1}^{6} G^r(\mathbf{u}^{\star e}_{dh_t})$.

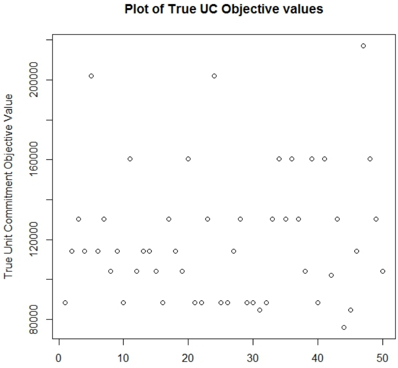
Data-driven SUC can lead to the lower expected cost than the empirical SUC, $\sum G_r^p \leq \sum G_r^e$.

Table 1: Aggregated total costs of October month operation.

	Total Cost
Data-Driven SUC with IMSAR model $\sum G_r^p$	2,822,705
Empirical SUC with Persistence model $\sum G_r^e$	2,969,178

Case Study of Two-Phase Optimization Procedure

We plot the scatter plot of $G^p(\widehat{\mathbf{u}}_1^\star), \ldots, G^p(\widehat{\mathbf{u}}_L^\star)$ with $\widehat{\mathbf{u}}_1^\star, \ldots, \widehat{\mathbf{u}}_L^\star$ obtained with SAA approximated SUC ($S = 50$ scenarios and $L = 50$ CPUs)



	mean	SE
Classical SUC approach	113480	6474
Our procedure with $T = 500$ and $\Delta T = 100$	100370	4685
Our procedure with $T = 1000$ and $\Delta T = 50$	101044	4776
Our procedure with $T = 1000$ and $\Delta T = 100$	94081	3851
Our procedure with $T = 1000$ and $\Delta T = 200$	100414	4846
Our procedure with $T = 2000$ and $\Delta T = 100$	99715	4753

The average running time used to solve for each $\widehat{\mathbf{u}}_\ell^\star$ is 1341 seconds, while the time for the second phase selection is around 50 seconds, which is negligible.

Conclusions

- We propose a data-driven SUC and optimization procedure that leads to the optimal unit commitment decision hedging against
  - wind power inherent stochastic uncertainty,
  - input model estimation uncertainty,
  - finite sampling error induced by SAA.
- The case studies demonstrate:
  - The proposed data-driven SUC has better performance than the empirical SUC.
  - The proposed two-phase optimization procedure can efficiently use parallel computing to control the impact of finite sampling error.