Power system restoration through mixed integer linear programming

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Outline

1. Motivation and contributions
2. MILP approximation of AC power flow equations
3. Integer L-shaped method for power system restoration
4. Numerical results
5. Conclusions and future work

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▷ Motivation and contributions

MILP approximation of AC power flow equations

Integer L-shaped method for power system restoration

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Motivation and contributions

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Blackouts and power system restoration

- Ohio blackout of 2003
  - Initiated by fault of HV power line
  - 50 million affected, 11 deaths, $6 billion
- European blackout of 2006
  - Initiated by routine disconnection of HV power line
  - 10 million affected, 100 trains delayed
  - Forced operators to improve their **resynchronization procedures**
- Chilean blackout of 2011
  - Initiated by fault at HV capacitor
  - Restoration delayed because of **procedural** and equipment issues

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Literature on power system restoration

- (Hou *et al.*, 2011) and (Liu *et al.*, 2012) propose a step-wise stategy based on achieving pre-specified milestones
- (Sun *et al.*, 2011) optimizes the startup strategy for generators considering only active power in a copper-plate model of the grid
- (Chou *et al.*, 2013) formulates a mixed-integer non-linear problem for restoration and provides solutions using an heuristic algorithm
- (Coffrin and Van Hentenryck, 2014) solve the optimal restoration problem using a LP approximation of the AC power flow equations on a network with 266 components

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Contributions

- We propose an MILP approximation of the AC power flow equations based on minimum square-error
  - LP relaxation is sufficient under scarcity of reactive power
  - MILP necessary under excess of reactive power
- We specialize the integer L-shaped method for solving optimal power system restoration on realistic instances
  - Stronger island-based no-good cuts
  - Hybrid Benders-no-good cuts
- We present numerical results for IEEE test systems and for the Chilean power grid with 3696 components (buses, branches, compensation, generators) – one order of magnitude larger than state-of-the-art

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MILP approximation of AC power flow equations

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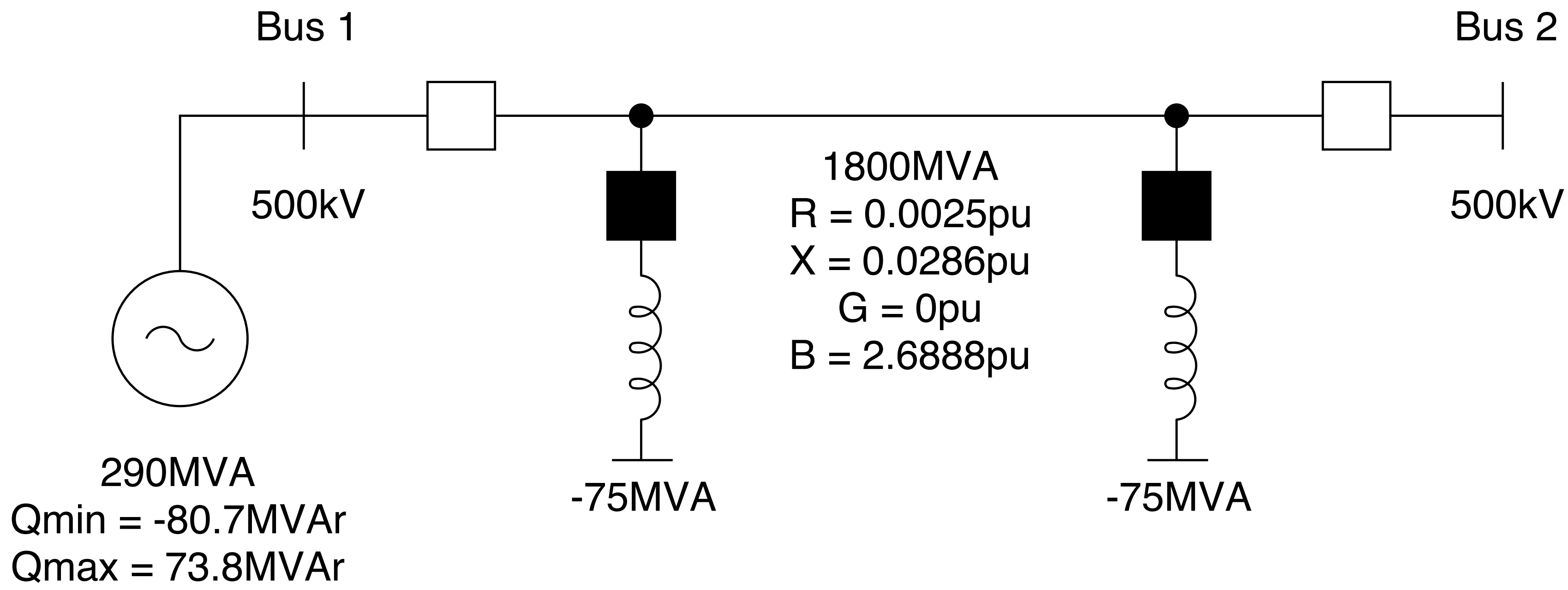
AC relaxations in power system restoration: an example



Is it possible to energize line 1-2?

Full AC answer: **No**, excess of reactive power

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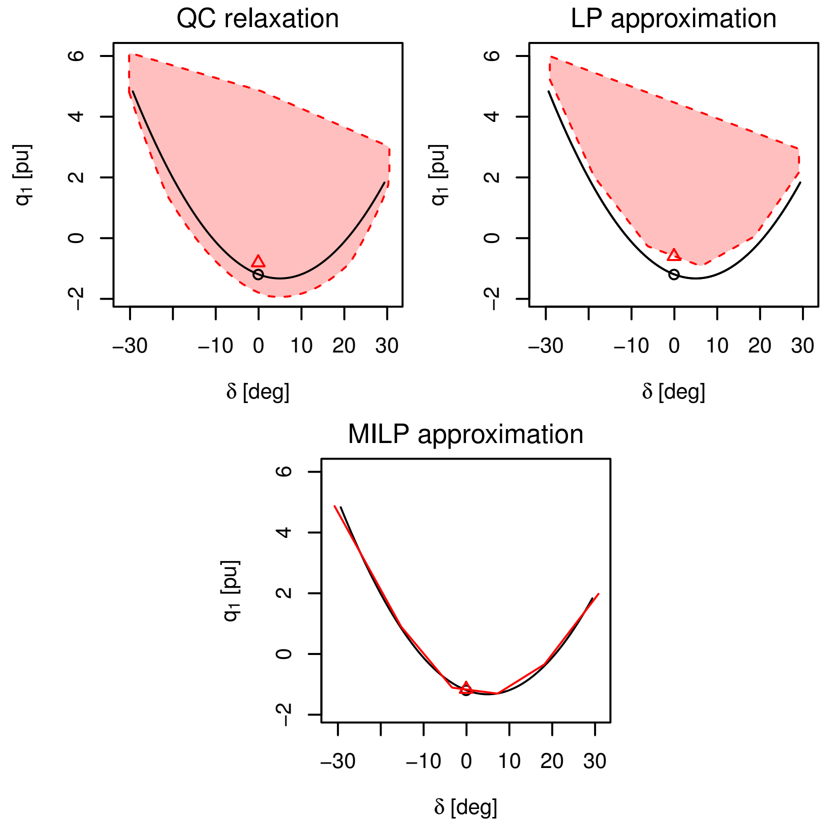
AC relaxations in power system restoration: an example

Feasible domains for reactive power entering a terminal as a function of angle difference δ for:

1. the QC relaxation (Coffrin *et al.*, 2016)
2. LP approximation (cold-start LPAC, 5 intervals) (Coffrin and Van Hentenryck, 2014)
3. MILP approximation (5 intervals)



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Piece-wise linear approximations of continuous functions

- (Toriello and Vielma, 2012): a partition I of the interest domain for the approximation and a set of slopes $\boldsymbol{a}_i$ and intercepts b_i

Problem: we have a continuous function $f : \mathbb{R}^n \to \mathbb{R}$ that we wish to approximate with a piecewise linear function $\hat{f} : \mathbb{R}^n \to \mathbb{R}$ defined as

$$\hat{f}(\boldsymbol{x}) := \boldsymbol{a}_i^T \boldsymbol{x} + b_i \text{ if } \boldsymbol{x}_i^L \preccurlyeq \boldsymbol{x} \preccurlyeq \boldsymbol{x}_i^U, i \in I,$$

- Determine $\boldsymbol{x}_i^L, \boldsymbol{x}_i^U \ \forall i \in I \to$ equal **curvature** intervals
- Determine $\boldsymbol{a}_i^T, b_i \ \forall i \in I \to$ solve:

$$\begin{aligned} \min_{\boldsymbol{a},b} \quad & \sum_{i \in I} \int_{[\boldsymbol{x}_i^L, \boldsymbol{x}_i^U]} (\boldsymbol{a}_i^T \boldsymbol{x} + b_i - f(\boldsymbol{x}))^2 d\boldsymbol{x} \\ \text{s.t.} \quad & \boldsymbol{a}_i^T \boldsymbol{x} + b_i = \boldsymbol{a}_j^T \boldsymbol{x} + b_j \quad \forall x \in F(i,j), \forall (i,j) \in \mathcal{F} \end{aligned}$$

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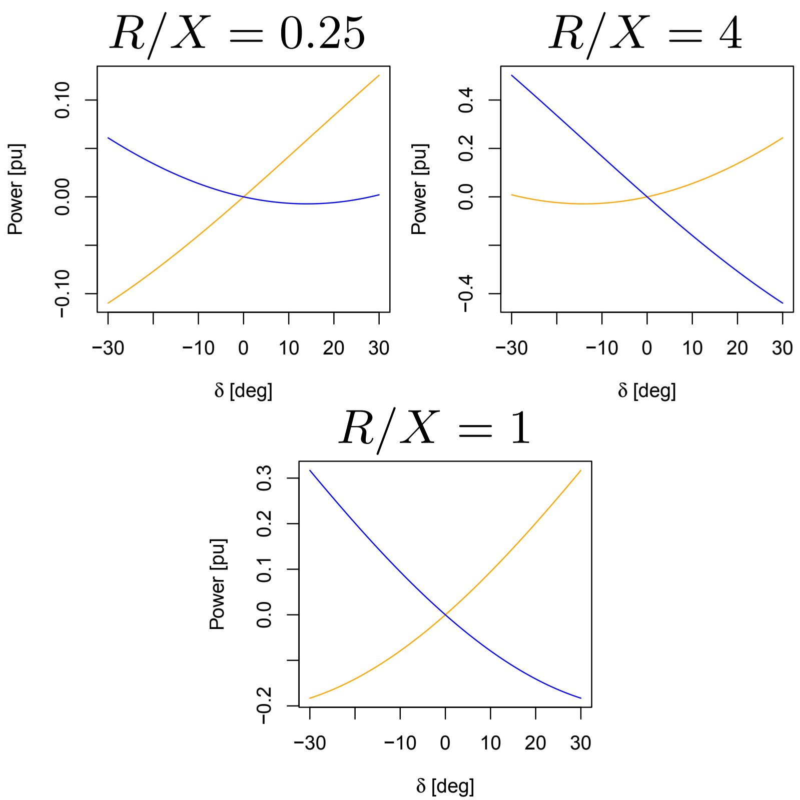
Approximating AC power flow equations

$$p_1(v_1, v_2, \delta) = v_1^2 g_{12} - v_1 v_2 g_{12} \cos(\delta) - v_1 v_1 b_{12} \sin(\delta)$$
$$q_1(v_1, v_2, \delta) = -v_1^2 b_{12} + v_1 v_2 b_{12} \cos(\delta) - v_1 v_2 g_{12} \sin(\delta)$$

- Linear approximation in terms of voltages
- Piece-wise linear approximation in terms of angle difference, **only if necessary**
- Enforcing convexity or concavity depending on g_{12}, b_{12}



p blue, q orange

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Power system components

- Power transformers:
  - N-windding transformers $(N > 2) \rightarrow$ star model
  - Phase shifters $\rightarrow$ adder to angle difference
  - Voltage tap-changers $\rightarrow$ adder to voltage magnitude
- Shunt compensators: sectionalized and variable compensators (FACTS)
- Voltage-responsive loads:

$$
\begin{aligned}
p + s &= P \cdot v^{\alpha} \\
q &= Q \cdot v^{\beta} \cdot p/(P \cdot v^{\alpha})
\end{aligned}
$$

Linear approximation of $p + s$, linear $+$ McCormick for q

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▷ Integer L-shaped method for power system restoration
Numerical results
Conclusions and future work

Integer L-shaped method for power system restoration

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Optimal power system restoration

- Objective: return the system to **normal operation**
- Critical loads must be energized, **normal load is a tool**

$$\max \sum_t \Big(\text{inertia}(t) + \#\ \text{branches energized}(t)\Big)$$

s.t. **generators**: off, cranking, normal operation

branches: can be energized at t only if a terminal is energized on $t-1$

buses: can be energized only if connected to energized line or generator*

grid: must be AC feasible for all t

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Integer L-shaped algorithm

- **Idea: exploit procedure followed by operators**
- Master problem: energization decisions for all components (ILP)
- Slave problem: AC feasibility for all t (NLP, approximated by MILP)

1. Start solving integer *master*
2. Whenever a **new incumbent** of the *master* is found:
   (a) Detect islands (**connected components**) of the grid for all t
   (b) Check feasibility of **each island** (highly parallelizable)
   (c) Add island feasibility cuts to *master*
3. Return to 2.

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No-good island cuts

- Islands are specified by:
  - Subset of buses, N_i
  - Non-energized/energized branches connected buses in N_i, L_i^{OFF}/L_i^{ON}
  - Non-energized/cranking/energized generators connected to buses in N_i, $G_i^{OFF}/G_i^{CRANK}/G_i^{ON}$
- No-good island cut (if island is infeasible)

$$\sum_{l \in L_i^{OFF}} u_l + \sum_{l \in L_i^{ON}} (1 - u_l) + \sum_{g \in G_i^{OFF}} u_g + \ldots \geq 1$$

- Feasibility cuts can be shared across all periods, limited to a window for practical performance

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Hybrid Benders-no-good island cuts

- Solving convex relaxation of MILP approximation, similar to LP approximation (Coffrin and Van Hentenryck, 2014)
- If convex relaxation is infeasible, the following cut is valid

$$V + \sum_{g \in G_i} (W_g u_g + X_g v_g) \leq M \cdot \left(\sum_{l \in L_i^{OFF}} u_l + \sum_{l \in L_i^{ON}} (1 - u_l) \right)$$

  where V, W, X describe an unbounded ray of the dual of the underlying convex relaxation and M is computed for each cut
- **Prevents arbitrarily large parameters (transmission switching) from affecting cuts**
- Using locally ideal formulation for MILP approximation (Sridhar *et al.*, 2013) $\rightarrow$ convex relaxation corresponds to LP relaxation

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Numerical results

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Simulation setting

Test instances.

Name	# buses	# branches	# shunts	# gens	# loads	T
IEEE-39*	39	46	0	10	21	50
IEEE-118*	118	186	14	55	99	50
Chilean SIC	1548	1954	159	297	605	100

- Implemented using Julia, JuMP, LightGraphs (island detection), Ipopt (approximation) and Cplex (LP/MILP solves)
- Solving restoration with DC oracle (*DC*), convex relaxation of MILP AC approximation (*Cvx AC*) and MILP AC approximation* (*AC*)
- Each experiment ran on single node of `cab` cluster: 16 cores, 32 GB of memory (no parallelization at this point)
- Lazy piece-wise modelling of AC power flow with up to 4 pieces, cut window: 1 period backward, 5 periods forward

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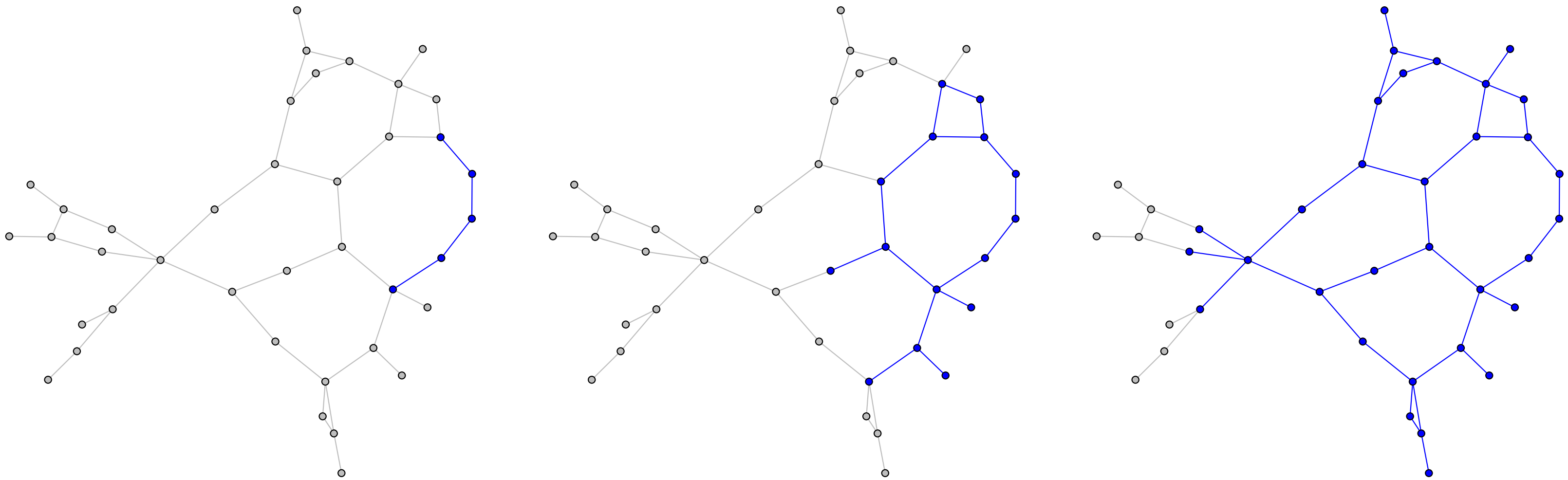
Performance on IEEE-39

- Identical optimal restoration plan for *DC*, *Cvx AC* and *AC*
  - $P_g^{\text{min}} = 0 \quad \forall g \rightarrow$ generators can be turned on without load
  - Short lines (small shunt capacitances) $\rightarrow$ no need for picking up load
- Optimal plan is **first incumbent** of the *master* (no cuts added), solved in 8.5 (*DC*) – 16.6 (*AC*) secs



$t = 5$ $t = 7$ $t = 10$

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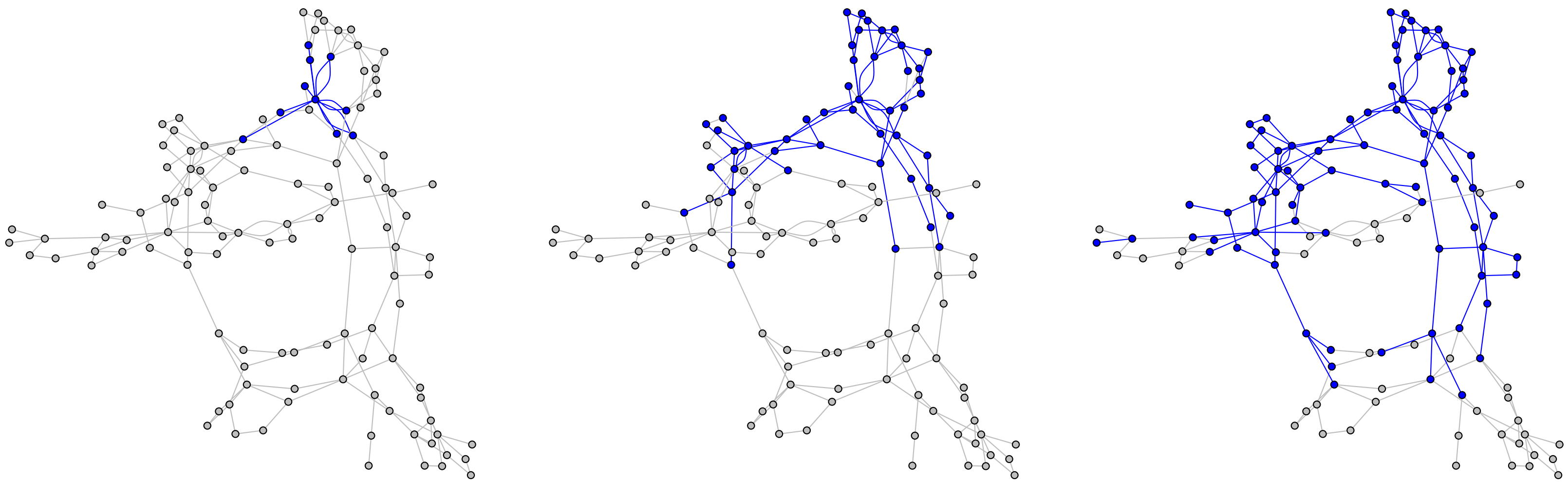
Performance on IEEE-118

- Identical optimal plans for *DC* and *Cvx AC*, different for *AC*
- Solved in 10.3 secs for *DC*, 15.8 secs for *Cvx AC* and **772.5 secs** for *AC*
- AC feasibility **delays energization** of several branches because of **excess of reactive power**

Snapthots of *AC* restoration plan



$t = 10$ $t = 12$ $t = 14$

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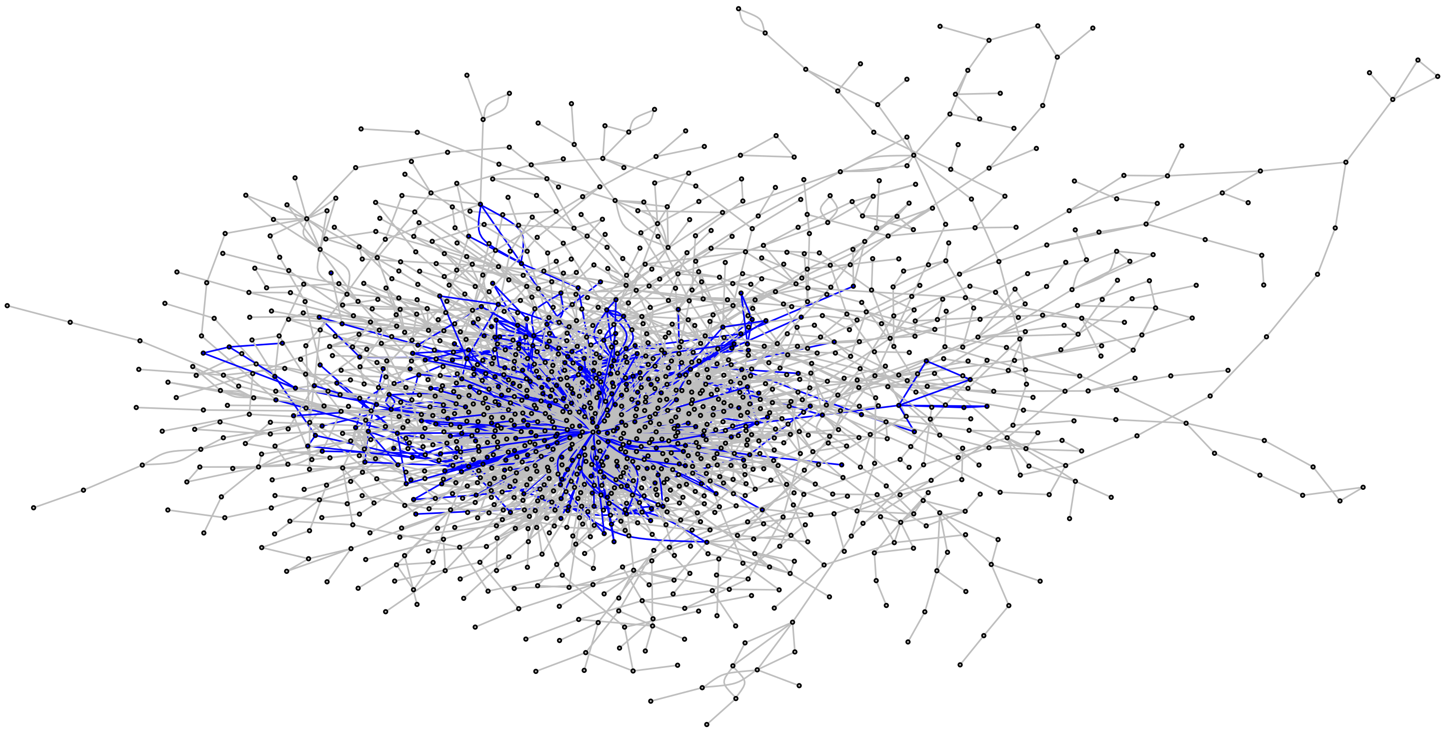
Performance on the Chilean SIC

- Slightly different optimal restoration plans for *DC* and *Cvx AC*, while *AC* runs out of time (16hrs)
- Hundreds to thousands of cuts applied to the *master*, solved in 707.1 secs (*DC*) and 838.2 secs (*Cvx AC*)



Cvx AC restoration plan at $t = 5$

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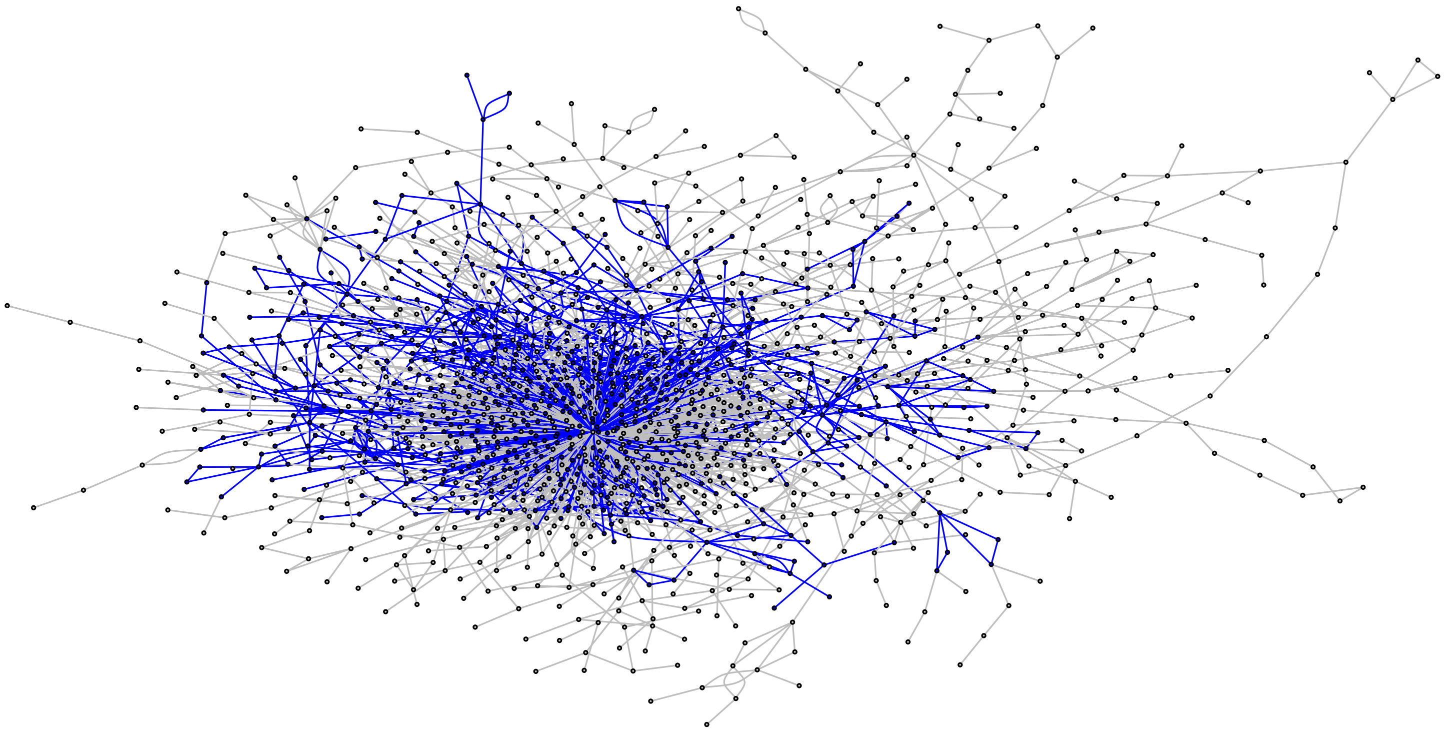

Cvx AC restoration plan at $t = 7$

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Performance on the Chilean SIC

- Slightly different optimal restoration plans for *DC* and *Cvx AC*, while *AC* runs out of time (16hrs)
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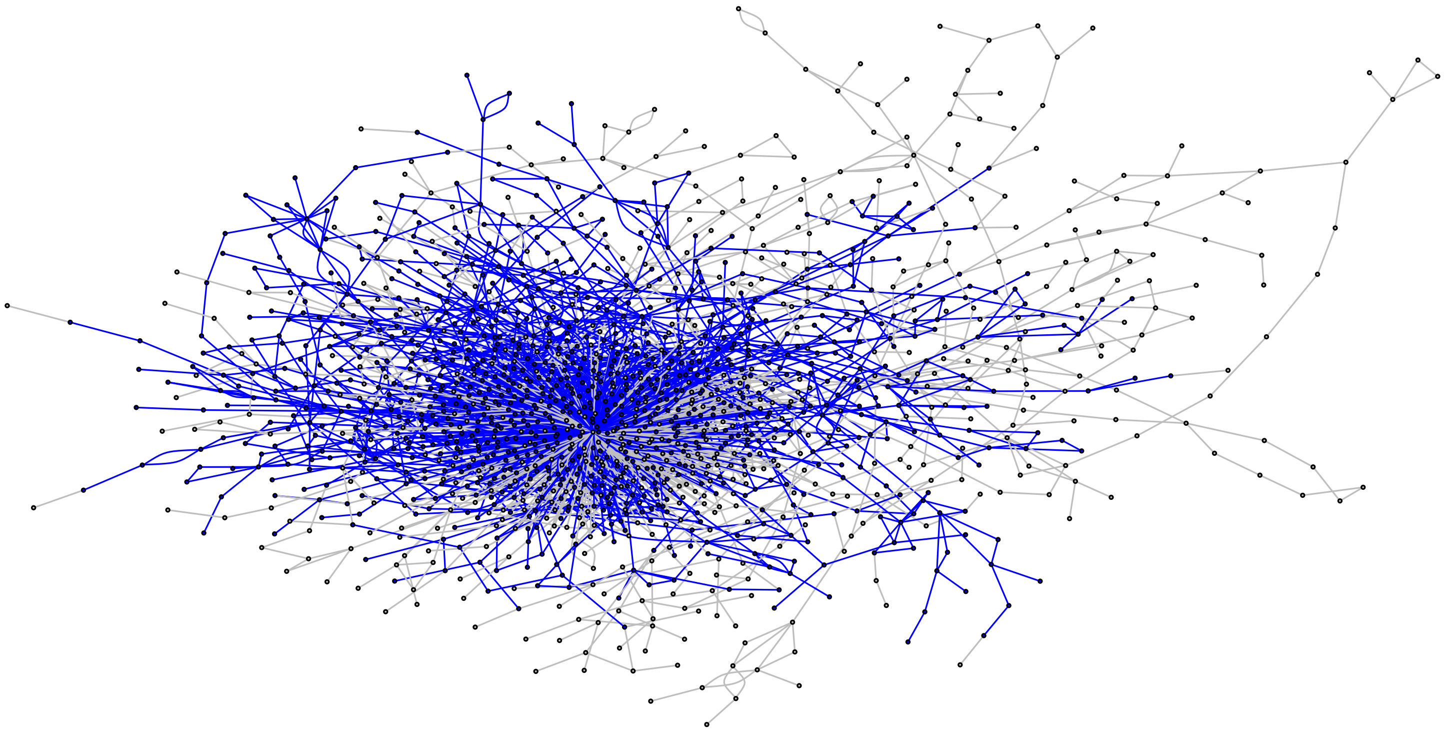

Cvx AC restoration plan at $t = 9$

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Performance on the Chilean SIC

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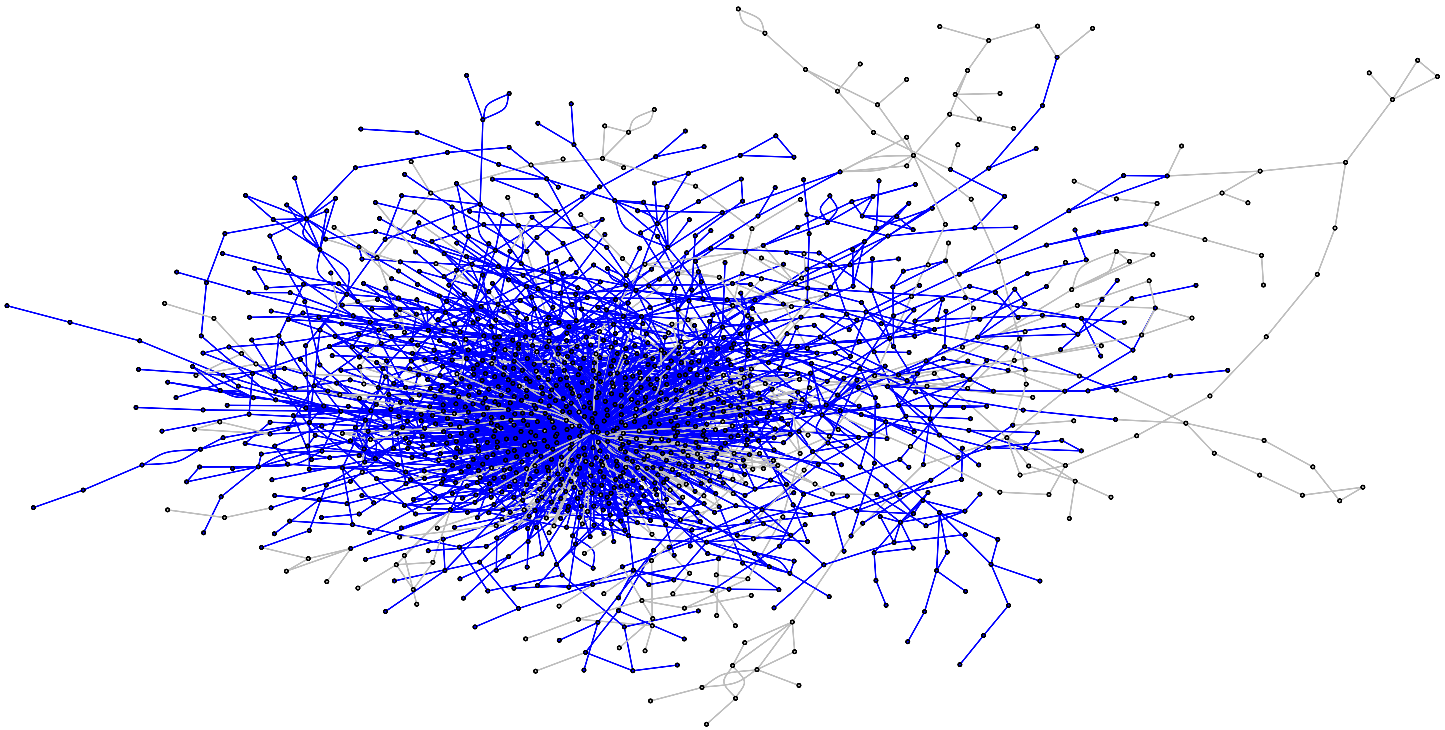
- Hundreds to thousands of cuts applied to the *master*, solved in 707.1 secs (*DC*) and 838.2 secs (*Cvx AC*)



Cvx AC restoration plan at $t = 11$

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Conclusions

- Convex relaxations of AC power flow fail in situations of excess of reactive power
- Hybrid cuts: stronger than no-good cuts and (likely) to be applicable in other contexts
- L-shaped approach allows to solve systems **one order of magnitude larger** than state-of-the-art using convex relaxation

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Future extensions

- Parallelization of feasibility check – expected speedups of 20-50
- Speeding-up solution of AC MILP feasibility check
- Primal heuristics: rolling horizon and convex relaxation
- Solving (stochastic) optimal black-start allocation, optimal islanding and green power system planning

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Thank you

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