Convex Hull, IP and European Electricity Pricing in a European Power Exchanges setting

with efficient computation of Convex Hull Prices

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Expected take-aways

- Insights on European day-ahead markets and bidding products
- Convex Hull Pricing: efficient computation with EU-like bids with startup costs, ramp constraints and min. output levels
- (Numerical) comparison of key pricing rules: CHP, IP and EU for two-sided day-ahead electricity auctions with EU-like non-convex demand/offer bids (source code in Julia online)

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European day-ahead markets and bidding products

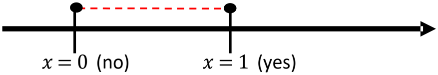
Context

- Guideline on Capacity Allocation & Congestion Management (Commission Regulation (EU) 2015/1222)
- Nominated Market Operators: “Power Exchanges”, entities like ISOs, privately owned:
  e.g. EPEX Spot (France, Germany, Belgium, etc.)
- A single integrated market: bidding zones = countries
- A single market clearing algorithm, EUPHEMIA: handles the bidding products/market rules of the different Power Exchanges
- Two-sided auctions with non-convex demand and offer bids

Non-convexities in day-ahead markets

Main non-convexities

Binary variables introduce non-convexities
Classical economic/strong duality results do not hold anymore.

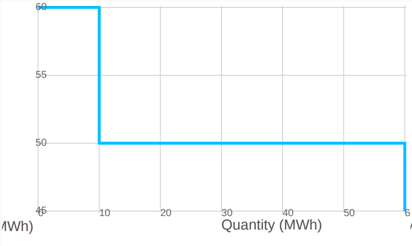


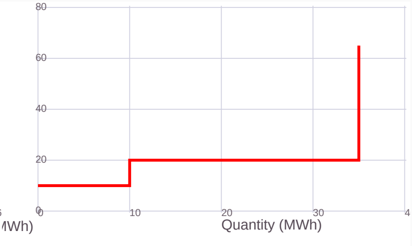
1. Technical constraints
   - Minimum power output levels
   - Minimum up and down times

2. Costs structure
   - Start up costs / shut down costs

Main bidding products in Europe and market rules

- Classical bid curves
  - Users: all Power Exchanges
  - “marginal costs/utility” without technical conditions
  - should be ’at equilibrium’: e.g. fractionally accepted bids/steps set the price





Main bidding products in Europe and market rules

- Block orders
  - Users: EPEX and Nord Pool (France, Germany, Belgium, Norway, The Netherlands, etc.)
  - Indivisibilities: minimum power output levels over several hours

    e.g. “fill-or-kill” for regular block bids: yes/no for all quantities over time horizon
  - Can be “paradoxically rejected” (profitable yet rejected) but cannot cause losses (if min. acceptance ratio: set the price if marginal)
  - Can be “linked” or be mutually exclusive

Main bidding products in Europe and market rules

- "Complex Orders with a Minimum Income Condition" (MIC)
  - Users: Spanish and Portuguese day-ahead markets (OMIE)
  - Input data for MICs
    - marginal cost curves for each hour
    - start up cost
    - ad hoc variable cost (on top of the marginal cost curves…)
    - ramp constraints, called "load gradients"
    - (Scheduled stop)

Minimum income condition: basic formulation

(quantities)(market prices) $\geq$ start up costs + (quantities)(variable cost)

$$(u_c = 1) \quad \Longrightarrow \quad \sum_t \pi_t \left(\sum_i -Q^{c,t,i} x_{c,t,i}\right) \geq F_c + V_c\left(\sum_{t,i} -Q^{c,t,i} x_{c,t,i}\right)$$

Exact linearization without any aux. var. in Madani and Van Vyve, A MIP framework for non-convex uniform price day-ahead electricity auctions, EURO Journal on Computational Optimization, 2017

General “EU-like” bidding products and welfare maximization

$$\max_{(u,x)} \sum_c \left(\sum_{t,i} P^{c,t,i} Q_{c,t,i} x_{c,t,i}\right) - F_c u_c$$

$$\sum_c \sum_i Q_{c,t,i} x_{c,t,i} = 0 \qquad \forall t \; [\pi_t]$$

$$r_{c,t,i} u_c \leq x_{c,t,i} \leq u_c \qquad \forall c, t, i$$

$$\sum_i (-Q^{c,t+1,i}) x_{c,t+1,i} - \sum_i (-Q^{c,t,i}) x_{c,t,i} \leq RU_c \, u_c \qquad \forall c, t$$

$$\sum_i (-Q^{c,t,i}) x_{c,t,i} - \sum_i (-Q^{c,t+1,i}) x_{c,t+1,i} \leq RD_c \, u_c \qquad \forall c, t$$

$$u_c \in \{0, 1\}$$

$Q < 0$ for sell orders, $Q > 0$ for buy orders,

$r_{ic} \in [0; 1]$ min. acceptance ratio

Primal welfare maximization program

$$\max_{(u,x)} \sum_c B_c(u_c, x_c) \tag{1}$$

s.t.

$$\sum_c \sum_i Q_{c,t,i} x_{c,t,i} = 0 \qquad \forall t \in T \;\; [\pi_t] \tag{2}$$

$$(u_c, x_c) \in X_c \qquad \forall c \in C \tag{3}$$

- $B_c(u_c, x_c) < 0$ for sell orders, $B_c(u_c, x_c) > 0$ for buy orders
- $Q < 0$ for sell orders, $Q > 0$ for buy orders

Convex Hull Pricing: efficient computation with EU-like bids

Uplifts

Given an optimal solution (u^*, x^*) and market prices π_t, the uplift of participant $c \in C$ is defined as:

$uplift_{(u_c^*, x_c^*)}(\pi) :=$

$$\left(\max_{(u_c, x_c) \in X_c} \left[B_c(u_c, x_c) - \sum_t \pi_t \sum_i Q_{c,t,i} x_{c,t,i} \right] \right) - \left(B_c(u_c^*, x_c^*) - \sum_t \pi_t \sum_i Q_{c,t,i} x_{c,t,i}^* \right)$$

at the given market prices π:

maximum profit participant c could get with its own decisions
− actual profit/losses with the Market Operator decisions

Convex Hull Pricing: key theorem

Theorem (Gribik et al. (2007))

Let π^* solve the Lagrangian dual where the balance constraint(s) have been dualized:

$$\min_{\pi} \left[\max_{(u_c, x_c) \in X_c, c \in C} \left[\sum_c B_c(u_c, x_c) - \sum_t \pi_t \sum_i Q_{c,t,i} x_{c,t,i} \right] \right] \tag{4}$$

Then, π^* solves:

$$\min_{\pi} \sum_c uplift_{(u_c^*, x_c^*)}(\pi) \tag{5}$$

The "primal approach"

Van Vyve (2011), Schiro et al. (2016) , Hua and Bowen (2016)

Under mild conditions, convex hull prices can be computed via:

$$\max \sum_c B_c(u_c, x_c) \tag{6}$$

$$\sum_c \sum_i Q_{c,t,i} x_{c,t,i} = 0 \qquad \forall t \quad [\pi_t] \tag{7}$$

$$(u_c, x_c) \in Conv(X_c) \qquad \forall c \in C \tag{8}$$

Polyhedral studies of $conv(X_c)$

With min. power output and min up/down times:

- D. Rajan and S. Takriti (2005) (3-bin unit commitment model)
  - Tight formulation, i.e. describing the Convex Hull
  - Used in Hua and Baldick (2016) for their "primal approach"

With min. power output, ramp constraints and min up/down times:

- Damcı-Kurt, Küçükyavuz, Rajan and Atamtürk (2015):
  - Convex Hull for two periods ramp up (resp. ramp down) polytopes
- Guan, Pan and Zhou (2018):
  - Convex hull for three periods
- Knueven, Ostrowski and Wang (2017):
  - Tight compact extended formulation for the multiperiod case
  - Proved via a Thm. on constrained Minkowski sums of polyhedra
  - Tractable to compute CH Prices for medium scale instances (big LP to solve), memory limitation for very large instances
- Gentile and Frangioni, results related to Knueven et al. (2017)

Easy Convex Hull Pricing with EU-like bids

With min. power output, ramp constraints and startup costs given by $F_c u_c$... but without min up/down times

X_C:

$$r_{c,t,i} u_c \leq x_{c,t,i} \leq u_c \qquad \forall c, t, i$$

$$\sum_i (-Q^{c,t+1,i}) x_{c,t+1,i} - \sum_i (-Q^{c,t,i}) x_{c,t,i} \leq RU_c \, u_c \qquad \forall c, t$$

$$\sum_i (-Q^{c,t,i}) x_{c,t,i} - \sum_i (-Q^{c,t+1,i}) x_{c,t+1,i} \leq RD_c \, u_c \qquad \forall c, t$$

$$u_c \in \{0, 1\}$$

- X_c of the form $\{(0,0)\} \cup \{(1,x), \text{with } x \mid Ax \leq b\}$
- $conv(X_c) = \{(u,x) \in \mathbb{R} \times \mathbb{R}^n \mid 0 \leq u \leq 1, Ax \leq bu\}$
- $conv(X_c)$: continuous relaxation of X_c, $\cancel{u_c \in \{0,1\}}$, $u \in [0,1]$

Easy Convex Hull Pricing with EU-like bids

$$\max_{(u,x)} \sum_c \Big(\sum_{t,i} P^{c,t,i} Q_{c,t,i} x_{c,t,i}\Big) - F_c u_c$$

$$\sum_c \sum_i Q_{c,t,i} x_{c,t,i} = 0 \qquad \forall t \; [\pi_t] \; CH\ Prices$$

$$r_{c,t,i} u_c \leq x_{c,t,i} \leq u_c \qquad \forall c,t,i$$

$$\sum_i (-Q^{c,t+1,i}) x_{c,t+1,i} - \sum_i (-Q^{c,t,i}) x_{c,t,i} \leq RU_c \, u_c \qquad \forall c,t$$

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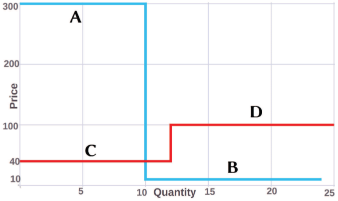
$$\cancel{u_c \in \{0,1\}} \qquad u_c \in [0,1]$$

$Q < 0$ for sell orders, $Q > 0$ for buy orders,

$r_{ic} \in [0;1]$ min. acceptance ratio

Convex Hull Pricing: basic example

Welfare Maximizing Solution:
Fully accept A + 10MW from C



« Welfare = ▇ - 200 € »

Bids	Quantity (MW)	Limit Price (€/MW)	Start up costs
A (buy)	10	300	-
B (buy)	14	10	-
C (sell)	12	40	200 €
D (sell)	13	100	-

Convex Hull Pricing

- market price = 56.6... € /MW
- Actual losses
  - 10(56.6... - 40) - 200 = - 33.33...€
- Opportunity costs
  - C: (56.6... – 40)x(12-10) = 33.33...€
- Deviation from equilibrium:
  - 33.33 € < 200 € (IP pricing)

IP Pricing and “EU Pricing”

$$\max_{(u,x)} \sum_c \Big(\sum_{t,i} P^{c,t,i} Q_{c,t,i} x_{c,t,i} \Big) - F_c u_c$$

$$\sum_c \sum_i Q_{c,t,i} x_{c,t,i} = 0 \qquad \forall t \ [\pi_t]$$

$$r_{c,t,i} u_c \leq x_{c,t,i} \leq u_c \qquad \forall c, t, i \ \ [s^{max}_{c,t,i}]$$

$$\sum_i (-Q^{c,t+1,i}) x_{c,t+1,i} - \sum_i (-Q^{c,t,i}) x_{c,t,i} \leq RU_c \ u_c \qquad \forall c, t$$

$$\sum_i (-Q^{c,t,i}) x_{c,t,i} - \sum_i (-Q^{c,t+1,i}) x_{c,t+1,i} \leq RD_c \ u_c \qquad \forall c, t$$

$$u_{c_a} = 1 \qquad \forall c_a \in \{c | u^*_c = 1\} := C_a \subseteq C \qquad [\delta_{c_a}]$$

$$u_{c_r} = 0 \qquad \forall c_r \in \{c | u^*_c = 0\} := C_r \subseteq C \qquad [\delta_{c_r}]$$

IP Pricing and EU-like market rules

C_a, C_r: partition given by the optimal u_c^* !

$$u_{c_a} = 1 \qquad \forall c_a \in \{c | u_c^* = 1\} := C_a \subseteq C \qquad [\delta_{c_a}]$$

$$u_{c_r} = 0 \qquad \forall c_r \in \{c | u_c^* = 0\} := C_r \subseteq C \qquad [\delta_{c_r}]$$

π, δ_c equilibrium prices for an appropriately defined settlement rule with payments depending on π, δ_c (R. P. O'Neill et al., EJOR, 2005)

Actually, for *any* commitment decisions $\overline{u_c}$ determining $C_a := \{c | \overline{u_c} = 1\}$ and $C_r := \{c | \overline{u_c} = 0\}$:

- $\delta_{c_a} =$ profit/loss of $c_a = \sum_{t,i} -Q^{c_a,t,i}(\pi_t - P^{c_a,t,i}) x_{c_a,t,i} - F_{c_a}$
- $\delta_{c_r} =$ upper bound on the opportunity costs (missed profits)
- $x_{c,t,i}$ optimal decisions, for fixed $\overline{u_c}$ and market prices π_t

IP Pricing and EU-like market rules

Revisiting Minimum Profit Conditions in Uniform Price Day-Ahead Electricity Auctions (Madani and Van Vyve, EJOR, 2018):

- Minimum Profit/Maximum Payment conditions revisited: only consider commitment decisions $\overline{u_c}$ determining $C_a := \{c|\overline{u_c} = 1\}$ and $C_r := \{c|\overline{u_c} = 0\}$ such that: $\delta_{c_a} \geq 0$: no losses for selected bids/committed plants

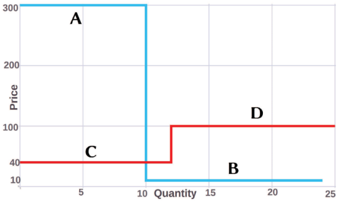
- European block orders clearing conditions turn out to be just a special case of this

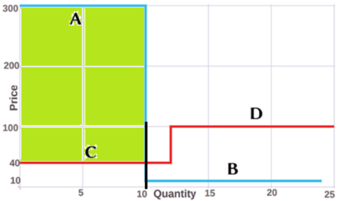
- the way to go to reformulate orders with a “Minimum Income Condition” used in Spain and Portugal: includes marginal costs and startup costs recovery conditions

Bids more general than block orders, and variant of MIC orders: hence, called “EU-like” bids and market rules.

IP Pricing: a basic example

Welfare Maximizing Solution:
Fully accept A + 10MW from C





« Welfare = ■ - 200 € »

Bids	Quantity (MW)	Limit Price (€/MW)	Start up costs
A (buy)	10	300	-
B (buy)	14	10	-
C (sell)	12	40	200 €
D (sell)	13	100	-

IP Pricing

- market price = 40 € /MW
- $\delta_c = -200$: C is compensated for its losses: 200 €, the start up costs

IP Pricing and EU-like rules: basic example

EU-like rules (min. proft cond. with uniform prices)

(a) Less Welfare (b) no losses incurred

(No “make-whole payments” required)

(c) **C is now paradoxically rejected**

Paradoxical rejecton only allowed for non-convex bids

only deviaton from equilibrium allowed

Bids	Quantty (MW)	Limit Price (€/MW)	Start up costs
A (buy)	10	300	-
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D (sell)	13	100	-

IP Pricing and EU-like market rules

Revisiting Minimum Profit Conditions in Uniform Price Day-Ahead Electricity Auctions (Madani and Van Vyve, EJOR, 2018):

”primal-dual” MILP formulation *without any auxiliary variables or compl. constraints for EU-like rules*

Benders decomposition derived from the MILP formulation

- globally valid ”no-good” cuts (also by Martin, Muller and Pokutta in a related context):

$$\sum_{c|u_c^*=1} (1-u_c) + \sum_{c|u_c^*=0} u_c \geq 1$$

- *locally* valid strengthened Benders cuts:

$$\sum_{c|u_c^*=1} (1-u_c) \geq 1$$

Numerical experiments

Comparing welfare, welfare loss and side payments

Table 1: Welfares and uplifts (euros). The “Welfare Loss (EU rules)” column indicates how much welfare is lost with European Pricing.

Inst	# Non-Convex bids	#Steps	Welfare (IP & CHP)	Welfare Loss (EU rules)	upliftsCHP	upliftsIP (make-whole)
1	90	14309	115426705.6	11084.8536	288.7258	7393.944
2	91	13986	107705738.5	5003.636	439.193	5000.8
3	91	14329	113999405.5	2141.15356	1030.314	6648.373
4	92	14594	109951139.7	9466.60112	603.5169	5827.93
5	89	14370	107172393.2	7754.3366	72.63568	867.284
6	87	14389	123823967.6	3377.139199	239.3088	1835.88
7	89	14783	119386085.4	6964.017	329.5143	3116.86
8	86	14414	105372099.8	2187.674081	72.25676	951.5828
9	88	14860	96023475.04	2046.41408	778.3553	5275.138
10	86	14677	98212635.81	2597.8314	401.637	2313.78

Paradoxically rejected/accepted bids

Table 2: Number of paradoxically accepted (resp. rejected) non-convex bids for each pricing rule

Inst	# Non-Convex bids	pabEU	prbEU	pabIP	prbIP	pabCHP	prbCHP
1	90	0	2	1	0	1	1
2	91	0	1	1	0	0	0
3	91	0	5	1	0	0	1
4	92	0	2	1	0	1	5
5	89	0	4	1	0	0	0
6	87	0	1	2	0	1	1
7	89	0	2	1	0	1	1
8	86	0	2	1	0	0	2
9	88	0	2	2	0	0	3
10	86	0	2	1	0	0	1

Total run times for each pricing rules - easy instances

Table 3: Run times for each pricing rule (in seconds)

Inst	# Non-convex bids	# Steps	runEU	runIP	runCHP
1	90	14309	4.047098	2.202199	2.073478
2	91	13986	4.648906	2.081456	2.065098
3	91	14329	4.231441	2.294439	2.102532
4	92	14594	4.82378	2.050598	2.345987
5	89	14370	4.410432	1.860187	1.819655
6	87	14389	3.78953	1.907919	2.25707
7	89	14783	4.631189	2.104128	2.149526
8	86	14414	3.8165	1.842994	2.142367
9	88	14860	4.603193	1.943571	2.043593
10	86	14677	3.73881	2.0862	1.897801

Total run times for each pricing rules - hard instances

Table 4: Run times for each pricing rule (in seconds)

Inst	# Non-convex bids	# Steps	runEU	runIP	runCHP
1	456	5274	≥ 300	21.96525	21.39609
2	660	7161	≥ 300	51.89202	59.04887
3	533	5373	≥ 300	24.74296	24.71822
4	487	4949	≥ 300	19.23026	18.69612
5	618	5905	74.10239	33.18137	35.50521
6	535	5148	41.32513	22.05355	20.78923
7	546	5394	29.91423	22.00446	21.20238
8	540	5395	31.25016	22.02144	22.29015
9	506	5473	30.5554	21.64758	21.26571
10	479	6537	76.04344	27.91634	26.5238

Conclusions

- EU-like rules:
  - avoids the use of any side payments
  - much more difficult to compute for large hard instances
  - rather small welfare losses compared to the real welfare max. sol.
- IP Pricing:
  - more welfare and easier to compute
  - Less “paradoxically rejected bids” and “paradoxically accepted bids” receive make-whole payments
- Convex Hull Pricing:
  - more welfare and easier to compute (for EU-like bids)
  - Less “paradoxically rejected bids”
  - smaller (smallest...) deviations from a market equilibrium

N.B. The three pricing rules can give surprising outcomes.

Thank you!