A Toolbox for Exploring AC OPF Formulations, Datasets and Solution Methods

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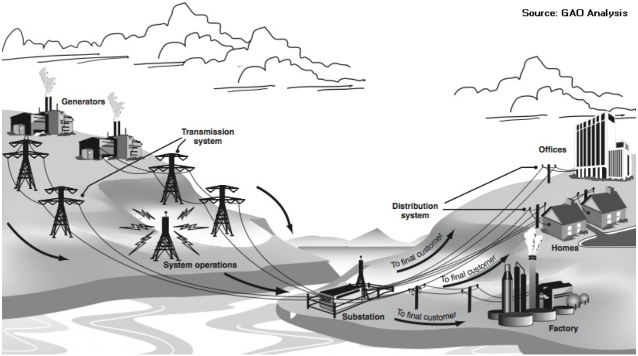
Power generation, transmission and distribution



- Today's electricity market encounters many complex questions, revolving around $\sum$ Gen MW $= \sum$ Load MW, at all times.
  - Operational, planning
  - Flexibility in face of new technologies
- Many interacting levels, multiple time-scales and agents, increasing levels of stochasticity.
- Larger network $\rightarrow$ Larger models

Optimal Power Flow Models

Optimal Power flow (OPF) models are at the heart of it all.

- Well researched, standard models have not changed much.
- Significant progress in decomposition and stochastic methods.
- Lacks cohesiveness in comparative literature due to different models, file formats, software, solvers.
- Core difficulty has always been solving large-scale models.

Consider the following data set:

- Network: 13867 buses, 18790 lines
- Generation capabilities: 1043 generators
- Non-zero Loads: 3753 nodes
- Time periods: 24 hours

How far can we take the modeling experience in a dataset of this size?

Outline of topics

- Introduction to the OPF Toolbox
- Observations and results
  - Initial conditions for AC models
  - Solver comparisons
  - Formulation comparisons
  - D-curves
- Compiling and solving realistic large-scale datasets
  - Realistic reactive demand profile
  - Solution Process
- Stochastic Extensions
  - Stochastic unit commitment
  - Value at Risk
- Conclusions

Objective: Bridge the gap between different software, solvers, data formats, industry and academics.

Toolbox: A general overview

Models and testcases are open source, and written primarily in GAMS.
The toolbox consists of:

- Optimization models for different OPF formulations.
- Testcase archive includes IEEE testcases, inlcuding Polish testcases (2000-3000+ buses), RTS-96 (6 files containing seasonal 24-hour demand data).
- Data management utilities for format conversion, data generation, and easy viewing output.
- Downloadable at `http://www.neos-guide.org/content/optimal-power-flow`
- A collection of large-scale datasets is available under Critical Energy Infrastructure Information (CEII) usage and agreement terms (not publicly available).

Why GAMS?

- One system
  Designed for modeling multiple types of problems
  e.g. linear, non-linear, mixed integer, stochastic.
- Write once
  Integrates multiple high-performance solvers
  e.g. CPLEX, CONOPT, IPOPTH, BARON, GUROBI, LINDO, PATH.
- Flexible
  Portable between different platforms, models are easily extensible, solver integration taken care of on the back-end.
- Reusability
  Models and data are easily saved and re-used in future applications. Generic GDX data interface.

Toolbox: Standard OPF Models

Core Models:

- Direct current (DC) OPF, with and without shift matrices
- Alternating current (AC) OPF models
  - Polar Power-Voltage Formulation
  - Rectangular Power-Voltage Formulation
  - Rectangular Current-Voltage Formulation
  - Y-bus formulations
- Decoupled OPF
- Unit commitment models, both AC and DC

Stochastic Model extensions:

- Stochastic unit commitment
- Security constrained unit commitment
- Value at Risk

Testcases

Data files include the following information:

- Network, including power and current limits, interfaces, tap transformers
- Generator, including operational, cost, and fuel (where available)
- Active and reactive demand
- Multiple time periods (where available)

Standard data enhancements in testcase archive include:

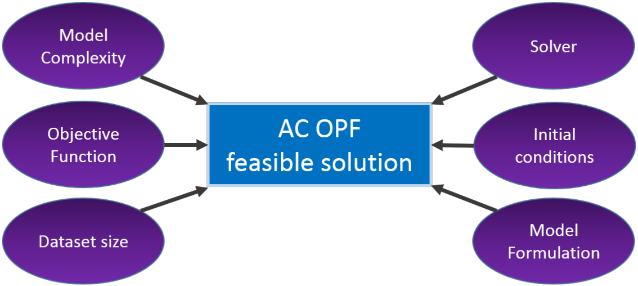
- Cost function approximations: Linear, quadratic, piecewise linear
- Demand bidding: Bid to shed load
- Generator Capability curves (D-curves)
- Lineflow limit approximation: Alternative approximations
- Generator ramp-rate approximation: Alternative approximations

Data management utilities

- Utilities to facilitate conversions between the following three formats
  - GAMS formatted input files (.gdx)
  - Matpower formatted input files (.m)
  - PSS^{TM}E-31 power flow raw data file (.raw)
- Compute Shift Matrix for a system
- Output data into Excel spreadsheet for easy viewing

	A	B	C	D	E
1	NAME	TYPE	Dim	Count	Explanatory Text
2	baseMVA	parameter	0	1	
3	branchinfo	parameter	5	4249	
4	branch_s	set	1	7	Set of branch info selectors
5	branch_t	set	1	16	Set of branch info data types
6	bus	set	1	118	Set of buses
7	businfo	parameter	3	1147	
8	bus_s	set	1	26	Set of bus info selectors
9	bus_t	set	1	25	Set of bus info data types
10	circuit	set	1	2	Indices for multiple lines between buses
11	demandbid	set	1	118	Set of buses
12	demandbidinfo	parameter	4	1881	
13	demandbidinfo_1	parameter	4	0	
14	demandbidinfo_2	parameter	4	0	
15	demandbidmap	set	2	99	Mapping of demand bid identifier to bus (demandbid,bus)
16	demandbid_s	set	1	11	Set of demandbid info selectors
17	demandbid_t	set	1	3	Set of demandbid info data types
18	fuelinfo	set	2	0	
19	fuel_s	set	1	3	Set of fuel info selectors
20	fuel_t	set	1	1	Set of fuel types
21	gen	set	1	54	Set of generators
22	geninfo	parameter	3	3151	

Using the OPF toolbox



Examples of model options include:

- Time: Select which time periods(s)
- Objective: Feasibility, linear, quadratic, piecewise linear functions.
- Initial conditions: Starting point methods for AC OPF.
- D-curve: Enforce reactive power limits as D-curve circle constraints.
- Demand bidding: Incremental elastic demand bidding is considered.

Initial conditions for AC OPF models

Within the toolbox, AC OPF models provide multiple starting point options. Some examples are listed below.

- ic=0 Midpoint
  All variables initialized at the midpoint between variable bounds.
- ic=1 Random
  All variables initialized using random draws between variable bounds.
- ic=2 Flat
  Voltage magnitude =1, voltage angle = 0. Real, reactive power = 0.
- ic=3 Random/inferAC
  Voltage magnitude & voltage angle variables are random draws. Real, reactive power are inferred using AC transmission line equations.
- ic=4 DC/inferAC
  Real power and voltage angle values are initialized using a DCOPF model. Voltage magnitudes are initialized at 1 and reactive power is inferred using AC transmission line equations.

Question: What expectations would we have?

Comparisons between initial conditions

Dataset	ic=0	ic=1	ic=2	ic=3	ic=4
case14	0.174s	0.1734s	0.1902s	0.2658s	0.3548s
case118	0.4822s	0.5572s	0.7646s	infeas	1.1414s
case300	1.0392s	1.0866s	1.633s	22.6518s	1.2314s
case2737sop	6.5286s	45.3575s	14.5468s	infeas	7.9692s
case3120sp	8.9356s	41.009s	16.9014s	infeas	11.9338s
case3375wp	14.2058s	93.9364s	infeas	infeas	16.9038s

Table: Comparison of initial conditions

Comparison of solvers

Dataset	CONOPT	Knitro	IPOPTH
case118	1.150s	0.687s	0.702s
case300	3.881s	1.057s	1.2314s
case2737sop	25.458s	8.736s	7.9692s
case3120sp	56.837s	4m 27s	11.9338s
case3375wp	1m 58s	12.657s	16.9038s
rts96_winter_wend (UC Polar)	8m 50s	14.806s	15.931s

Table: Comparison of Solvers

Comparison of OPF formulations

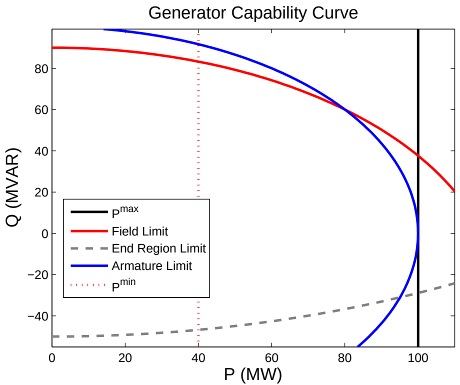
- Polar power-voltage formulation uses polar form of complex quantities and explicitly uses sines and cosines.
- Rectangular power-voltage formulation uses the rectangular form of complex quantities, resulting in quadratic power flow constraints.
- Rectangular current-voltage formulation models current flow instead of power on a line. Also uses rectangular form of complex quantities, but has linear current flow equations.

Dataset	**Polar**	**Rect-PV**	**Rect-IV**
case118	0.702s	0.757s	0.843s
case300	1.2314s	1.339s	1.369s
case2737sop	7.9692s	9.483s	9.357s
case3120sp	11.9338s	14.411s	12.269s
case3375wp	16.9038s	15.553s	36.412s
rts96_winter_wend (UCAC)	15.931s	33.981s	infeas

Table: Comparison of OPF formulations

D-curve constraints

- Generator models primarily use "rectangular constraints" for active and reactive output limits.
- A more detailed model is necessary to accurately characterize generator capability curves, which are also called "D-curves".



$$P^2 + Q^2 \leq (R^{\max})^2 \qquad (1)$$

$$P^2 + (Q - Q_0^{\text{field}})^2 \leq (r^{\text{field}})^2 \qquad (2)$$

$$P^2 + (Q - Q_0^{\text{end}})^2 \leq (r^{\text{end}})^2 \qquad (3)$$

Model result using D-curve constraints

Dataset	Time		Objective	
	Standard	D-curves	Standard	D-curves
case14	0.3548s	0.424s	8.08153e+03	8.09162e+03
case118	0.734s	0.704s	1.29661e+05	1.29913e+05
case300	1.2314s	0.871s	7.19725e+05	7.20176e+05
case2737sop	7.9692s	8.689	7.77629e+05	7.77649e+05
case3120sp	11.9338s	11.279s	2.14270e+06	2.15042e+06
case3375wp	16.9038s	23.046s	7.41203e+06	7.43363e+06

Table: Rectangular vs. D-curve constraints

FERC: Winter and Summer datasets

Dataset Profile:

- Large scale: 13867/13981 buses and 18790/18626 lines for Winter/Summer datasets respectively.
- Datasets compiled using CEII network information and public information provided on FERC e-Library website.
- Includes information on prime movers, tap transformers, interfaces, fuel.
- *Datasets are non-publicly available and part of Critical Energy Infrastructure Information (CEII).*

Reactive demand

Question: What is the definition of a “realistic” reactive demand profile?

- Good power factor values at each load bus.
  - $PF = \frac{P}{S} = \frac{P}{\sqrt{P^2+Q^2}}$
  - Given P and S limits, can provide bounds on Q
  - What about when P=0?
- Feasiblity of values in the ACOPF model.
- A “reasonable” number of buses with non-zero reactive demand values.
  - L-2 or L-1 norm in objective function
- A larger ratio of withdrawals to injections in the overall system.

Our Solution: Minimize reactive demand with respect to AC OPF constraints.

Solution Process

When considering large scale datasets in the AC models, regular solution practices may be insufficient in finding solutions. Large-scale AC models are much harder, if not impossible to solve without good initial conditions.

Is the toolbox useful for a dataset this size?

Procedure 1: Feasibility methodology

$(\tilde{P}, \tilde{\theta}, U) \leftarrow$ Solve `UC_DC --lineloss=1.055`

$(P, Q, \theta, V) \leftarrow$ Solve `polar_acopf(`$\tilde{P}, \tilde{\theta}, U$`) --ic=#`

$(P, Q, \theta, V, U) \leftarrow$ Solve `UC_AC(`$\tilde{P}, \tilde{Q}, \tilde{V}, \tilde{\theta}, \tilde{U}$`)`

Dataset	ic=0	ic=4	ic=7
Winter, t=20	14m 28.69s	4m 27.31s	4m 35.52s
Summer, t=18	infeas	4m 34.74s	3m 50.66s

Handling uncertainty

Stochastic models are becoming increasingly important in today's electricity delivery landscape.

- Uncertainty stemming from wind forecasts
- Contingency planning
- Simple to model using GAMS EMP, with `randvar`

Stochastic unit commitment model

$$\min \mathbb{E}_s[\mathrm{cost}(\mathrm{P_s},\ \mathrm{U})] \tag{4}$$

$$g(P_s, \theta_s, U) = D_s \tag{5}$$

$$h(P_s, \theta_s, U) \leq 0 \tag{6}$$

Stochastic Unit Commitment

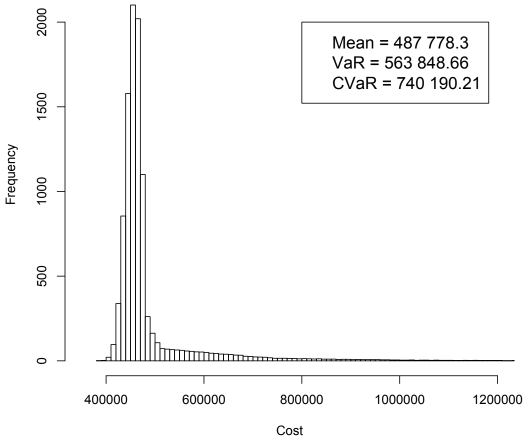
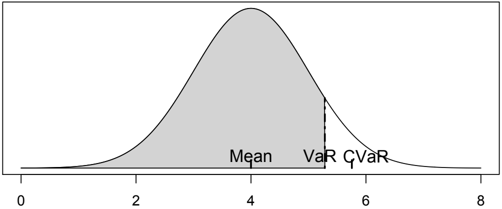


Figure: Validation using 10000 independent samples

Value at Risk



- Value at Risk (VaR) and Conditional VaR (CVaR) are risk measures, designed to evaluate effects of uncertainty on the outcomes of interest.
- $\overline{\text{VaR}}_\alpha$ is the Value at Risk at the upper α percentile.

VaR Model

$$\min \overline{VaR}_\alpha[\text{cost}(P_s, U)] \tag{7}$$

$$\text{and } (5-6) \tag{8}$$

Comparing Stochastic UC with VaR and CVaR

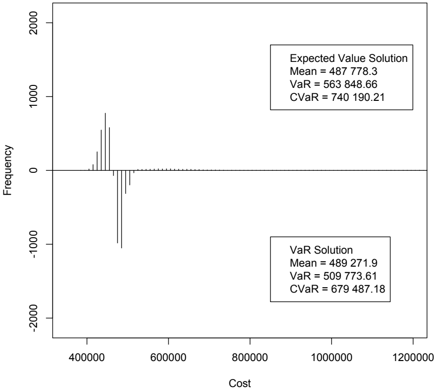


Figure: Difference:EV-VaR

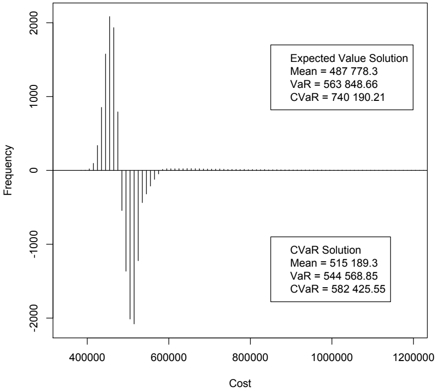


Figure: Difference:EV-CVaR

Other GAMS extensions

- equilibrium
- vi (agents can solve min/max/vi)
- bilevel (reformulate as MPEC or SOCP)
- dualvar (use multipliers from one agent as variables for another)
- Benders decomposition (available in LINDO)
- Distribution sampling (available in LINDO)
- Conversion techniques to PYOMO, AMPL

Conclusions

- OPF Toolbox as an analytical and solution tool:
  - Bridges the gap between work done on different software, solvers, formats.
  - Facilitates structured use and analysis of algorithms for solving large-scale and complex problems.
  - Provides access to powerful, established solvers.
  - Enables us to model complex new devices, test policy.
  - Deal with incomplete data.
- Domain knowledge/expertise is important, e.g. good starting points, solvers.
- Ongoing and future work include:
  - Exploring structured methods to solve large-scale models.
  - Incorporate/test decomposition methods.
  - Further research into stochastic models and solution methods.