NYU TANDON SCHOOL OF ENGINEERING

Pricing Under Uncertainty: A Chance Constraint Approach to a Robust Competitive Equilibrium

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June 27, 2018

Overview: Why Chance Constraints?

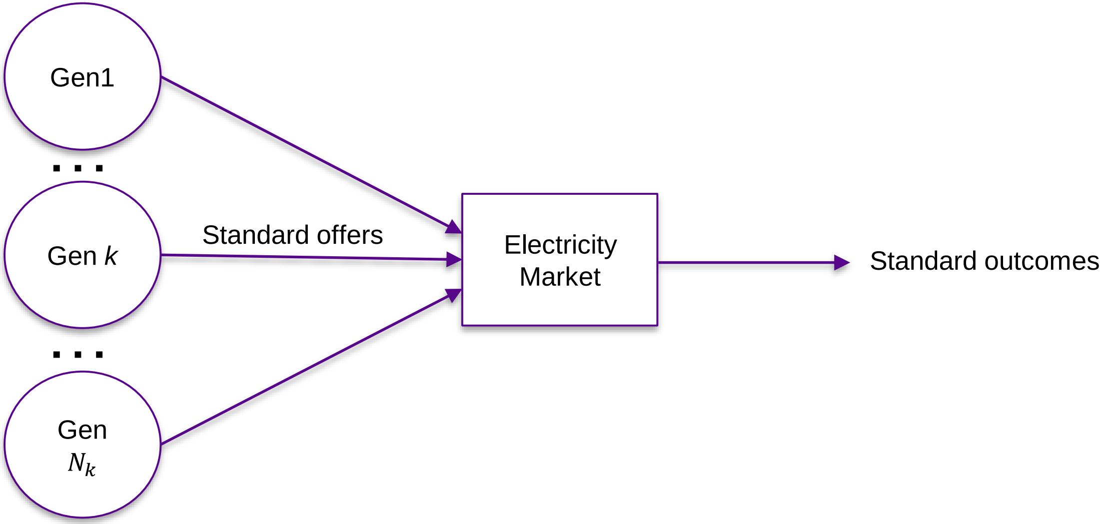
- **Power grid operations have shifted to uncertainty-aware decision-making frameworks**
  - Scenario/interval-based optimization
  - Robust optimization
  - Chance constraints
  - Real-life OPF/UC applications: China, Switzerland, Russia (hydro+nucs)
- **Electricity markets are largely lagging far behind**
  - No consensus contract design
  - Uncertainty factors are not explicitly internalized the price formation process
  - No systematic framework to map uncertainty to a given network
  - “Stochasticity” concerns: What does the “stochastic” pay off actually mean? How to resolve the risk versus expectation dilemma? And how to explain it to a generation owners?
  - Lack of data or format dependencies on third-party providers (e.g. NOAA)
  - Scalability concerns

Chance constraints can be just the right framework to address the key issues

- **Feasibility and effectiveness of chance constraints have been well-established**
  - Demonstration of cost-efficient and tractable reformulation (Bienstock et al, 2014) applied to a network with a 2,000+ nodes with location-specific treatment of uncertainty
  - Discriminatory treatment of small and large constraint violations (Roald et al, 2015; Dvorkin et al, 2017) for non-affine control policies and separating primary, secondary, and tertiary reserve needs
  - Scalable extensions to distributionally robust formulations, both algorithmically (Lubin, 2016) and via exact, or almost, convex reformulations (Xie et al, 2018)
  - Enable a “complete” electricity market design via a linearization of ac power flows (Lubin, 2018) or a convex relaxation (Halilbasic et al, 2018)
  - Support contingency-constrained formulations (Roald et al, 2016)
- **Can leverage existing results**
  - The exact SOCP reformulation is convex
  - Results obtained using the LP duality (deterministic markets) can be extended to a more general SOCP case (with some modifications)
  - SOCP duality ensures compatibility with legacy electricity market designs (important for the successful transition; Kuhn, 1962)

Chance constraints ensure high performance, modeling fidelity and compatibility with legacy market designs

- **Contract design & market equilibrium with chance constraints**
  - Single node case
  - Contract design with chance constraints
  - Market equilibrium under chance constraints
- **Extensions to network-specific pricing with chance constraints**
  - How to enforce the chance-constrained apparent power flow limits?
  - Implications on pricing
  - Contract design feasibility: is possible with the single-node contract?
- **Not in this presentation**
  - Explicit treatment of non-convexities
  - Can be internalized using previous results for deterministic markets (using a connection between the LP – SOCP duality)



- **Contract design = {Standard offers, Standard outcomes}**
- **Standard offers include energy and reserve offers (capacity, price)**
- **Standard outcomes include cleared offers and prices**

Contract Design with Chance Constraints

- **Chance-constrained, single-node, single-period unit commitment problem**

$$\min \quad \mathbb{E}\left(\sum_k c_k(p_k - \alpha_k \boldsymbol{\Omega}) + f_k z_k\right)$$

Incremental cost

Fixed cost

Affine control with the power output p_k, participation factor α_k and system-wide uncertainty $\boldsymbol{\Omega}$

- Factors in the cost of real-time output: $\boldsymbol{p}_k = p_k - \alpha_k \boldsymbol{\Omega}$
- Real-time system-wide uncertainty: $\boldsymbol{\Omega} \sim N(0, \sigma^2)$
- Affine response and Gaussian, zero-mean assumptions are for the sake of convenience; can be revisited

- **Chance-constrained, single-node, single-period unit commitment problem**

$$\min \quad \mathbb{E}\left(\sum_k c_k(p_k - \alpha_k \boldsymbol{\Omega}) + f_k z_k\right)$$

$$\text{s.t.} \quad \sum_k p_k + W^f = D,$$

System-wide power balance constraint (deterministic)

$$p_k^{\min} z_k \le p_k \le p_k^{\max} z_k, \qquad \forall k,$$

Output limits on generators (deterministic)

$$\mathbb{P}\left(p_k - \alpha_k \boldsymbol{\Omega} \le p_k^{\max} z_k\right) \ge 1 - \epsilon, \qquad \forall k,$$

$$\mathbb{P}\left(p_k - \alpha_k \boldsymbol{\Omega} \ge p_k^{\min} z_k\right) \ge 1 - \epsilon, \qquad \forall k,$$

Chance constrained output limits on generators

$$\sum_k \alpha_k = 1,$$

Constraint on the system-wide response

$$\alpha_k \ge 0, z_k \in \{0,1\}, \qquad \forall k,$$

- **Chance-constrained, single-node, single-period unit commitment problem**

$$
\begin{aligned}
\min \quad & \mathbb{E}\left(\sum_k c_k(p_k - \alpha_k \boldsymbol{\Omega}) + f_k z_k\right) \\
\text{s.t.} \quad & \sum_k p_k + W^f = D, \\
& p_k^{\min} z_k \le p_k \le p_k^{\max} z_k, && \forall k, \\
& \mathbb{P}\left(p_k - \alpha_k \boldsymbol{\Omega} \le p_k^{\max} z_k\right) \ge 1 - \epsilon, && \forall k, \qquad \text{(CCUCP)} \\
& \mathbb{P}\left(p_k - \alpha_k \boldsymbol{\Omega} \ge p_k^{\min} z_k\right) \ge 1 - \epsilon, && \forall k, \\
& \sum_k \alpha_k = 1, \\
& \alpha_k \ge 0, z_k \in \{0, 1\}, && \forall k,
\end{aligned}
$$

This problem can be reduced to an LP (using the zero-mean assumption + fixing binary decisions)

- **Deterministic LP for the chance-constrained, single-node, single-period unit commitment problem**

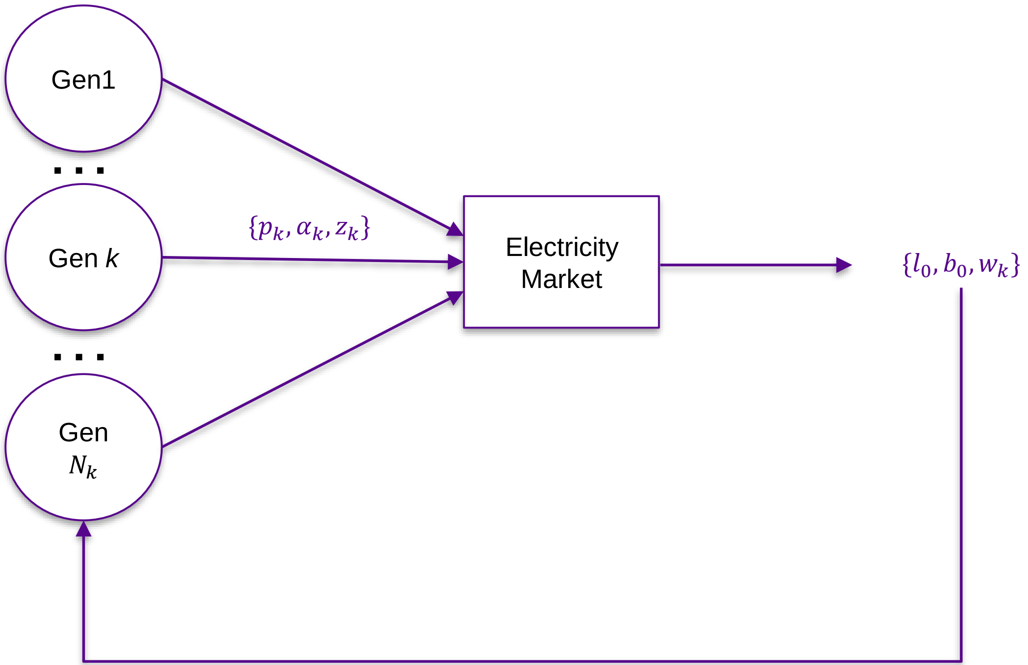
$$
\begin{aligned}
\min \quad & \sum_k c_k p_k + f_k z_k \\
\text{s.t.} \quad & \sum_k p_k + W^f = D, \\
& p_k^{\min} z_k - \Phi_\epsilon^{-1} \sigma \alpha_k \le p_k \le p_k^{\max} z_k + \Phi_\epsilon^{-1} \sigma \alpha_k, \quad \forall k, \\
& \sum_k \alpha_k = 1, \\
& \alpha_k \ge 0, \quad \forall k, \\
& (\text{CCUCP}_{\text{IP}})
\end{aligned}
$$

← Φ_ϵ^{-1} can be scaled to represent non-Gaussian distribution

  - This LP can be then decomposed into "generators" problem (O'Neil, 2005)

$$
\begin{aligned}
\min \quad & c_k p_k + f_k z_k - l_0 p_k - b_0 \alpha_k - w_k z_k \\
\text{s.t.} \quad & p_k^{\min} z_k - \Phi_\epsilon^{-1} \sigma \alpha_k \le p_k \le p_k^{\max} z_k + \Phi_\epsilon^{-1} \sigma \alpha_k, \\
& \alpha_k \ge 0, \quad (\text{CCUCP}k_{\text{IP}})
\end{aligned}
$$

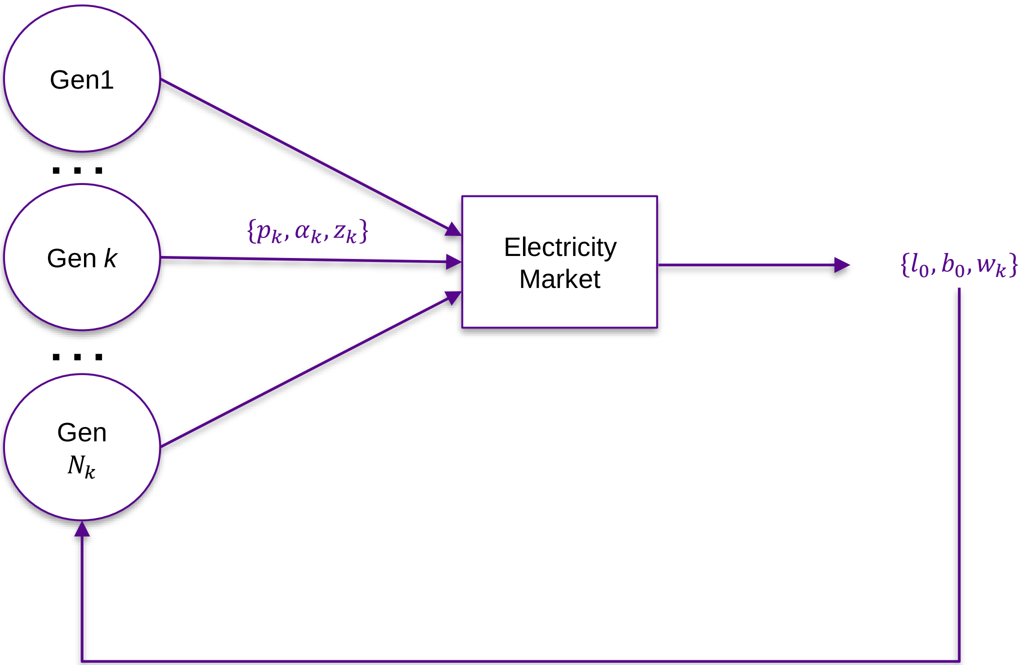
← $\{l_0, b_0, w_k\}$ define the compensation of each generator for the power price, ramp power price, and commitment compensation



- Contract design

Let T_k be a contract between the power market operator and generator k with the following terms: (1) Generator's k decision is given by $\{p_k, \alpha_k, z_k\}$, and (2) Generator k receives an amount from the power market operator equal to the following payment function: $l_0 p_k + b_0 \alpha_k + w_k z_k$.

- This contract design leads to a stable market equilibrium



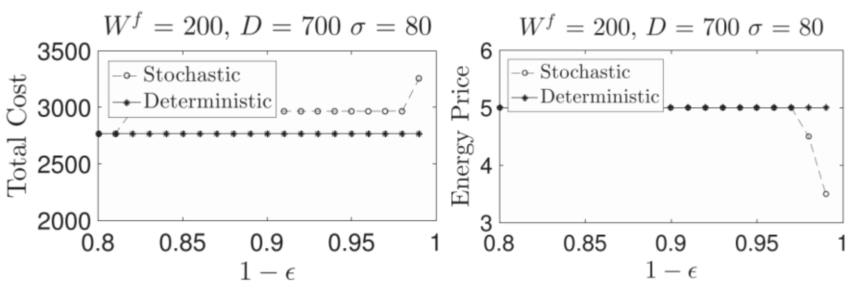
- Market equilibrium must satisfy two conditions:

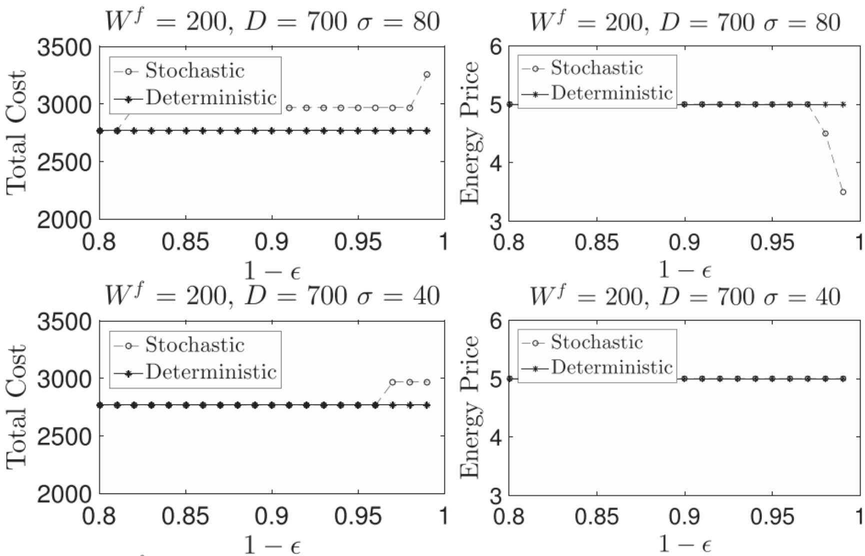
$$\sum_k p_k + W^f = D,$$

$$\sum_k \alpha_k = 1$$

Theorem 1: Let $\{p_k^*, \alpha_k^*, z_k^*\}$ for all k be an optimal solution of (CCUCP) (or equivalently, of (CCUCP$_{\mathrm{IP}}$)), and let $\{\lambda_0^*, \beta_0^*, \{\omega_k^* \ \forall k\}\}$ for all k be an optimal solution of the dual LP of (CCUCP$_{\mathrm{LP}}(z^*)$). Then the prices $l_0 = \lambda_0^*, b_0 = \beta_0^*, w_k = \omega_k^*$ for all k, and the decisions $p_k' = p_k^*, \alpha_k' = \alpha_k^*$, and $z_k' = z_k^*$ for all k represent a robust competitive equilibrium.

- Our proof exploits LP duality (as in O'Neil, 2005)
- Still it works for a single-node case, transmission constraints need to be accounted for additionally
- See our proof in Kuang, 2018.





- The price formation process adequately reflects uncertainty (ϵ, σ)
- Externalities (ϵ, σ) can be related to power grid operations and have well-defined temporal and spatial interpretations (important for transmission-constrained extensions)
- Provides a high customization level for the assumptions on uncertainties, but does not increase computational complexity
- Has connections to the existing practice
  - One bid, no multiple bids for multiple scenarios
  - Easy interpretation + deterministic dc network constraints can be factored in straightforwardly

- **How to enforce power flow constraints?**
  - AC power flows (e.g. voltage + reactive power limits are accounted for)
  - Apparent power limits → no exact reformulation
  - Voltage limits → reformulated into linear deterministic constraints
- **A few modeling choices:**
  - Power flow linearization around an given operating point (an feasible AC power flow solution exists)
  - Affine response policies
  - Zero-mean, Gaussian uncertainty
  - Single-period optimization

Power Flow Linearization

- **AC power flow equations can be linearized or relaxed**

$$f^p_{ij}(v,\theta) = v_i v_j \left[G_{ij}\cos(\theta_i - \theta_j) + B_{ij}\sin(\theta_i - \theta_j)\right]$$
$$f^q_{ij}(v,\theta) = v_i v_j \left[G_{ij}\sin(\theta_i - \theta_j) - B_{ij}\cos(\theta_i - \theta_j)\right],$$

- **Linearization is based on the Taylor's approximation**
  - Can be solved sequentially to improve accuracy of the approximated solution
- **Even linearized AC power flow equations are difficult due to the quadratic dependency on uncertainty (ω)**

$$\mathbb{P}((f^p_{ij}(\omega))^2 + (f^q_{ij}(\omega))^2 \le (s^{max}_{ij})^2) \ge 1 - \epsilon_I$$

Active flow (f^p_{ij})

Reactive flow (f^q_{ij})

- **Inner approximation of the quadratic dependency (Lubin et al, 2018)**

$$\mathbb{P}((f_{ij}^p(\omega))^2 + (f_{ij}^q(\omega))^2 \leq (s_{ij}^{max})^2) \geq 1 - \epsilon_I$$

$$\Downarrow$$

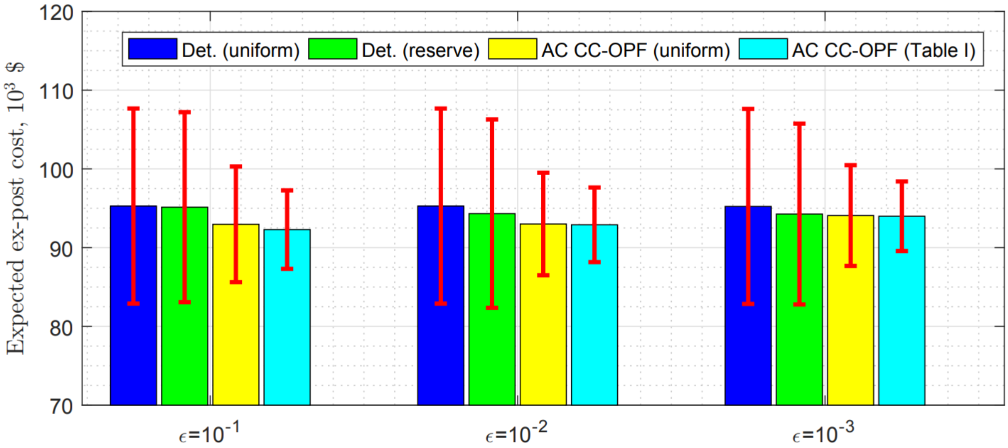
$$\mathbb{P}(|f_{ij}^p(\omega)| \leq t_{ij}^p) \geq 1 - \frac{\epsilon_I}{2}, \forall ij \in \mathcal{L}$$
$$\mathbb{P}(|f_{ij}^q(\omega)| \leq t_{ij}^q) \geq 1 - \frac{\epsilon_I}{2}, \forall ij \in \mathcal{L}$$
$$(t_{ij}^p)^2 + (t_{ij}^q)^2 \leq (s_{ij}^{max})^2, \forall ij \in \mathcal{L},$$

$$\Downarrow$$

Approximate absolute values with:

$$-t_{ij}^* - f_{ij}^*(0) \leq \Phi^{-1}(\frac{\epsilon_I}{2.5})\,\mathrm{Stdev}[f_{ij}^*(\omega)]$$

- **Inner approximation of the quadratic dependency (Lubin et al, 2018) works quite well**



- **However, the resulting problem is not an LP anymore due to the approximation:**

$$(t^p_{ij})^2 + (t^q_{ij})^2 \leq (s^{max}_{ij})^2, \forall ij \in \mathcal{L},$$

- **However, the resulting problem is not an LP anymore due to the approximation:**

$$(t_{ij}^{p})^2 + (t_{ij}^{q})^2 \leq (s_{ij}^{max})^2, \forall ij \in \mathcal{L},$$

- **However the program is still convex and the convex duality can be used in this case**
- **The same contract design can be used**
- **New proof is work in progress**

Summary

- **Chance constraints offer a great deal of modeling flexibility at an acceptable computational cost**
- **Can be used for pricing under uncertainty**
  - At least, for the single-node case or for the transmission-constrained case with DC assumptions or with deterministic power flow limit
  - Explicit consideration of uncertainty & risk tolerance on the price formation process
- **Can be built on existing practices**
- **More info:**
  - M. Lubin, Y. Dvorkin, and L. Roald, "Chance Constraints for Improving the Security of AC Optimal Power Flow," under review, 2018. Available at: https://arxiv.org/abs/1803.08754
  - X. Kuang, Y. Dvorkin, A. J. Lamadrid, M. Ortega-Vazquez, and L. Zuluaga, "Pricing Chance Constraints in Electricity Markets," IEEE Transactions on Power Systems, early access, 2018.


THANK
YOU
GRACIAS
ARIGATO
SHUKURIA
DANKSCHEEN
TASHAKKUR ATU
YAQHANYELAY
SUKSAMA
EKHMET
GRAZIE
MEHRBANI
BOLZÏN
MERCI
BÏYAN
SHUKRIA
TINGKI
GOZAIMASHITA
EFCHARISTO
JUSPAXAR
KOMAPSUMNIDA
PALDIES
MAAKE

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[2] L. Roald, S. Misra, M. Chertkov, G. Andersson, “Optimal power flow with weighted chance constraints and general policies for generation control”, 2015 IEEE 54th Annual Conference on Decision and Control (CDC), Osaka, Japan, 2015, pp. 6927-6933.

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[8] R. P. O’Neill et al., “Efficient market-clearing prices in markets with nonconvexities,” Eur. J. Oper. Res., vol. 164, no. 1, pp. 269–285, 2005

[9] X. Kuang, Y. Dvorkin, A. J. Lamadrid, M. Ortega-Vazquez, and L. Zuluaga, “Pricing Chance Constraints in Electricity Markets,” IEEE Transactions on Power Systems, early access, 2018.