



Multiport Element Models and Sparse Tableau Network Representation for Security Constrained Optimal Power Flow

Professor Chris DeMarco

Department of Electrical & Computer Engineering

University of Wisconsin-Madison

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Acknowledgement

- *Today's tutorial includes a Sparse Tableau formulation of the Optimal Power Flow, based on work in collaboration with University of Wisconsin-Madison colleague Professor **Michael Ferris**, and PhD candidates **Byungkwon Park** and **Jayanth Netha**.*
- *These formulations is being employed in the construction of large-scale synthetic grid models for OPF, as part of the EPIGRIDS project under the ARPA-E GRID DATA program. This work is supported by the Advanced Research Projects Agency-Energy (ARPA-E), U.S. Department of Energy, under Award Number DEAR0000717.*
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Synopsis

Three related topics, each with a “revisionist” twist.

- **Component modeling:** *Use multi-ports as ideal circuit elements in model, maintaining element port voltages and currents as explicit variables in OPF.*
- **Network representation:** *Abandon “Ybus”; i.e. ditch bus-branch model. Advocate Sparse Tableau Analysis (STA) for network constraints with node-breaker detail.*
- **Coordinate choice:** *With complex phasor bus voltages and powers, one obviously has choice of polar or rectangular coordinates. STA approach extends this coordinate frame choice to a more complete set of bus **and component** currents, voltages, and powers.*

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Multi-port Component Modeling

- *Claim: texts often “handicap” the power flow model development in choice of admissible ideal elements. Case in point: Overhead three-phase transmission lines (below: 69kV line at Madison, WI Blount St power plant)*





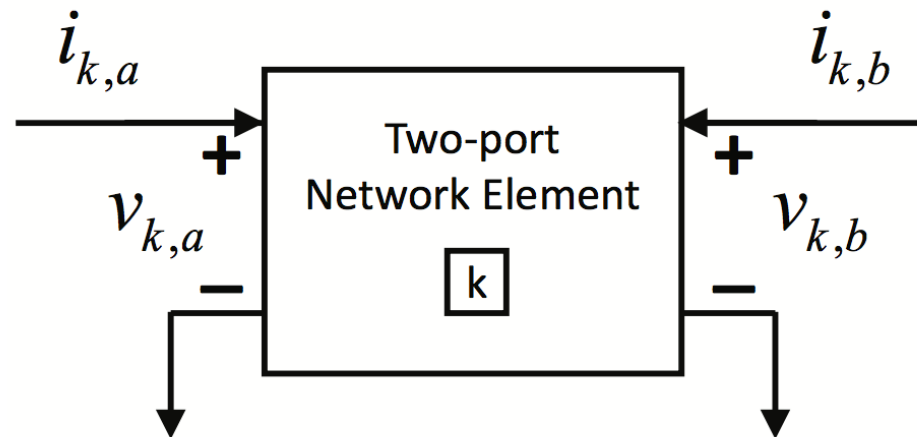
Multi-port Component Modeling

- *Transmission lines are perhaps the most ubiquitous power system component, and the very definition of distributed-parameter.*
 - *Consider typical textbook's first analysis steps:*
 - (i) Begin from pde's describing distributed behavior.*
 - (ii) Impose assumptions of balanced three phase operation, in sinusoidal steady state (SSS).*
 - (iii) Focus on relation between "sending end" and "receiving end" voltage-current pairs.*
- (BTW – these first steps are perfectly ok, when assumptions hold appropriately)*



Transmission line as a two-port

- Assumption (ii) provides per-phase algebraic relations;
(iii) dictates structure of relation is **naturally** a two-port.

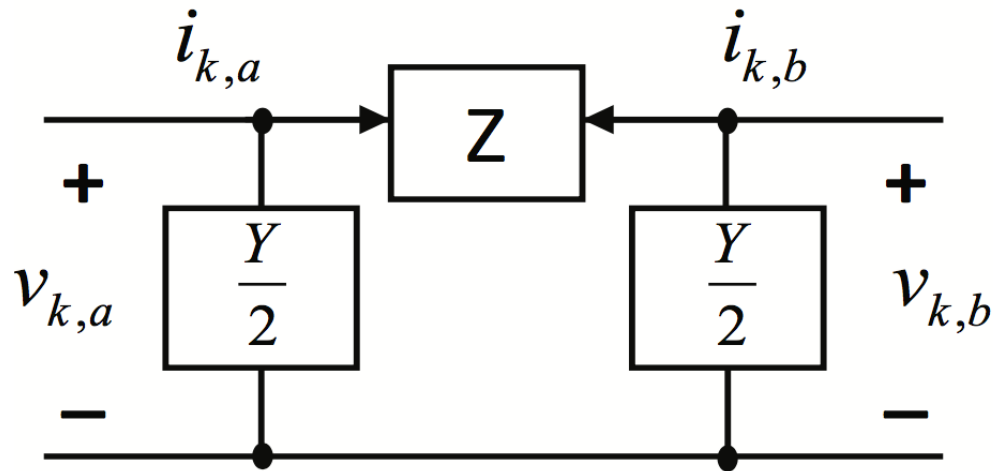


- The handicap (IMO) standard power systems formulation occurs in next step: creation of an equivalent circuit for this two-port, constructed of **strictly two-terminal admittances** (instead of keeping the two-port model)



Shortcoming of Pi-equivalent for Transmission Line Two-port

- *Characteristics of overhead line yield Y and Z below, that match behavior of distributed model in SSS, at terminals.*



- *Standard practice line specifies data for OPF via Y and Z . Shortcoming: Otherwise “reasonable-looking” (Y, Z) can fail to be realizable from physical parameters of kmil conductor diameter, permeability, inter-phase conductor distance.*



Ideal Transformer as a Two-Port

- An ideal transformer the poster-child for two-port analysis. For transformer having transformation gain “k,” the two constitutive relations among the port variables are simply*

$$v_b = kv_a, i_b = (1/k^*)i_a$$

(for phase shifting transformer, k may be complex)

- But standard practice in specifying power flow/OPF data disallows a “pure,” ideal transformer as a power systems element. Non-ideal series reactance **must** be included to represent the physical effect of leakage flux.*
- Why?*



Ideal Transformer as a Two-Port

- *Here the problem involves both the nature of the individual component's analysis, and the formulation of the overall network constraints.*
- *While details will follow, the insistence on Ybus analysis (strict nodal analysis) requires that constitutive relations for every component must have the property that the component's current(s) be expressible in terms of the component's voltage(s). From a two-port perspective, the component must permit an admittance representation.*
- *An ideal transformer does not have this property.*



General Two-Port Element Representation

- *General two-port is straightforward. In the a nonlinear case, with phasor quantities, imposes two complex constraints on the four complex port variables, i.e.*

$$f_k : \mathbb{C}^4 \longrightarrow \mathbb{C}^2$$
$$f_k(v_{k,a}, i_{k,a}, v_{k,b}, i_{k,b}) = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \quad (1)$$

- *For affine-linear case, most prevalent in power systems, the general two-port written as: (for strictly linear, $u_s=0$)*

$$F_v v + F_i i = u_s \quad (2)$$

- *Ybus-based OPF formulations restrict to linear elements, **with restriction that F_i must be invertible.***



The Role of Circuit Breakers

- *Circuit breakers (i.e., switches) are also ubiquitous throughout the power grid. In recent years, considerable attention in OPF literature on treatment of line switching.*
- *In line switching, position of circuit breakers on select lines allowed to be integer decision variables.*
- *Easily accommodated in Ybus/admittance formulation: one simply sets admittances of a line's pi-equivalent to zero when that line's breaker is open.*
- *But circuit breakers have other important roles in reconfiguring substations in event of a contingency.*



The Role of Circuit Breakers

- *Circuit breakers “sectionalize” buses in a substation.*





The Role of Circuit Breakers

- *Breaker Closed: two sections of bus held to equal voltage, function as a single node in the idealized circuit.*
- *Breaker Open: zero current flows through breaker, two sections of bus function as two independent nodes (and will appear as two separate buses in a bus-branch Y_{bus} model).*



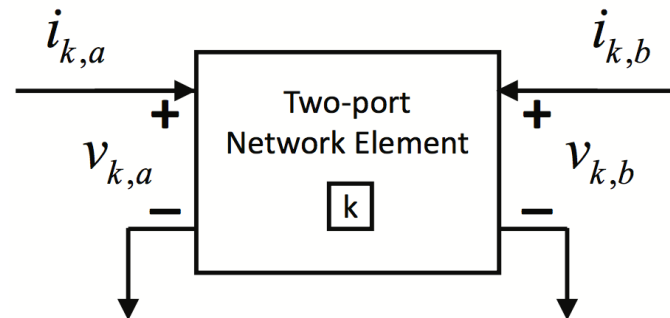
The Role of Circuit Breakers

- *Standard power flow/OPF models, based on strict nodal analyses, use **ONLY** node voltages as fundamental circuit variables. Hence, they change dimension of model between the two breaker positions.*
- *In power systems parlance, a “topology processing” algorithm rebuilds a distinct Ybus admittance matrix for each configuration.*
- *Editorial comment: IMO, this is dumb. Opening or closing breaker **does not change network topology** – it changes the voltage/current behavior of **one element!***



Circuit Breaker as ANOTHER Natural Two-Port

- (i) breaker position indicated by binary variable γ ;*
- (ii) maintain port voltage/current pairs as explicit variables;*
- (iii) as previously described, don't insist on F_i invertible.*



$$\text{Circuit breaker closed, } \gamma = 1 : \begin{cases} v_a - v_b = 0 \\ i_a + i_b = 0 \end{cases}$$

$$\Rightarrow \underbrace{\begin{bmatrix} 1 & -1 \\ 0 & 0 \end{bmatrix}}_{\mathbf{F}_v} \begin{bmatrix} v_a \\ v_b \end{bmatrix} + \underbrace{\begin{bmatrix} 0 & 0 \\ 1 & 1 \end{bmatrix}}_{\mathbf{F}_i} \begin{bmatrix} i_a \\ i_b \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$



Circuit Breaker as ANOTHER Natural Two-Port

$$\text{Circuit breaker open, } \gamma = 0 : \begin{cases} i_a = 0 \\ i_b = 0 \end{cases}$$
$$\Rightarrow \underbrace{\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}}_{\mathbf{F}_v} \begin{bmatrix} v_a \\ v_b \end{bmatrix} + \underbrace{\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}}_{\mathbf{F}_i} \begin{bmatrix} i_a \\ i_b \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

and as single description, in terms of γ :

$$\underbrace{\begin{bmatrix} \gamma & -\gamma \\ 0 & 0 \end{bmatrix}}_{\mathbf{F}_v} \begin{bmatrix} v_a \\ v_b \end{bmatrix} + \underbrace{\begin{bmatrix} (1-\gamma) & 0 \\ \gamma & 1 \end{bmatrix}}_{\mathbf{F}_i} \begin{bmatrix} i_a \\ i_b \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$



Network Interconnection Constraints: KCL and KVL

- *Observe that thus far we have described only the constitutive relations for a set of ideal elements. **These pertain to the elements themselves, independent of interconnection topology.***
- *When elements are interconnected in a network, linear KCL and KVL constrain those elements' port currents and voltages, and relate them to nodal quantities:*
 - (i) node voltages, V (in the grid, busbar voltages);*
 - (ii) currents externally injected at nodes, I ;*
(these represent externally injected currents, supplied by generation, or withdrawn by load. Descriptions of I behavior to follow)



Network Interconnection Constraints: KCL and KVL

- *Familiar mechanism to express KCL and KVL in compact form is that of node-to-element incidence matrix, here denoted A .*
- *Combining KCL, KVL, and linear constitutive relations, Sparse Tableau formulation is extraordinarily simple:*

$$Ai = I$$

$$v = A^T V$$

$$F_v v + F_i i = 0$$

- *If generation and load behaved as constant current sources/sinks, with fixed I , we'd be done now.*



Network Interconnection Constraints: KCL and KVL

- *The strict nodal analysis of Ybus is easily recovered as a special case reduction of the Sparse Tableau.*
- *One simply eliminates the “intermediate variables” of elements’ port currents and voltages, i and v , to obtain the relation between externally injected currents and bus voltages as:*

$$I = \underbrace{-A \cdot (F_i)^{-1} \cdot F_v \cdot A^T}_{Y_{bus}} V$$



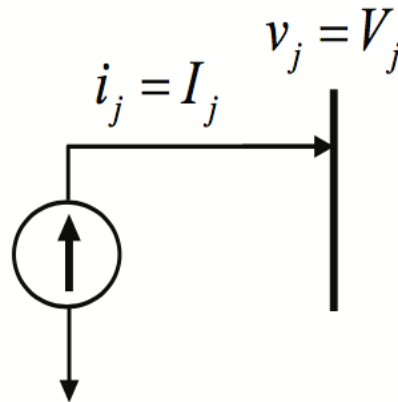
Generation and Load Behavior

- *One may treat generation and load as general nonlinear one-ports, connecting each node/bus to ground. Here we choose to restrict to a special case, treating them as nonlinear voltage controlled current sources, setting I .*
- *Physical equipment that dominates power production and consumption today is largely inductive in nature (e.g., coupled windings of a synchronous generator or induction motor). Therefore, voltage controlled current source is natural.*
- *Caveat: As voltage source inverters play growing role interfacing future generation to grid, we'll want to discard this voltage-controlled current source restriction.*



Generation and Load Behavior

- In OPF, most common nonlinear behavior associated with generation or loads is that of constant complex powers, denoted S , either as fixed parameters, or as decision variables to be solved via optimization.*



Here $I_j - \frac{S_j^*}{V_j^*} = 0$ or power balance form, $S_j = V_j I_j^*$



Real-Valued Coordinate Choices for Phasor Based Models

- *Phasor-based analysis offers inherent flexibility in choice of real-value coordinate representation of complex quantities. Most naturally: polar versus rectangular.*
- *Ybus/Nodal analysis formulations use only bus voltages as key network electrical variables. Hence in OPF literature to date, pros/cons of polar versus rectangular coordinate choice has tended to focus on bus voltages.*
- *However, because it maintains “intermediate” port currents and voltages, Sparse Tableau offers this polar versus rectangular choice over larger set of variables.*



Real-Valued Coordinate Choices for Phasor Based Models

- *Choice of coordinate system can have significant impact on geometry of the feasible region for the OPF, sometimes significantly impacting performance of optimization algorithms.*
- *At risk of preaching to the choir here... better understanding of geometry of feasible region for the OPF, and the interplay between this geometry and modeling/coordinate system choices, is (IMO) a critical area for research.*



Experience with Sparse Tableau Formulation OPF

*Sparse Tableau offers very simple (dare I say elegant?)
formulation of OPF, as summarized below:*

$$\min_{P, Q, v, i, V, I} \sum_{j \in \mathbf{G}} \tilde{c}_j (P_{g,j}) \quad \text{subject to}$$

$$\text{Linear Element: } F_v v + F_i i = 0$$

$$\text{KCL: } I - Ai = 0$$

$$\text{KVL: } v - A^T V = 0$$

$$\text{Nonlinear Element: } S - V \odot (I)^* = 0$$

$$\begin{aligned} \text{Gen. Limit: } P_j^{min} &\leq P_{g,j} \leq P_j^{max} \\ Q_j^{min} &\leq Q_{g,j} \leq Q_j^{max}, \forall j \in \mathbf{G} \end{aligned}$$

$$\text{Vol. Limit: } V_j^{min} \leq |V_j| \leq V_j^{max}, \forall j \in \mathbf{N}$$

$$\text{Line Limit: } |i_{k,a/b}|^2 \leq i_k^{max}, \forall k \in \mathbf{L}$$



Computational Experience with Sparse Tableau Formulation OPF

- *Experiments comparing Sparse Tableau to traditional Ybus OPF formulations are very preliminary, and to date have been performed only in the GAMS general purpose optimization environment, primarily with KNITRO solver.*
- *In several test systems from the MATPOWER distribution, up to several thousand buses, experience so far shows Sparse Tableau very comparable in speed.*



Computational Experience with Sparse Tableau Formulation OPF

	POLAR- Y_{bus}		REC-IV- Y_{bus}		REC-STF	
	Obj	Time	Obj	Time	Obj	Time
case118	129660.68	0.3sec	129660.68	0.3sec	129660.68	0.3sec
case300	719725.07	0.6sec	719725.07	2sec	719725.07	0.6sec
case2383wp	1868511.82	5.6sec	1862367.02	5.2sec	1862367.02	6.3sec
case3012wp	2591706.57	5.8sec	2582670.47	5.9sec	2582670.47	6sec
case3120sp	2142703.76	5.8sec	2141532.10	6.3sec	2141532.10	6.5sec
case3375wp	7412030.67	54sec	7404635.99	11.7sec	7404637.15	11.4sec



Take Away Points

- *Many parts of power grid transmission network are fundamentally simple circuits, often linear.*
- *Many of the historic “tricks”/reductions in power grid model formulations are arguably becoming less advantageous, because of advances in computational tools, and because new component technologies undermine assumptions needed for these shortcuts.*
- *Sparse Tableau formulation facilitates model construction that is versatile for representing node-breaker detail, allowing model to easily capture substation reconfiguration in contingencies, and (in first experiments) just as fast as traditional Ybus.*