TOWARDS A PRICE-BASED DISPATCH FORMULATION IN RTO

How can a nodal-branch price equality constraint help?

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Content

- Background and Inspirations
- Development of New Formulation
- Examples
- Applications
- Further Research

Fundamentals of Markets Optimization Software

Objective
- Surplus
- Payment
- Costs

Controls
- Resources
- Load
- [Prices]

Constraints
- Power flow
- Transmission
- Resources
- [Prices]

Data & Parameter
- Network
- Contingency
- Rules
- FERC questions in this Docket

Solution
- LR
- MIP
- SQP
- Others

Applications
- UC
- Dispatch
- Pricing
- Operator vs. self schedule
- Capacity market
- CRR

Background and Inspirations

Search for Better Software?

- Convex hull and payment minimization
- Alternatives (APPA, EEI, Litvinov, etc.)
- This presentation is about a new formulation
  - Not a solution method
  - New constraint (nodal-branch price equality constraint) and decision variables
  - Discussions of new ways of applying the formulation
  - Inspired by dual solution and ELMP, with a transmission network

Background and Inspiration

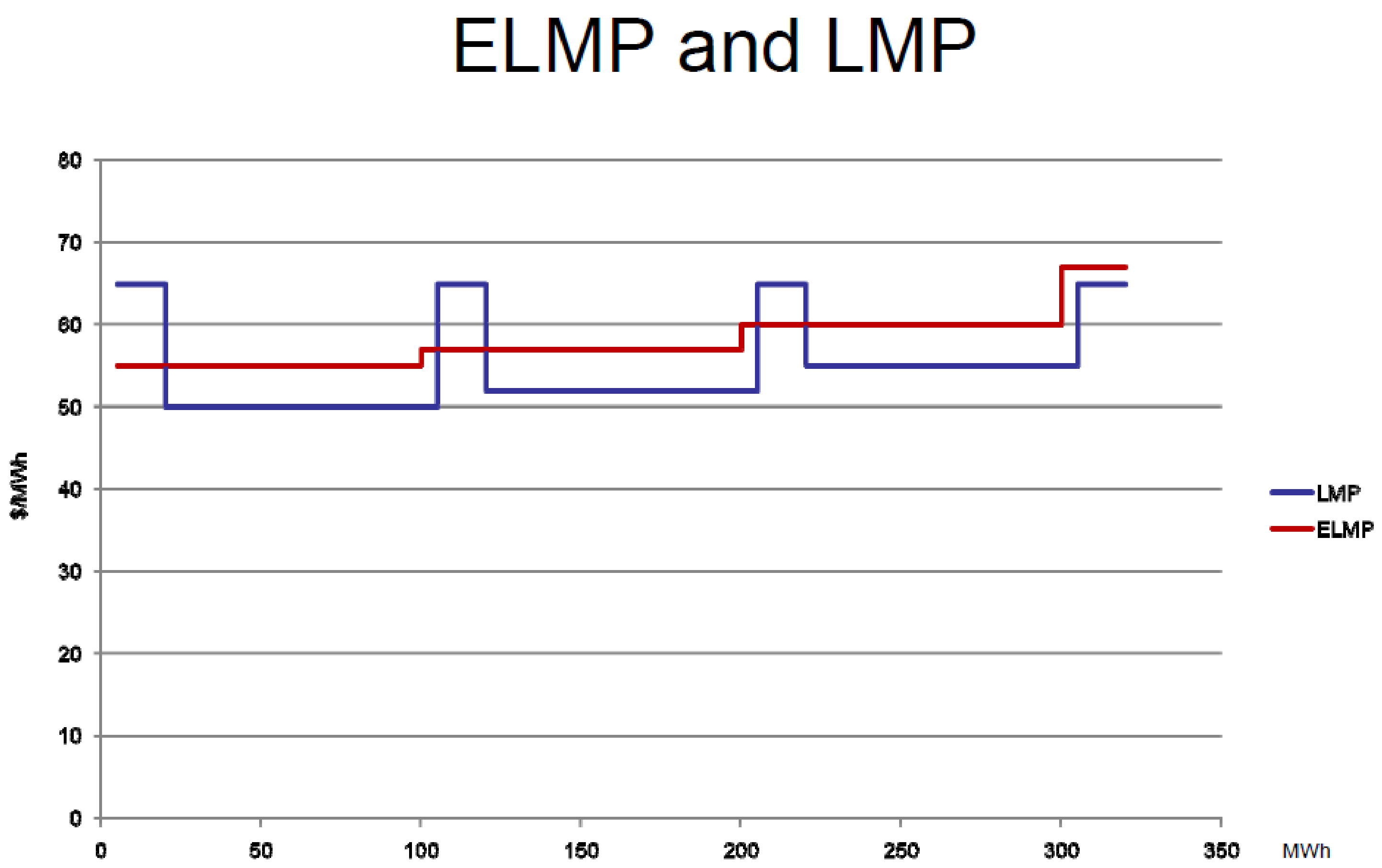
Background

- FERC's reports and related proceedings: importance of price formation
- Pricing has been based a principle of "marginal cost of load": many issues under non-convexities
- Alternatives: Non-intuitive pricing

Inspirations

- Payment minimization and self-commitment
- Dual solution, but with locational aspects of LMP
- Previous studies

ELMP* (Convex Hull) and LMP



*Paul Gribik et al. of MISO

Problem Overview

Assuming a DC OPF problem where transmission network model is considered

New Formulation

Max f(x, price)

g(x, price) ≤0

h(x, price) =0

New constraints and explicit price as decision variables

Traditional Formulation

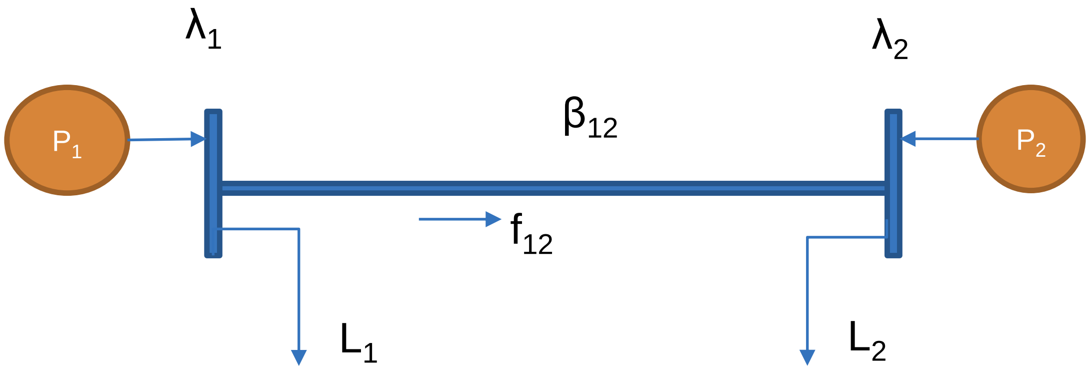
Max f(x)

g(x) ≤0

h(x) =0

Prices are by-products of solution

Illustrative Example



Variable	λ_1 λ_2	β_{12}	P_1, P_2	L_1, L_2	f_{12}
Definition	LMP at 1 and 2	Branch congestion price	Injection at 1 and 2	Load at 1 and 2	Flow from 1 to 2

Find λ_1, λ_2 , β_{12}, P_1, P_2, L_1 and L_2 to optimize an objective function and satisfy a sets of bidding rules/system requirements

However, λ_1, λ_2 and β_{12} are interdependent and are also dependent upon the flow f_{12}.

Development of a Price Based Formulation

Approach Outline

- Review optimality conditions to a traditional dispatch problem
- Analyze the optimality conditions
- Derive a special relation between node and branch price
- Make a new formulation with newly derived relation between node and branch prices by adding it as new constraint and allowing price to be decision variables
- Consider pricing rules and implementation issues

Development of Nodal-branch Price Equality Constraint

DC OPF Solution

Consider a dispatch optimization problem with a DC power flow network,

$$min\ f(x)$$

s.t.

$B\theta - G(x) = 0$	DC power flow vector
$T(\theta) \leq 0$	Branch flow constraint
$R(x,\theta) \leq 0$	Other inequality constraints
$H(x,\theta) = 0$	Other equality constraints

where

x: Control variable

B is the admittance matrix and $B=B^t$

G(x) is the power injection vector

T(θ) is the inequality constraint vector for line flow,

R(x) is the rest inequality constraint vector,

H(x) is the rest equality constraint vector, and

θ is the bus angle vector.

Development of Nodal-branch Price Equality Constraint

Optimality Conditions

Let

$$L(x, \theta, \lambda, \nu) = f(x) + \lambda^T (B\,\theta - G(x)) + \beta^T T\ \ (\theta) + \nu^T R(x) + \phi^T H(x)$$

Where $\beta \geq 0,\ \ \nu \geq 0.$

In RTO markets:

λ = LMP

β = Branch shadow price

Karush Kuhn Tucker (KKT) optimality necessary conditions:

$$\frac{\partial L}{\partial x} = \frac{\partial f}{\partial x} - \frac{\partial G}{\partial x}\lambda + \frac{\partial R}{\partial x}\upsilon + \frac{\partial H}{\partial x}\phi^T = 0$$

$$\frac{\partial L}{\partial \theta} = B^t \lambda + \frac{\partial T}{\partial \theta}\beta = 0$$

$$B\theta - G(x) = 0$$
$$T(\theta) \leq 0$$
$$R(x) \leq 0$$
$$H(x) = 0$$

In general, the active set of binding constraints is updated to manage complementary slackness condition that is in the form of

$$\beta^T T = 0$$

Development of Nodal-branch Price Equality Constraint

Analysis of the Relation of Branch Shadow Price and LMP

Incident Matrices:

A is incidence matrix for a give network topology (direction is arbitrary),

Based on circuit analysis, we can rewrite

$$B = A^t DA$$

$$T = DA\theta - T_{max}$$

Where D is the diagonal matrix of the line admittance for the monitored branches.

As a result, the highlighted equation from the last slide becomes

$$B^t\lambda + \frac{\partial G}{\partial \theta}\beta = B^t\lambda + A^t D\beta = 0$$

Development of Nodal-branch Price Equality Constraint

Revenue Sufficiency

Take inner product of λ and G(x),

$\lambda^t G(x) = \lambda^t B\theta = \theta^t B^t \lambda = -\theta^t A^t D\beta = flow^t \beta$

Namely, the net payment from all injections are the same of total congestion revenues.

No congestion

$\beta = 0, B^t \lambda = 0$

Rank(B) = N-1, $\lambda_1 = \lambda_2 = \lambda_3 \ldots\ldots = \lambda$

N is the number of node.

Development of Nodal-branch Price Equality Constraint

Price Based Formulation

Consider a dispatch optimization problem of DC power flow

$$Min\ f(x, \lambda, \beta)$$

s.t.

$B\theta - G(x, \lambda) = 0$	DC power flow Vector
$T(\theta) \leq 0$	Branch flow constraint
$R(x, \theta) \leq 0$	Other inequality constraints
$H(x, \theta) = 0$	Other equality constraints
$B^t \lambda + A^t D\beta = 0$	Branch-node price Constraint
$\beta^T T = 0$	Complementary slackness

$\lambda \geq$ Awarded Offer

$\lambda \leq$ Awarded Bid

With the Branch-Node Price constraint being added, the new formulation allows use of LMP and branch prices explicitly and directly.

Example

2 bus

2 Bus Example

Let b_{12} is the line admittance between 1 and 2, T_{12max} is the limit flow between 1 and 2,

$$L(x,\theta,\lambda,v) = f(x) + \lambda^t(B\,\theta - G(x)) + \beta^T T(\theta) + v^T R(x) + \phi^T H(x)$$

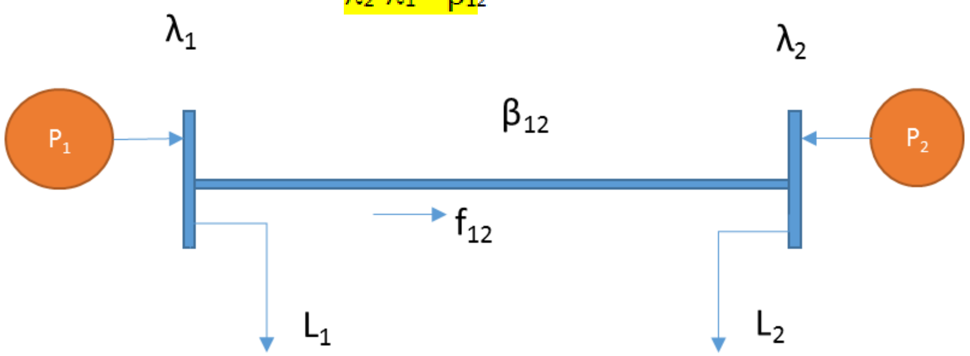
$$L(x,\theta,\lambda,v) = f(x) + \lambda_1(G_1 - b_{12}(\theta_2-\theta_1)) + \lambda_2(G_2 - b_{12}(\theta_1-\theta_2)) - \beta_{12}(T_{12max} - b_{12}(\theta_1-\theta_2)) + v^T R(x) + \phi^T H(x)$$

Set slack bus $\theta_1 = 0$

$$L(x,\theta,\lambda,v) = f(x) + \lambda_1(G_1 - b_{12}\theta_2) + \lambda_2(G_2 + b_{12}\theta_2) - \beta_{12}(T_{12max} + b_{12}\theta_2) + v^T R(x) + \phi^T H(x)$$

$$\frac{\partial L}{\partial \theta_2} = -\lambda_1 b_{12} + \lambda_2 b_{12} - \beta_{12} b_{12} = 0$$

$\lambda_2 - \lambda_1 = \beta_{12}$



$G_i = P_i - L_i$,

$\lambda_2 - \lambda_1 = \beta_{12}$

LMP difference =Branch congestion price

Example

3 bus

3 Bus Example

Set slack bus θ_1=0, and assuming one flow constraint between 1 and 2,

$$L(x, \theta, \lambda, \nu) = f(x) + \lambda_1(G_1 - b_{12}\theta_2 - b_{13}\theta_3) + \lambda_2(G_2 + b_{12}\theta_2 - b_{23}(\theta_3 - \theta_2)) + \lambda_3(G_3 + b_{13}\theta_3 - b_{23}(\theta_2 - \theta_3)) - \beta_{12}(T_{12max} + b_{12}\theta_2) + \nu^T R(x) + \phi^T H(x)$$

$$\frac{\partial L}{\partial \theta_2} = -\lambda_1 b_{12} + \lambda_2 b_{12} + \lambda_2 b_{13} + \lambda_3 b_{23} - \beta_{12} b_{12} = 0$$

$$\frac{\partial L}{\partial \theta_3} = \lambda_1 b_{13} - \lambda_2 b_{23} + \lambda_2 b_{13} + \lambda_3 b_{13} - \lambda_3 b_{23} = 0$$

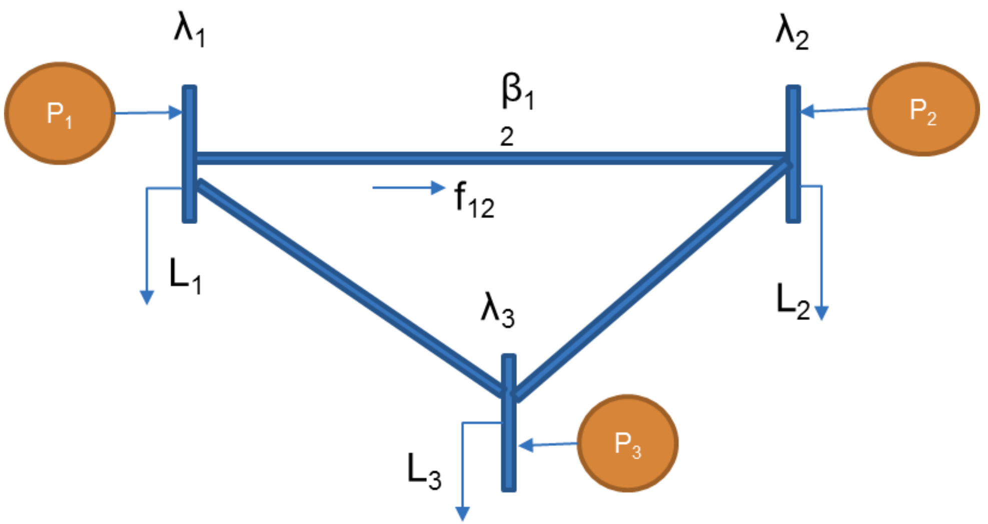
Assuming $b_{12} = b_{13} = b_{23}$

$$-\lambda_1 + \lambda_2 + \lambda_2 - \lambda_3 - \beta_{12} = 0$$

$$-\lambda_1 - \lambda_2 + \lambda_3 + \lambda_3 = 0$$

$$\lambda_2 = \lambda_1 + \frac{2}{3}\beta_{12}$$

$$\lambda_3 = \lambda_1 + \frac{1}{3}\beta_{12}$$



$G_i = P_i - L_i$,

Numerical Example

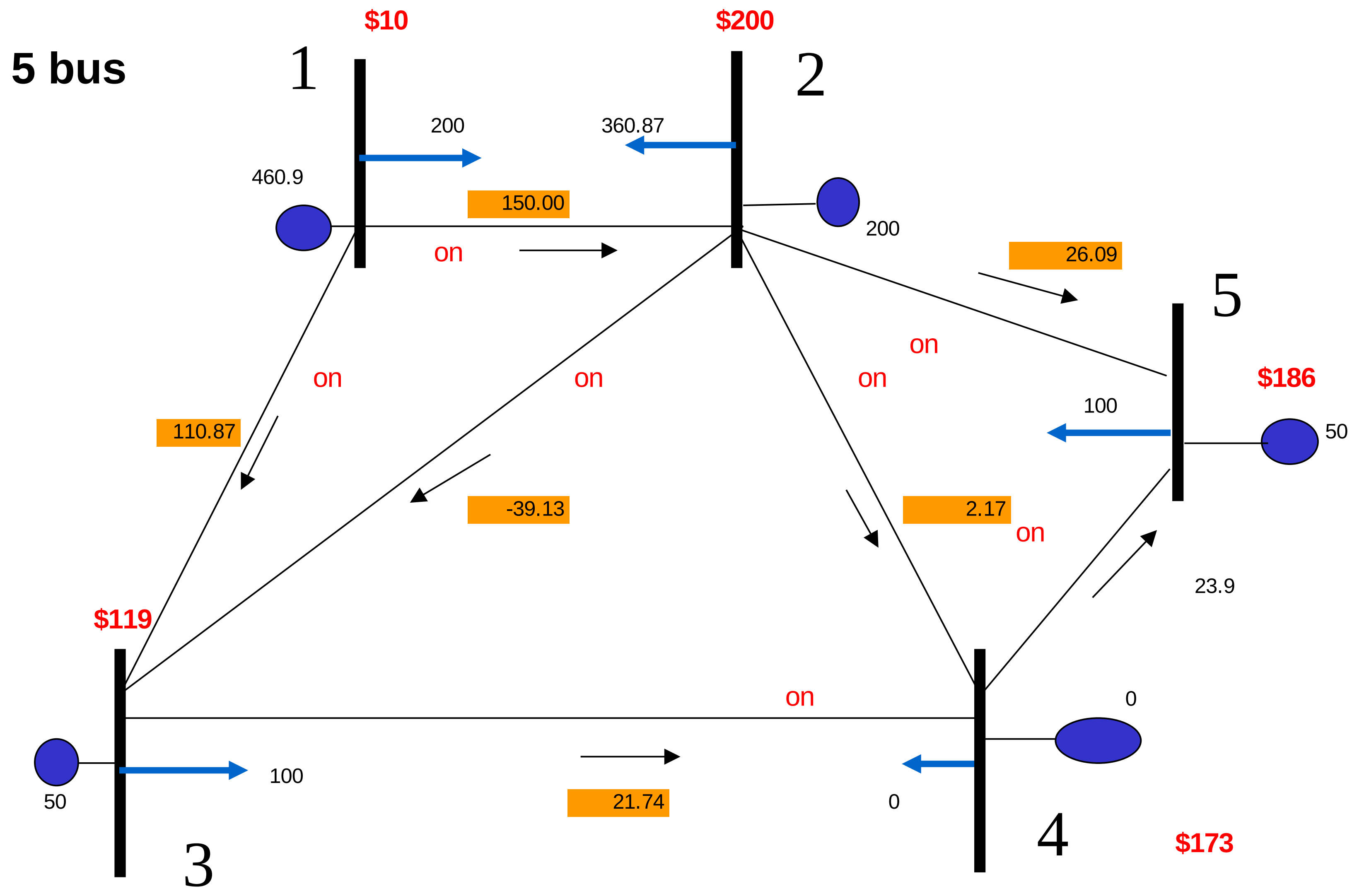
- **Excel Macro Solver**
- **5 buses**
- **8 resources:** flat bid segment
- **Objective:** Total surplus
- **Constraints:** DC Load flow

  Node and branch prices

  Bidding constraints

  Complementary Slackness
- **Control:** Load and gen schedules, λ vector (LMP), β (line shadow price)
- **Transmission**: 7 links of different line admittances, limited from 1 to 2

Example



5 bus
$10
1
$200
2
200
360.87
460.9
150.00
200
on
26.09
5
on
on
on
on
$186
100
50
110.87
-39.13
2.17
on
23.9
$119
on
0
100
50
21.74
0
4
$173
3

Example

5 bus

Resource	Bus	Type	Bid Price	Bid Qauntity	Cleared	LMP
1	**1**	G	10.00	750	460.87	10.00
2	**2**	G	20.00	200	200.00	200.00
3	**3**	G	50.00	50	50.00	118.66
4	**5**	G	40.00	50	50.00	186.02
5	**1**	L	200.00	200	200.00	10.00
6	**2**	L	200.00	400	360.87	200.00
7	**3**	L	200.00	100	100.00	118.66
8	**5**	L	200.00	100	100.00	186.02

Path constraint: 150 MW from 1 to 2

This example serves as validation of the proposed formulation. It gives for the same solution under convex condition.

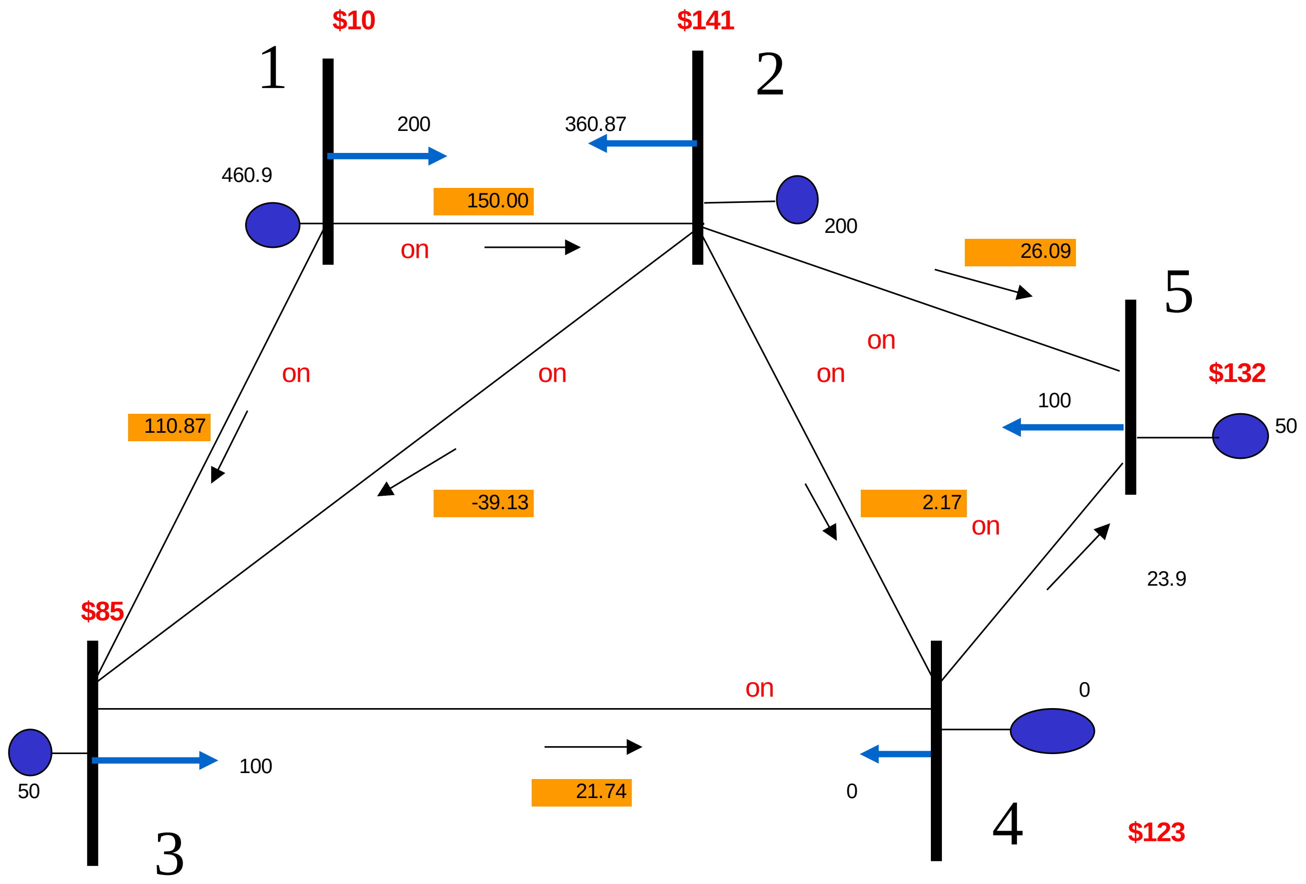
Payment Minimization

Objective
Load Payments

Add constraints
Gen surplus>0

Decision
Fix load

Solution
MW are the same
Prices are lower



This example serves as another validation. It gives the same scheduling solution, but lower price under payment minimization and fixed load.

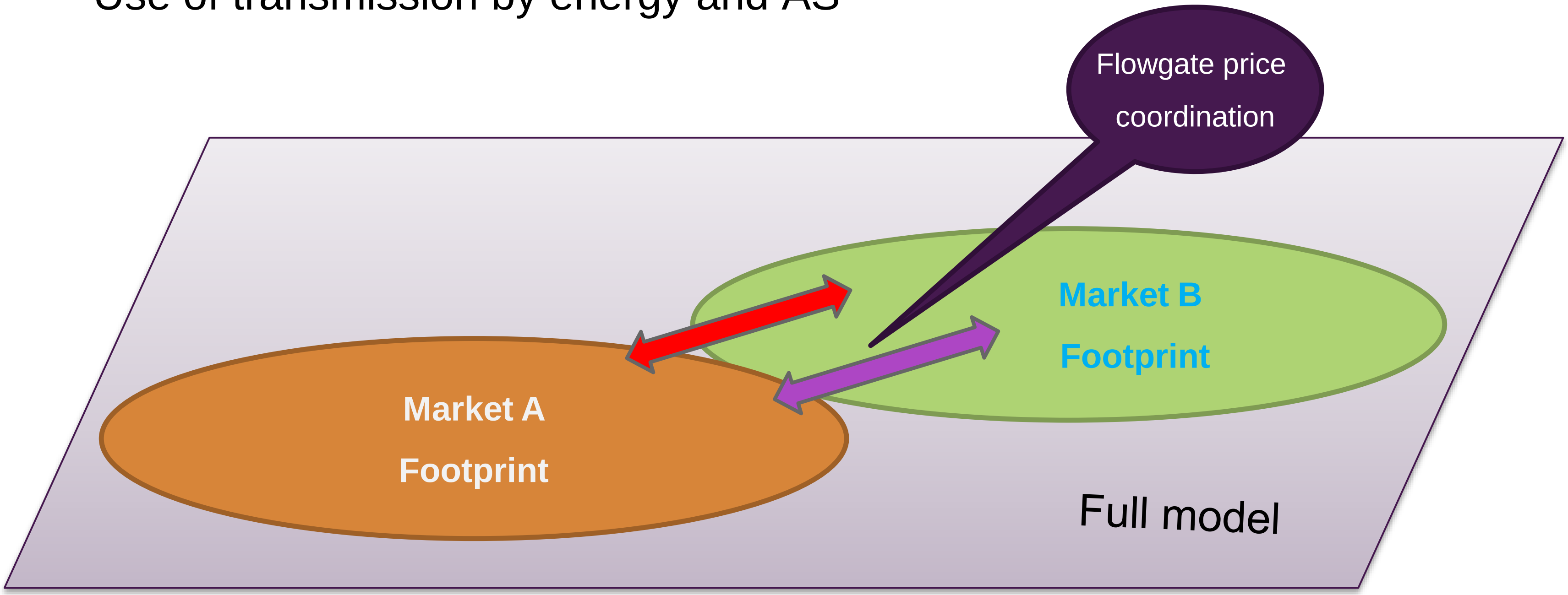
New Applications

Uplift Minimization/Elimination!

- Uplift is no-longer a post-solution product if price-based optimization is used!
- With prices as decision variables, supplier shortfall associated with 3-part bids can be expressed directly *a priori* for the scheduling intervals
  - Shortfall = output*energy bids + startup + no-load -output* LMP
- New pricing rules can be modeled in the optimization model
  - Limit shortfall to be smaller than a threshold value
  - Consider shortfall as part of the objective

New Applications

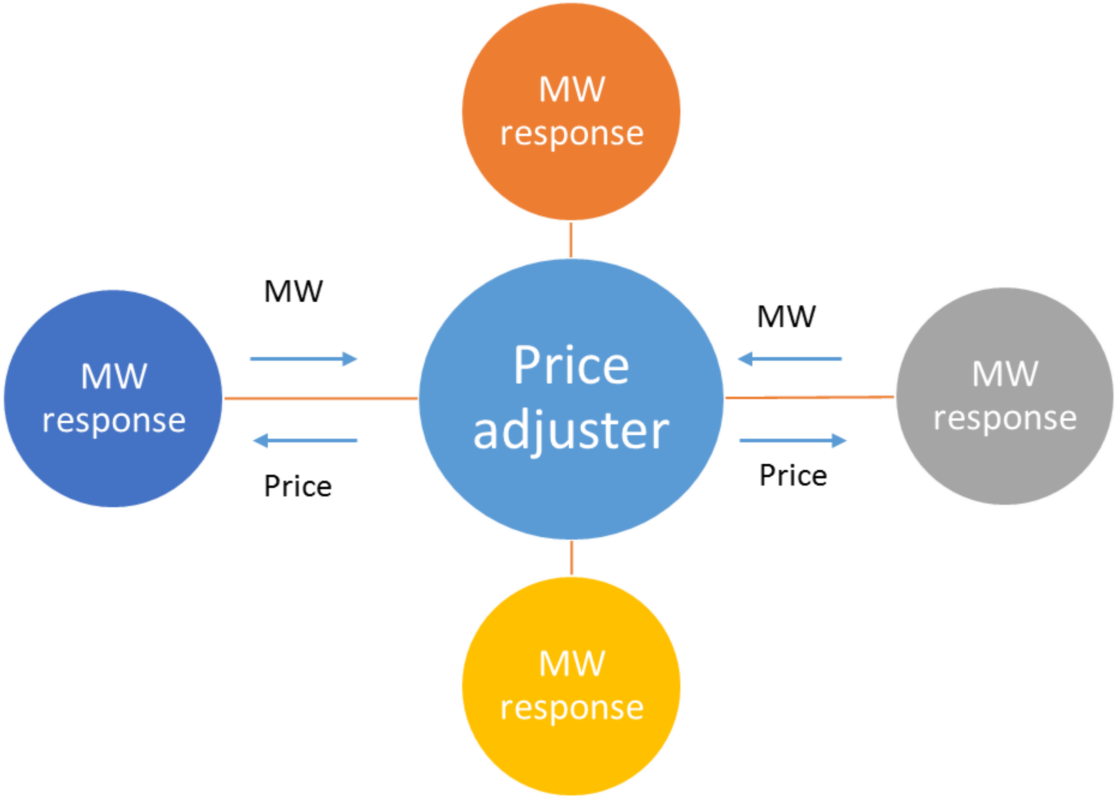
- Seams coordination between multiple markets
  - Flowgate prices are decision variables and contribution to flowgates by each market can be modeled
- Coordinated scheduling of multi-products
  - Use of transmission by energy and AS



New Applications

Market structure changes: Potentials for a price-based market

- Decentralized market operation such as self commitment or new computational sub-process
  - Operator adjusts prices (LMP/Shadow)
  - A market participant or a sub-process can optimize its own resources



New Applications

Simpler market process

- Possible to have one market engine without multiple/inconsistent modules or runs?
- Improve market efficiency?

Other applications are be discovered.

Key Message and Conclusion

- Price based dispatch with a transmission network is demonstrated in theory and in sample system
- Potentials of applying price-based formulation are to be discovered: new solutions may yield innovative and non-obvious applications.

Practical Issues and Further Research

Challenges:

- Local solutions
- Solutions for large-scale system
- Solution properties

More research efforts are needed:

- Three-part bids and commitments, temporal constraints
- Solution properties and market efficiency

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