Department of
Electrical & Computer Engineering
COLLEGE OF ENGINEERING | THE UNIVERSITY OF UTAH

# Towards Continuous-time Optimization Models for Power Systems Operation

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# Motivation and Background

- Power system operation optimization problem: stochastic, continuous-time, mixed-integer

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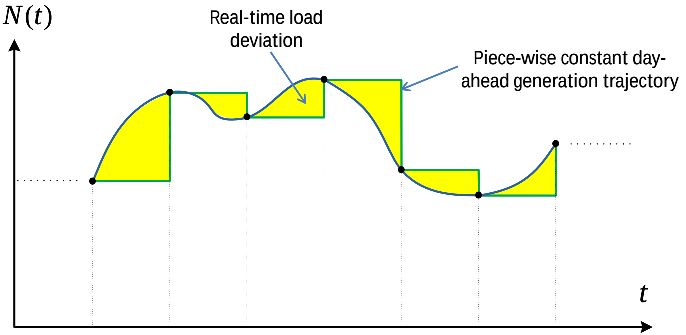
- Power system operation optimization problem: stochastic, continuous-time, mixed-integer
- Current practice: break down the problem into different time scales, from several days ahead to real-time operation, solving discrete-time optimization problems for each time scale.



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- If the real-time ramping requirement is beyond the available ramping capacity $\rightarrow$ ramping scarcity event

# Continuous-time Generation and Ramping Trajectories

- Instead of discrete-time schedules, assume that a set of $K$ generating units are modeled by:
  - Continuous-time generation trajectories: $\mathbf{G}(t)=(G_1(t),\ldots,G_K(t))^T$
  - Continuous-time commitment variables: $\mathbf{I}(t)=(I_1(t),\ldots,I_K(t))^T$

$$I_k(t) = \sum_{h=1}^{H_k} \left( u(t - t_{k,h}^{(\mathtt{SU})}) - u(t - t_{k,h}^{(\mathtt{SD})}) \right)$$

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- We define the **continuous-time ramping trajectory** of unit $k$ as the time derivative of its continuous-time generation trajectory:

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- Explicit definition of ramping trajectories allows us to define cost functions that are also functions of ramping trajectories:

$$C_k(G_k(t), \dot{G}_k(t), I_k(t))$$

# Continuous-time Unit Commitment Model

- Continuous-time Unit Commitment:

$$\min \quad \sum_{k=1}^{K} \int_{\mathcal{T}} C_k(G_k(t), \dot{G}_k(t), I_k(t))dt$$

$$\text{s.t.} \quad \sum_{k=1}^{K} G_k(t) = N(t) \qquad \forall t \in \mathcal{T}$$

$$\underline{G}_k I_k(t) \leq G_k(t) \leq \overline{G}_k I_k(t) \quad \forall k, t \in \mathcal{T}$$

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- The continuous-time UC model is a *constrained variational problem* with infinite dimensional decision space
  $\Rightarrow$ We need to *reduce dimensionality* of the problem. Idea:
  1. Subdivide $\mathcal{T}$ into $M$ intervals: $\mathcal{T}_m = [t_m, t_{m+1}), \mathcal{T} = \cup_{m=0}^{M-1} \mathcal{T}_m$.
  2. Map the parameters and decision variables in each interval into a finite-dimensional function space.

# Continuous-time Trajectories in a Function Space

- Assume that in $\mathcal{T}$, except for a small residual error, the continuous-time load trajectory $N(t)$ lies in a countable and finite function space of dimensionality $P$, spanned by a set of basis functions $\mathbf{e}(t) = (e_1(t), \ldots, e_P(t))^T$, that is:

$$N(t) = \sum_{p=1}^{P} N_p e_p(t) + \epsilon_N(t) = \mathbf{N}^T \mathbf{e}(t) + \epsilon_N(t)$$

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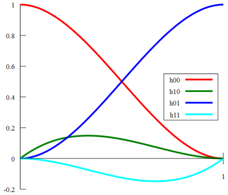
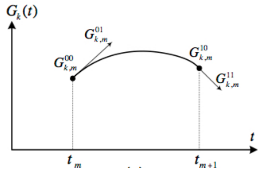
- To ensure the power balance in continuous-time, any generation trajectory should have a component that lies in the same subspace spanned by $\mathbf{e}(t)$ and one that is orthogonal to it, i.e.,:

$$G_k(t) = \sum_{p=1}^{P} G_{k,p} e_p(t) + \epsilon_{G_k}(t) = \mathbf{G}_k^T \mathbf{e}(t) + \epsilon_{G_k}(t).$$

# Spline Representation using Cubic Hermite Polynomials

- Cubic Hermite Polynomials: four polynomials in $t \in [0, 1)$, forming the vector: $\mathbf{H}(t) = (H_{00}(t), H_{01}(t), H_{10}(t), H_{11}(t))^T$



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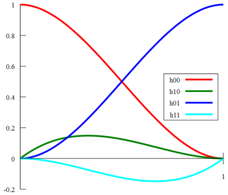
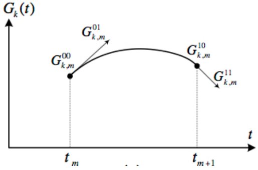



- Modeling the continuous-time load and generation trajectories in spline function space of cubic Hermite:

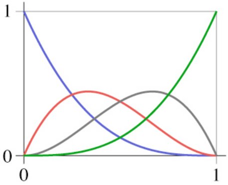
$$\hat{N}(t) = \sum_{m=0}^{M-1} \mathbf{H}^T(\tau_m)\mathbf{N}^H_m \quad , \quad G_k(t) = \sum_{m=0}^{M-1} \mathbf{H}^T(\tau_m)\mathbf{G}^H_{k,m}$$

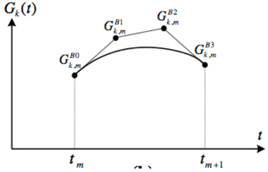
$\mathbf{N}^H_m$ and $\mathbf{G}^H_{k,m}$ are the vectors of Hermite coefficients.

# Spline Representation using Bernstein Polynomials

- Bernstein Polynomials of degree $Q$: $Q+1$ polynomials in $t \in [0,1)$, forming the vector $\mathbf{B}_Q(t) = (B_{0,Q}, ..., B_{q,Q}, ..., B_{Q,Q})^T$, where $B_{q,Q}(t) = \binom{q}{Q} t^q (1-t)^{Q-q}$
- Modeling the continuous-time load and generation trajectories in spline function space of Bernstein polynomials of degree 3:

$$\hat{N}(t) = \sum_{m=0}^{M-1} \mathbf{B}_3^T(\tau_m)\mathbf{N}_m^B \ , \quad G_k(t) = \sum_{m=0}^{M-1} \mathbf{B}_3^T(\tau_m)\mathbf{G}_{k,m}^B$$



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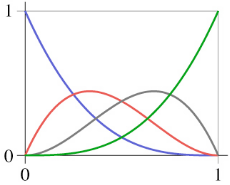


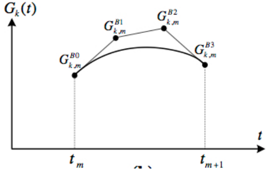


- The Bernstein and Hermite coefficients are linearly related:

$$\mathbf{G}_{k,m}^B = \mathbf{W}^T\mathbf{G}_{k,m}^H \ , \quad \mathbf{N}_{k,m}^B = \mathbf{W}^T\mathbf{N}_{k,m}^H$$

# Why Bernstein Polynomials?

- Bernstein coefficients of the derivative of generation trajectory are linearly related with the coefficients of the generation trajectory:

$$\dot{G}_k(t) = \sum_{m=0}^{M-1} \mathbf{B}_2^T(\tau_m)\dot{\mathbf{G}}_{k,m}^B \ , \quad \dot{\mathbf{G}}_{k,m}^B = \mathbf{K}^T\mathbf{G}_{k,m}^B = \mathbf{K}^T\mathbf{W}^T\mathbf{G}_{k,m}^H$$

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- **Convex hull property** of the Bernstein polynomials: trajectories are bounded by the convex hull formed by the four Bernstein points:

$$\min_{t_m \leq t \leq t_{m+1}} \{\mathbf{B}_3^T(\tau_m)\mathbf{G}_{k,m}^B\} \geq \min\{\mathbf{G}_{k,m}^B\}$$

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- Piecewise linear continuous-time cost function can be written in terms of the spline coefficients of generation and ramping trajectories:

$$\int_{\mathcal{T}} C_k(G_k(t), \dot{G}_k(t), I_k(t))dt = C_k(\mathbf{G}_k, \dot{\mathbf{G}}_k, \mathbf{I}_k).$$

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- Continuous-time power balance is ensured by balancing the four cubic Hermite coefficients of the continuous-time load and generation trajectory in each interval:

$$\sum_{k=1}^{K} G_k(t) = N(t) \quad \forall t \in \mathcal{T} \qquad \rightarrow \qquad \sum_{k=1}^{K} \mathbf{G}_{k,m}^{H} = \mathbf{N}_m^{H} \quad \forall m$$

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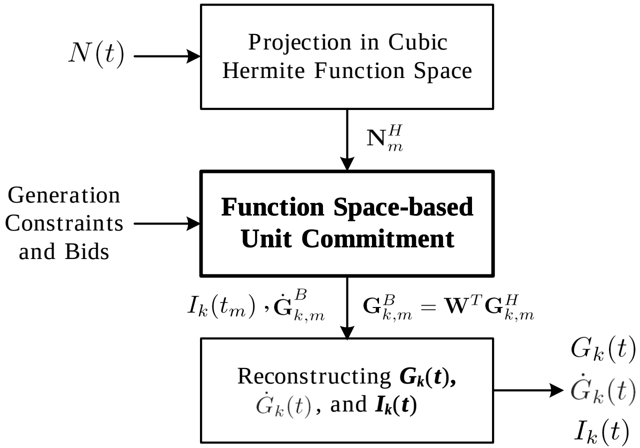
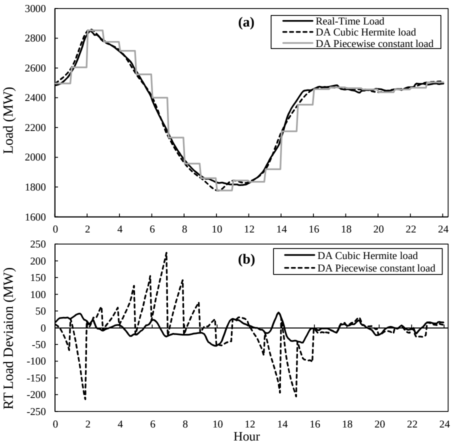
- DC power flow constraints can be modeled similarly.

# Continuous-time UC Solution



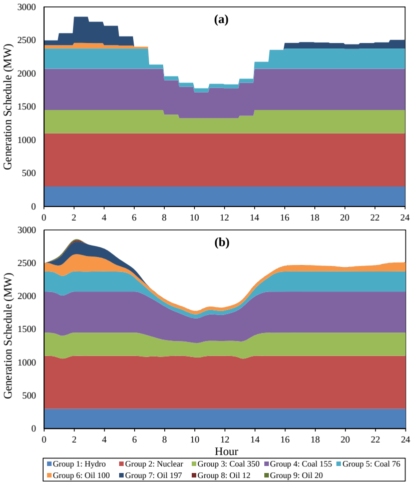
# Simulation Results: IEEE-RTS + CAISO Load

- The data regarding 32 units of the IEEE-RTS and load data from the CAISO are used here.
- Both the day-ahead (DA) and real-time (RT) operations are simulated.
- The five-minute net-load forecast data of CAISO for Feb. 2, 2015 is scaled down to the original IEEE-RTS peak load of 2850MW, and the hourly day-ahead load forecast is generated where the forecast standard deviation is considered to be %1 of the load at the time.



# Reduced Operation Cost and Ramping Scarcity Events

- Case 1: Hourly UC Model
- Case 2: Continuous-time UC Model



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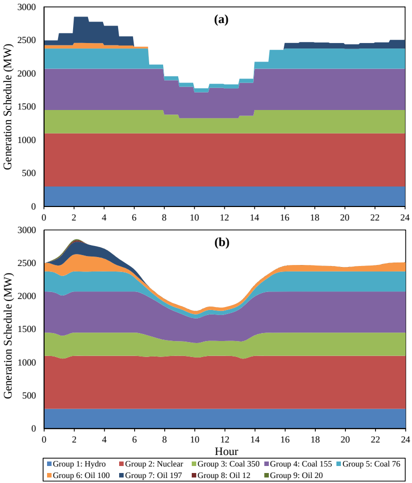


Total operation cost and ramping scarcity events are reduced in Case 2

| Case | DA Operation Cost ($) | RT Operation Cost ($) | Total DA and RT Operation Cost ($) | RT Ramping Scarcity Events |
|---|---|---|---|---|
| Case 1 | 471,130.7 | 16,882.9 | 488,013.6 | 27 |
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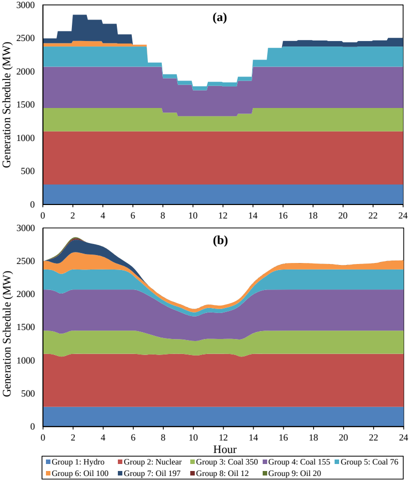


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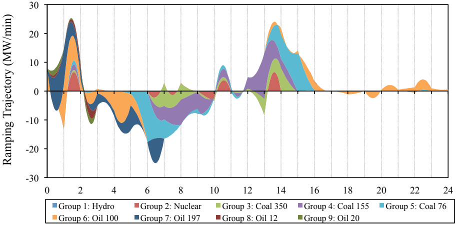
Continuous-time ramping trajectories



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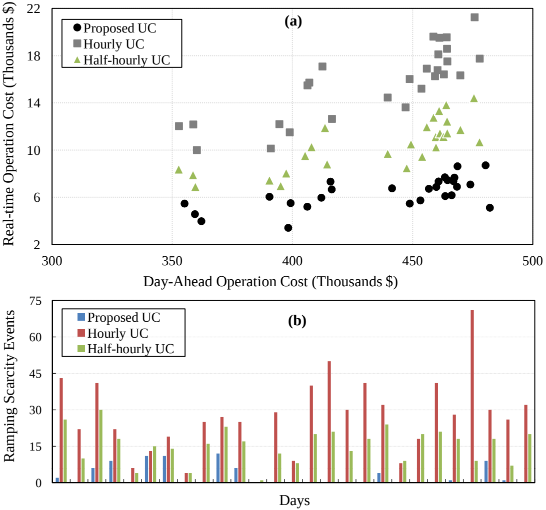

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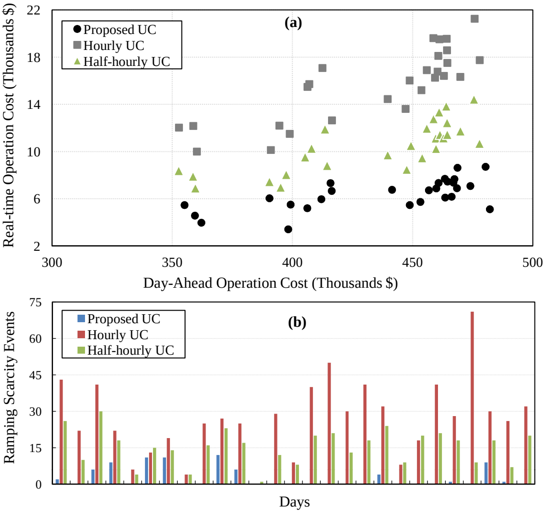
- Computation time for Feb. 2, 2015 load data:
  - Hourly UC: 0.257*s*
  - Half-hourly UC: 0.572*s*
  - Proposed UC: 1.369*s*



# Conclusions

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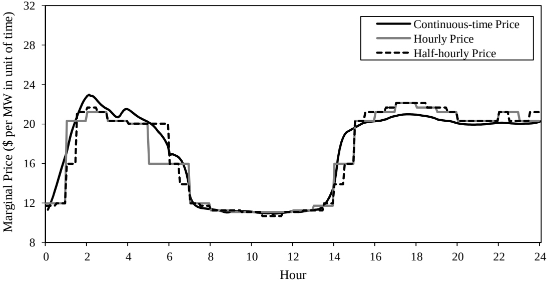
- Enabling the definition of *continuous-time marginal electricity price*:



# Further Reading


- M. Parvania, A. Scaglione, "Unit Commitment with Continuous-time Generation and Ramping Trajectory Models," *IEEE Transactions on Power Systems*, vol. 31, no. 4, pp. 3169-3178, July 2016.
- M. Parvania, A. Scaglione, "Generation Ramping Valuation in Day-Ahead Electricity Markets," in Proc. *49th Hawaii International Conference on System Sciences (HICSS)*, Kauai, HI, Jan. 5-8, 2016.