Fast and Accurate Calculation of Dynamics Sensitivities Using a Discrete-Adjoint Approach

June 29, 2016

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Outline

1. Sensitivity analysis for power grid simulations: local discrete adjoint sensitivity

2. Software infrastructure: Efficient implementation and accurate calculations of dynamic sensitivities

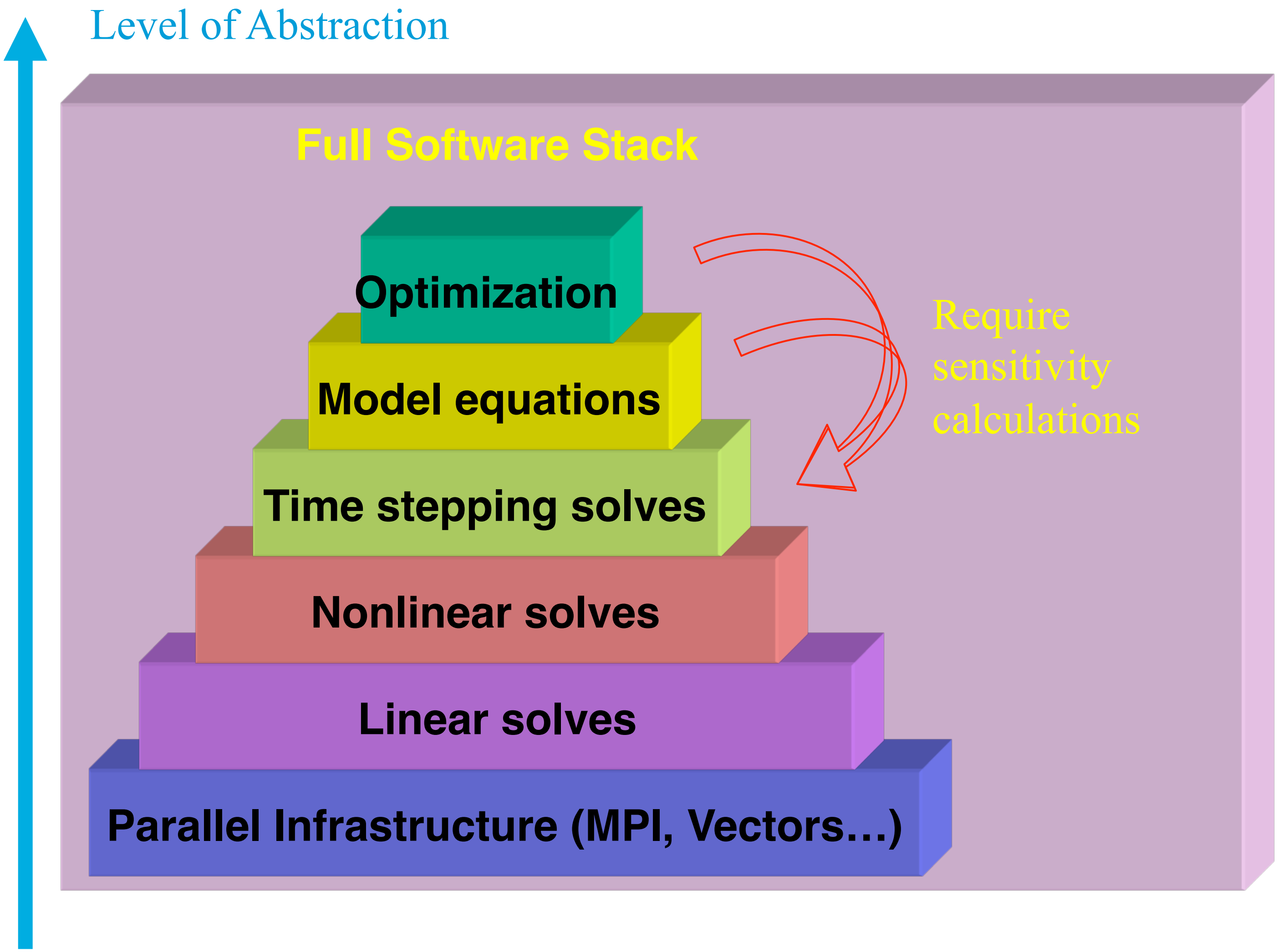
3. Applications: optimization, parameter estimation, uncertainty quantification

Sensitivity Analysis for Dynamic Power Grid Simulations

- Sensitivity analysis: describe the behavior of functionals that depend on dynamic variables with respect to system parameters
- Applications: sensitivity analysis in power grid simulations:
  - Optimization: security constrained OPF, economic dispatch
  - Impact or apportionment assessment,
  - Uncertainty quantification, parameter estimation
- Two types of sensitivities: local and global
- Local sensitivities: discrete and continuous
- Local sensitivity can be computed by: finite difference, forward, and adjoint

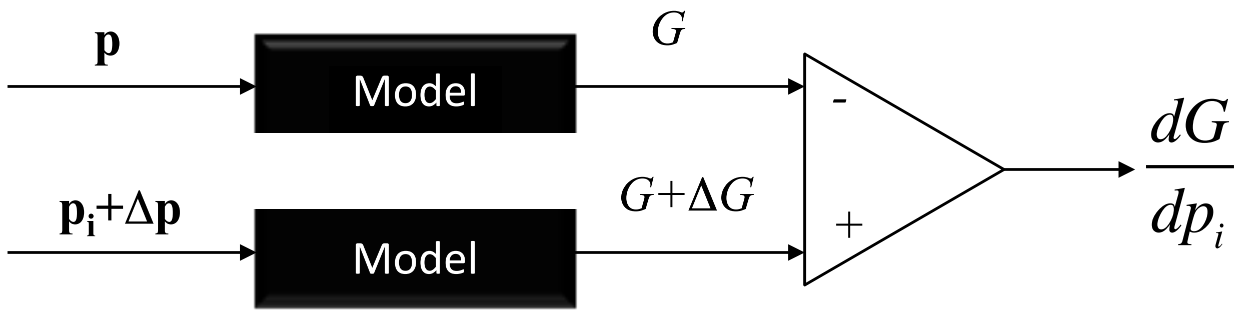
- System: $M\frac{dy}{dt} = f(t, y, p)$
- Numerical model: $y_{n+1} = \mathcal{N}_n(y_n)$
- Cost function, e.g.,: $G = g(y(t_F))$
- Sensitivity: $\mathcal{S} = \frac{dG()}{dp}$

Numerical Software Stack

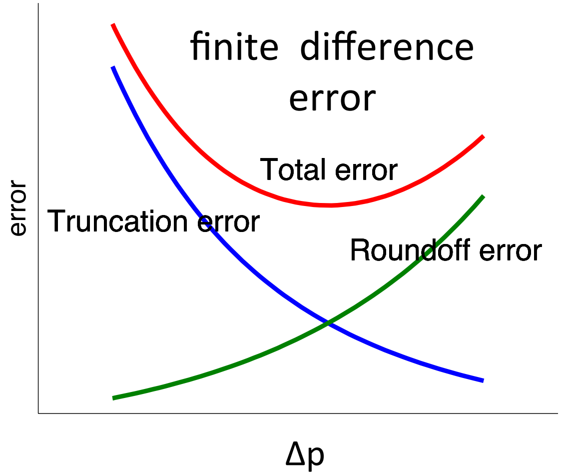


Computing Sensitivities: Finite Differences

- Easy to implement



- Inefficient for many parameter case, due to one-at-a-time
- Error depends critically on the perturbation value Δp



Computing Sensitivities: The Forward Approach

- Governing equation

$$M\frac{dy}{dt} = f(t, y), \quad y(t_0) = y_0(p)$$

- Discretization with a time stepping algorithm, (e.g. backward Euler)

$$My_{n+1} = My_n + h\left(f(t_{n+1}, y_{n+1})\right)$$

- Differentiate the equation on parameters $\mathcal{S}_{\ell,N} = dG/dp_\ell = dy_N/dp_\ell$

$$M\mathcal{S}_{\ell,n+1} = M\mathcal{S}_{\ell,n} + h\left(\mathbf{f}_{\mathbf{y}}(t_{n+1}, y_{n+1})\mathcal{S}_{\ell,n+1} + \mathbf{f}_{\mathbf{p}}(t_{n+1}, y_{n+1})\right)$$

- Solve one full (linear) system for each parameter

Computing Sensitivities: The Adjoint Approach

- Numerical one-step integrator:

$$y_{n+1} = \mathcal{N}_n(y_n), \quad n = 0, \ldots, N-1, \quad y_0 = \gamma(p)$$

- Enforce sensitivity equation through Lagrange multipliers, then differentiate:

$$\frac{d}{dp}\mathcal{L} = \frac{d}{dp}\left\{G - (\lambda_0)^T (y_0 - \gamma) - \sum_{n=0}^{N-1} (\lambda_{n+1})^T (y_{n+1} - \mathcal{N}(y_n))\right\}$$

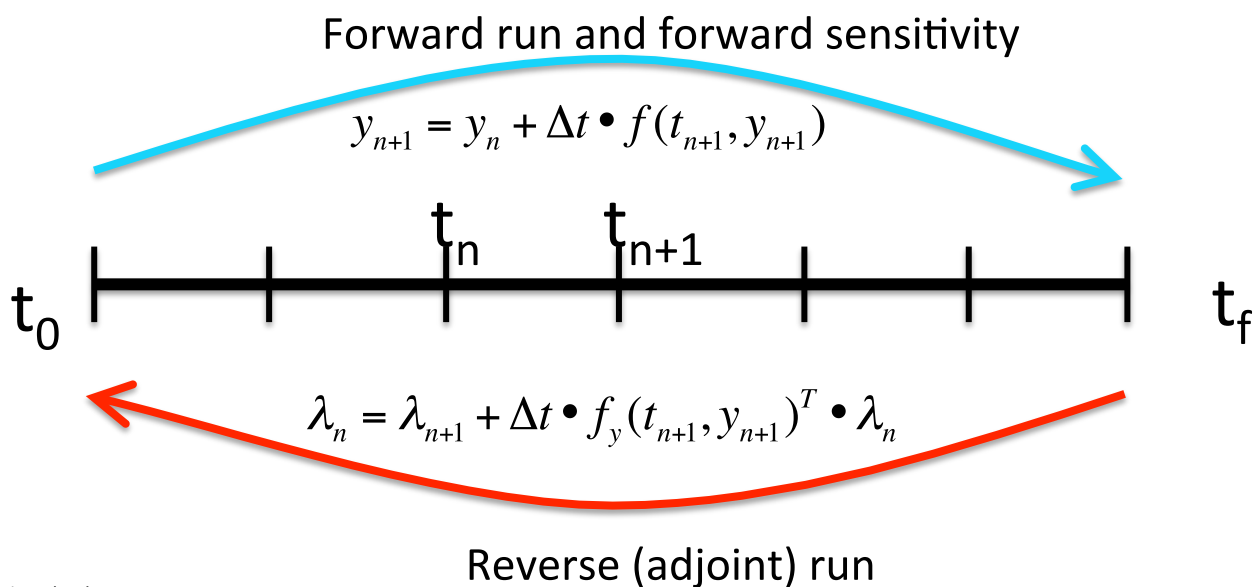
- Solve the linear sensitivity equations for all parameters in one-shot:

$$\lambda_N = \left(\frac{dG}{dy}\right)^T, \lambda_n = \left(\frac{d\mathcal{N}}{dy}(y_n)\right)^T \lambda_{n+1}, \; n = N-1, \ldots, 0,$$

$$\nabla_p G = \left(\frac{d\gamma}{dp}\right)^T \lambda_0$$

Sensitivity calculations: Forward or Adjoint?

	Forward	Adjoint
Best to use when the number of	parameters << functionals	parameters >> functionals
Complexity	O(# of parameters)	O(# of functionals)
Checkpointing	No	Yes
Implementation	Medium	High



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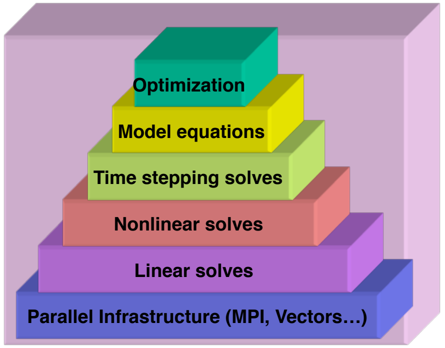
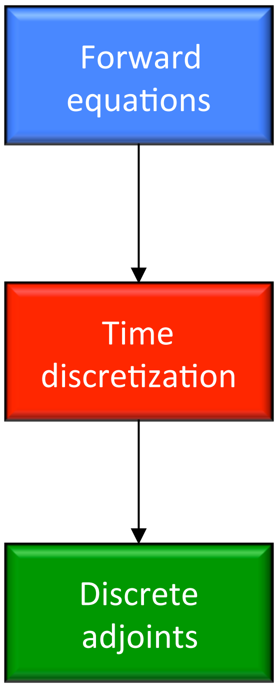
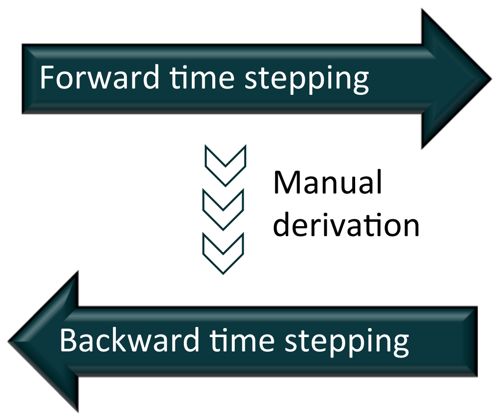
Adjoint Integration with Portable, Extensible Toolkit for Scientific Computation (PETSc)

- PETSc: Open-source numerical library for large-scale parallel computation
- Portability
  - 32/64 bit, real/complex,
  - single/double/quad precision
  - Unix, Linux, MacOS, Windows
  - C, C++, Fortran, Python, MATLAB
  - GPGPUs and support for threads
- Extensibility
  - ParMetis, SuperLU, SuperLU_Dist, MUMPS, HYPRE,UMFPACK, Sundials, Elemental, Scalapack, UMFPack, …
- Toolkit
  - Sequential and Parallel vectors
  - Sequential and Parallel matrices
  - **Iterative solvers and preconditioners**
  - **Parallel nonlinear solvers**
  - **Adaptive time stepping (ODE and DAE) solvers**

Time-stepping level adjoint implementation







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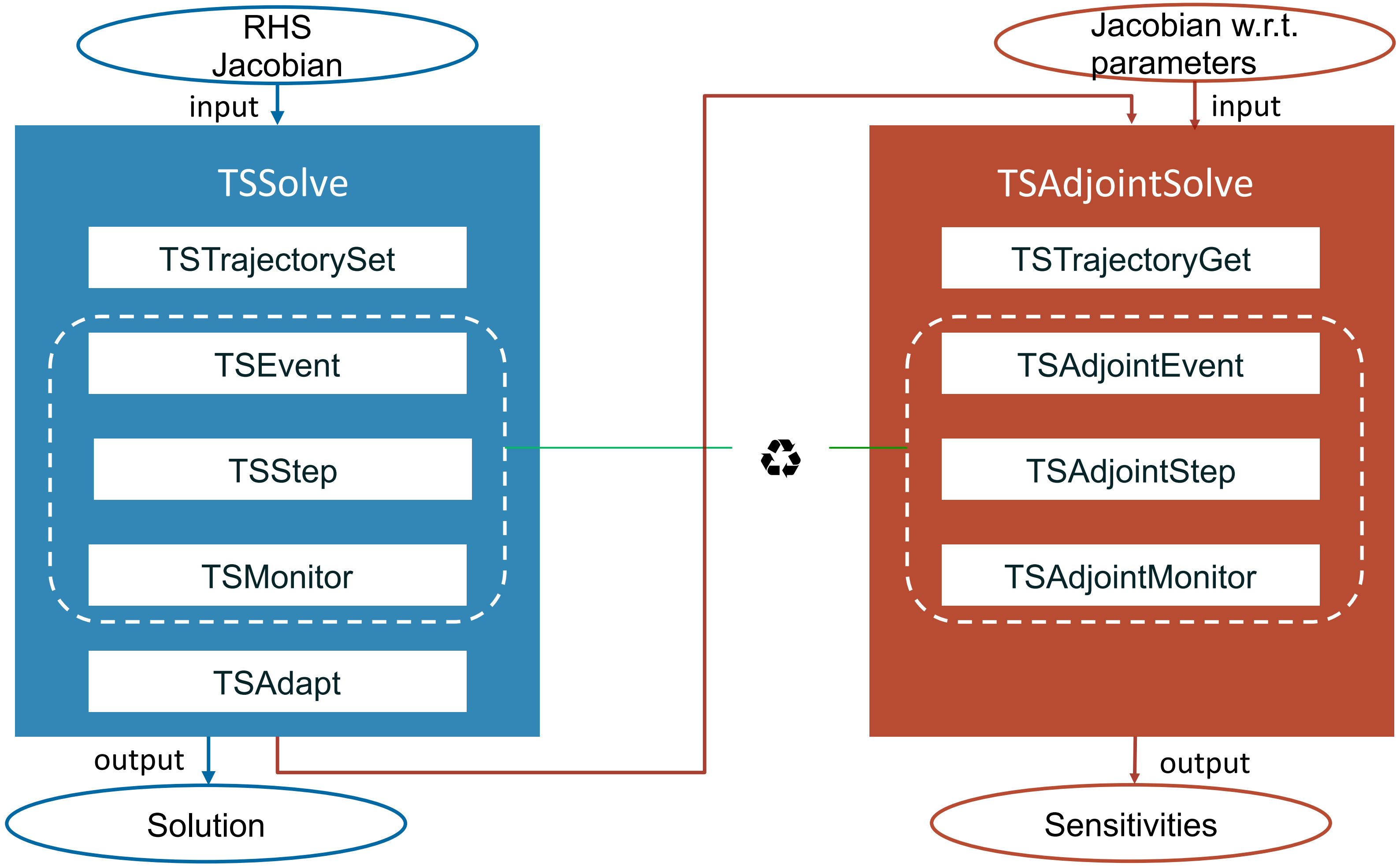
PETSc Design Goals and Implementation

1. Minimize intrusion
2. Reuse functionalities (already implemented in PETSc or provided by users)
3. Aim for general-purpose solutions and support for switching



Optimal Checkpointing

- Minimize the number of recomputations and the number of reads/writes by using exiting library **revolve** a less intrusive way
  - **revolve** is designed as a top-level controller for time stepping
  - TSTrajectory consults **revolve** about when to restore/restore/recompute
- Incorporate a variety of single-level and two-level schemes for offline and online checkpointing
  - Existing algorithms work great for RAM only checkpointing
  - Our extension is optimal for RAM+disk

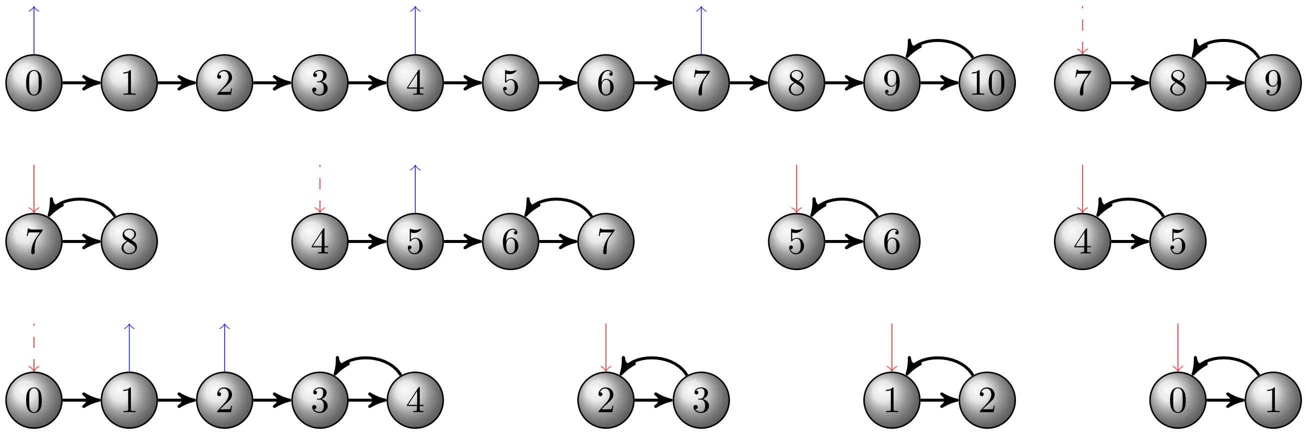
An optimal schedule given 3 allowable checkpoints in RAM:

blue arrow: store a checkpoint

red arrow: restore a checkpoint

black arrow: a step

circle: solution



How Precision of the Gradients Affects Optimization

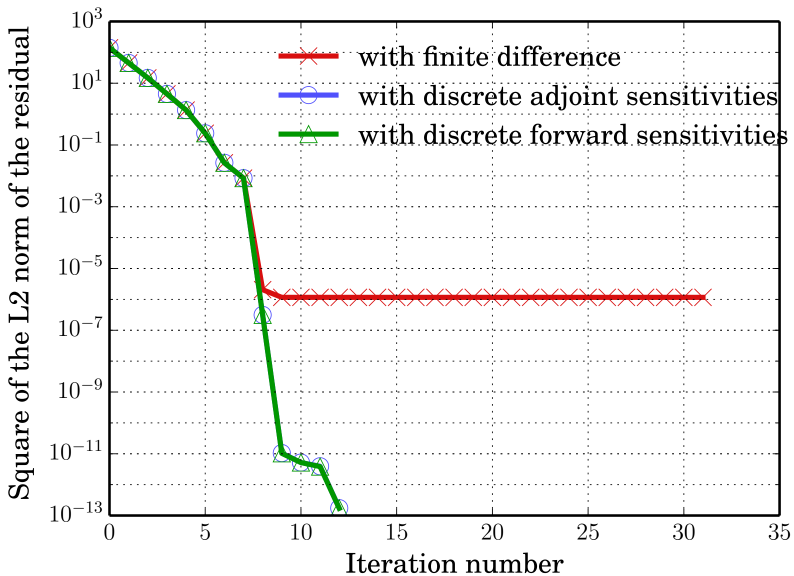
- Maximize mechanical power input, subject to the generator swing equations and a constraint on the maximum rotor angle deviation:

$$\min_{P_m} -P_m + \sigma \int_{t_0}^{t_F} \max(0, \delta - \delta_{max})^{\eta} \mathrm{dt}$$

s.t.

$$\frac{d\delta}{dt} = \omega_B(\omega - \omega_s)$$

$$\frac{d\delta}{dt} = \frac{\omega_s}{2H}\left(P_m - P_{max}\sin(\delta) - D\,(\omega - \omega_s)\right)$$



- optimization process using the forward and adjoint sensitivities converge after 13 iterations

- optimization using the finite-difference approximations stall with a residual of 10^{-6}

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Sensitivity of Dynamic Security Metric to System Dispatch Parameters

- Dynamic security metric for each generator

$$H_i(x,y) = \sigma \int_0^T [max(0, \omega_i - \omega^+, \omega^+ - \omega_i)]^\eta \, dt$$

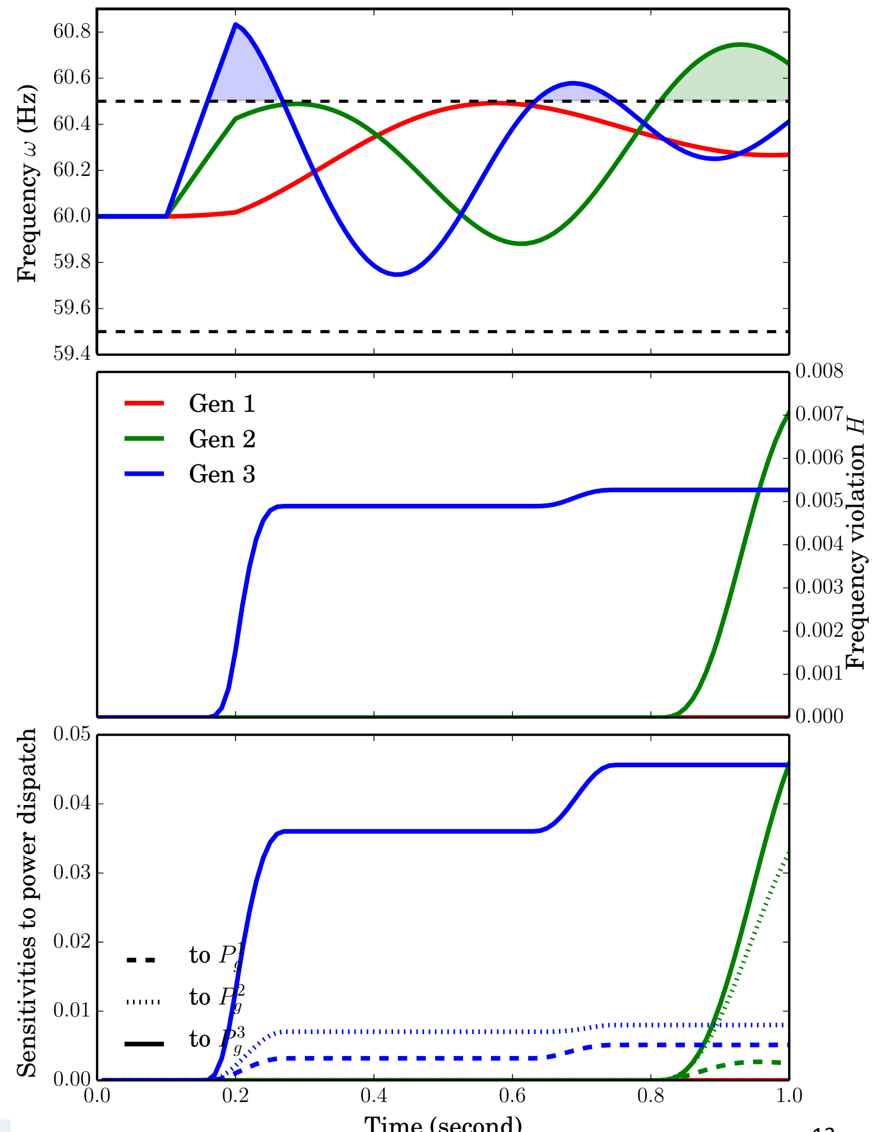
  - ω_i: the frequency of the generator i
  - ω+/ω−: the max and min freq limits

- Compute sensitivity of each H_i w.r.t. generator active and reactive dispatch, and the bus voltage magnitudes and angles at initial time

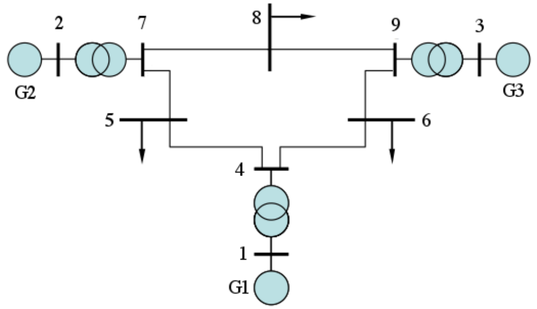
	No. of Variables	No. of Parameters	No. of Functions
9 bus	42	24	3
118 bus	884	344	54

	Forward	Adjoint	Simulation
9 bus	0.12 s	0.05 s	0.03 s
118 bus	14.00 s	1.82 s	0.33 s

The adjoint method is faster than the forward method by 2.4X and 7.7X for the 9-bus and 118- bus systems



Bayesian Approach for Parameter Estimation



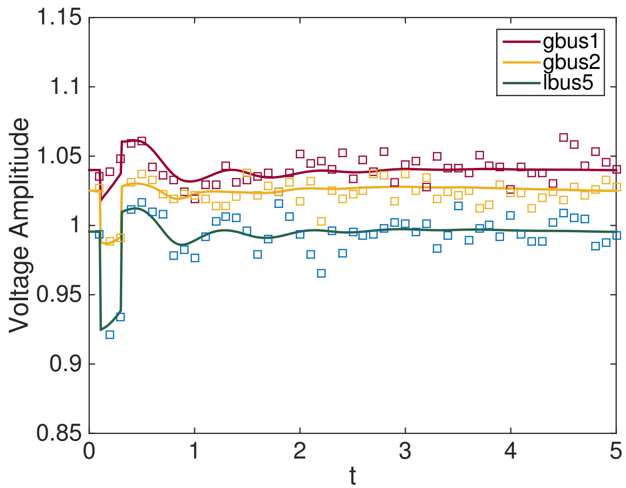
- Estimate generator inertias during dynamic transient generated by inducing a load disturbance

$$d = H(\mathcal{N}(p)) + \varepsilon\,,\ \varepsilon \sim \mathrm{N}(0, \Gamma_{\text{noise}})$$

- Measurements: voltage phase and amplitude

$$\pi_{\text{like}}(d|p) = \exp(-\frac{1}{2}(H(\mathcal{N}(p)) - d)^T \Gamma_{\text{noise}}^{-1}(H(\mathcal{N}(p)) - d))$$

$$\log \pi_{\text{post}}(p) \propto -||H(\mathcal{N}(p)) - d||_{\Gamma_{\text{noise}}^{-1}} - ||p - p_{\text{prior}}||_{\Gamma_{\text{prior}}^{-1}}$$



- Need to maximize the maximum aposteriori estimate:

$$p_{\text{MAP}} = \arg\max_{p} -\log(\pi_{\text{post}}(p))$$

Noemi Petra, Cosmin G. Petra, Zheng Zhang, Emil M. Constantinescu, and Mihai Anitescu, "A Bayesian approach for parameter estimation with uncertainty for dynamic power systems." IEEE Transactions on Power Systems, Submitted, 2016.

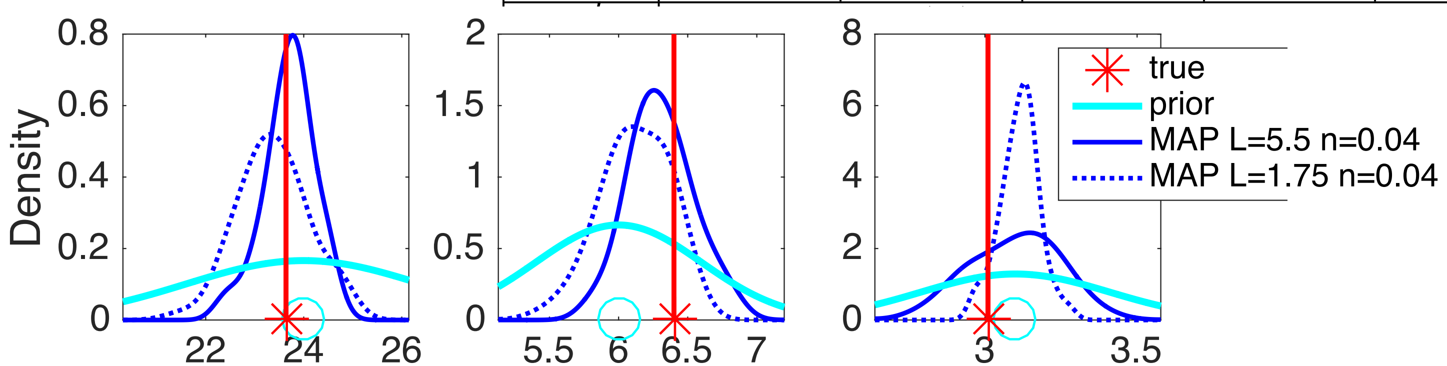
Bayesian Approach for Parameter Estimation

- Use adjoints to compute the gradient of the posterior distribution
- The optimization is solved with quasi-Newton in TAO/PETSc

$$p_{\mathrm{MAP}} = \arg\max_{p} - \log(\pi_{\mathrm{post}}(p))$$

$$\mathcal{S} = -\frac{\partial \log(\pi_{\mathrm{post}})}{\partial p}$$

t_f	m_1 **23.64**	m_2 **6.40**	m_3 **3.01**	#iter	τ
	(a) $\Delta_t = 0.01$, $\Delta_t^{obs} = 0.05$				
5.0	23.60	6.35	3.02	15	1.59e-02
3.0	23.79	6.39	3.06	9	1.85e-02
1.0	23.56	6.32	3.06	14	3.60e-02
0.8	23.67	6.54	2.95	11	5.81e-02
0.6	22.45	6.14	3.01	10	9.43e-02



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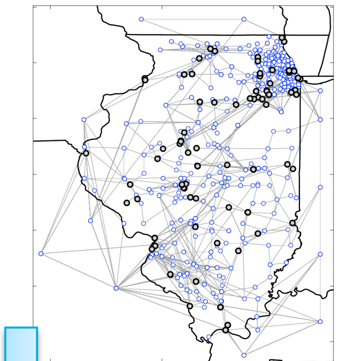
Adjoint Sensitivity Analysis for Targeted Generation Cost

- Sensitivity of [energy generation] cost functional with respect to ambient conditions:

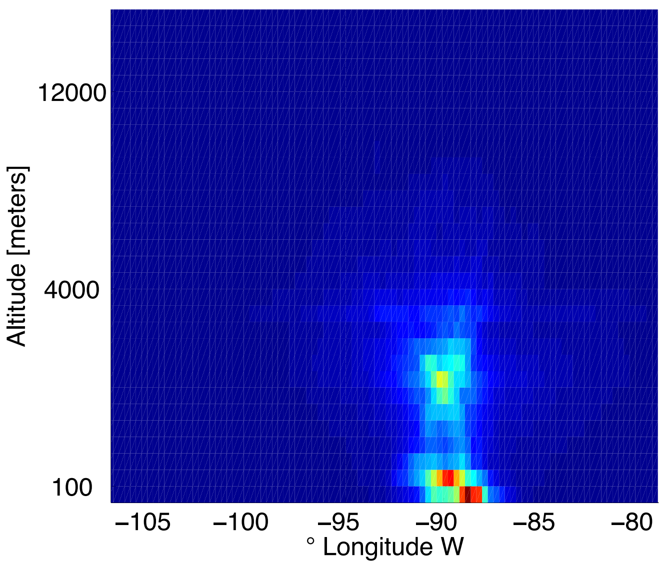
$$G(w(t))\,[\$] \;= c(t) + \lambda(t)^T \omega(w(t)) \longrightarrow \mathcal{S} = \frac{\partial G}{\partial \mathbf{W}(t)}$$

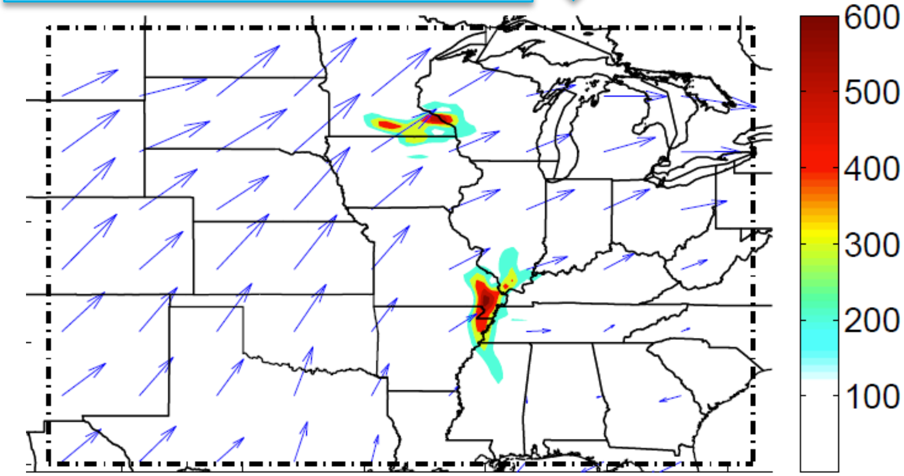


- Applications: sensor placement, reduce uncertainty with detailed simulation, reveals correlations among physical variables and economic functions

Sensitivity of grid operation costs with respect to weather conditions

target: cost in IL



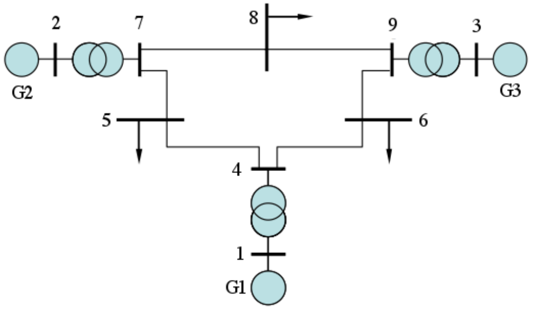


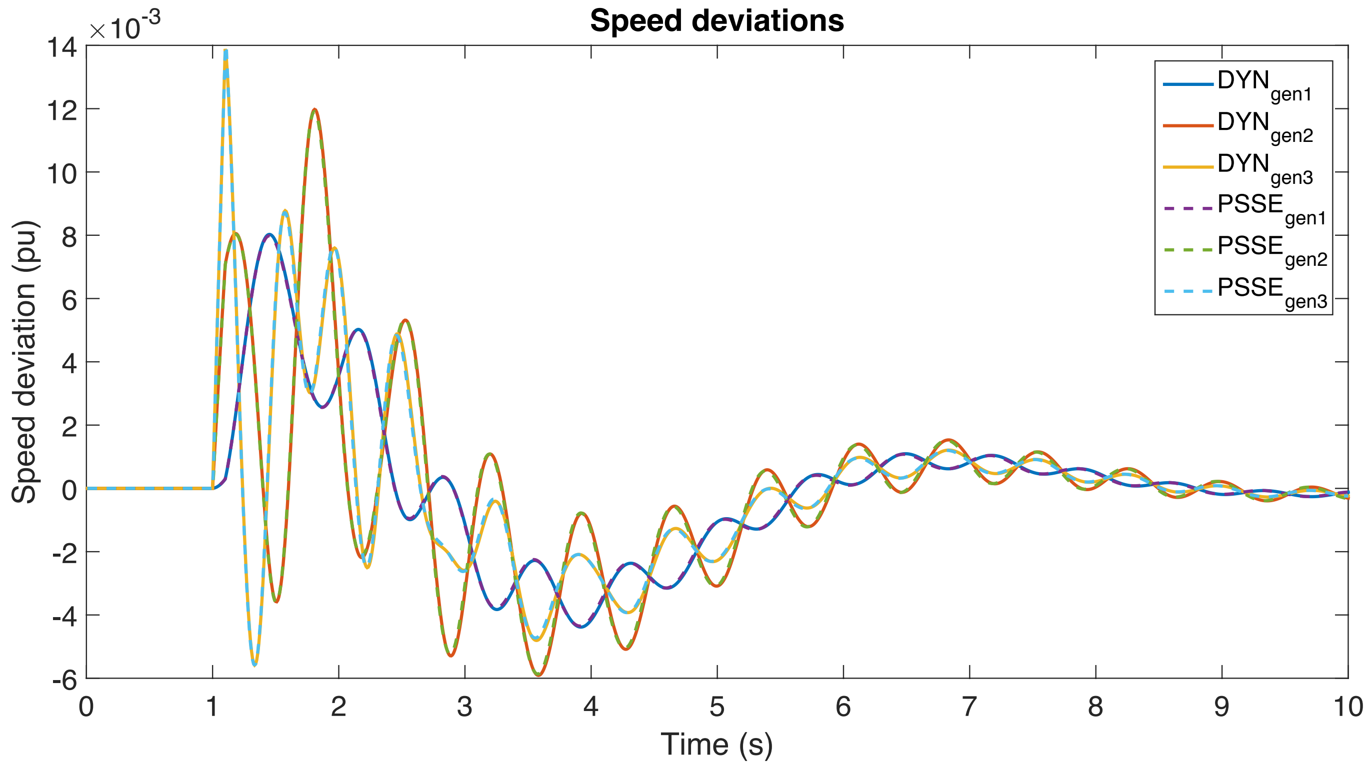
Alexandru Cioaca, Victor Zavala, and Emil M. Constantinescu, "Adjoint Sensitivity Analysis for Numerical Weather Prediction: Applications to Power Grid Optimization." Networking and Analytics for the Power Grid, 2011.

Dynamic Code Consistency (vs PSSE)

Transient stability analysis: IEEE 9bus, fault for 0.1 sec

1. Generators: GENROU
2. Exciters: IEEET1
3. Governors: TGOV1
4. Stabilizers: STAB1





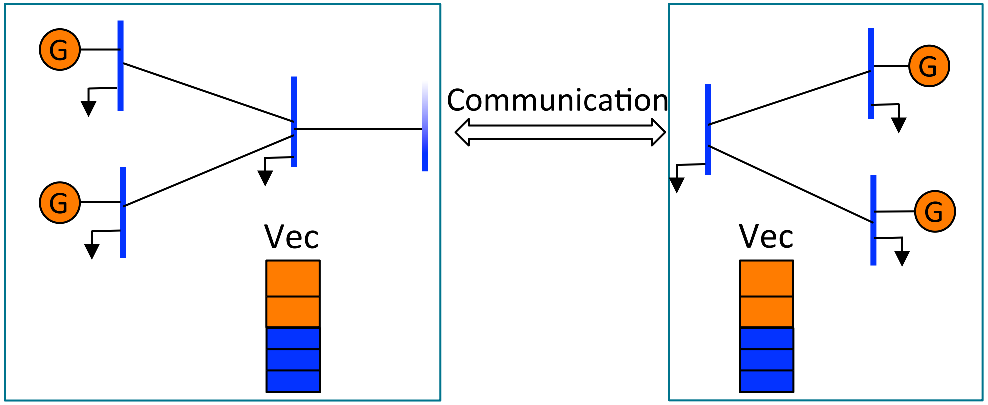
Large Scale Dynamic Simulations Using PETSc

- Dynamic power grid simulation: 10 second-simulation with a six cycle temporary three-phase fault applied at a bus for 1 second
- 16-core machine, peak speedups of about 3 or 4

CaseName	Buses	Generators	Branches
case2737sop	2737	399	3506
case9241pegase	9241	1445	16049
case22996	22996	2416	27408

Test System	PETSc
case2737sop	23.58
9241pegase	90.79
case22996	138.02

- Parallel direct solvers are not scalable: use MUMPS and SuperLU_Dist
- Preconditioned GMRES more scalable: use additive Schwarz
- Adaptive time stepping



Shrirang Abhyankar, Emil M. Constantinescu, Barry Smith, Alexander J. Flueck, and Daniel A. Maldonado "Acceleration of dynamic simulations using parallel Newton-GMRES-Schwarz methods." IEEE Transactions on Smart Grid Special Issue on High Performance Computing (HPC) Applications for a More Resilient and Efficient Power Grid, Under Review, 2015.

Summary

- Local sensitivity analysis using discrete adjoints
- Most efficient and accurate for problems with many decision parameters
- The implementation takes advantage of highly developed solver infrastructure: MPI, parallel vectors/matrices, domain decomposition, linear/nonlinear solvers
- Advanced checkpointing, transparent to the user
- Current implementation avoids complete algorithmic differentiation and requires minimal user input, reuses information provided for the forward simulation
- Implementation accommodates jumps/switches/discontinuities
- Experiments on parameter estimation, dynamic security constraints for IEEE 9-bus and 118-bus dispatch parameters; and other large scale problems

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