Modeling and Solving Battery Operation Problems

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FERC Software conference, 2016

Talk outline

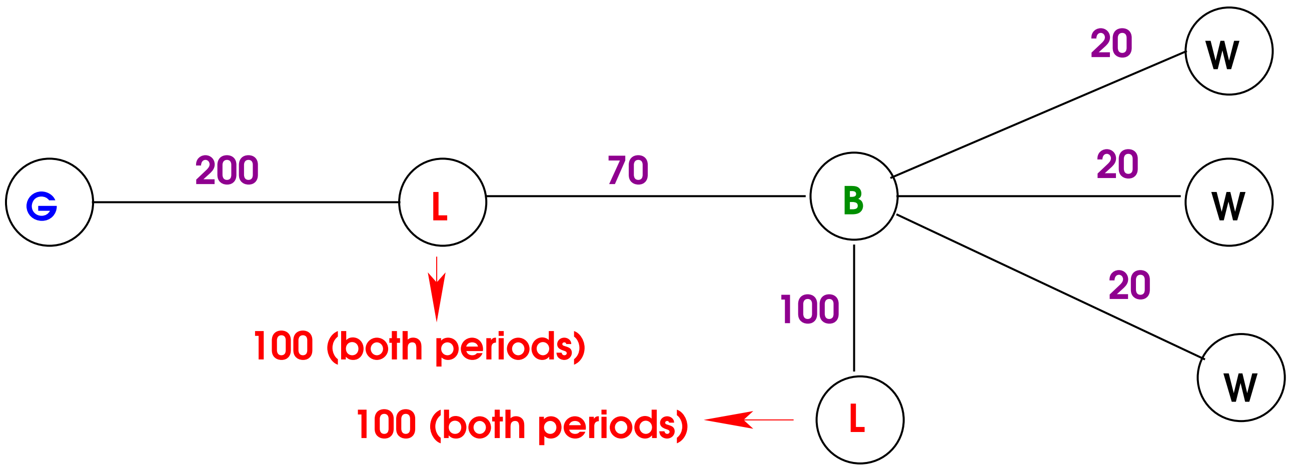
- Optimization framework for using storage to offset uncertainty in renewable forecast output
- "OPF-like" setup: minimize cost of generation
- Multiperiod model, with per-period renewable forecasts
- affine control for renewable output
- robust optimization used to handle forecast errors

Questions/issues

- Batteries not quite here yet?
- Transmission level or distribution level only?
- Can batteries be moved around? In what time frame?
- What is the cost of moving/installing batteries and how should that be factored into operations?
- What is the correct time frame for analyzing the benefit of using batteries?

A simple example on 2 periods

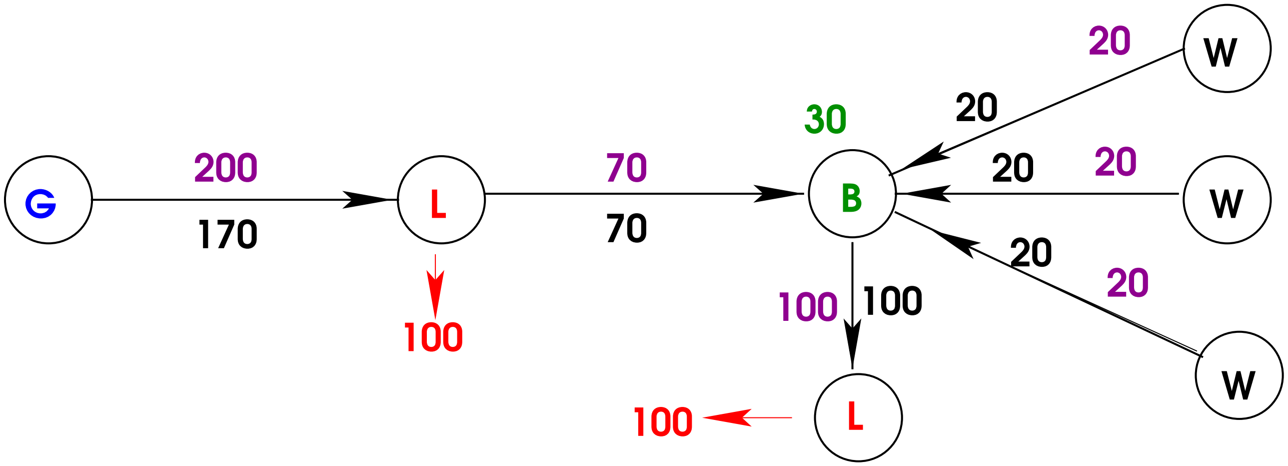
G = generator, L = load, B = battery, W = renewable



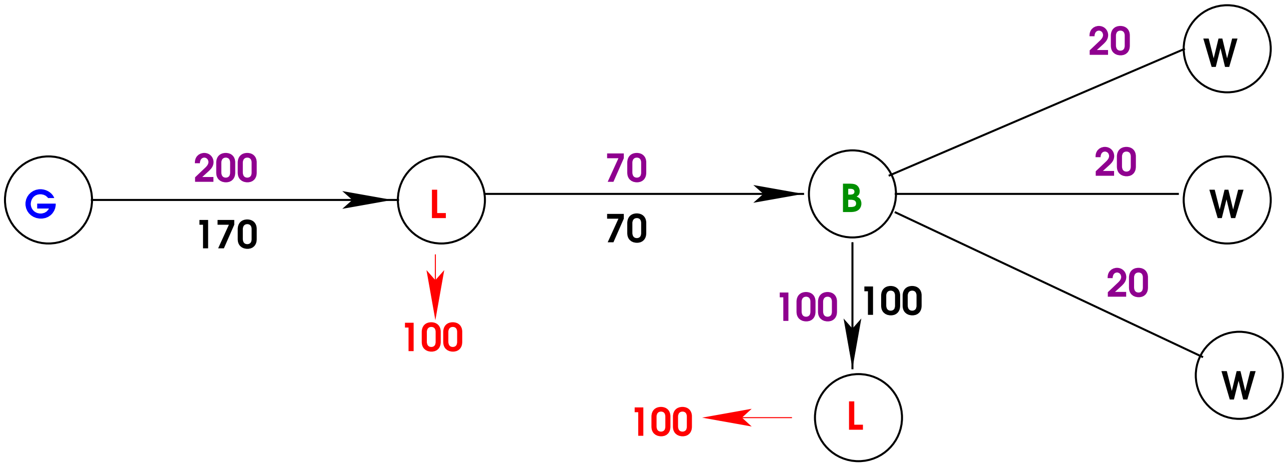
- **Period 1:** each renewable outputs **20**, no uncertainty.
- **Period 2:** each renewable outputs in the range $[\mathbf{0}, \mathbf{20}]$.
- Battery and generator are large, but battery starts **drained**.

- **Period 1:** each renewable outputs $\mathbf{20}$, no uncertainty.
- **Period 2:** each renewable outputs in the range $[\mathbf{0}, \mathbf{20}]$.

Period 1:



Period 2:



Notation

1. $\boldsymbol{T}$ time periods of equal length $\boldsymbol{\Delta}$. Assume $\boldsymbol{\Delta = 1}$.
2. $\boldsymbol{P_k^{g,t}}$ = output of generator at bus $\boldsymbol{k}$ at time $\boldsymbol{t}$ (decision variable)
3. $\bar{\boldsymbol{w}}_{\boldsymbol{i}}^{\boldsymbol{t}} + \boldsymbol{w_i^t}$ = output of renewable at bus $\boldsymbol{i}$ at time $\boldsymbol{t}$.
   $\bar{\boldsymbol{w}}_{\boldsymbol{i}}^{\boldsymbol{t}}$ = forecast, $\boldsymbol{w_i^t}$ = error (uncertain).
4. $\boldsymbol{\delta_j^t}$ = output of battery at bus $\boldsymbol{j}$ at time $\boldsymbol{t}$.
   Assumption: all batteries at a given bus $\boldsymbol{j}$ are of similar type.
5. $\boldsymbol{P_k^{d,t}}$ = load at bus $\boldsymbol{k}$ at time $\boldsymbol{t}$ (data).
6. **DC** power flow $\longrightarrow$ for all $\boldsymbol{t}$, and all $\boldsymbol{w}$,

$$\sum_k P_k^{g,t} + \sum_i (\bar{w}_i^t + \boldsymbol{w_i^t}) + \sum_j \delta_j^t = \sum_k P_k^{d,t}$$

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4. $\boldsymbol{\delta_j^t}$ = output of battery at bus $\boldsymbol{j}$ at time $\boldsymbol{t}$.
5. $\boldsymbol{P_k^{d,t}}$ = load at bus $\boldsymbol{k}$ at time $\boldsymbol{t}$ (data).
6. For all $\boldsymbol{t}$, and all $\boldsymbol{w}$, $\sum_k P_k^{g,t} + \sum_i (\bar{w}_i^t + \boldsymbol{w_i^t}) + \sum_j \delta_j^t = \sum_k P_k^{d,t}$

Generic affine control:

$$\boldsymbol{\delta_j^t} = \bar{\lambda}_j^t - \lambda_j^t \sum_{i \in R(j)} \boldsymbol{w_i}$$

- $\bar{\lambda}^t$, $\bar{\lambda}^t$: decision variables
- $\boldsymbol{R(j)}$: set of buses that battery at $\boldsymbol{j}$ **responds to**

Example 1: (bad?) a battery at each renewable, $\boldsymbol{R(j) = j}$ for all j

Example 2: for all j, $\boldsymbol{R(j) =}$ all renewables

Nominal case:

$$\sum_k P_k^{g,t} + \sum_i \bar{w}_i^t + \sum_j \bar{\lambda}_j^t = \sum_k P_k^{d,t}$$

Balance: $\sum_k P_k^{g,t} \ + \ \sum_i (\bar{w}_i^t + \ \boldsymbol{w_i^t}) \ + \ \sum_j \delta_j^t \ = \ \sum_k P_k^{d,t} \quad \forall \ \boldsymbol{w}$

Nominal case: $\sum_k P_k^{g,t} \ + \ \sum_i \bar{w}_i^t \ + \ \sum_j \bar{\lambda}_j^t \ = \ \sum_k P_k^{d,t}$

Generic affine control: $\boldsymbol{\delta_j^t} \ = \ \bar{\lambda}_j^t \ - \ \lambda_j^t \left(\sum_{i \in R(j)} \ \boldsymbol{w_i} \right)$

This talk: $\boldsymbol{R(j) =}$ all renewables, for each $\boldsymbol{j}$.

Affine control: $\boldsymbol{\delta_j^t} \ = \ \bar{\lambda}_j^t \ - \ \lambda_j^t \, (\sum_i \ \boldsymbol{w_i})$

Balance: $\sum_k P_k^{g,t} + \sum_i \bar{w}_i^t + \sum_j \bar{\lambda}_j^t + (\sum_i \ \boldsymbol{w_i}) \left(1 - \sum_j \lambda_j^t \right) \ = \ \sum_k P_k^{d,t}$

Together with nomimal case, implies: $\boldsymbol{\sum_j \lambda_j^t = 1}$

Other considerations

- What is the **sign** of $\boldsymbol{\bar{\lambda}_j^t}$? Of $\boldsymbol{\bar{\lambda}_j^t}$?
- Require that they have the same sign? That they **all** have the same sign, for any given $\boldsymbol{t}$?
- Force $\boldsymbol{\lambda_j^t \geq 0}$?
- Restrict the **number** of nonzero $\boldsymbol{\lambda_j^t}$, for any $\boldsymbol{t}$?

Battery model

- Discharge rate bounds. We will want to lower- and upper-bound

$$\boldsymbol{\delta_j^t} = \bar{\lambda}_j^t - \lambda_j^t \left(\sum_{i \in R(j)} \boldsymbol{w_i} \right)$$

for all batteries $\boldsymbol{i}$, time $\boldsymbol{t}$ and all $\boldsymbol{w}$

- Energy state bounds. Let E_j^0 = initial energy state of battery at site i. Then

$$\boldsymbol{E_j^t} = E_j^0 + \Delta \sum_{h=1}^{t} \boldsymbol{\delta_j^h}$$

is the energy state at the end of period t. (Δ = length of time periods) Must have lower- and upper-bounds on $\boldsymbol{E_j^t}$.

- Special bounds for $\boldsymbol{E_j^T}$?

Renewable forecast error model

Output of renewable $\boldsymbol{i}$ at time $\boldsymbol{t}$: $\bar{\boldsymbol{w}}_i^t + \boldsymbol{w}_i^t$

- $\bar{\boldsymbol{w}}_i^t$ = forecast output
- $\boldsymbol{w}_i^t$ = error; $\boldsymbol{w} \in \mathcal{W}$, where $\mathcal{W}$ = uncertainty model.

Linear error model: $\mathcal{W} \ = \ \{w \ : \ Cw \leq d\}$

Example: concentration model

$$\begin{aligned} |w_i^t| \ &\leq \ \gamma_i^t, \quad \text{all } t \text{ and } i \\ \sum_i \alpha_i^t |w_i^t| \ &\leq \ \Gamma^t \quad \text{all } t \end{aligned}$$

Here, the $\boldsymbol{\gamma}_i^t$, $\boldsymbol{\alpha}_i^t$ and $\boldsymbol{\Gamma}^t$ are parameters. **Special case:** $\boldsymbol{\alpha}_i^t = \mathbf{1}/\boldsymbol{\gamma}_i^t$

Many variations, e.g. time-correlated models

$$\begin{aligned} \sum_t \alpha_i^t |w_i^t| \ &\leq \ \Gamma_i \\ \sum_i \sum_t \alpha_i^t |w_i^t| \ &\leq \ \Gamma \end{aligned}$$

Extension: histogram models.

Formulation

Optimization problem: minimize generation cost subject to being feasible under all modeled renewable outputs. **Variables:** $\boldsymbol{P^g, \bar{\lambda}, \lambda}$

$$\min_{P^g, \bar{\lambda}, \lambda} \quad \sum_t \sum_k c_k^t(P_k^{g,t})$$

s.t. the following constraints being feasible at all times t, for all $\boldsymbol{w} \in \mathcal{W}$:

$$B\,\boldsymbol{\theta^t} \; = \; P^{g,t} \; + \; \overbrace{\bar{w}^t + \boldsymbol{w^t}}^{\text{renewables}} \; + \overbrace{\bar{\lambda}^t - \left(\sum_i \boldsymbol{w_i^t} \right) \lambda^t}^{\text{batteries}} \; - \; P^{d,t}$$

$$\frac{|\,\boldsymbol{\theta_k^t} - \boldsymbol{\theta_m^t}|}{x_{km}} \; \leq \; u_{km} \quad \text{for all } km \text{ (line limits at time } t\text{)}$$

$$r_j^{\min} \; \leq \; \bar{\lambda}_j^t \; - \; \lambda_j^t \left(\sum_i \boldsymbol{w_i^t} \right) \; \leq \; r_j^{\max} \quad \text{for all } j \;\; \text{(discharge rate limits)}$$

$$0 \; \leq \; E_j^0 \; + \; \Delta \sum_{h=1}^{t} \left[\bar{\lambda}_j^h \; - \; \lambda_j^h \left(\sum_i \boldsymbol{w_i^h} \right) \right] \; \leq \; E_j^{\max} \quad \text{all } j \quad \text{(energy state)}$$

$$\min_{P^g,\bar{\lambda},\lambda} \quad \sum_t \sum_k c_k^t(P_k^{g,t})$$

s.t. the following constraints being feasible at all times t, for all $\boldsymbol{w} \in \mathcal{W}$:

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$$\frac{|\,\boldsymbol{\theta_k^t} - \boldsymbol{\theta_m^t}|}{x_{km}} \;\leq\; u_{km} \quad \text{for all } km \text{ (line limits at time } t)$$

$$r_j^{\min} \leq \bar{\lambda}_j^t - \lambda_j^t\left(\sum_i \boldsymbol{w_i^t}\right) \leq r_j^{\max} \quad \text{for all } j \;\; \text{(discharge rate limits)}$$

$$0 \leq E_j^0 + \Delta \sum_{h=1}^{t}\left[\bar{\lambda}_j^h - \lambda_j^h\left(\sum_i \boldsymbol{w_i^h}\right)\right] \leq E_j^{\max} \quad \text{all } j \quad \text{(energy state)}$$

Is there a "compact" formulation?

- **What** is a compact formulation?
- For linear error model: $\mathcal{W} = \{w : Cw \leq d\}$, the answer is **yes**
- But it is going to large, and expensive.
- Factoid: modifications of DC model for large grids are ~~dangerous~~ hard for LP solvers.

How to build a cutting-plane procedure

Consider the balance equations at a given time $\boldsymbol{t}$ (removed from notation)

$$B\,\boldsymbol{\theta} \;=\; P^g + \bar{w} + \boldsymbol{w} + \bar{\lambda} - \lambda\left(\sum_i \boldsymbol{w_i}\right) - P^d$$

we can set $\boldsymbol{\theta_\rho = 0}$ where $\boldsymbol{\rho}$ = reference bus; flow balance can be rewritten

$$\boldsymbol{\theta} \;=\; V\left[P^g + \bar{w} + \boldsymbol{w} + \bar{\lambda} - \lambda\left(\sum_i \boldsymbol{w_i}\right) - P^d\right]$$

here V is a **pseudo-inverse** of $\boldsymbol{B}$

So for any line $\boldsymbol{km}$,

$$\boldsymbol{\theta_k - \theta_m} \;=\; (v_k - v_m)\left[P^g + \bar{w} + \boldsymbol{w} + \bar{\lambda} - \lambda\left(\sum_i \boldsymbol{w_i}\right) - P^d\right]$$

(v_h = row h of V)

How to build a cutting-plane procedure, 2

$$\boldsymbol{\theta_k} - \boldsymbol{\theta_m} \;=\; (v_k - v_m)\left[P^g + \bar{w} + \boldsymbol{w} + \bar{\lambda} - \lambda\left(\sum_i \boldsymbol{w_i}\right) - P^d\right]$$

Suppose that $(\boldsymbol{P^{g*}}, \boldsymbol{\bar{\lambda}^*}, \boldsymbol{\lambda^*})$ is a **proposed** solution that is **infeasible**

i.e. there is a line $\boldsymbol{km}$ and an **error vector** $\hat{\boldsymbol{w}}$ such that

$$\frac{(v_k - v_m)}{x_{km}}\left[\boldsymbol{P^{g*}} + \bar{w} + \hat{\boldsymbol{w}} + \boldsymbol{\bar{\lambda}^*} - \boldsymbol{\lambda^*}\left(\sum_i \hat{\boldsymbol{w}}_{\boldsymbol{i}}\right) - P^d\right] \;>\; u_{km}$$

Then the inequality

$$\frac{(v_k - v_m)}{x_{km}}\left[P^g + \bar{w} + \hat{\boldsymbol{w}} + \bar{\lambda} - \lambda\left(\sum_i \hat{\boldsymbol{w}}_{\boldsymbol{i}}\right) - P^d\right] \;\leq\; u_{km}$$

is **valid** and **cuts-off** $(\boldsymbol{P^{g*}}, \boldsymbol{\bar{\lambda}^*}, \boldsymbol{\lambda^*})$

A cutting-plane procedure

Start with a **relaxation** for the robust problem, e.g. the nominal problem (no errors), and then

1. Solve relaxation, with solution $(\boldsymbol{P^{g*}}, \boldsymbol{\bar{\lambda}^*}, \boldsymbol{\lambda^*})$
2. **Play adversary:** find a worst-case distribution $\hat{\boldsymbol{w}}$ for $(\boldsymbol{P^{g*}}, \boldsymbol{\bar{\lambda}^*}, \boldsymbol{\lambda^*})$

   Comment: Requires solving small LPs
3. Say, e.g. for a given line $\boldsymbol{km}$,

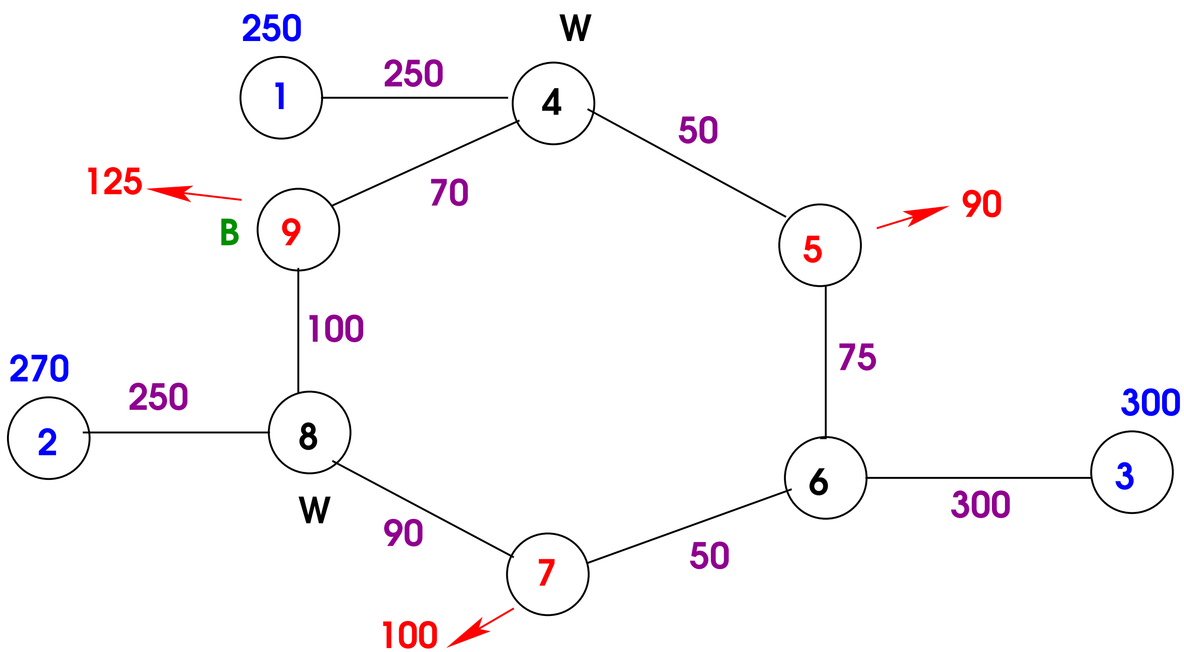
$$\frac{(v_k - v_m)}{x_{km}} \left[\boldsymbol{P^{g*}} + \bar{w} + \hat{\boldsymbol{w}} + \boldsymbol{\bar{\lambda}^*} - \boldsymbol{\lambda^*} \left(\sum_i \hat{\boldsymbol{w}}_{\boldsymbol{i}} \right) - P^d \right] > u_{km}$$

   Comment: could be the reverse flow
4. Then we add, to the relaxation, the cut

$$\frac{(v_k - v_m)}{x_{km}} \left[P^g + \bar{w} + \hat{\boldsymbol{w}} + \bar{\lambda} - \lambda \left(\sum_i \hat{\boldsymbol{w}}_{\boldsymbol{i}} \right) - P^d \right] \leq u_{km}$$

   and continue (goto **1**).
5. Else if the adversary fails, $(\boldsymbol{P^{g*}}, \boldsymbol{\bar{\lambda}^*}, \boldsymbol{\lambda^*})$ is **optimal**

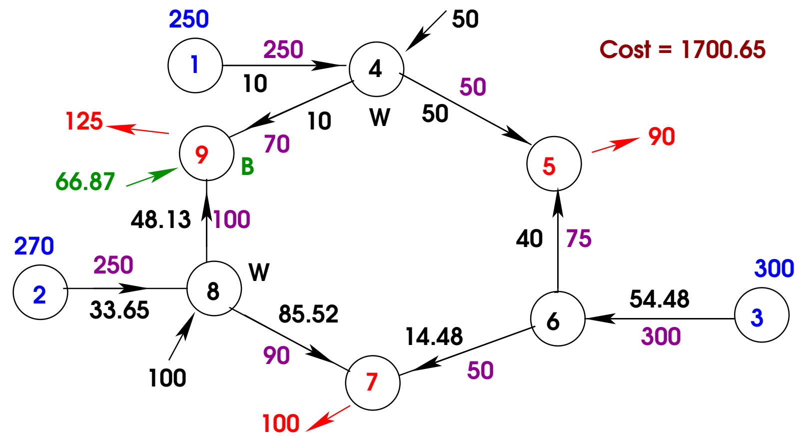
Modified Case9 example



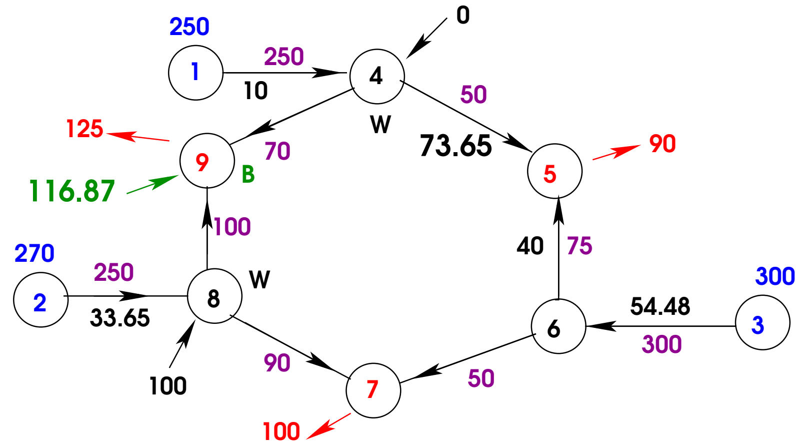
Renewables model:

- Output at **4**: $\mathbf{50 + w_4}$
- Output at **8**: $\mathbf{100 + w_8}$
- $\mathbf{-50 \leq w_4 \leq 0}$, $\mathbf{-100 \leq w_8 \leq 0}$,
- $\mathbf{2|w_4| \ + \ |w_8| \ \leq \ 100}$

Nominal case

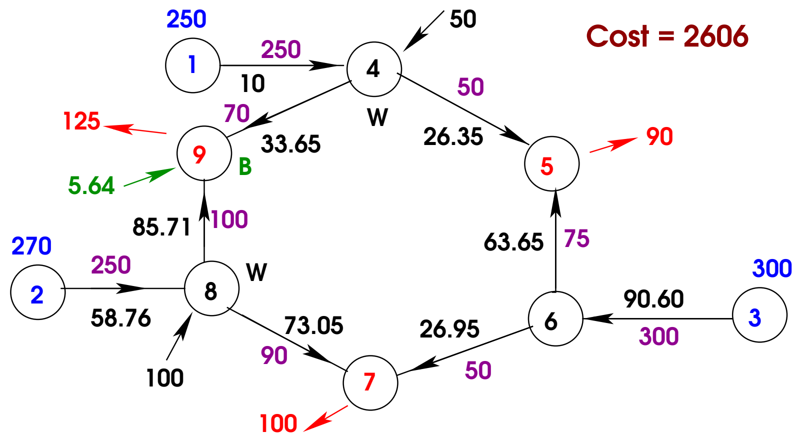


Worst case

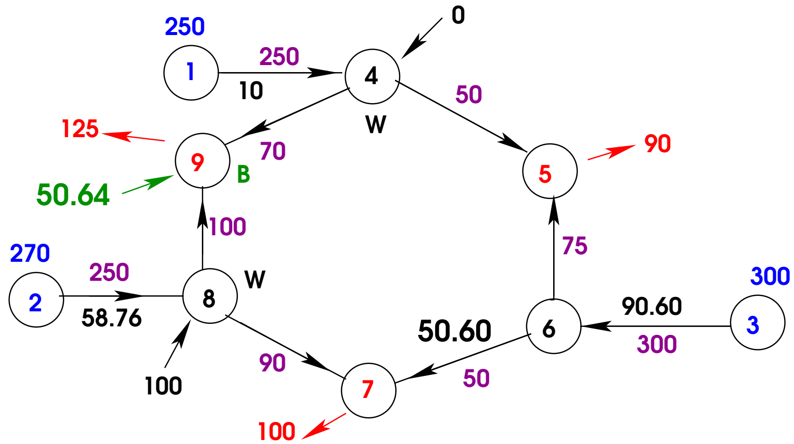


cut: $-0.36P_2^g - 0.62P_3^g - 0.125\bar{\lambda}_9 - 12.5\lambda_9 \leq -90.2$

Updated nominal case



Worst case



A compact formulation (abridged)

A solution $(\boldsymbol{P^g}, \boldsymbol{\bar{\lambda}}, \boldsymbol{\lambda})$ is **safe** for a line $\boldsymbol{km}$ if:

$$(v_k - v_m)\left[P^g + \bar{w} + \boldsymbol{w} + \bar{\lambda} - \lambda\left(\sum_i \boldsymbol{w_i}\right) - P^d\right] \leq x_{km}u_{km}$$

for all $\boldsymbol{w} \in \mathcal{W}$. (Dropped superscript $\boldsymbol{t}$, also should consider line $\boldsymbol{mk}$).

This is the same as

$$x_{km}u_{km} \geq \max_{\boldsymbol{w}\in\mathcal{W}} (v_k - v_m)\left[P^g + \bar{w} + \boldsymbol{w} + \bar{\lambda} - \lambda\left(\sum_i \boldsymbol{w_i}\right) - P^d\right]$$

In the linear model $\mathcal{W} = \{w : Cw \leq d\}$ this is the same as

$$\begin{aligned} x_{km}u_{km} \geq \max\ & (v_k - v_m)\left[P^g + \bar{w} + w + \bar{\lambda} - \lambda\left(\sum_i w_i\right) - P^d\right] \\ \text{s.t.}\ & Cw \leq d \end{aligned}$$

or,

$$\begin{aligned} x_{km}u_{km} \geq\ & (v_k - v_m)\left[P^g + \bar{w} + \bar{\lambda} - P^d\right] + \\ & \max \sum_i \left[(v_{ki} - v_{mi}) - (v_k - v_m)\lambda\right] w_i \\ & \text{s.t.}\ Cw \leq d \end{aligned}$$

A compact formulation (abridged)

A solution $(\boldsymbol{P^g}, \bar{\boldsymbol{\lambda}}, \boldsymbol{\lambda})$ is **safe** for a line $\boldsymbol{km}$ if:

$$(v_k - v_m)\left[P^g + \bar{w} + \boldsymbol{w} + \bar{\lambda} - \lambda\left(\sum_i \boldsymbol{w_i}\right) - P^d\right] \leq x_{km}u_{km}$$

for all $\boldsymbol{w} \in \mathcal{W}$. (Dropped superscript $\boldsymbol{t}$). Or,

$$x_{km}u_{km} \geq (v_k - v_m)\left[P^g + \bar{w} + \bar{\lambda} - P^d\right] + \max \sum_i \left[(v_{ki} - v_{mi}) - (v_k - v_m)\lambda\right] w_i \quad \text{s.t.} \quad Cw \leq d$$

which holds **if and only if** the following system is feasible

$$\pi \geq 0 \tag{1a}$$

$$[\pi^T C]_i = (v_{ki} - v_{mi}) - (v_k - v_m)\lambda \quad \text{for all } i \tag{1b}$$

$$\pi^T d \leq x_{km}u_{km} - (v_k - v_m)\left(P^g - P^d + \bar{\lambda} + \bar{w}\right) \tag{1c}$$

In system (1), the variables $\boldsymbol{\pi}$ should be indexed by $\boldsymbol{km}$ and $\boldsymbol{t}$.

The system must be added to the nominal formulation for each $\boldsymbol{km}$ and $\boldsymbol{t}$.
Total number of added variables: **(# of lines)** $\times$ **(# of renewables)**