Federal Energy Regulatory Commission Technical Conference
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Optimizing Sensor Type and Location for Rapid Restoration of Power Grids

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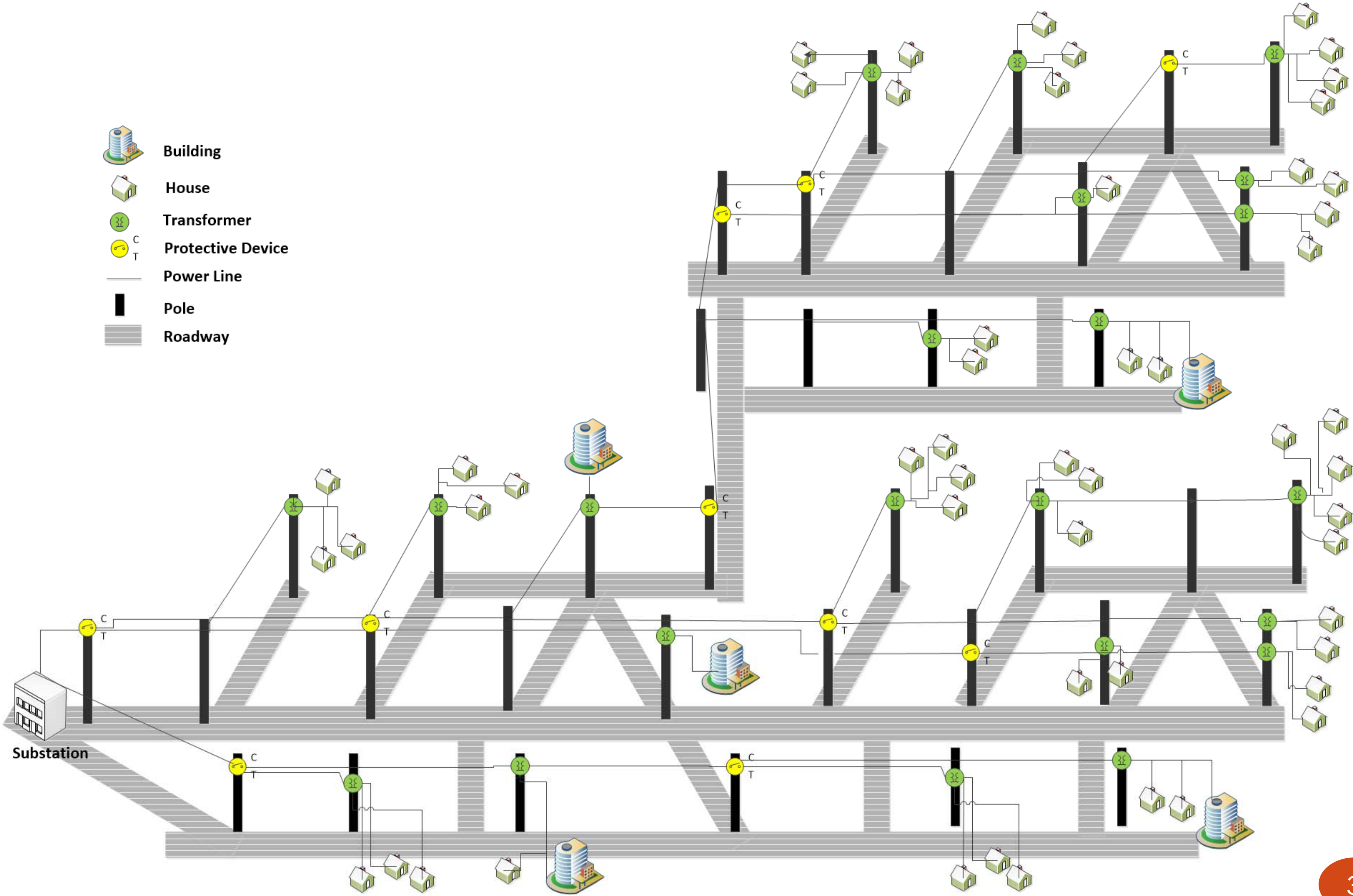
Outline

- Problem Description
- Probability Model for Grid Faults
- Sequential Stochastic Optimization Model for Utility Crew Routing
  - Multistage Lookahead Policy (Monte Carlo Tree Search)
- Stochastic Optimization Model for Sensor and Protective Device Placements

Problem Description – Power System



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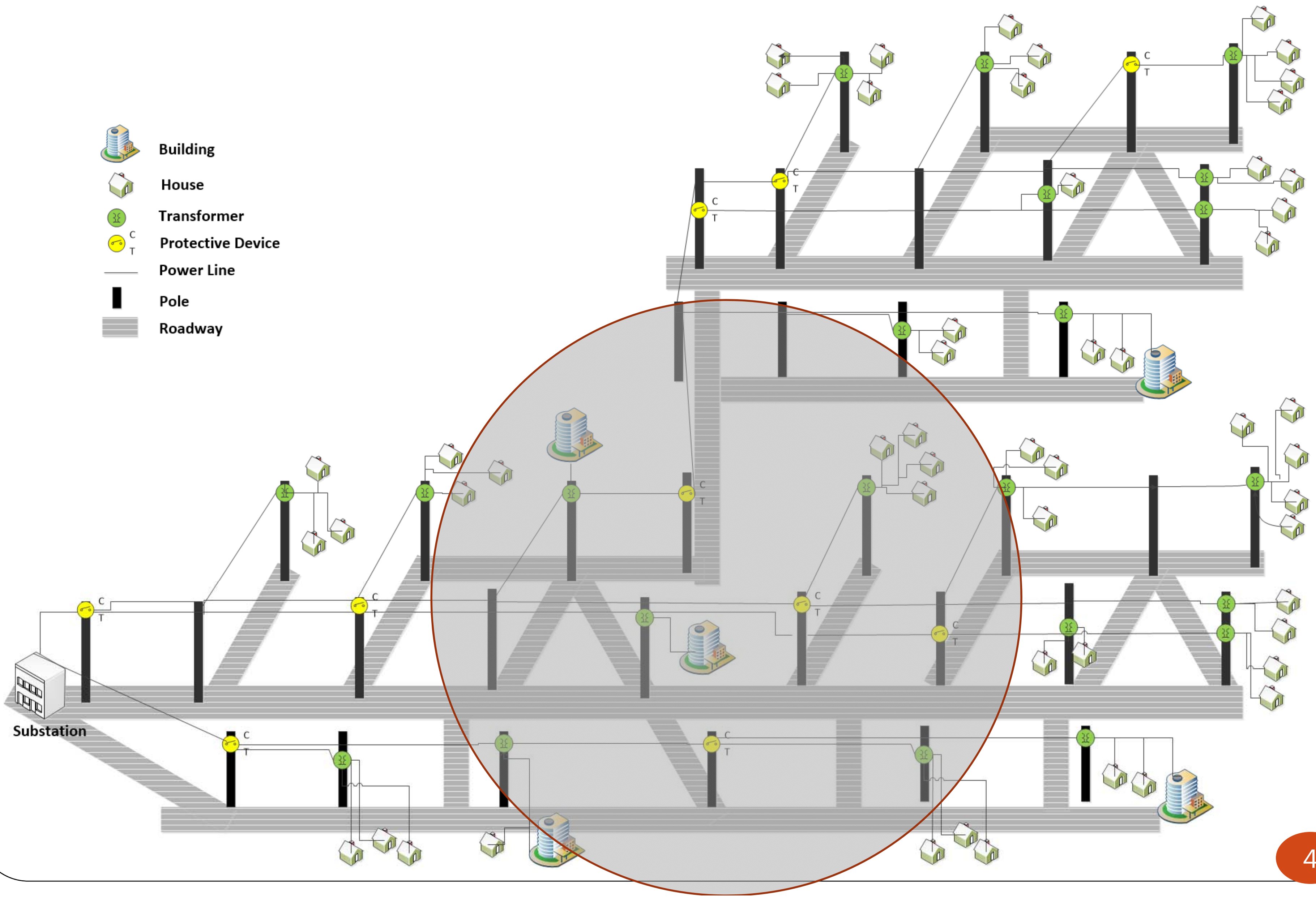

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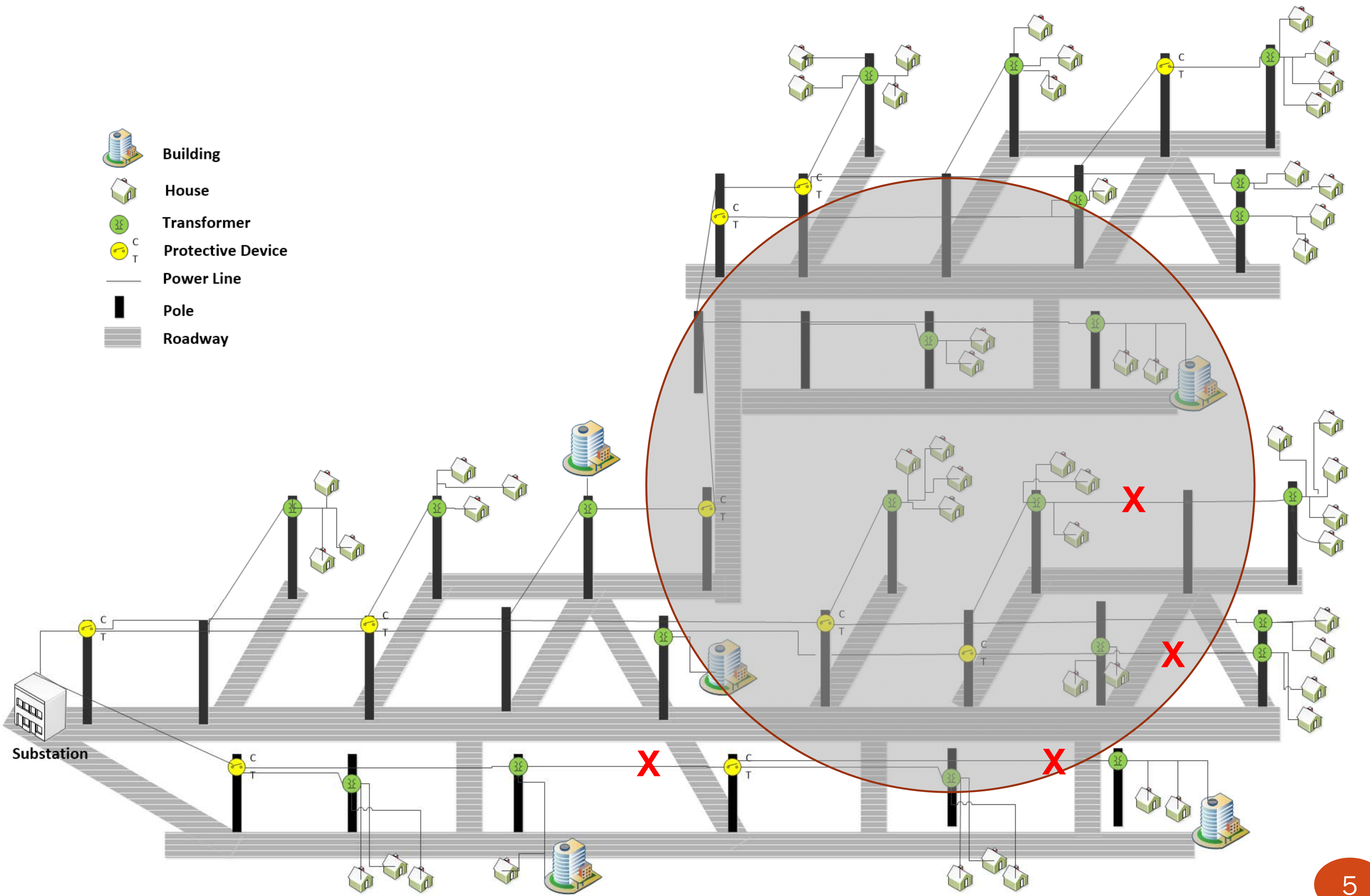

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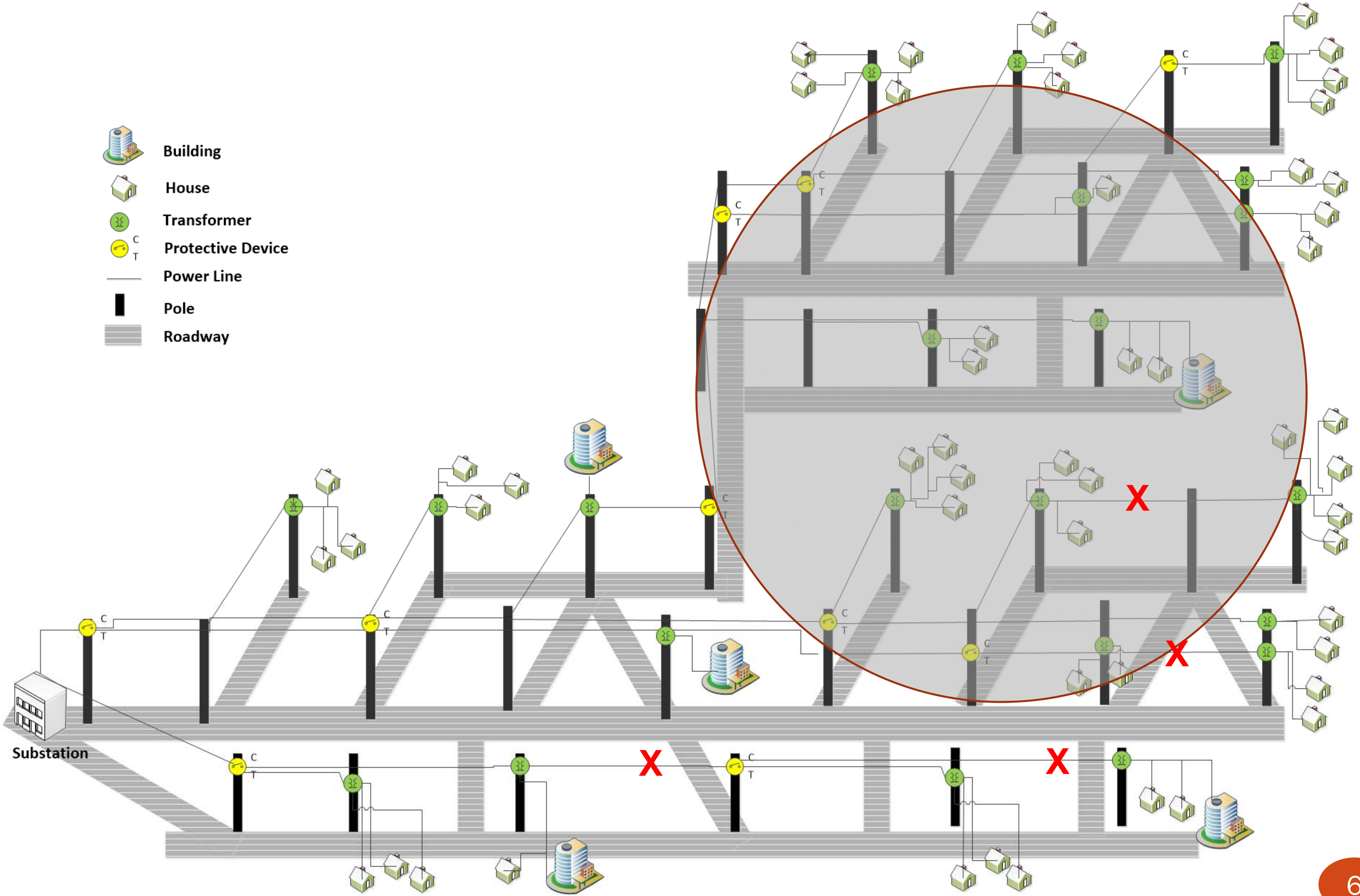

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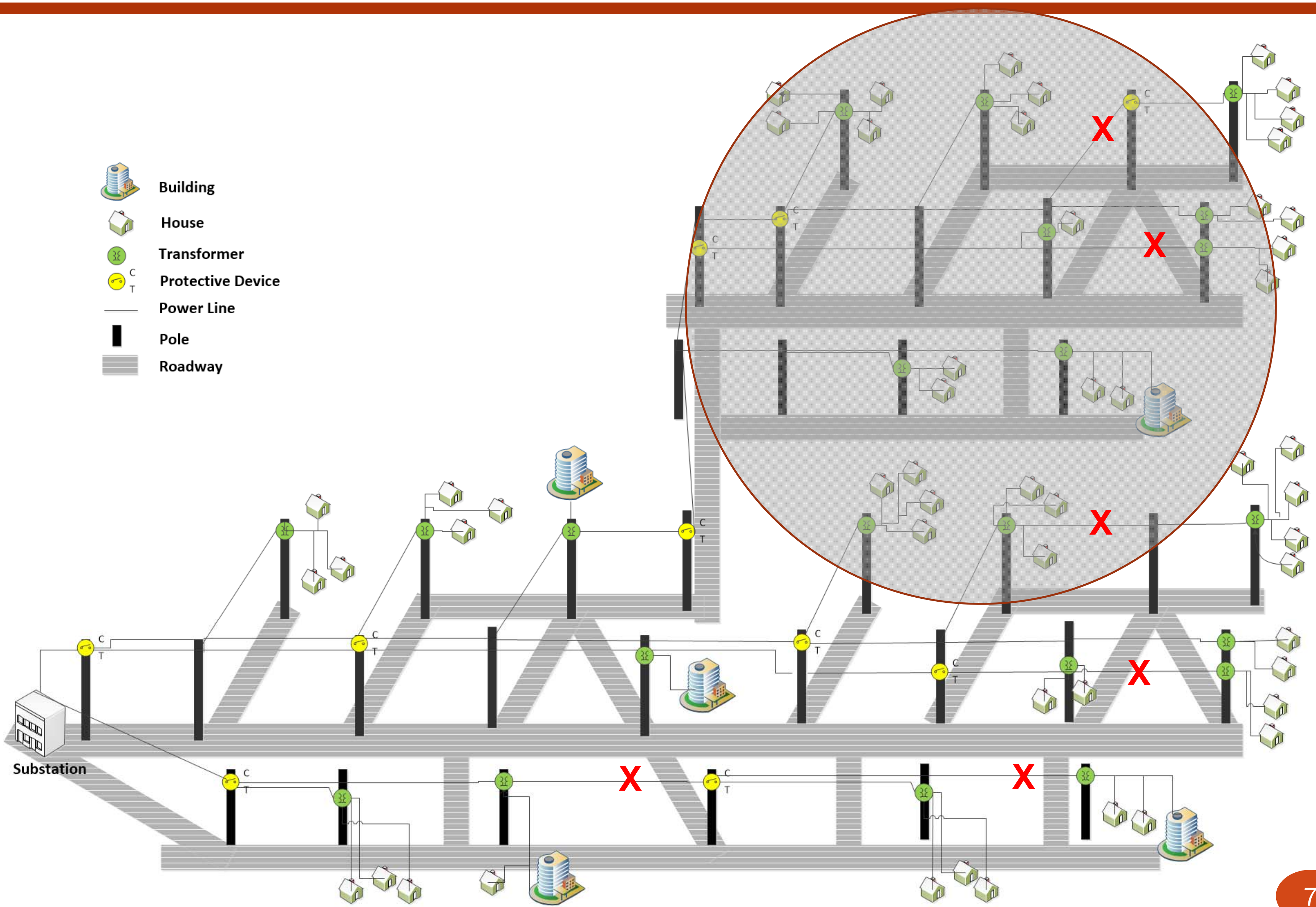
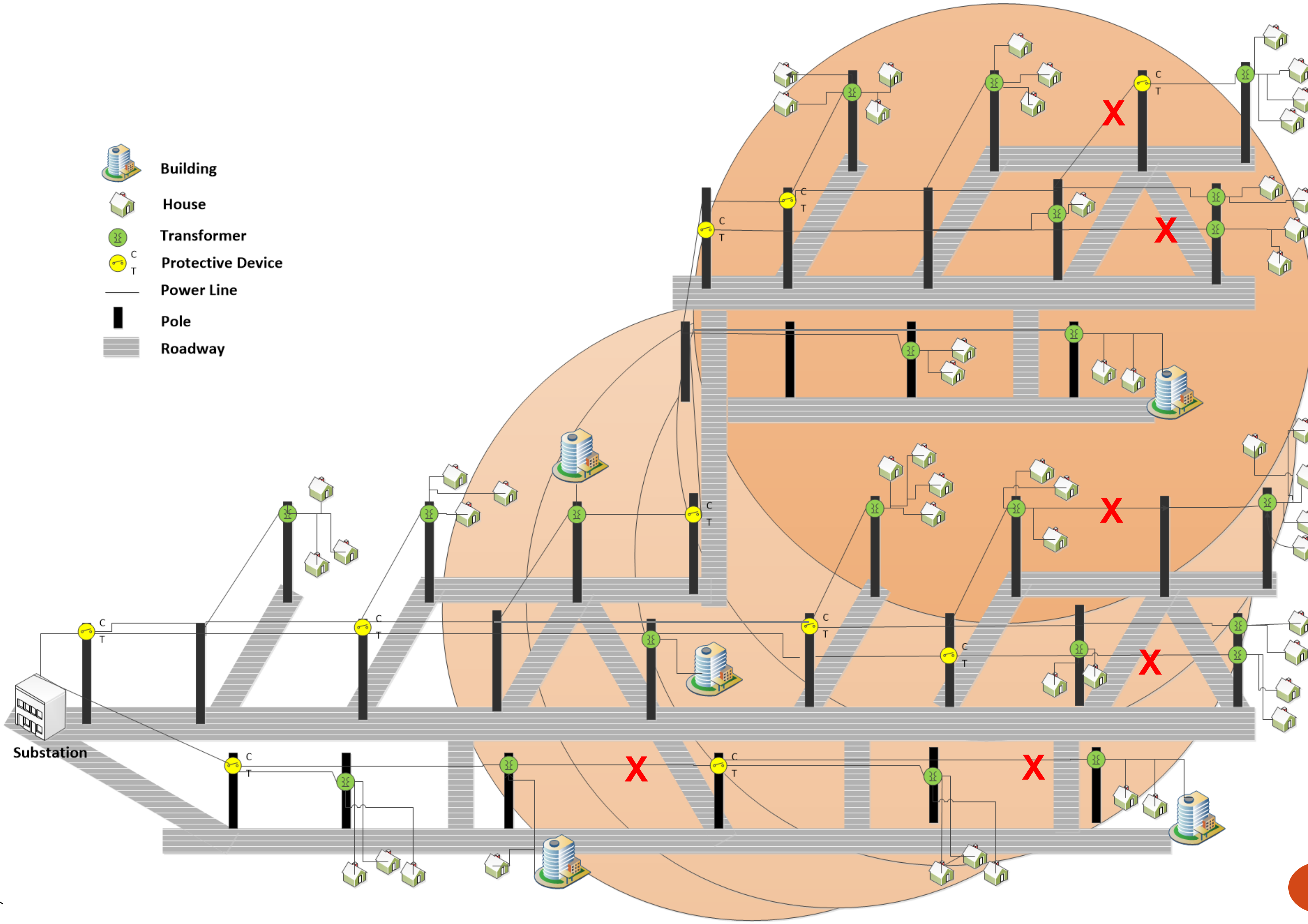


Problem Description - Emergency Storm Response



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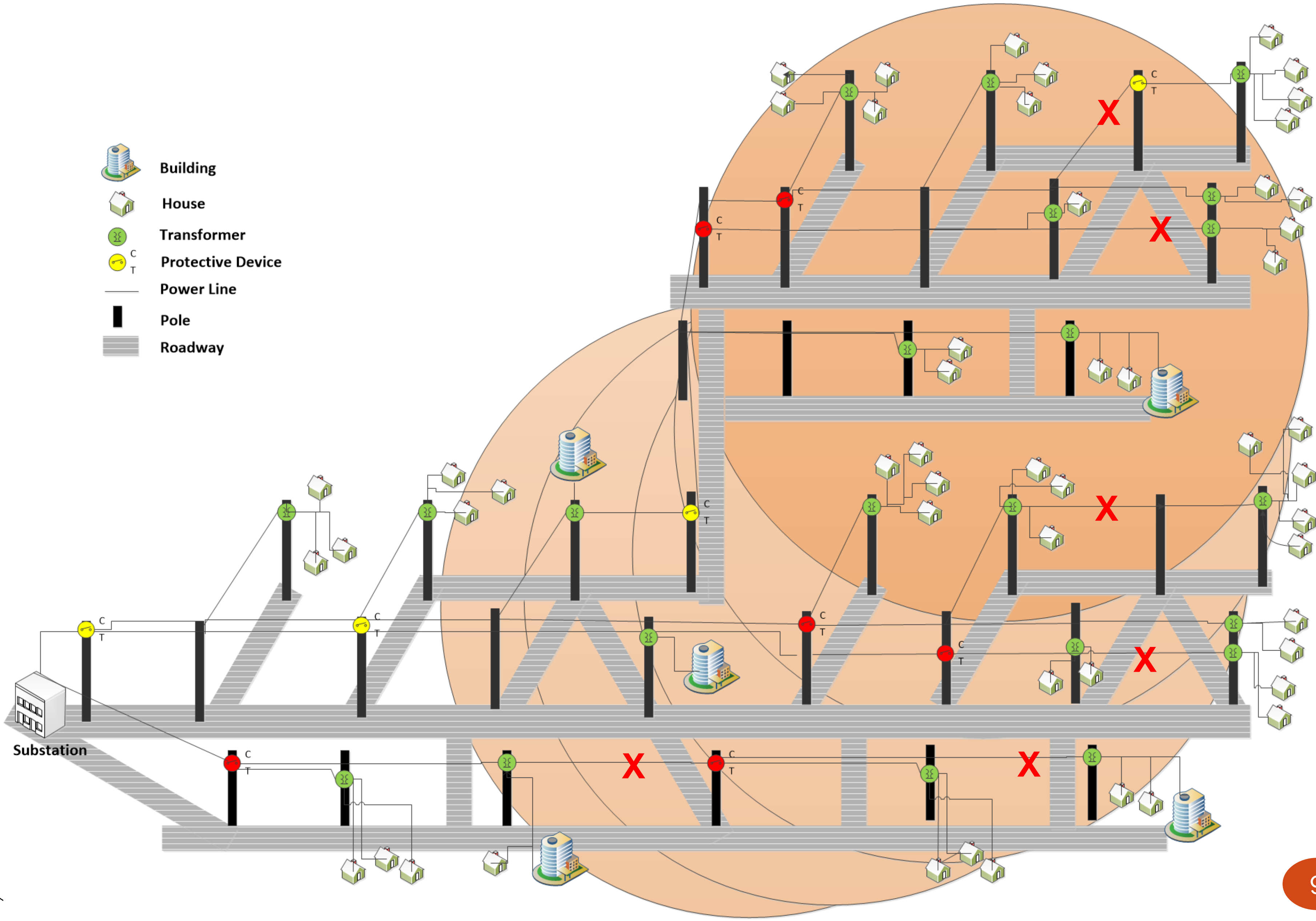

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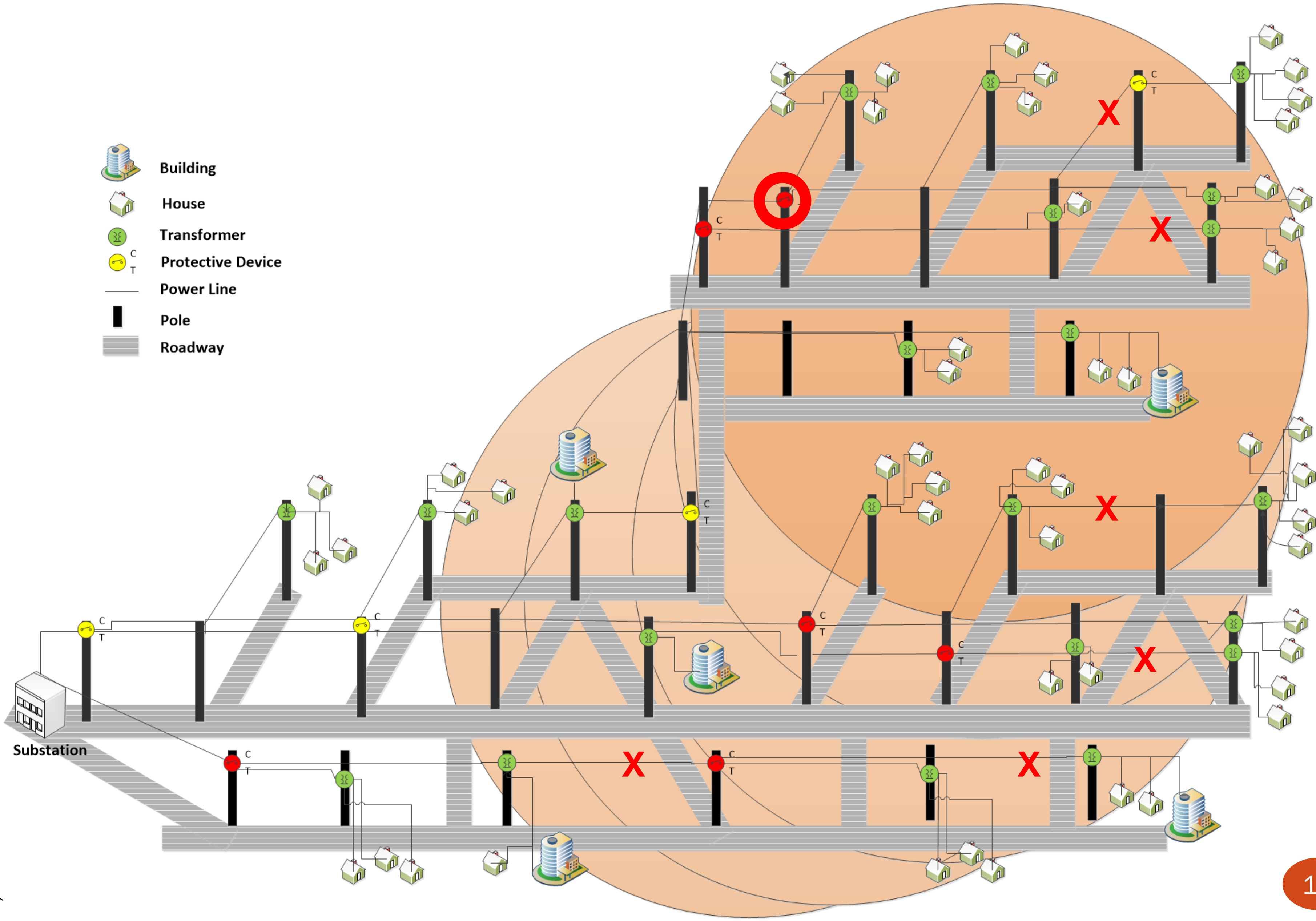

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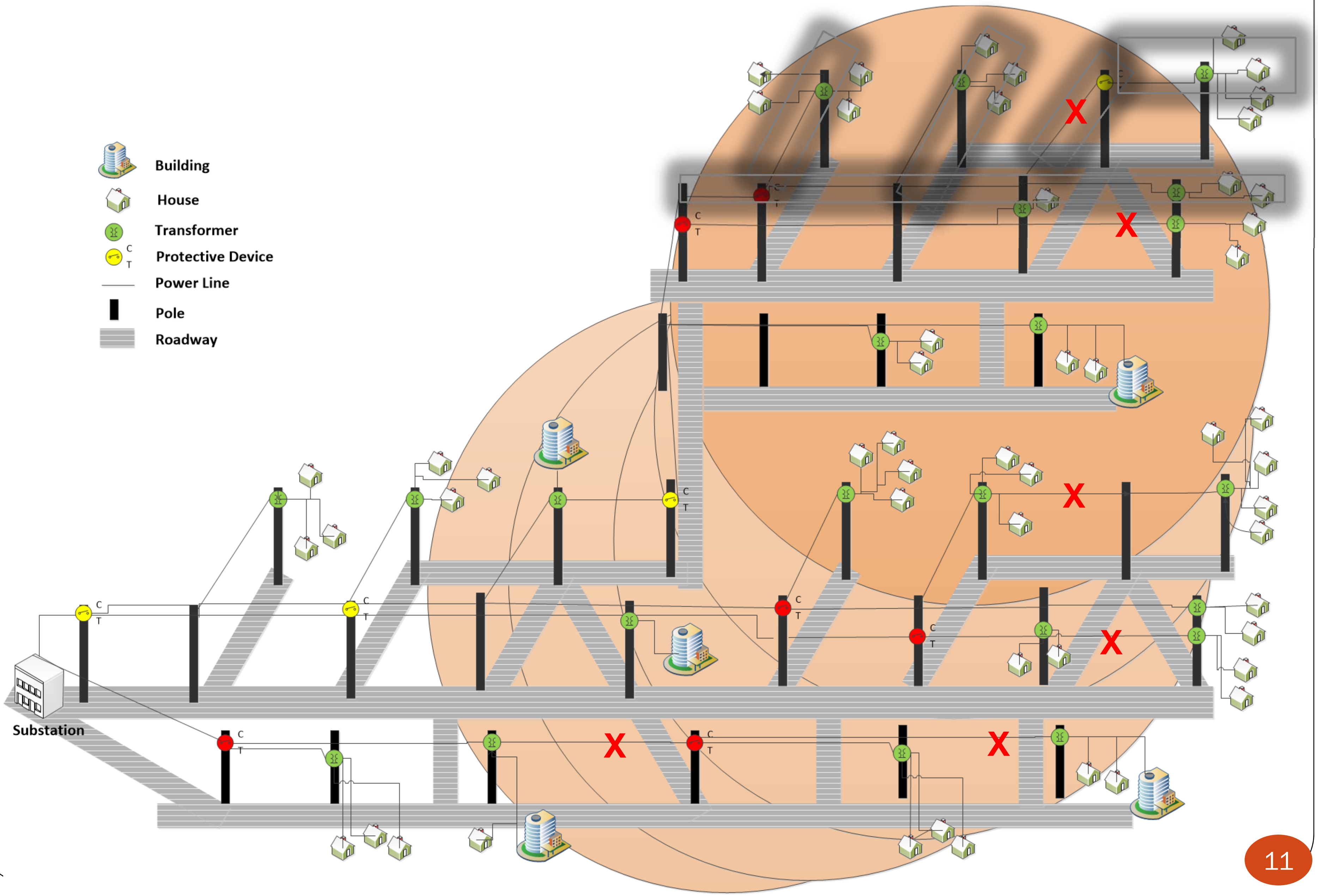

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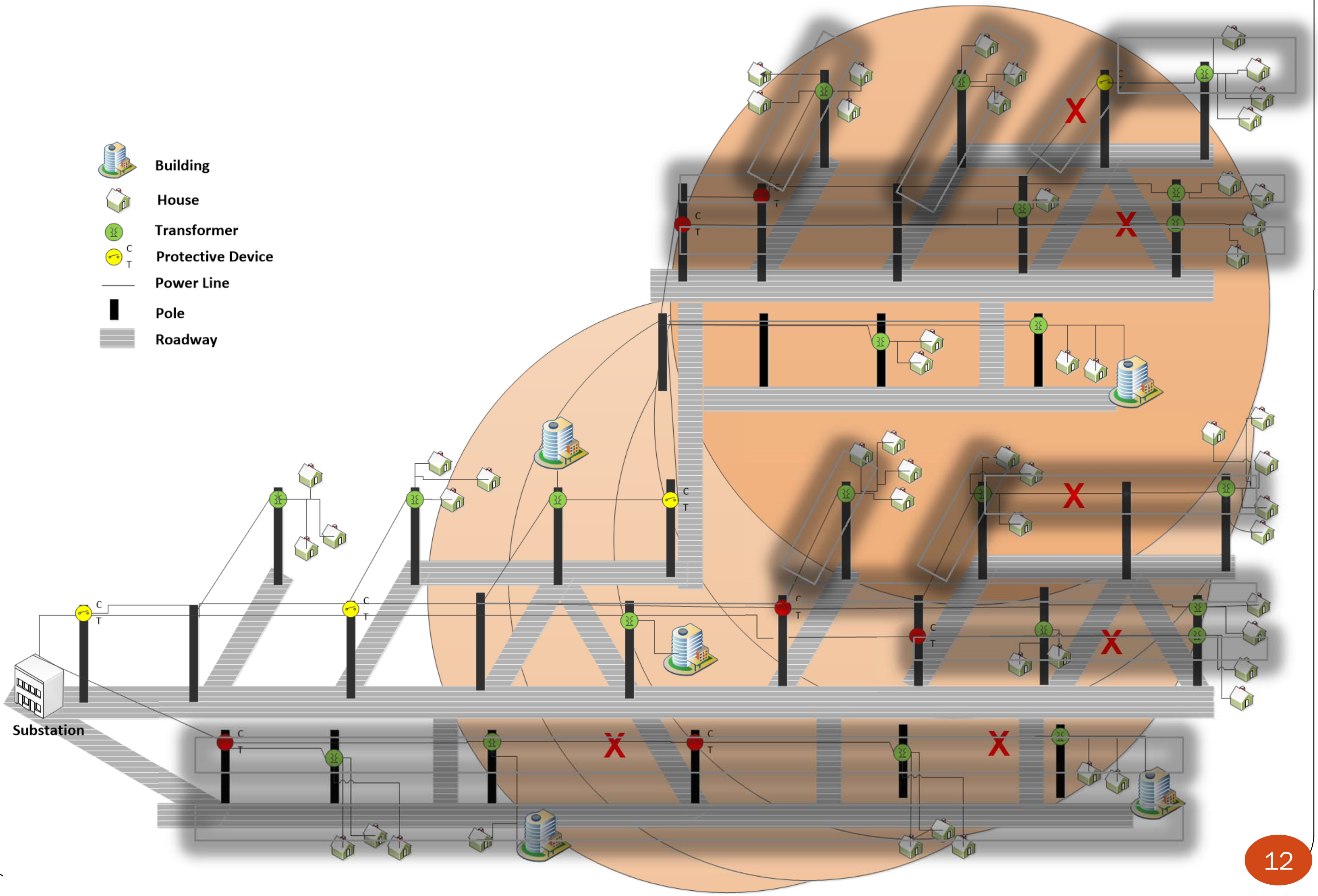

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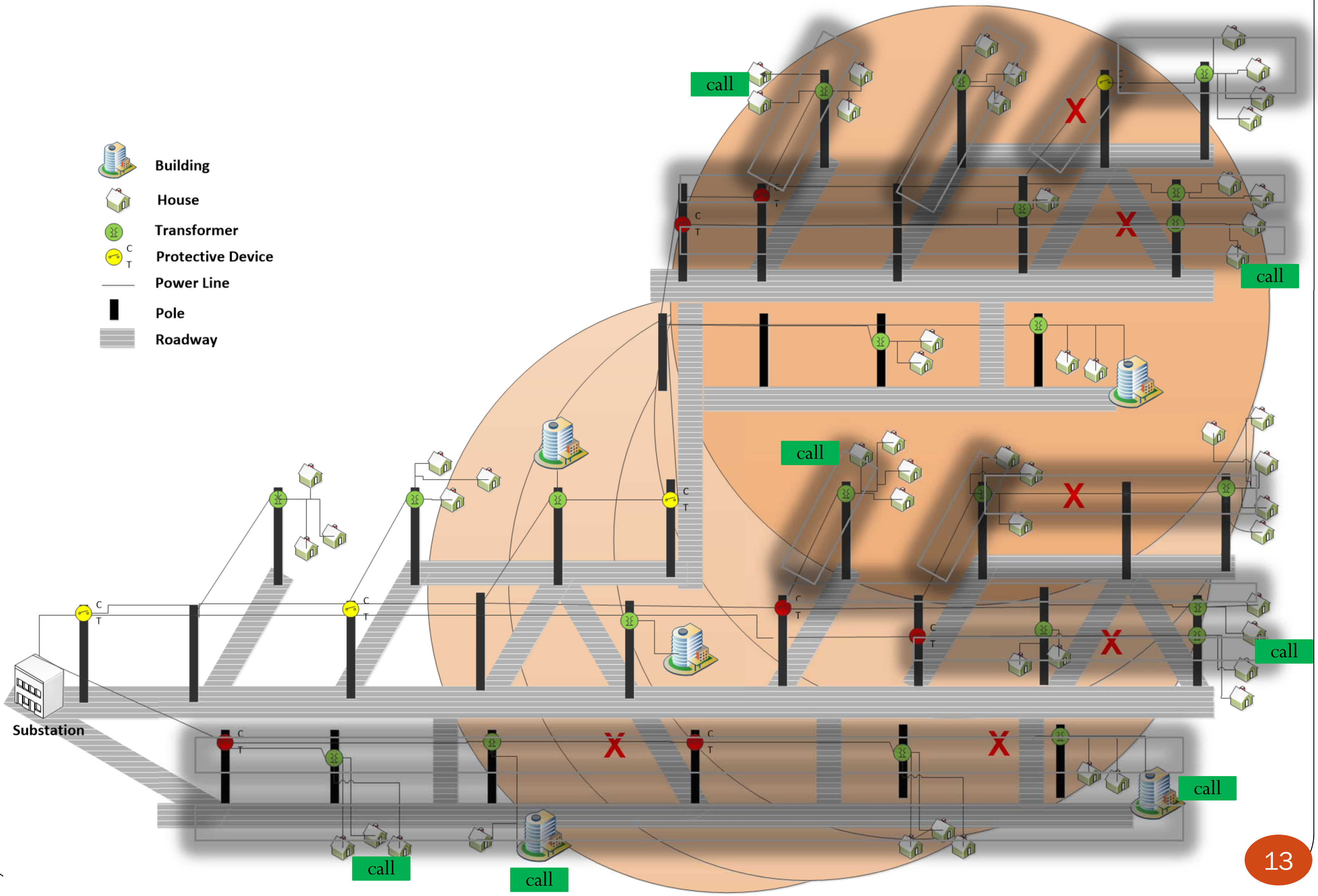

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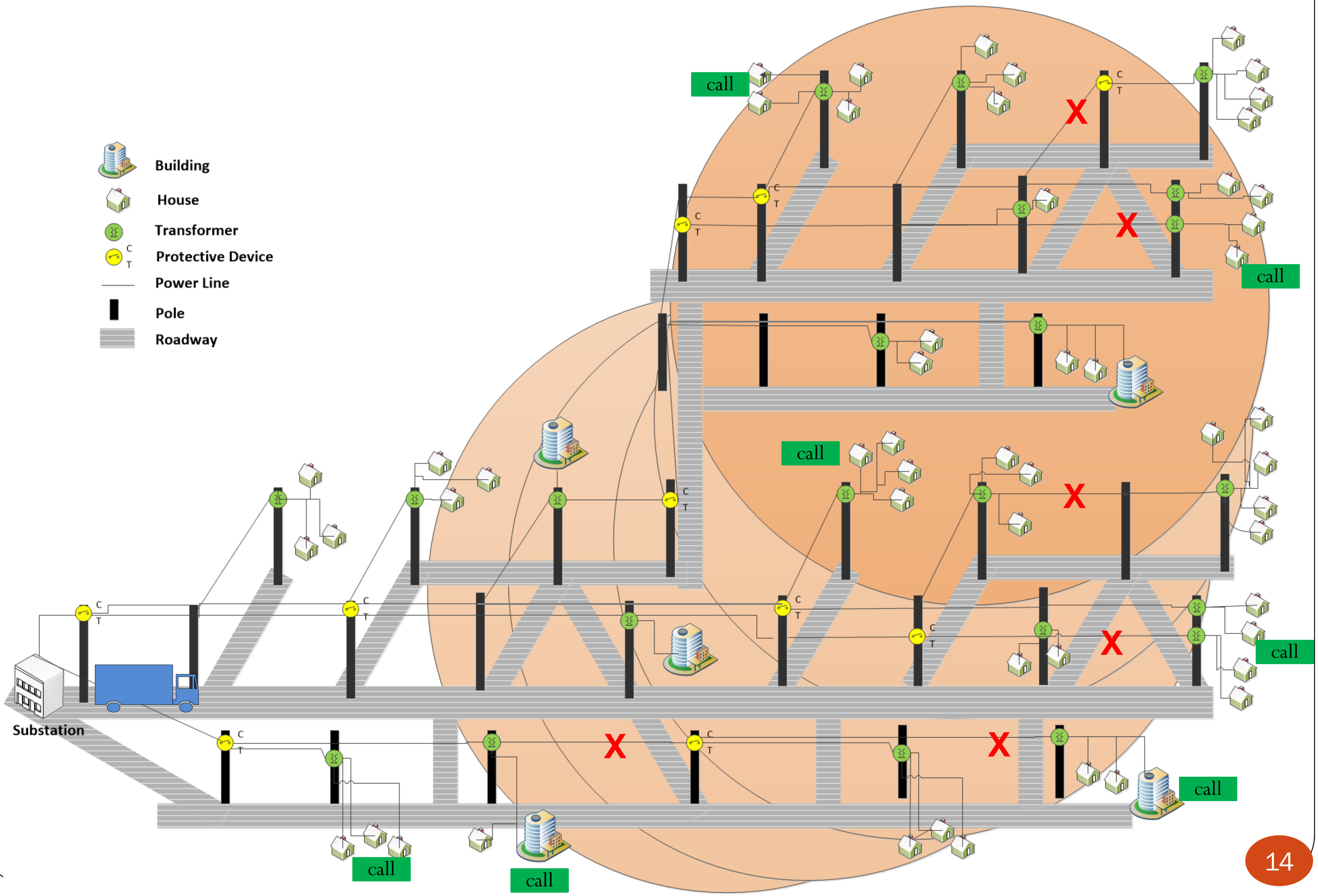

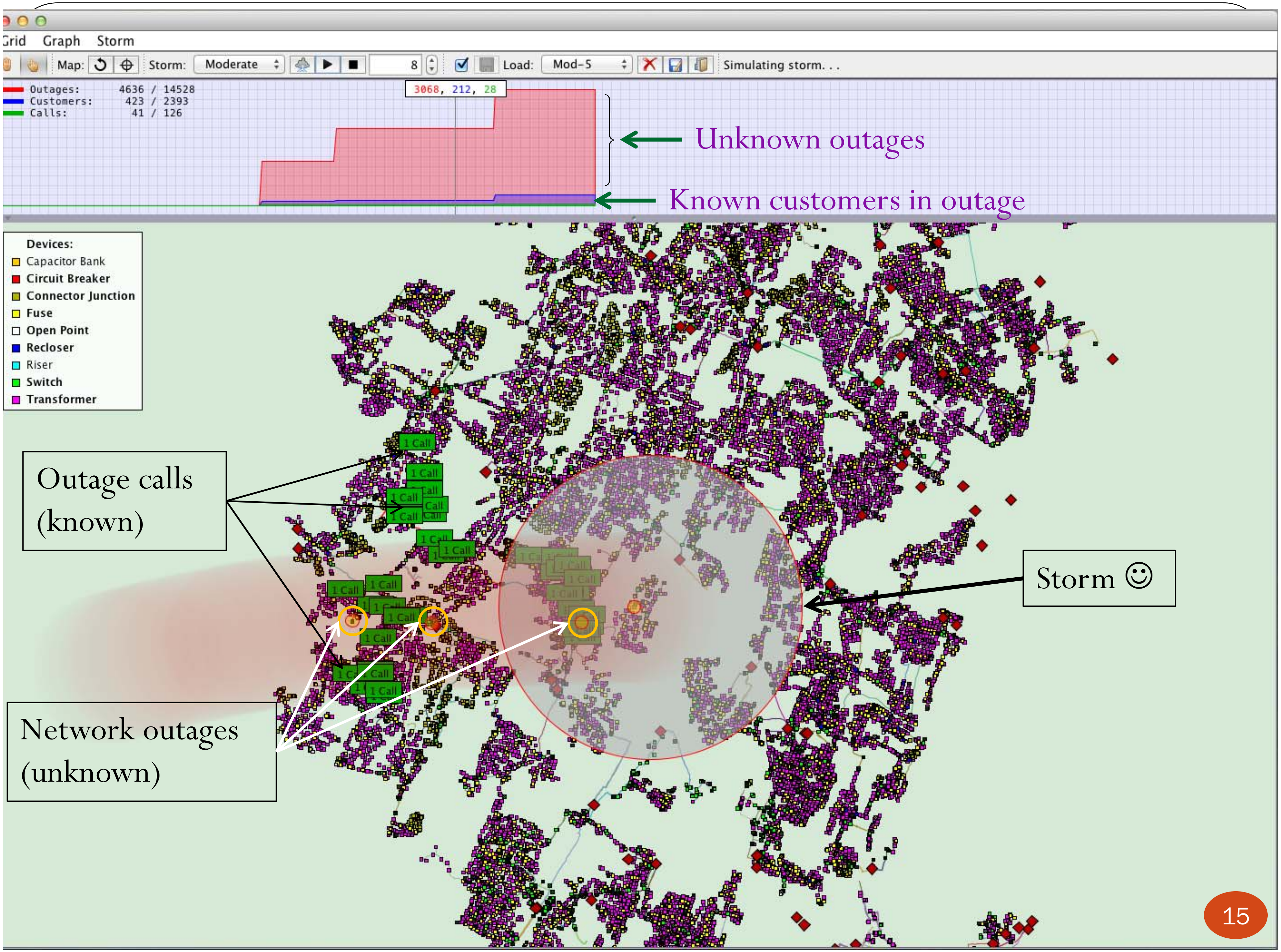
Grid Graph Storm
Map: Storm: Moderate 8 Load: Mod-5 Simulating storm. . .
Outages: 4636 / 14528
Customers: 423 / 2393
Calls: 41 / 126
3068, 212, 28
Unknown outages
Known customers in outage
Devices:
Capacitor Bank
Circuit Breaker
Connector Junction
Fuse
Open Point
Recloser
Riser
Switch
Transformer
Outage calls
(known)
Network outages
(unknown)
Storm ☺
1 Call

Problem Description – Research Goals

- How to create a belief about outages?
- How to reconstruct the grid to minimize customer outage-minutes?
- What is the value of a sensor/re-closer in terms of reducing customer outage-minutes?
- What is the best place to put a sensor or re-closer?
- What is the value of upgrades to SCADA systems?
- What is the value of investments in reconfigurable grids?

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Probability Model for Outage Identification

- Exploiting the information from phone calls
  - We have to blend the following….
    - What we knew before the phone calls came in – this is called the *prior belief*.
    - The outage calls – this is called *information*.
  - …to produce the updated estimates of outage – this is called *the posterior*.
- To compute this we have to use *Bayes theorem:*

$$\text{Prob}[\text{segment } l \text{ is out}|\text{lights-out calls}] = \frac{\text{Prob}[\text{lights-out calls} \mid \text{segment } l \text{ is out}]\text{Prob}[l \text{ is out}]}{\text{Prob}[\text{lights-out calls}]}$$

[1] L. Al-Kanj, B. Bouzaeini-Ayari and W. Powell, "A Probability Model for Grid Faults Using Incomplete Information", *IEEE Transactions on Smart Grids,* July 2015.

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The posterior

The information

The conditional outage distribution

The prior

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The posterior

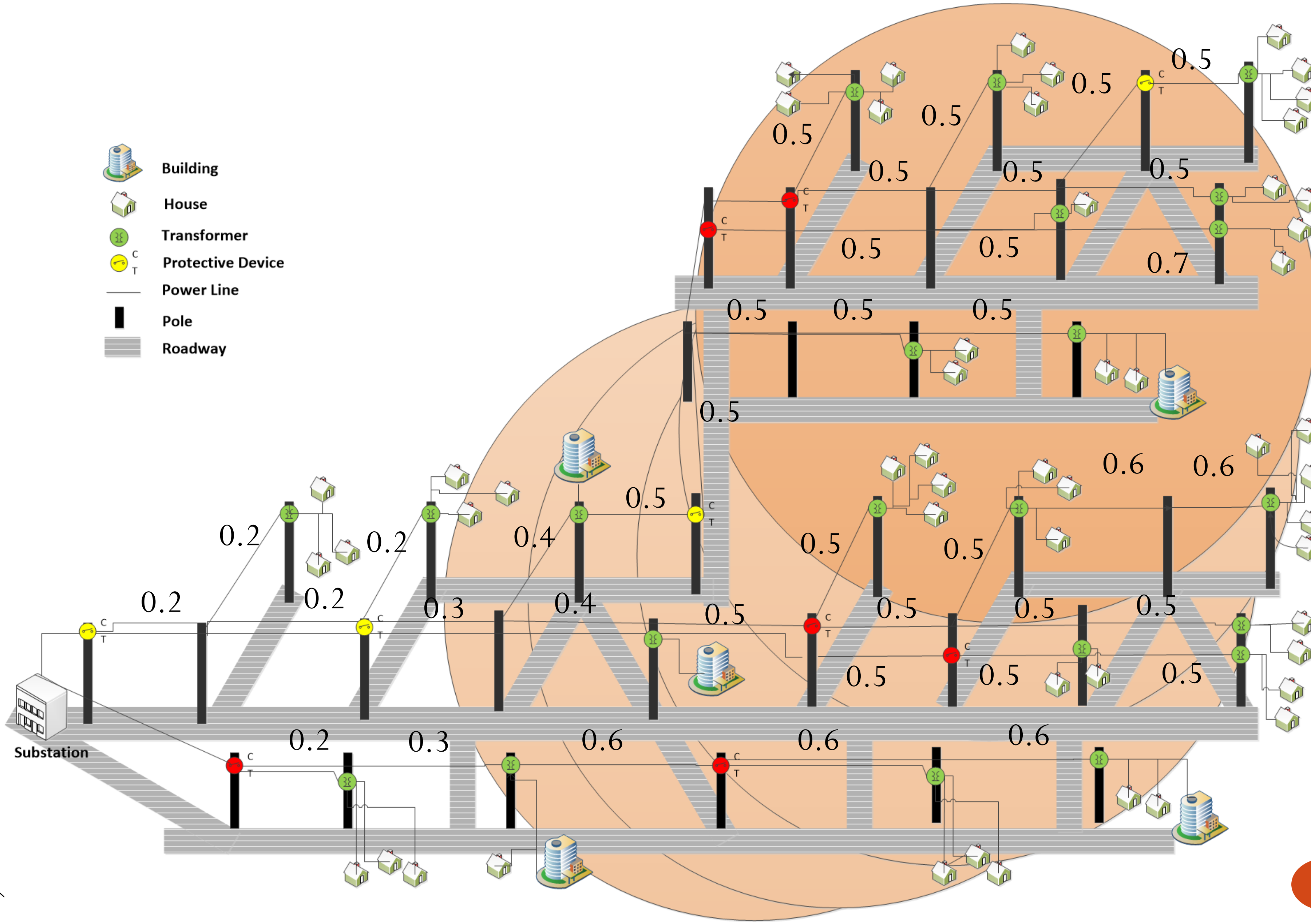
The information

The conditional outage distribution

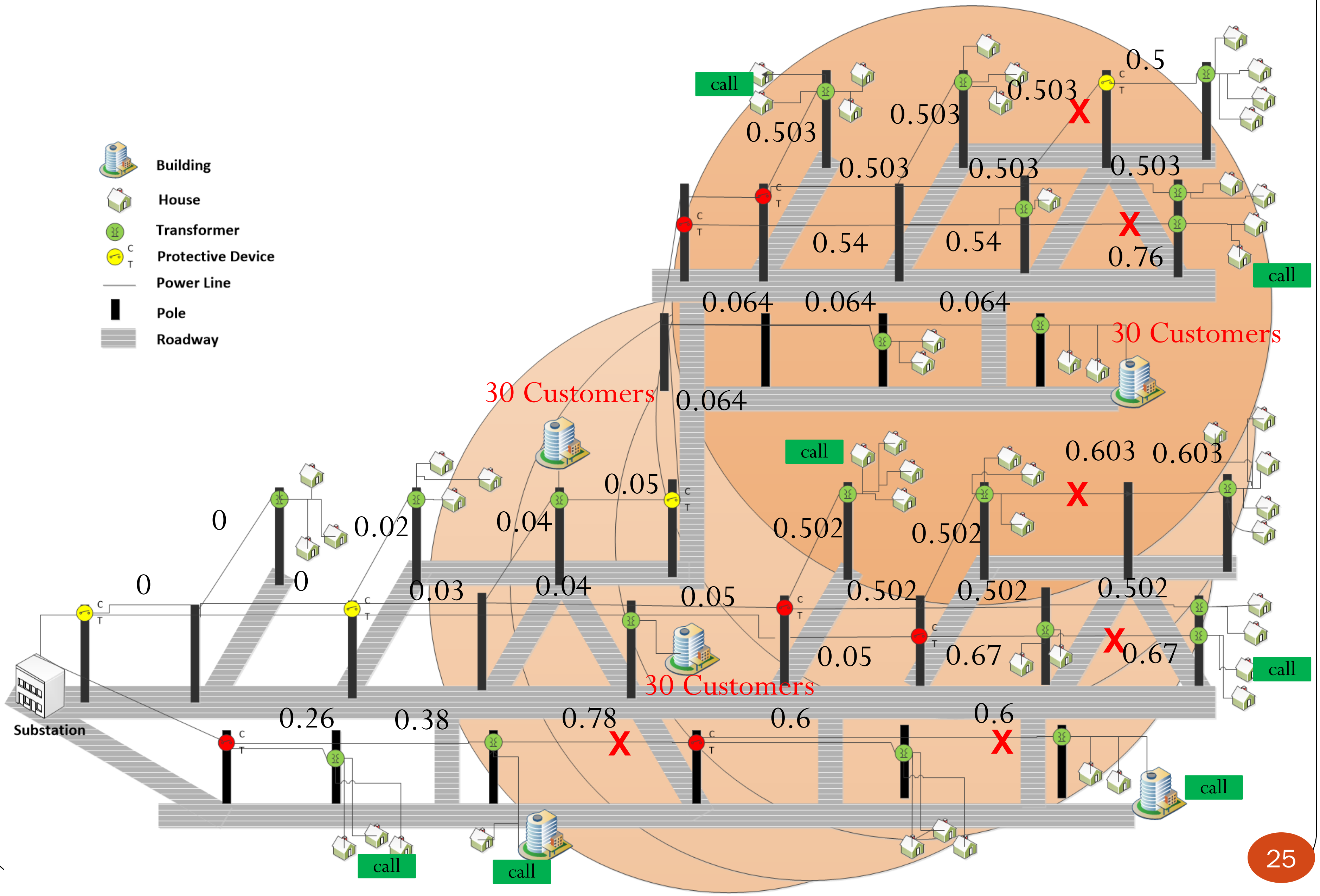
The prior

The phone call distribution

Probability Model – Prior Probabilities



Posterior Probabilities



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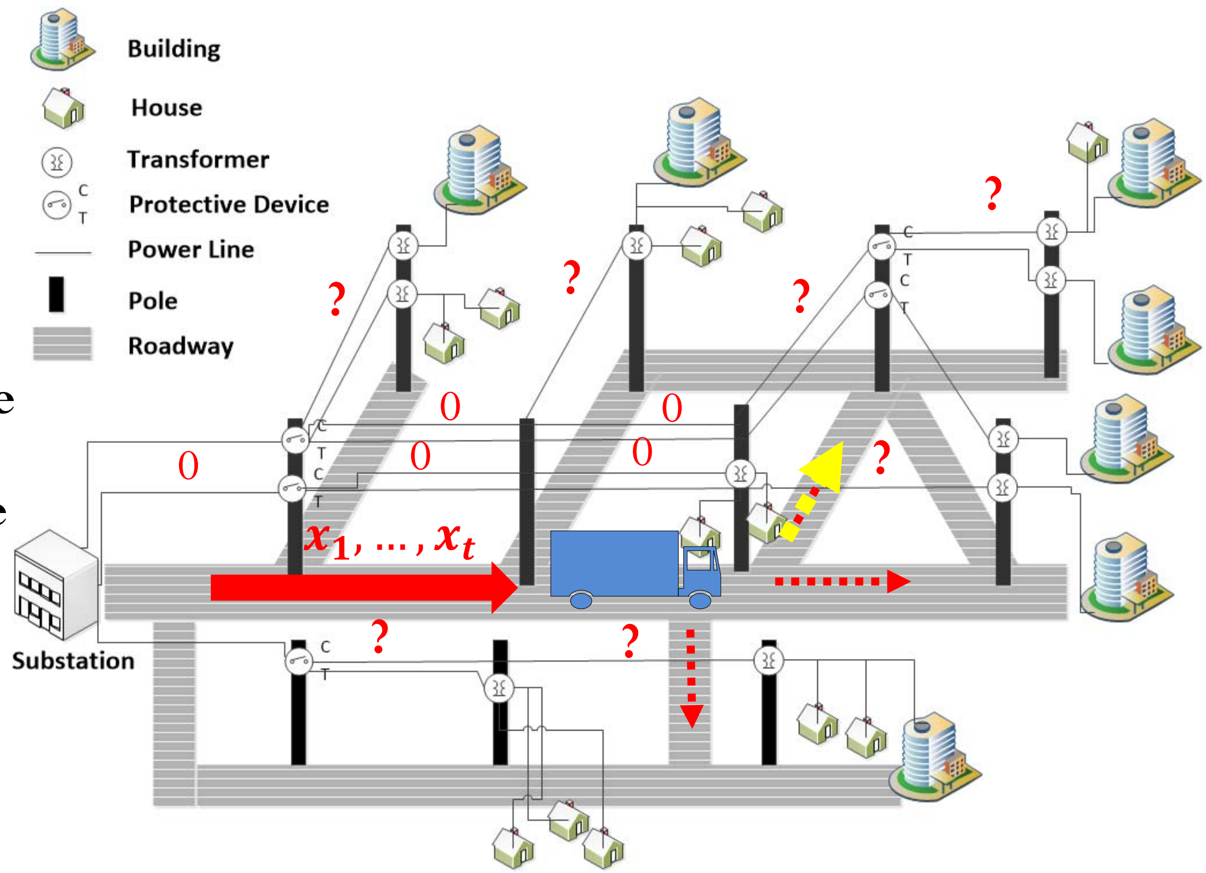
Grid Restoration Model

- Objective: Develop an optimal policy that routes the utility truck in order to minimize the number of customers in outage at any point in time.

R_t: Physical State

I_t: Information State

K_t: Knowledge State



Information-Collection utility truck routing

[4] **L. Al-Kanj,** W. Powell and B. Bouzaeini-Ayari, "The Information-Collecting Vehicle Routing Problem: Stochastic Optimization for Emergency Storm Response", *Submitted to Operations Research, http://arxiv.org/abs/1605.05711, 2016*.

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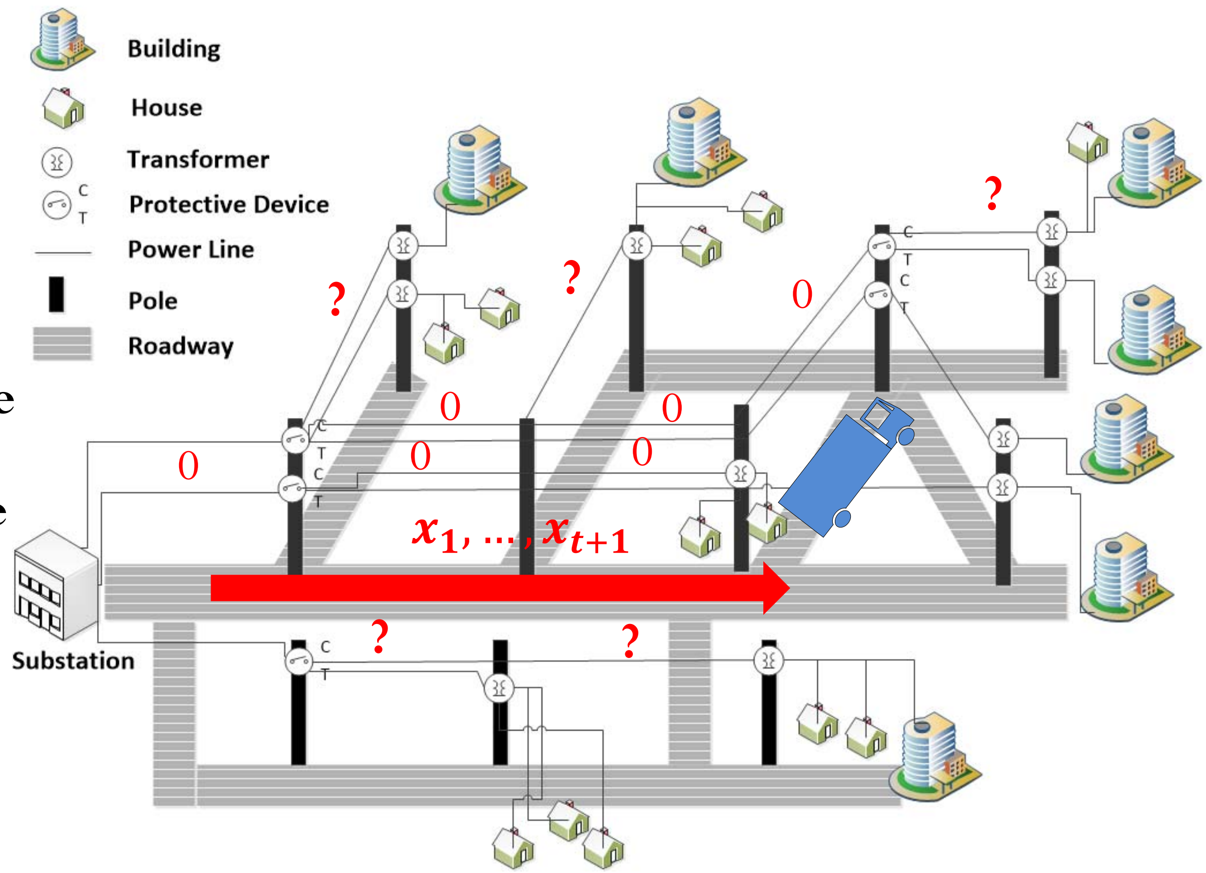

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[4] **L. Al-Kanj,** W. Powell and B. Bouzaeini-Ayari, "The Information-Collecting Vehicle Routing Problem: Stochastic Optimization for Emergency Storm Response", *In preparation*.

Sequential Stochastic Optimization Model

- Five fundamental elements of sequential stochastic optimization:
  - State S_t - information capturing what we know at time t;
    $S_t = (R_t, P_t^L, H_t)$ where R_t represents the physical state of the network.
  - Decision x_t - captures the decision made at time t;
    Let $X_t^\pi(S_t)$ be the policy that determines $x_t \in \mathcal{X}_t$ given S_t.
  - Exogenous information W_t - new information that arrives between $t-1$ and t;
    Includes new arriving calls, travel time, repair time and outages.
  - Transition function - $S_{t+1} = S^M(S_t, x_t, W_{t+1})$ represents the evolution of the states
    e.g., $H_{t+1} = H_t + \widehat{H}_{t+1}, p\left(L_{t+1,j} = 1 | x_{t,ij} = 1\right) = 0$ in addition to W_{t+1}.
  - Objective function - $\min_\pi \mathbb{E}^\pi[\sum_{t=0}^{T} C\,(S_t, X^\pi(S_t))]$;
    minimizes the number of customers in outage at any point in time.

Sequential Stochastic Optimization Model

- Cost function - $\sum_{t=0}^{T} C(S_t, X_t^{\pi}(S_t))$

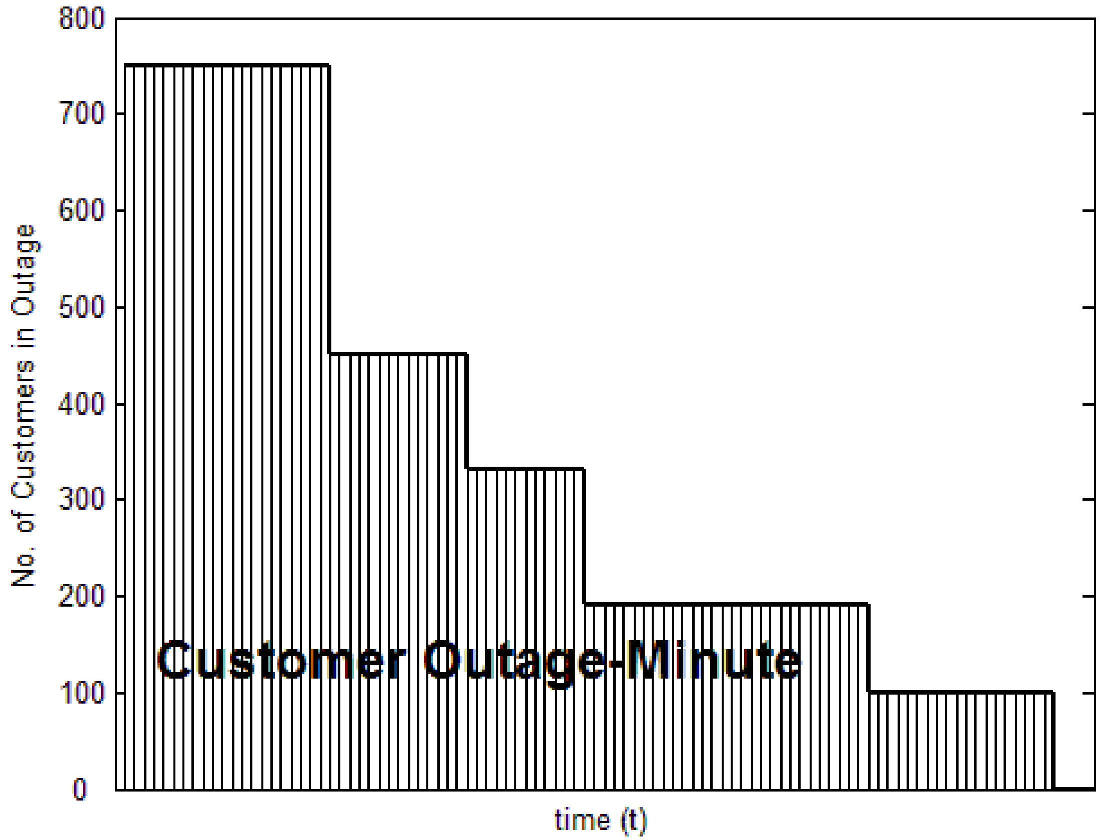


Figure 6: Objective function; Customer outage-minute is represented by the shaded area under the curve.

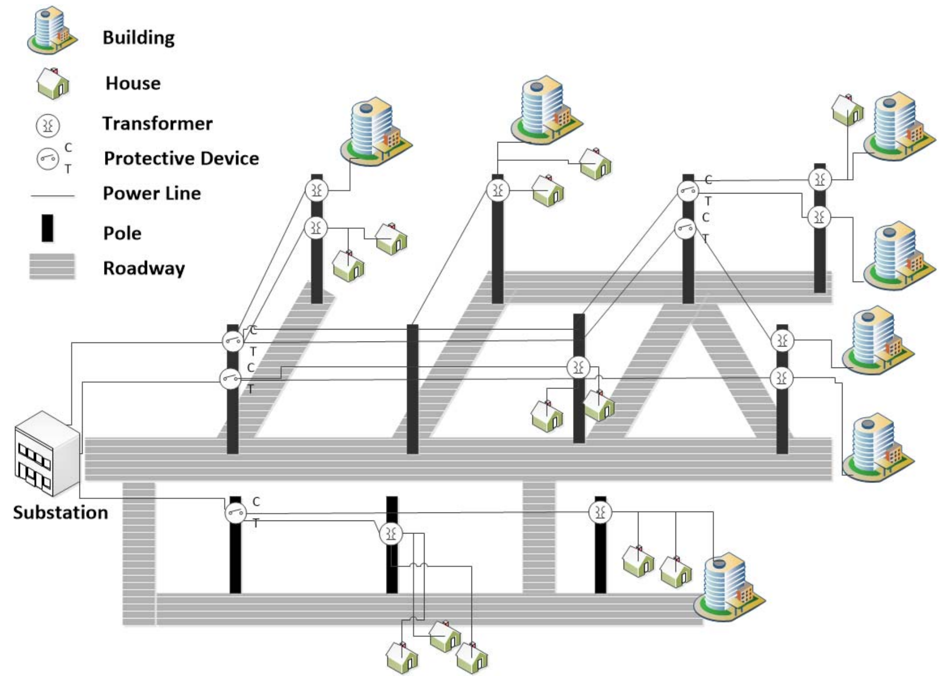
Sequential Stochastic Optimization Model

- Optimization problem

$$\min_{\pi} \mathbb{E}^{\pi}\left[\sum_{t=0}^{T} C\left(S_t, X^{\pi}(S_t)\right)\right]$$

where

$$S_{t+1} = S^M(S_t, x_t, W_{t+1})$$



- Two fundamental strategies for designing policies [5]:
  - **Policy Search**
  - **Lookahead Approximations**
    - Value function approximation
    - Direct lookahead

[2] **W. B Powell, A Unified Framework for Optimization under Uncertainty, Informs Tutorials, November 2016.**

Lookahead Approximations

- Lookahead approximations – Approximate the impact of a decision now on the future:
  - An optimal policy (based on looking ahead):

$$X_t^*(S_t) = \arg\max_{x_t} \left(C(S_t, x_t) + \mathbb{E}\left\{ \max_{\pi \in \Pi} \left\{ \mathbb{E} \sum_{t'=t+1}^{T} C(S_{t'}, X_{t'}^{\pi}(S_{t'})) \mid S_{t+1} \right\} \mid S_t, x_t \right\} \right)$$

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2a) Approximating the value of being in a downstream state using machine learning ("value function approximations")

$$X_t^*(S_t) = \arg\max_{x_t} \left(C(S_t, x_t) + \mathbb{E}\left\{ V_{t+1}(S_{t+1}) \mid S_t, x_t \right\} \right)$$

$$X_t^{VFA}(S_t) = \arg\max_{x_t} \left(C(S_t, x_t) + \mathbb{E}\left\{ \overline{V}_{t+1}(S_{t+1}) \mid S_t, x_t \right\} \right)$$

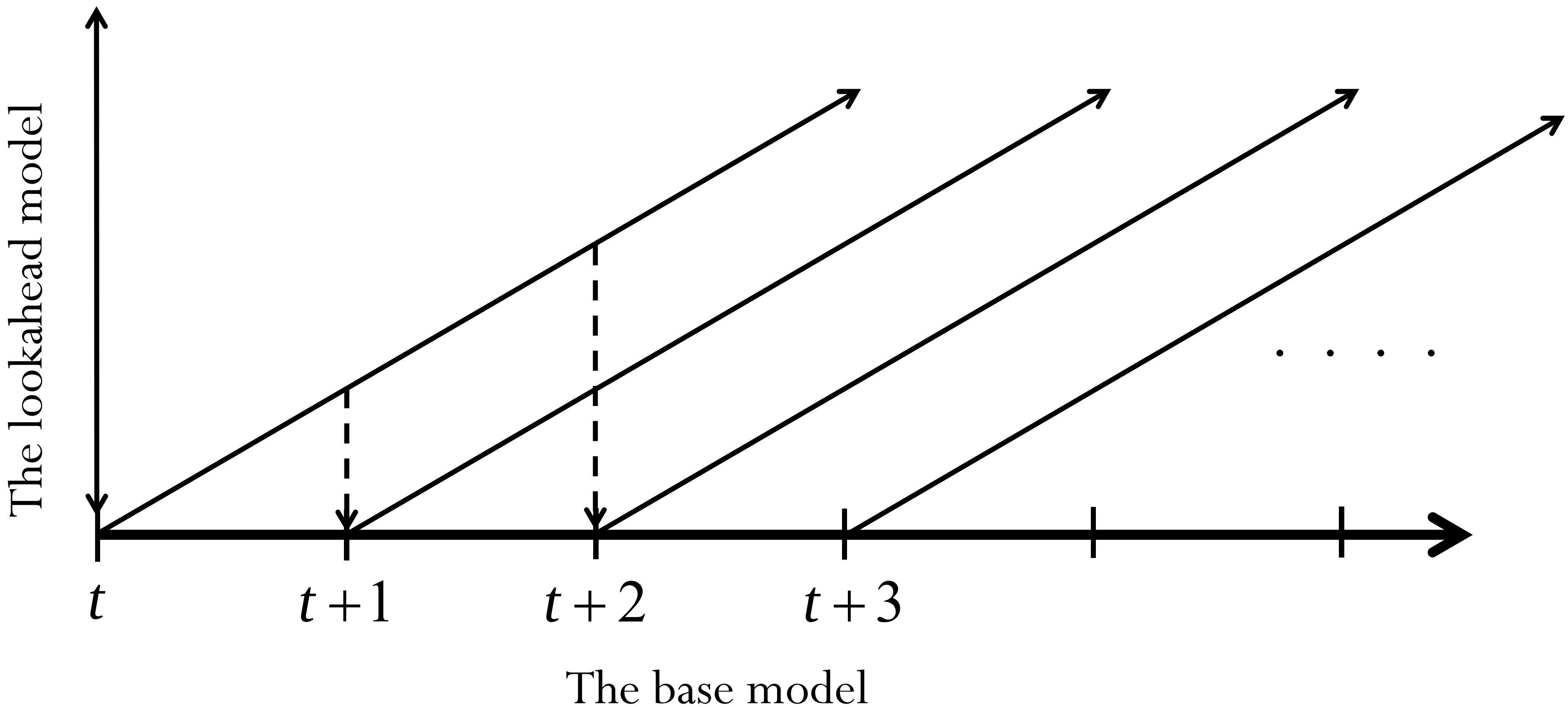
$$= \arg\max_{x_t} \left(C(S_t, x_t) + \overline{V}_t^x(S_t^x) \right)$$

2b) Approximate lookahead models – Optimize over an approximate model of the future:

$$X_t^{LA}(S_t) = \arg\max \left(C(S_t, x_t) + \tilde{\mathbb{E}}\left\{ \max_{\tilde{\pi}\in\tilde{\Pi}} \left\{ \tilde{\mathbb{E}} \sum_{t'=t+1}^{T} C(\tilde{S}_{tt'}, \tilde{X}^{\tilde{\pi}}(\tilde{S}_{tt'})) \mid \tilde{S}_{t,t+!} \right\} \mid S_t, x_t \right\} \right)$$

Multistage Lookahead Approximations

- We can then simulate this *lookahead policy* over time:



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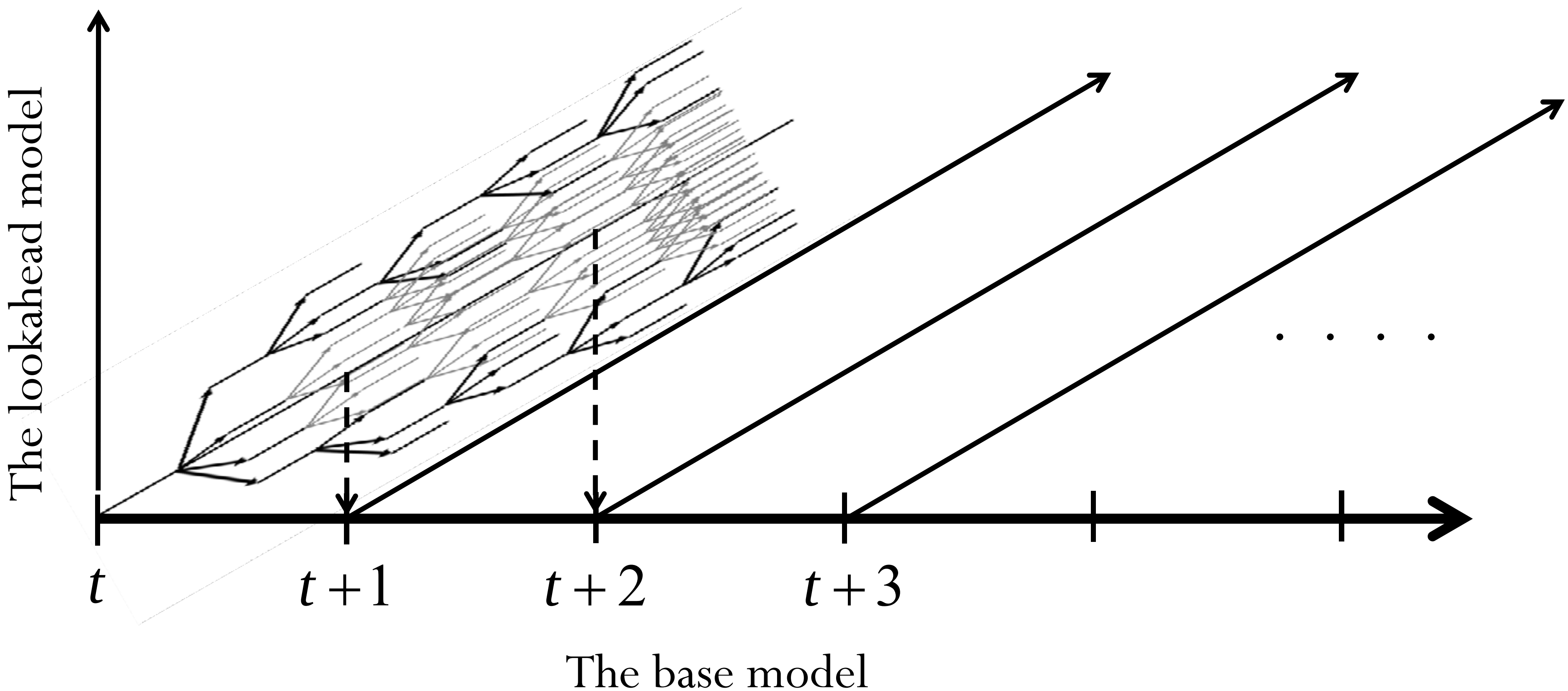

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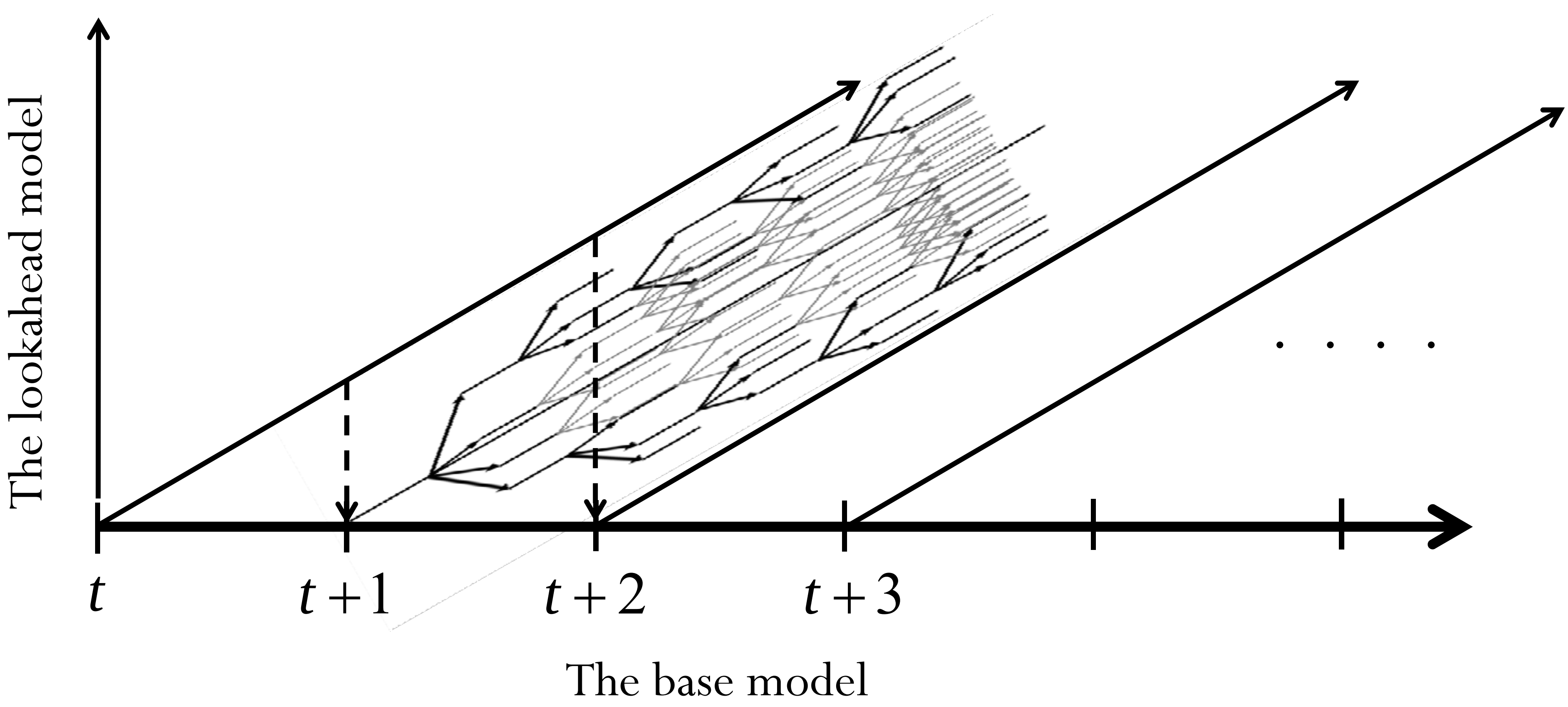

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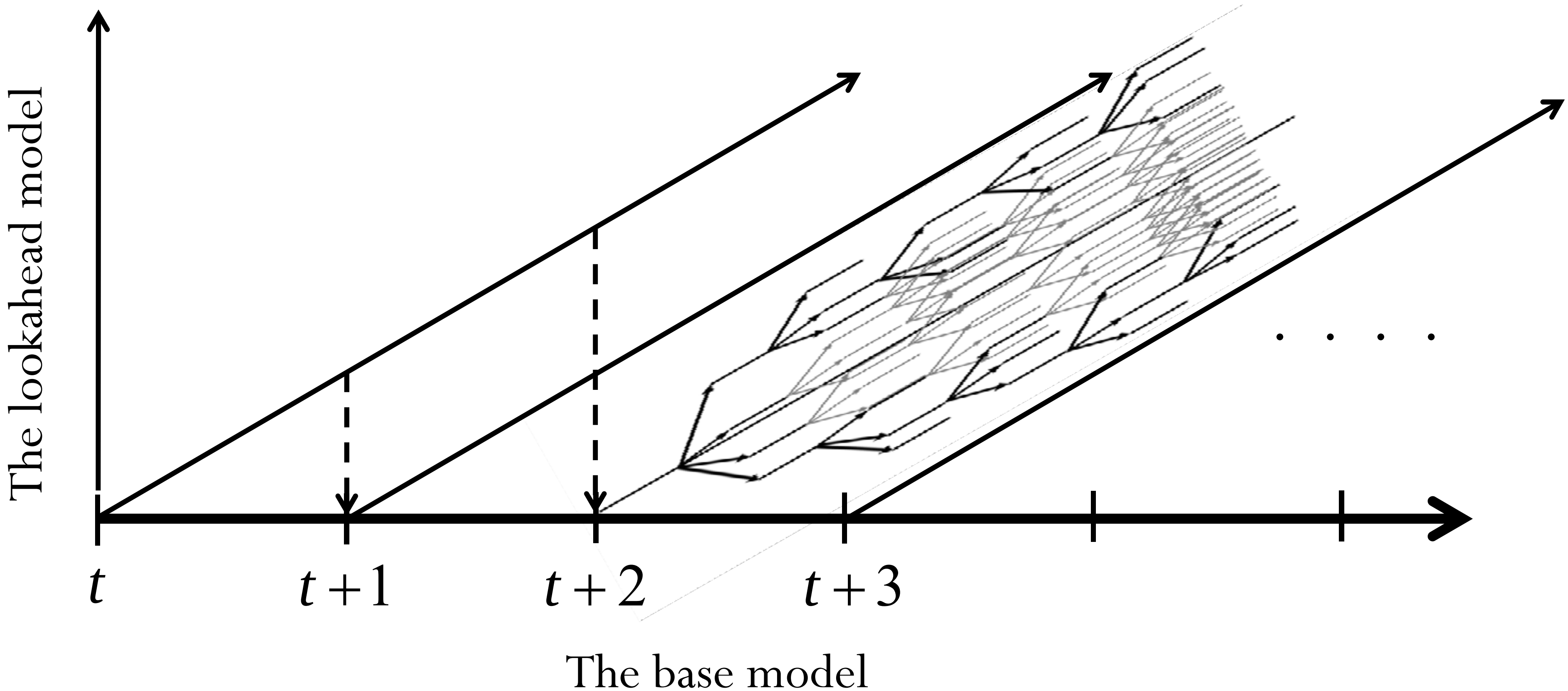
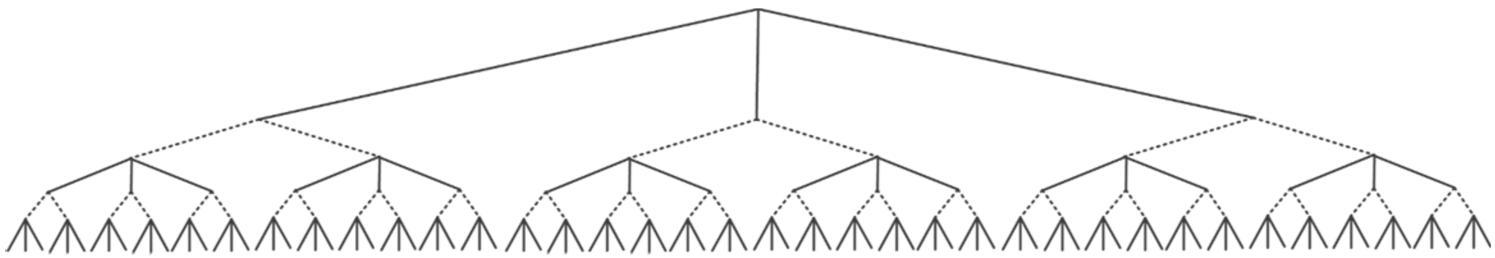


Multistage Lookahead Policy

- The optimal policy is computationally intractable, requiring approximations:

$$X_t^*(S_t) = \text{argmin}_{x_t \in \mathcal{X}_t(S_t)}\Big(C(S_t, x_t) + \widetilde{\mathbb{E}}_{\widetilde{W}_{t,t+1} \in \widetilde{\Omega}_{t,t+1}}\big[\min_{\tilde{x}_{t,t+1} \in \widetilde{\mathcal{X}}_{t,t+1}(\tilde{S}_{t,t+1})} \tilde{C}(\tilde{S}_{t,t+1}, \tilde{x}_{t,t+1}) + \widetilde{\mathbb{E}}_{\widetilde{W}_{t,t+2} \in \widetilde{\Omega}_{t,t+2}}[\ldots \widetilde{\mathbb{E}}_{\widetilde{W}_{t,t+H} \in \widetilde{\Omega}_{t,t+H}}[\tilde{C}(\tilde{S}_{t,t+H}) | \tilde{S}^x_{t,t+H-1}] \ldots] | \tilde{S}^x_{t,t+1}] | S^x_t]$$

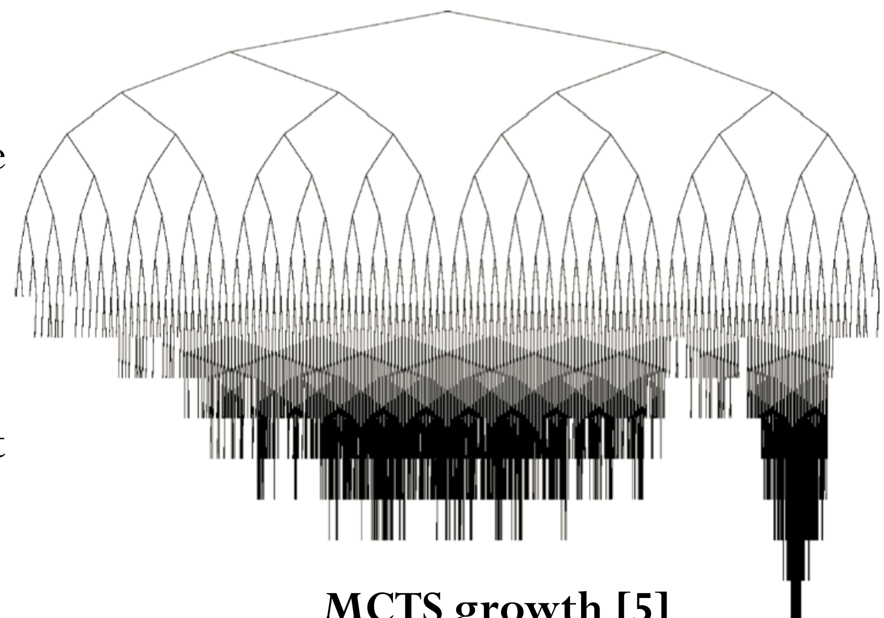
where $\tilde{S}_{t,t'+1} = S^M(\tilde{S}_{tt'}, \tilde{x}_{tt'}, \widetilde{W}_{t,t'+1}),\ t' = t, \ldots, t+H-1$



- Discretizing the time, states and decision
- Limiting the horizon from (t, T) to $(t, t+H)$
- Dimensionality reduction e.g., by limiting some variables (e.g., fixing the set of calls, limiting the fault types and travel times)
- Aggregating the outcome or sampling by using Monte Carlo sampling

Monte Carlo Tree Search (MCTS)

- MCTS is a recent research area; the first MCTS algorithm has been developed by Chang et al [2005].
- MCTS biases the growth of the tree towards the most promising moves which decreases the search space [3].
- MCTS mainly applied for deterministic problems but Coutoux et al [2011] extended it to stochastic problems based on *double progressive widening*.



MCTS growth [5]

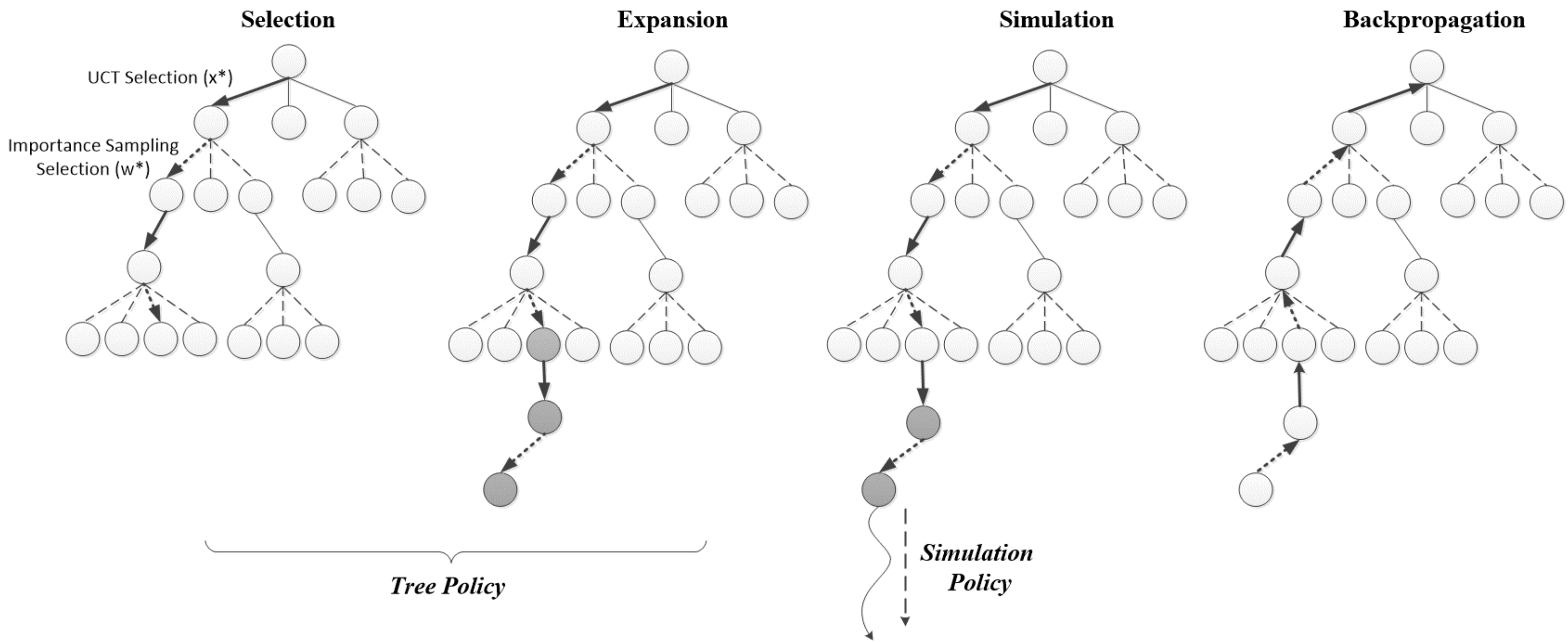
[3] H. S. Chang, M. C. Fu, J. Q. Hu, and S. I. Marcus. An adaptive sampling algorithm for solving Markov decision processes. Operations research, 53(1):126{139, 2005.

[4] Kocsis Levente and Csaba Szepesvári. "Bandit based monte-carlo planning“, Machine Learning: ECML 2006. pp. 282-293.

[5] R. Coquelin, Pierre-Arnaud and Munos, “Bandit Algorithms for Tree Search,” in Proc. Conf. Uncert. Artif. Intell. Vancouver, Canada: AUAI Press, 2007, pp. 67–74.

[6] A. Coutoux, J. B. Hoock, N. Sokolovska, O. Teytaud, and N. Bonnard. Continuous upper confidence trees. In International Conference on Learning and Intelligent Optimization, pages 433{445. Springer Bernlin Heidelberg, 2011.

Monte Carlo Tree Search



MCTS Construction Steps

- Decision selection based on UCB (Upper Confidence Bounds for Trees) [7]:

$$\tilde{x}^*_{tt'} = \text{argmax}_{\tilde{x}_{tt'}\in\tilde{\mathcal{X}}^e_{tt'}(\tilde{S}_{tt'})}\left(-\underbrace{\left(\tilde{C}(\tilde{S}_{tt'},\tilde{x}_{tt'})+\tilde{V}_{tt'}(\tilde{S}^x_{tt'})\right)}_{\text{Exploitation}} + \alpha\underbrace{\sqrt{\frac{\log N(\tilde{S}_{tt'})}{N(\tilde{S}_{tt'},\tilde{x}_{tt'})}}}_{\text{Exploration}}\right)$$

Visit count of state $\tilde{S}_{tt'}$

Visit count of decision $\tilde{x}_{tt'}$ from state $\tilde{S}_{tt'}$

[7] D. Auger, A. Couetoux, and O. Teytaud, "Continuous upper confidence trees with polynomial exploration consistency," in Proc. of the European Conference on Machine Learning and Knowledge Discovery in Databases, (Prague, Czech Republic), pp. 194–209, 20

Monte Carlo Tree Search – Convergence Theory

- For **deterministic MCTS**, the **UCB policy samples the actions infinitely often** and Kocsis and Szepesvari [2006] exploit this to show that the probability of selecting a suboptimal action converges to zero at the root of the tree.

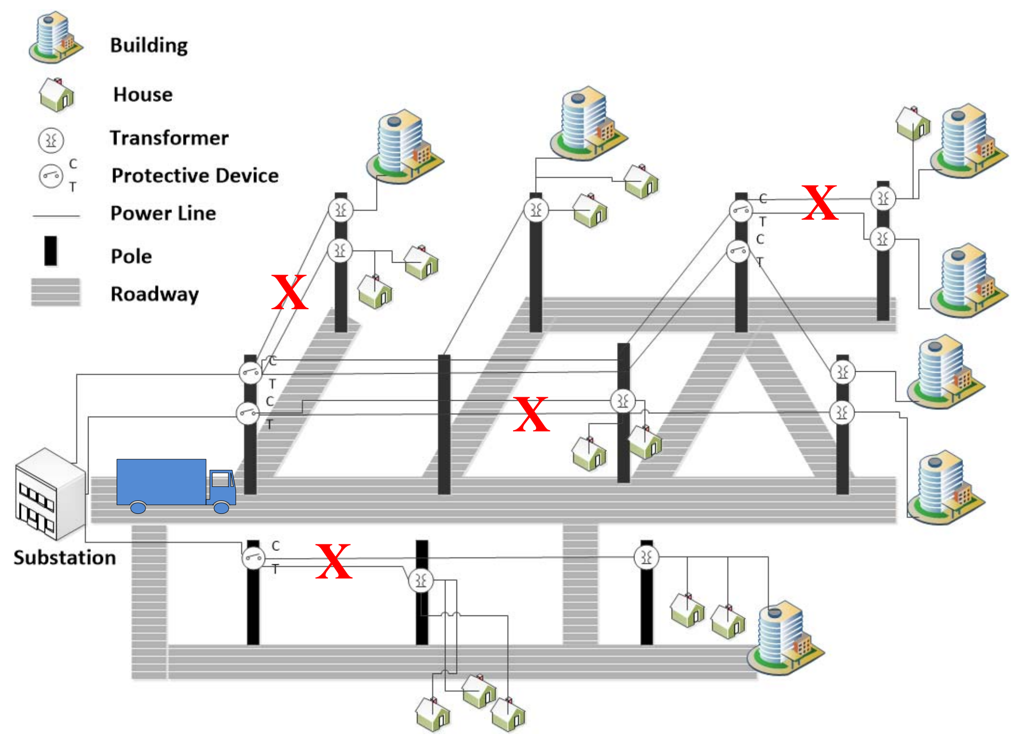
- Auger et al. [2013] provides convergence results for **stochastic MCTS** with *double progressive widening* under an action sampling assumption. The asymptotic convergence of also MCTS relies on some form of "**exploring every node infinitely often**".

- In Jiang et al. [2017], we design a version of **stochastic MCTS** that **asymptotically does not expand the entire tree**, yet is still optimal!

[6] Kocsis Levente and Csaba Szepesvári. "Bandit based monte-carlo planning", Machine Learning: ECML 2006. pp. 282-293.
[7] D. Auger, A. Couetoux, and O. Teytaud, "Continuous upper confidence trees with polynomial exploration consistency," in Proc The European Conference on Machine Learning and Knowledge Discovery in Databases, (Prague, Czech Republic), pp. 194–209,
[8] D. Jiang, L. Al-Kanj, W. B. Powell, "Monte Carlo Tree Search with Information Relaxation Dual Bounds", In preparation for Operations Research, 2017.

Simulation Policy

- Generate sample path $\widetilde{\omega}$ from $\widetilde{\Omega}_{t,t'}$ that determines all random events (faults, travel time, repair time).
- Utility truck should visit each fault once to repair it.
- Objective: find the optimal route that minimizes the customer-outage minute.
- In the worst case, computational complexity $O(n!)$.



Simulation Policy – Optimal Solution

- Define the graph $G(\mathcal{V},\mathcal{E})$ with connection costs T_{ij}.
- S: subset of nodes visited by the truck.
- $C(S,i)$: customer-outage minute starting from node 1 and ending at node i. .
- $f(S)$: function returning the number of customers in outage after visiting the nodes of S.
- The cost of moving from node i to node j is

$$C(S,j) = C(S-\{j\},i) + f(S-\{j\}) * (T_{ij} + R_j)$$

Dynamic Program

For all $j \in \mathcal{V}$ **do** $C(\{1,j\},j) = \sum_i n_i\,(T_{1j} + R_j)$

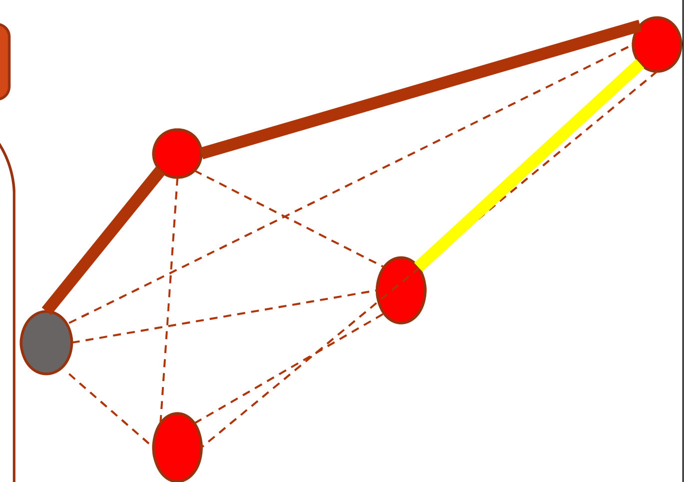
For $s = 3$ **to** n

For all Subsets S of $\mathcal{V}$ of size s **do**

For all $j \in S, j \neq 1$ **do**

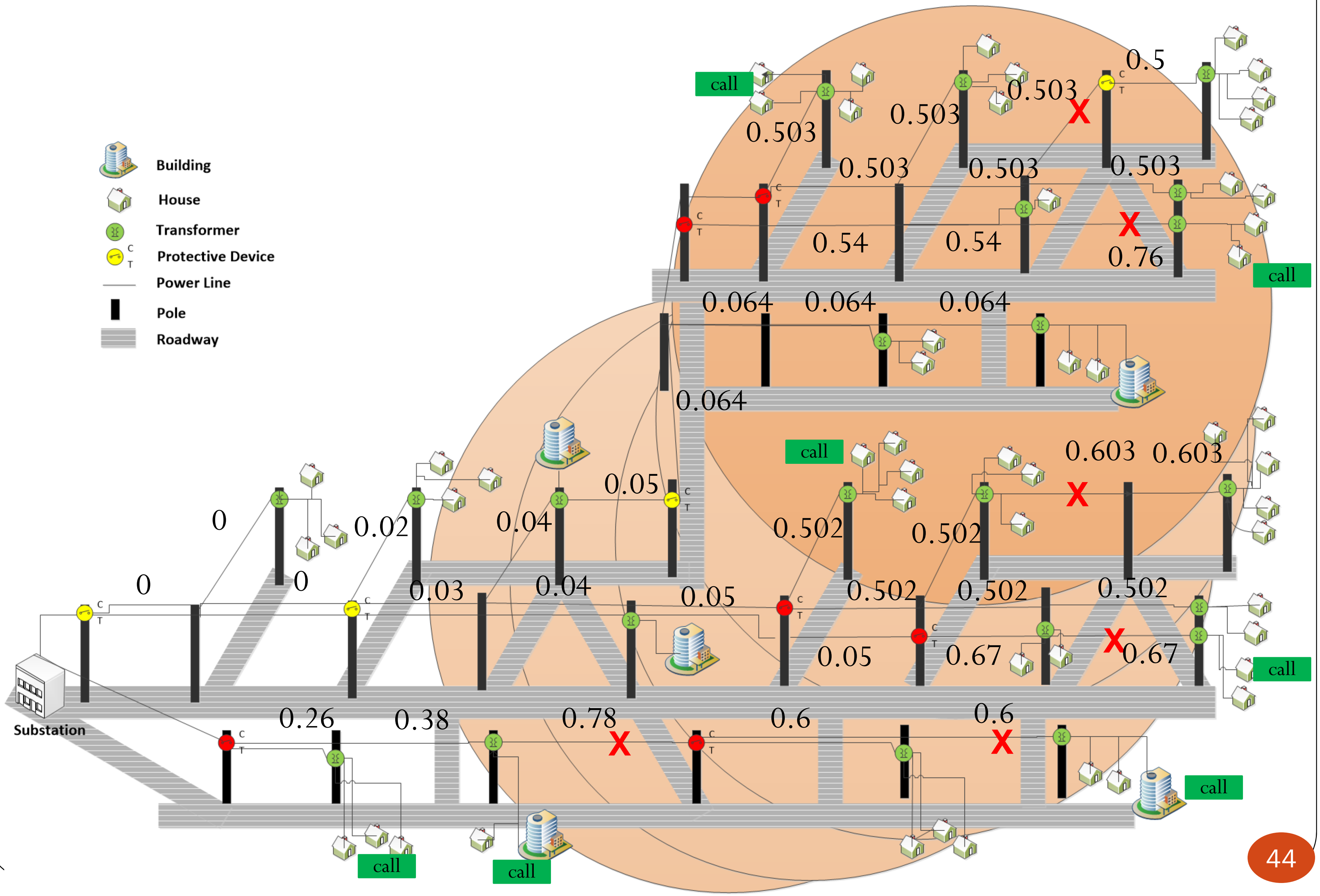
$$C(S,j) = \min_{i \in S, i \neq j} C(S-\{j\},i) + f(S-\{j\}) * (T_{ij} + R_j)$$

$$opt = \min_j C(\mathcal{V},j)$$

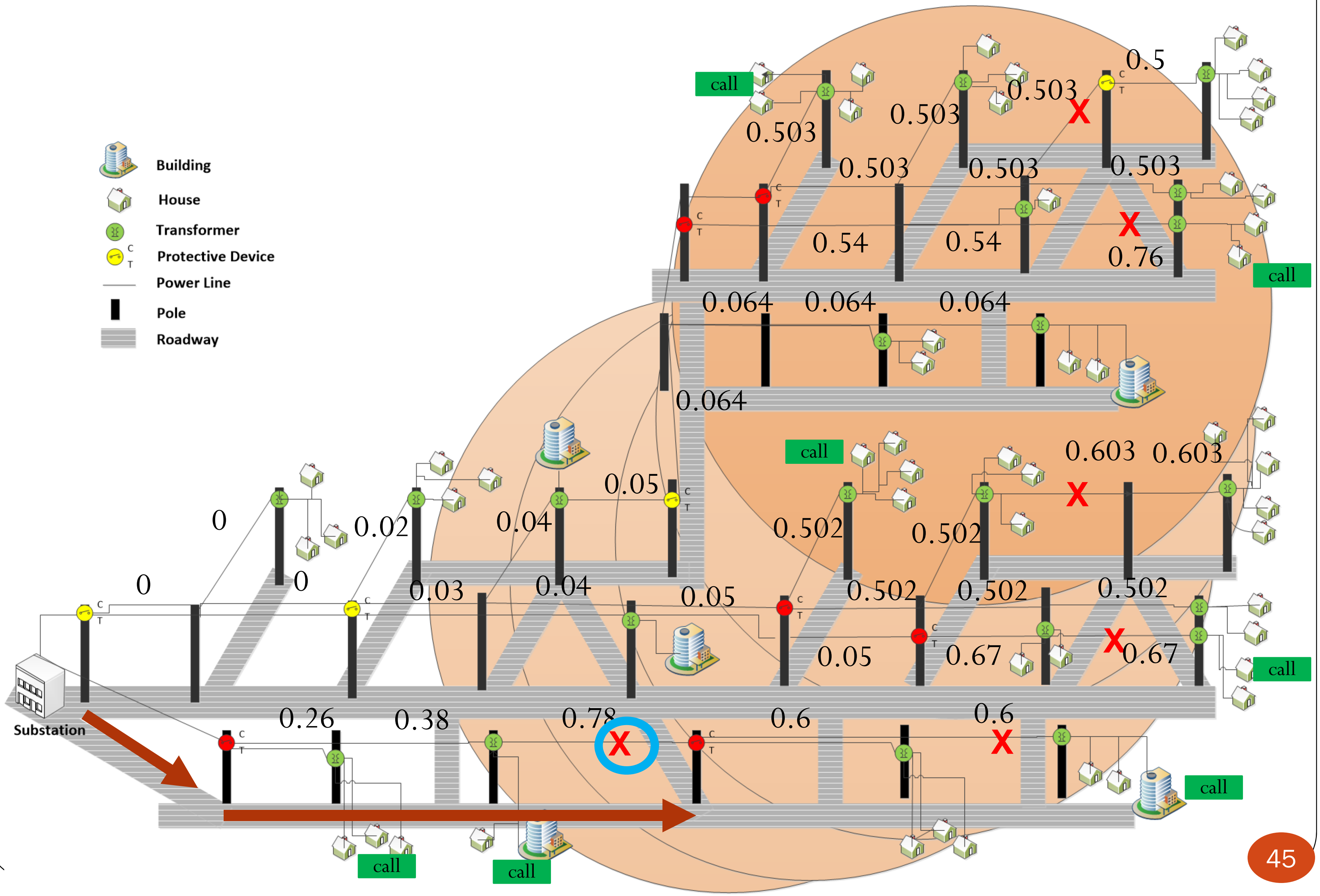


Computational Complexity
$O(n^2 2^n)$

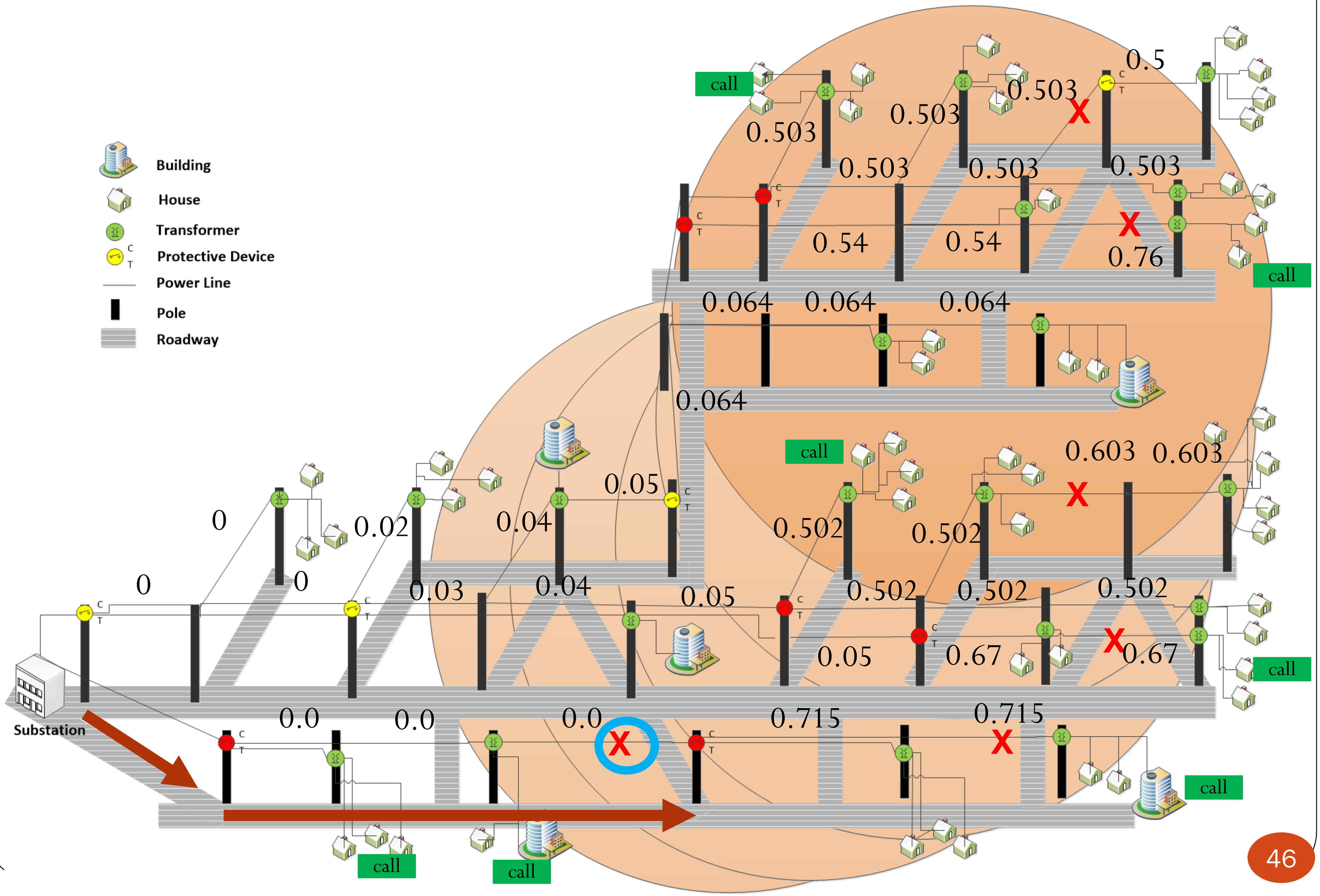
Lookahead Policy



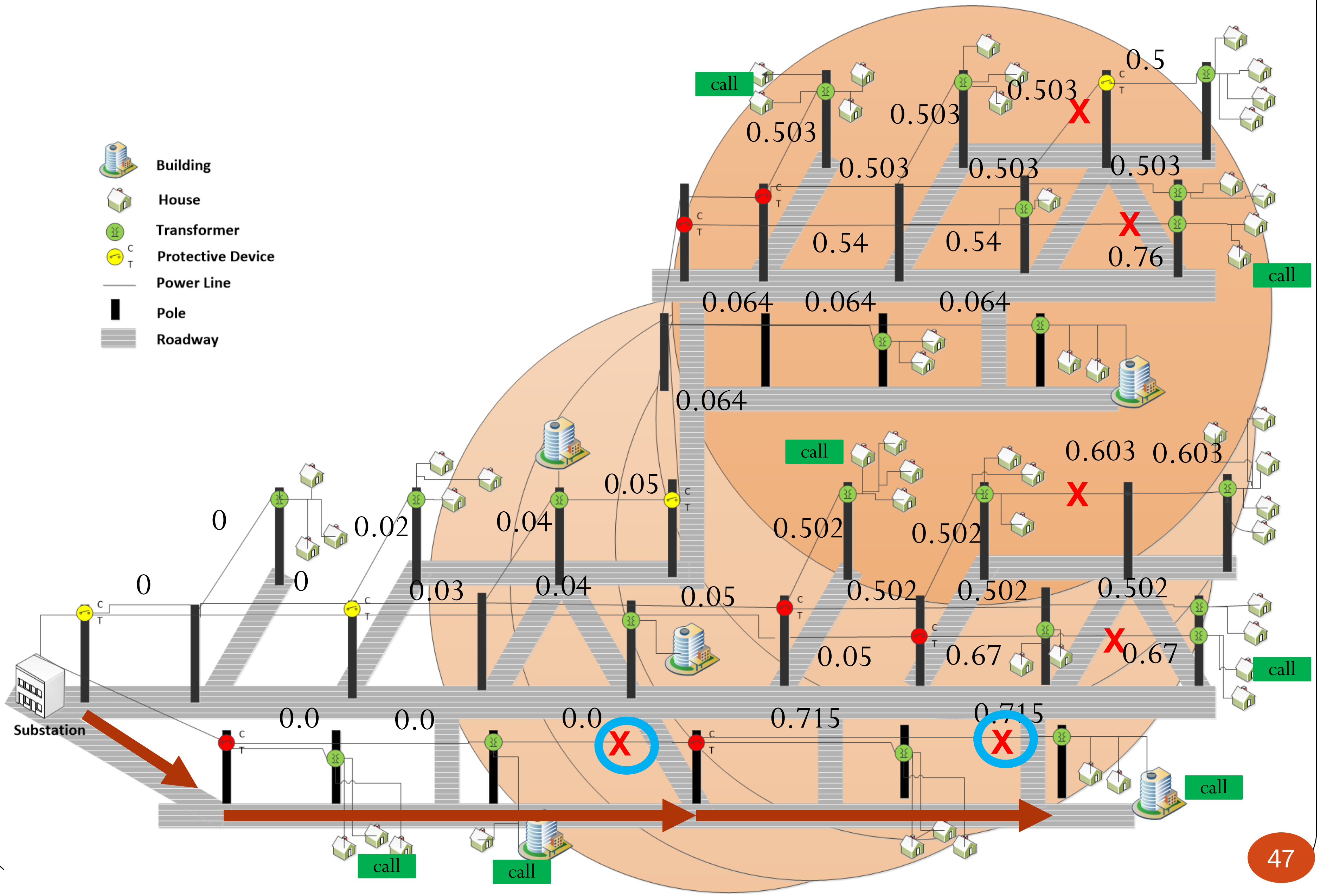
Lookahead Policy



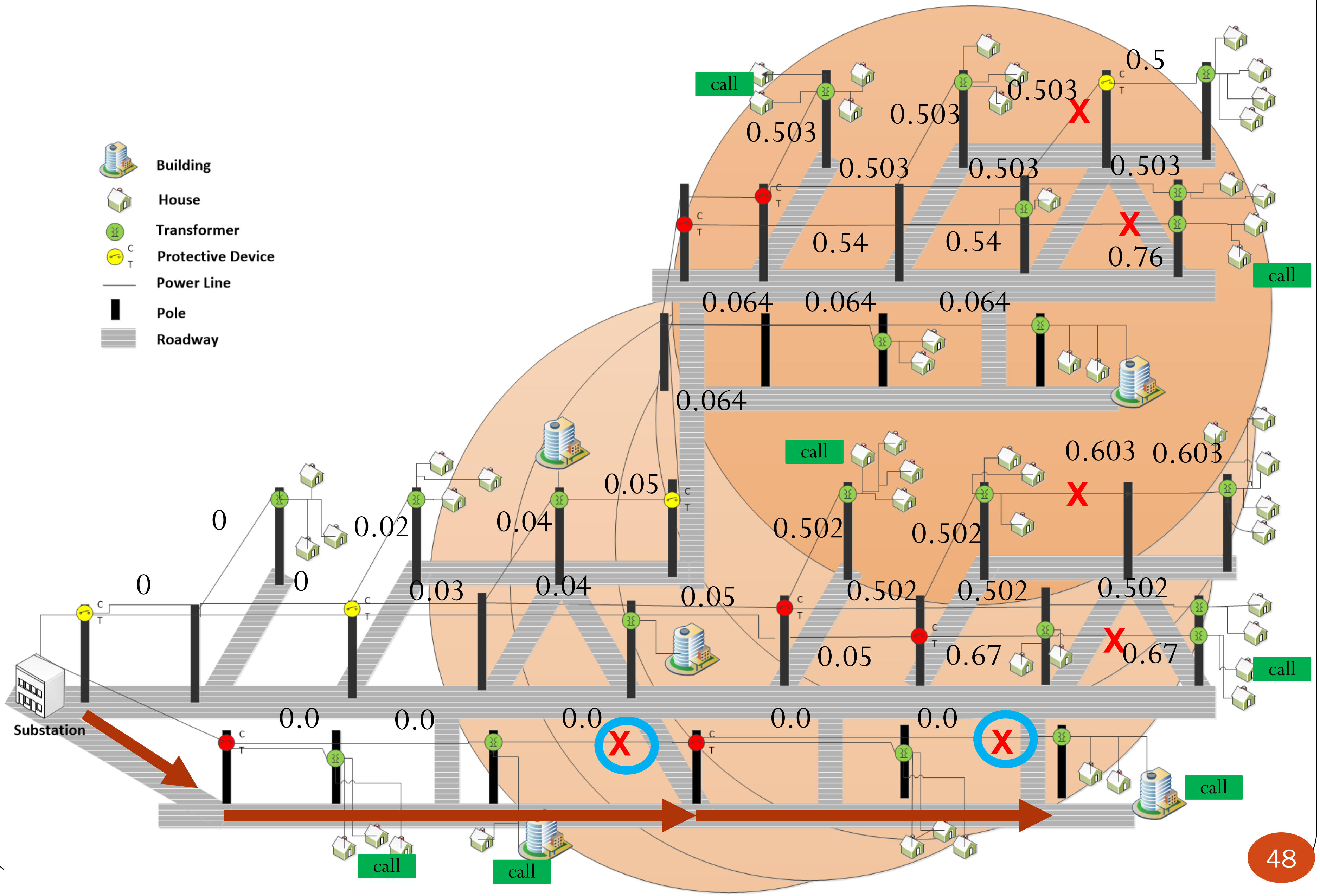
Lookahead Policy



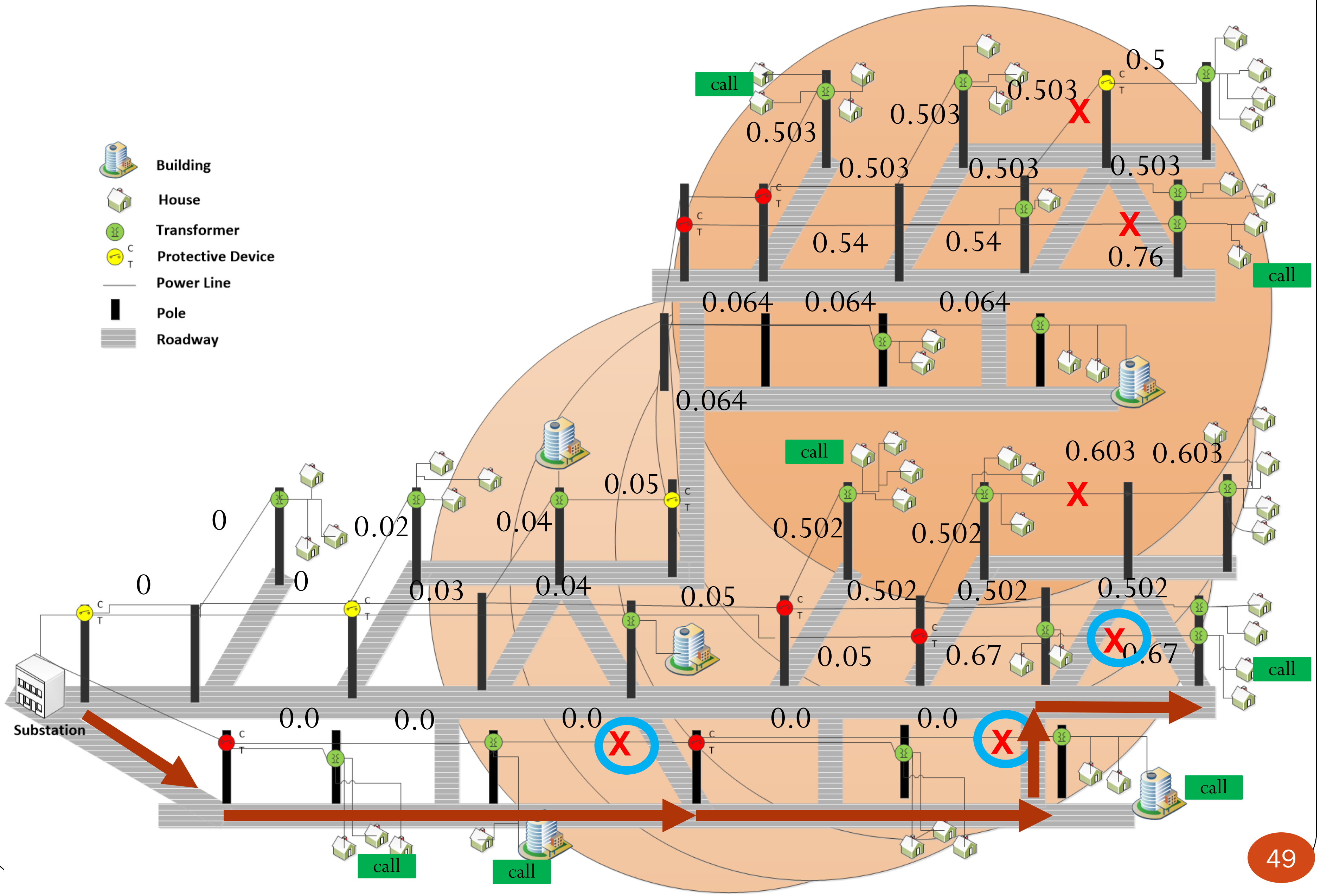
Lookahead Policy



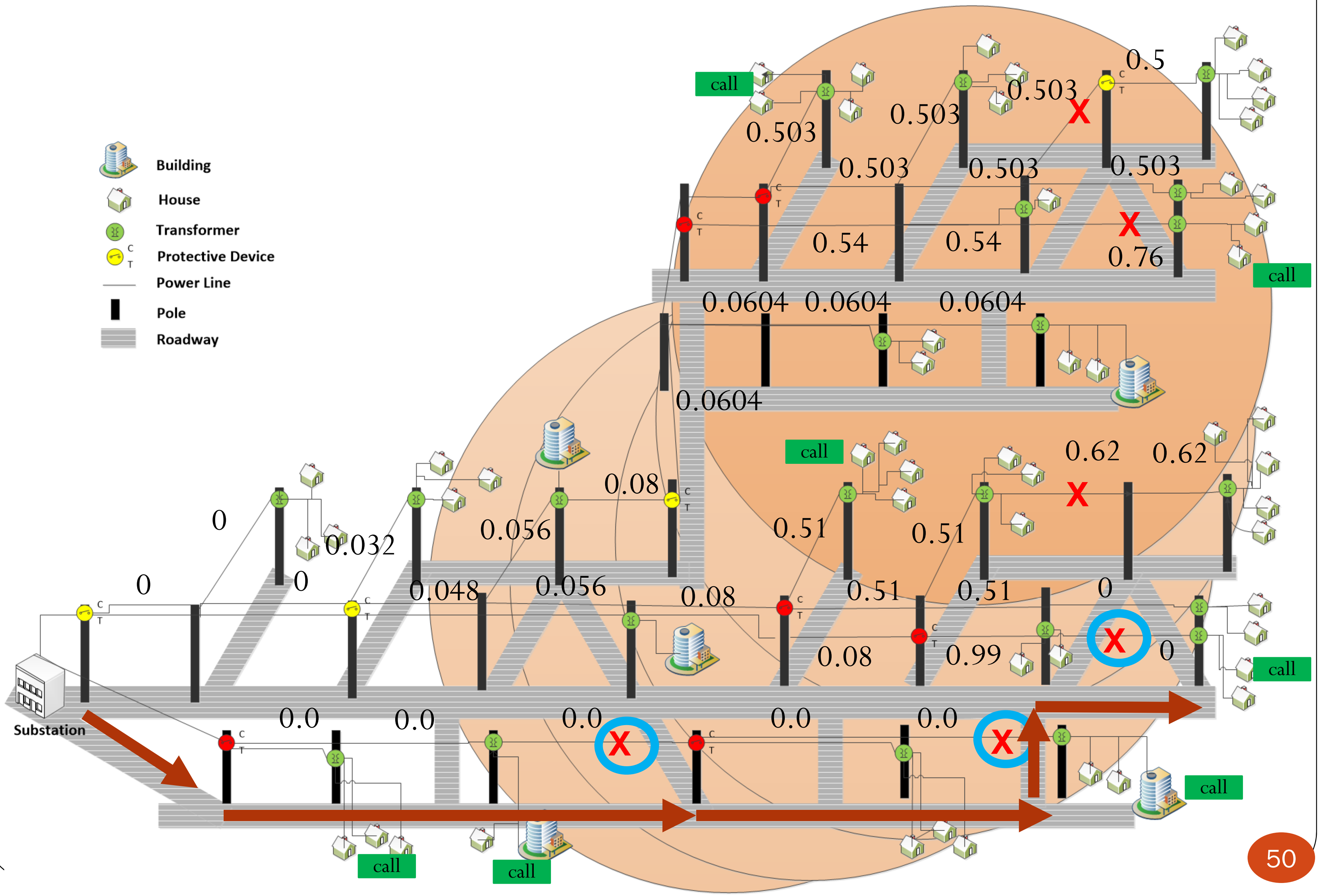
Lookahead Policy



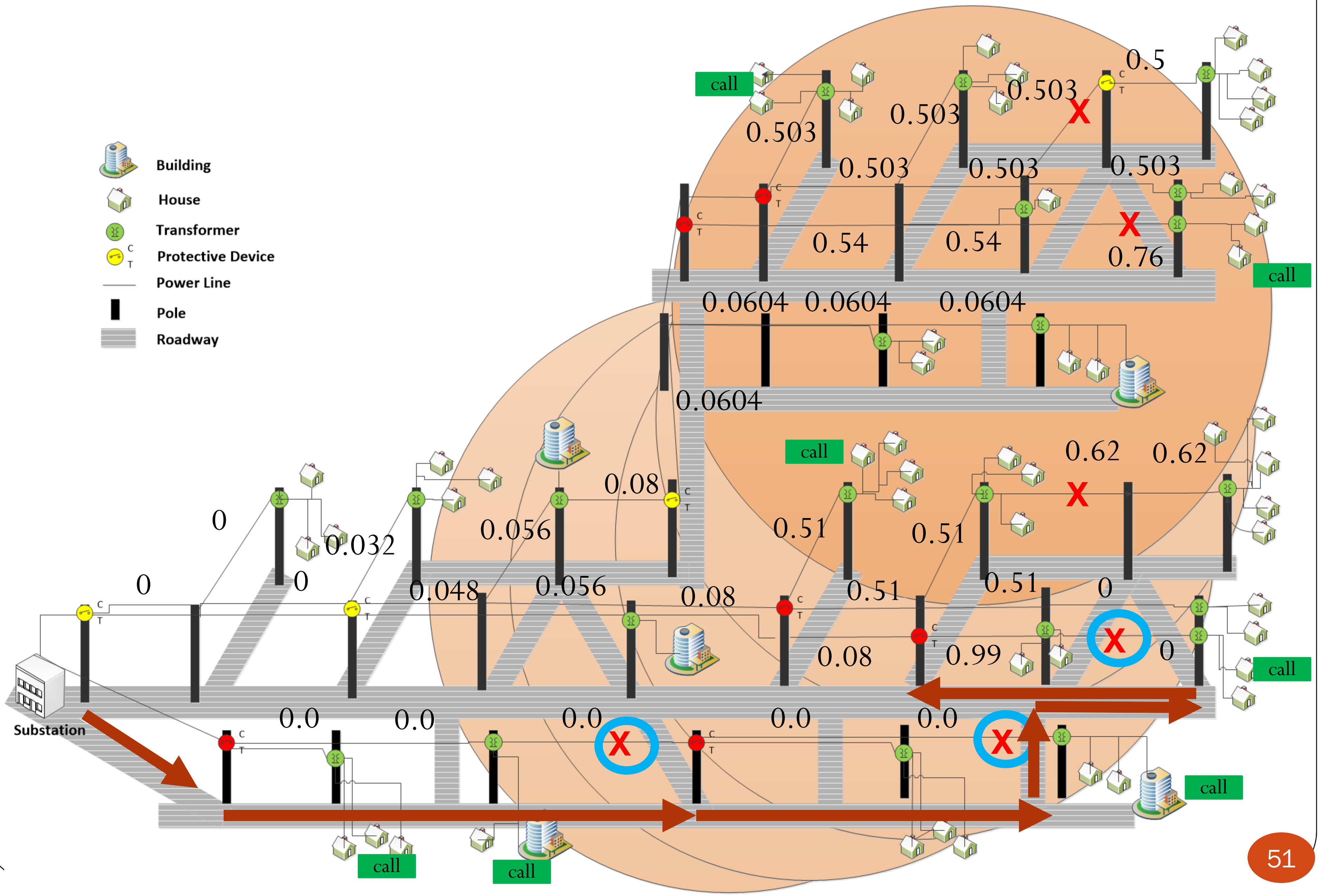
Lookahead Policy



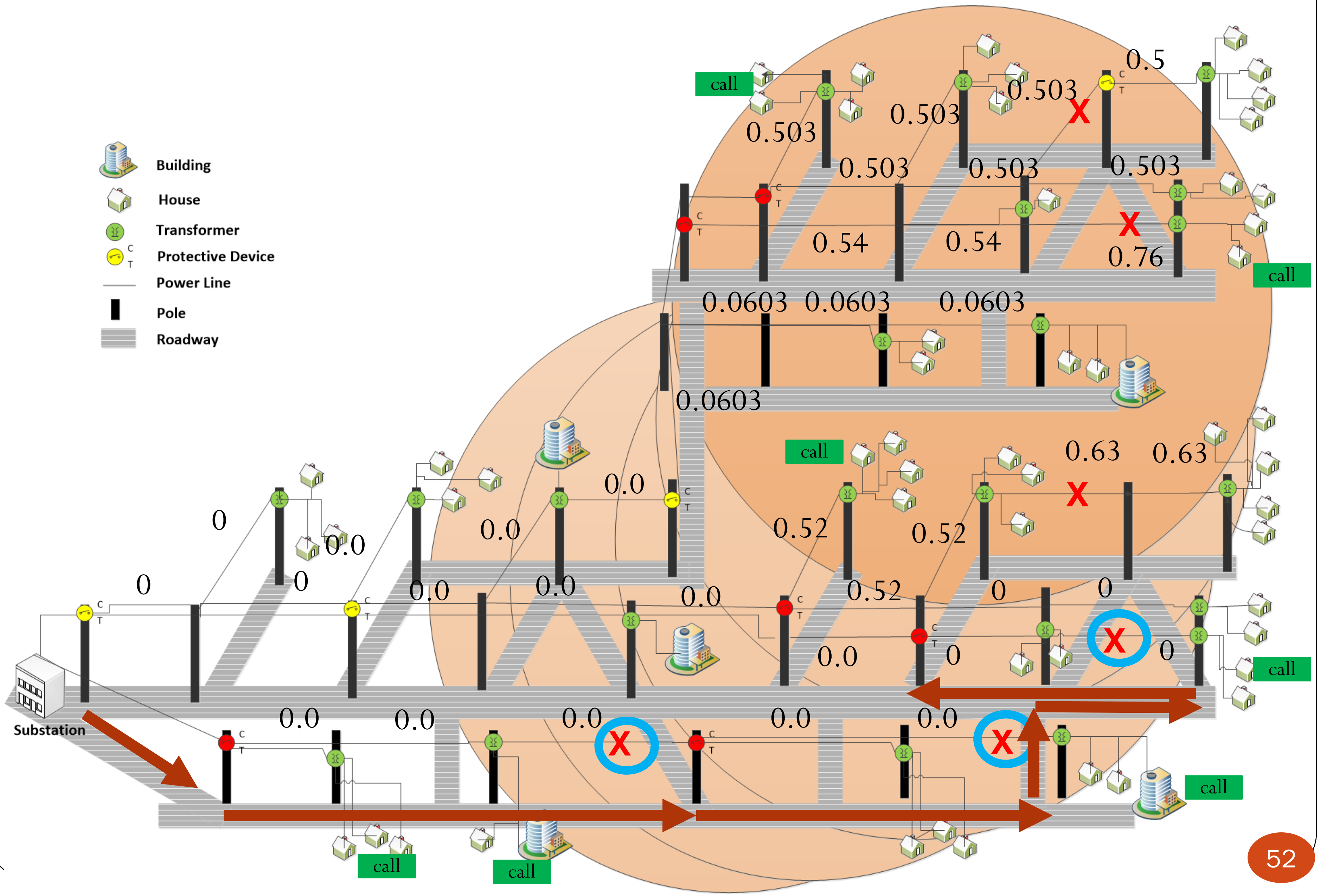
Lookahead Policy



Lookahead Policy



Lookahead Policy



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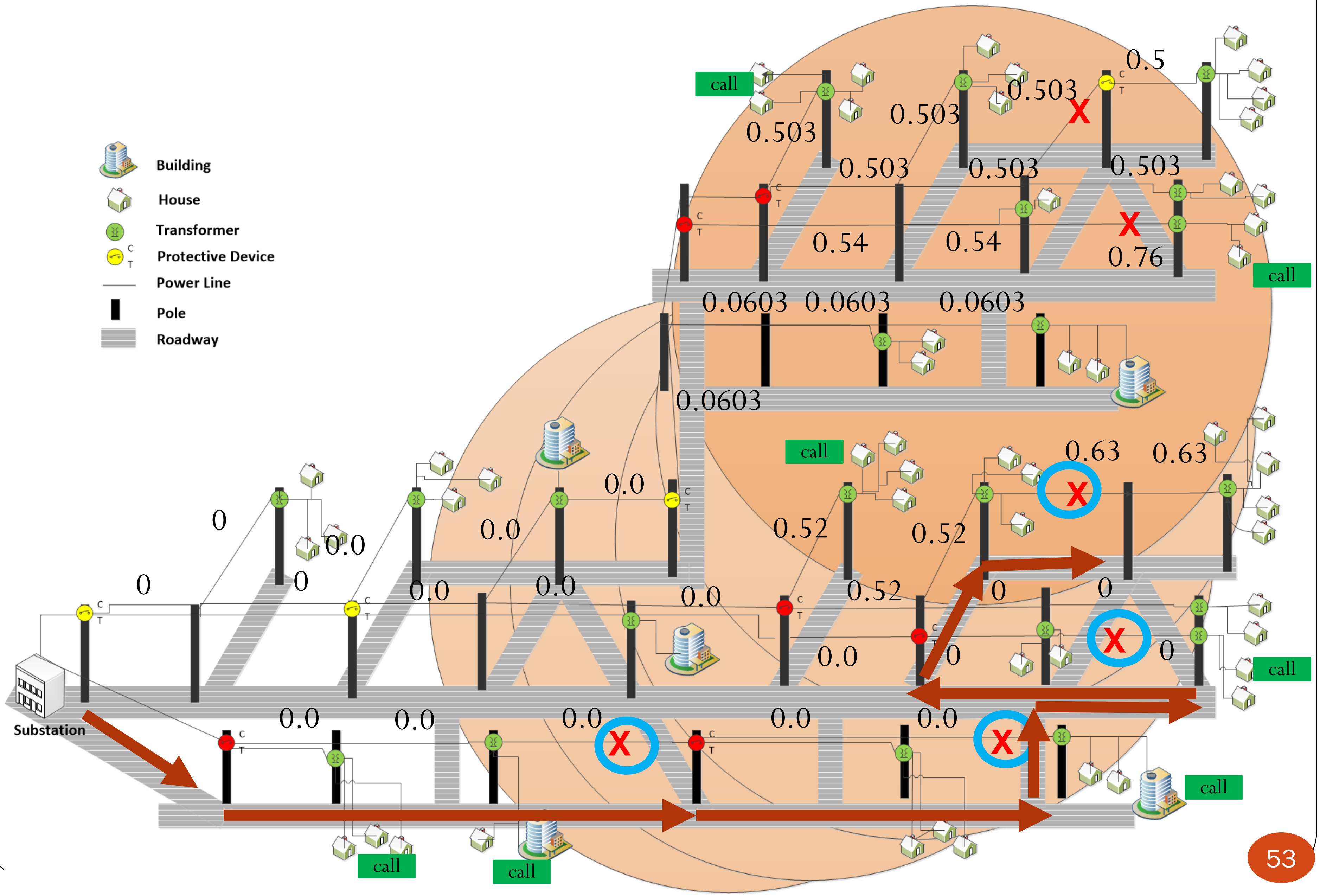

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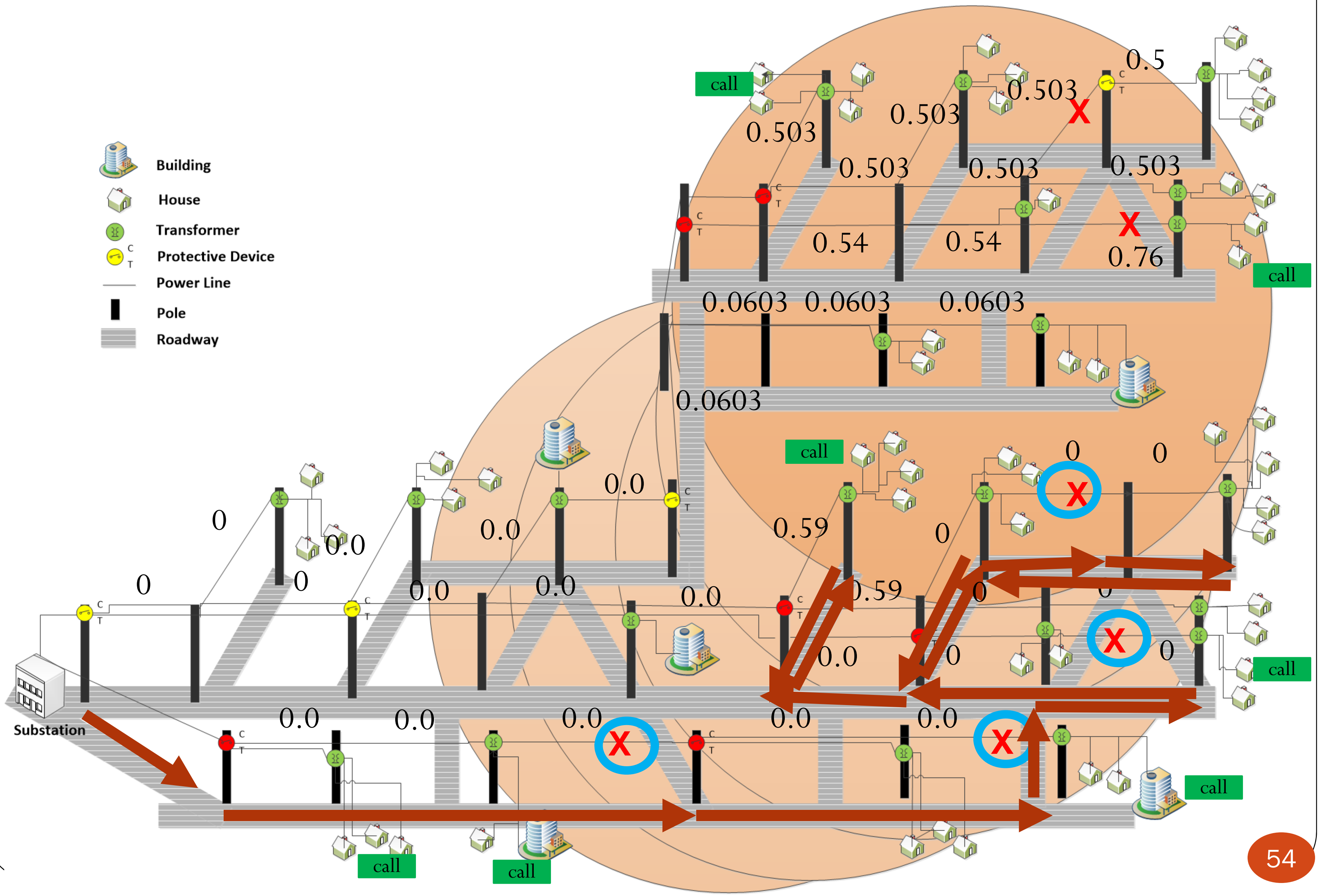

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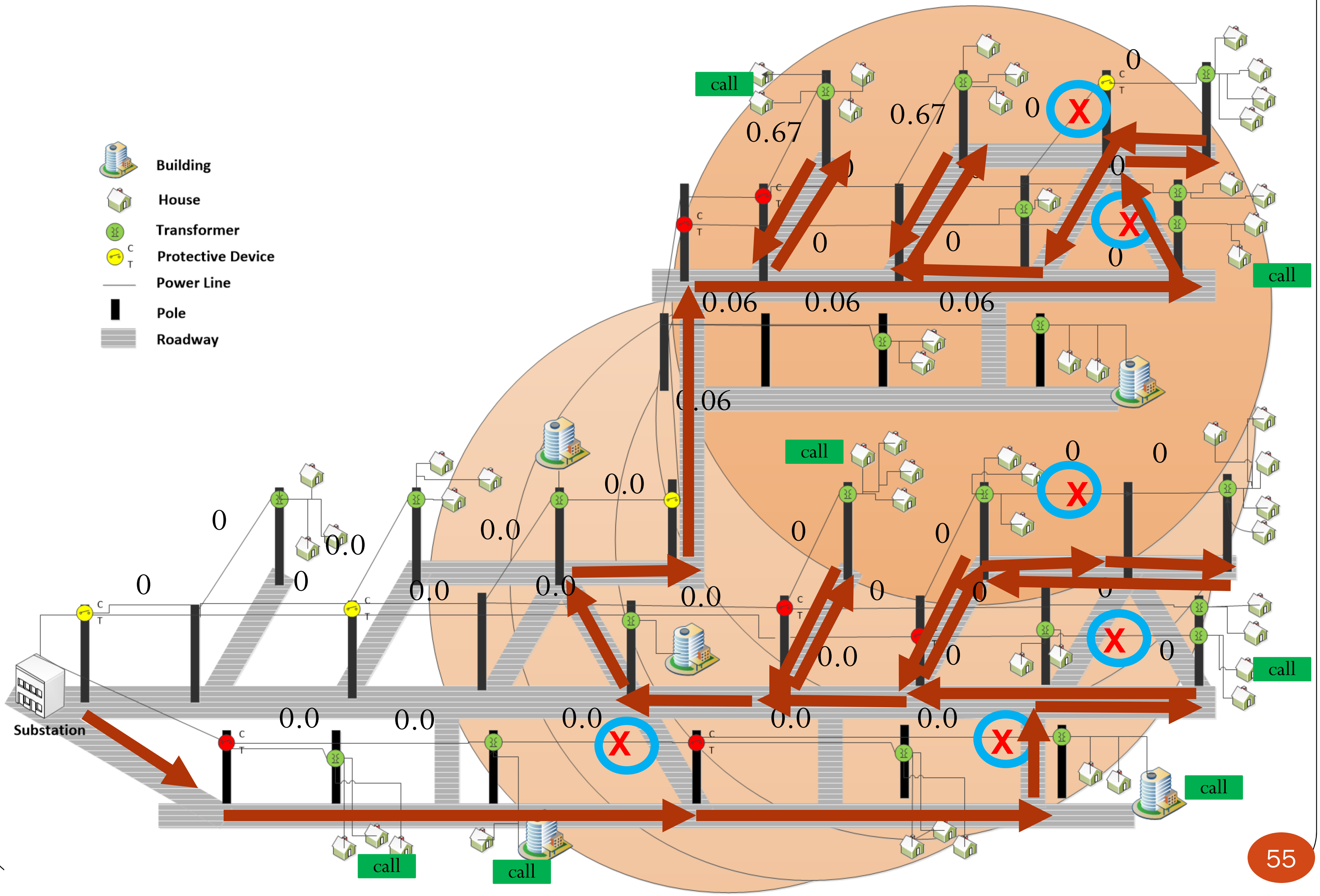


Industrial Heuristics

Escalation Algorithm

Step 1. **For** each circuit **do**

Step 1a. Collect all calls and back trace to find the first node that is common to all calls say node x.

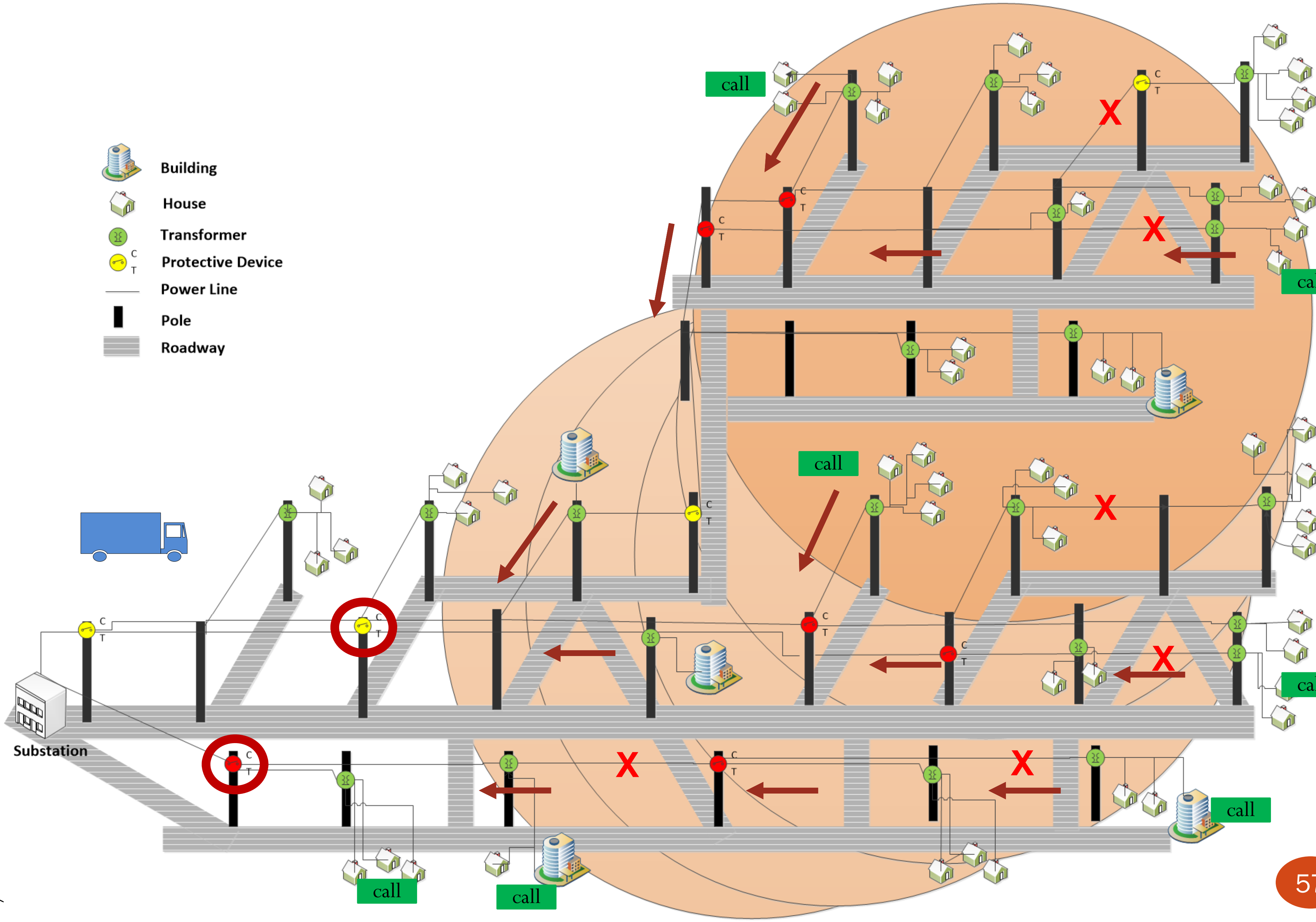
Step 1b. Send the truck to node x and then back trace to the substation to make sure that there is no upstream fault

Step 1c. From node x, perform down tracing to reach the first segment from which a call was initiated and place it in set D.

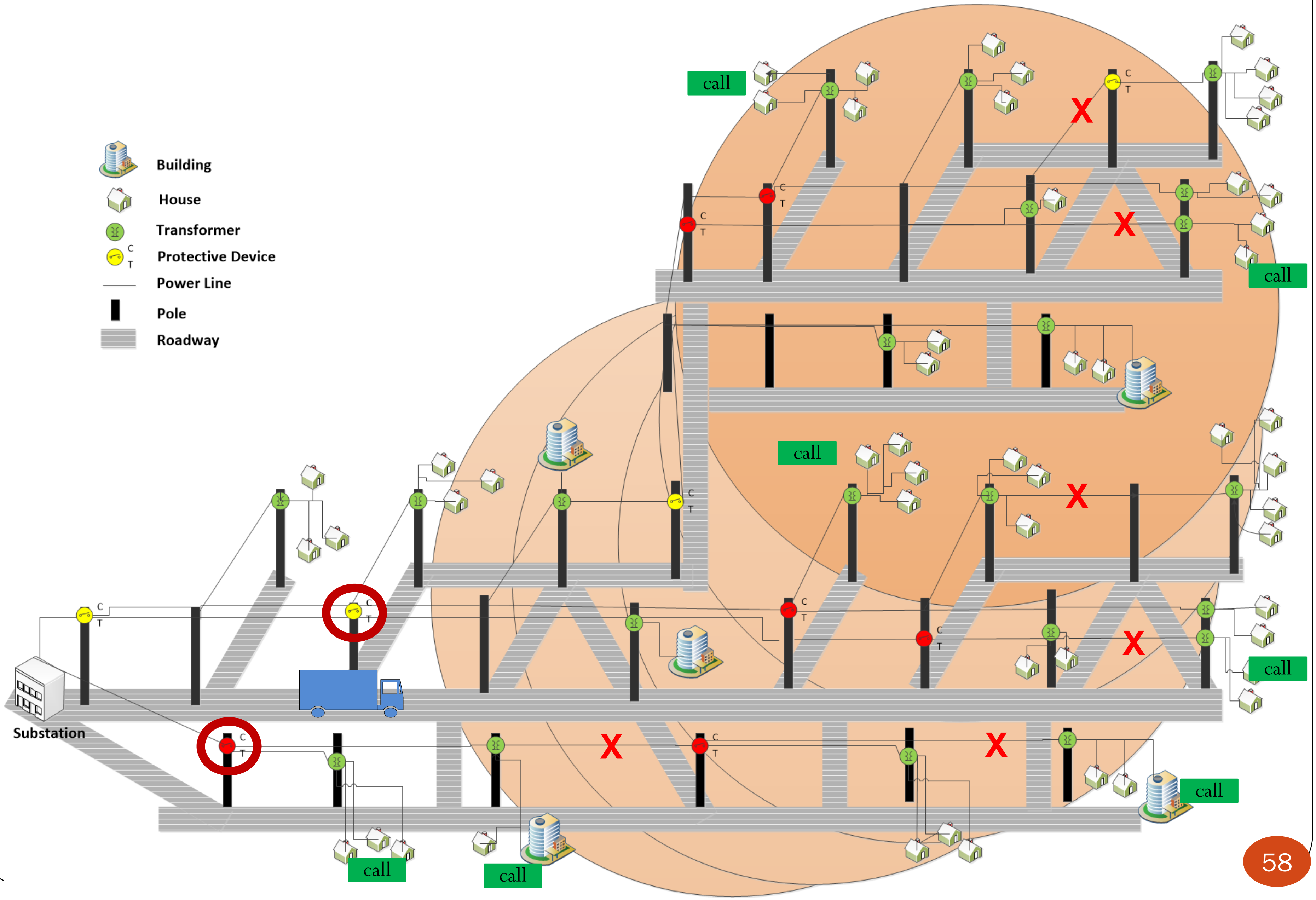
Step 2. **For** each segment in D **do**

Step 2a. Perform down tracing to cover all nodes that called.

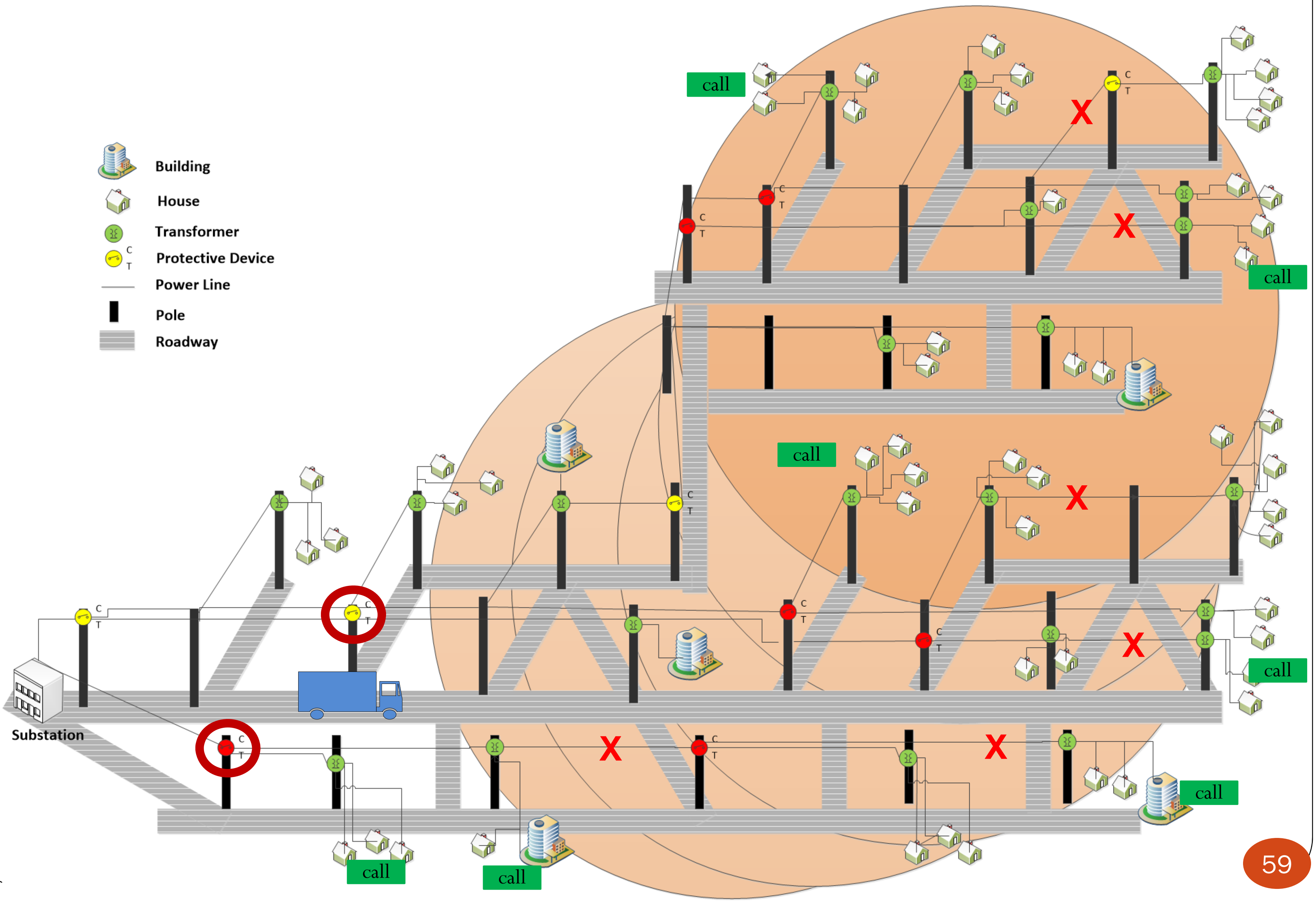
Escalation Algorithm



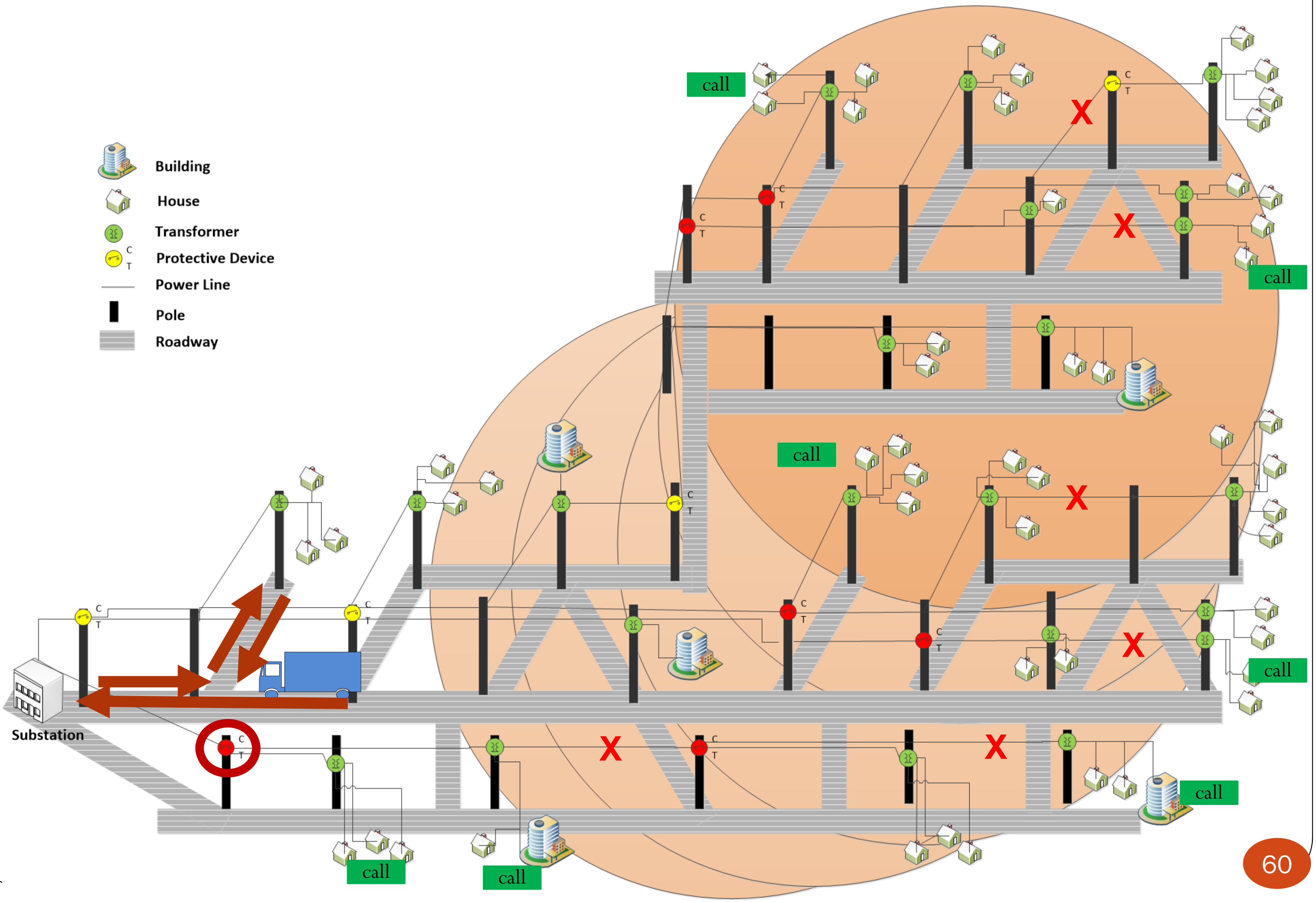
Escalation Algorithm



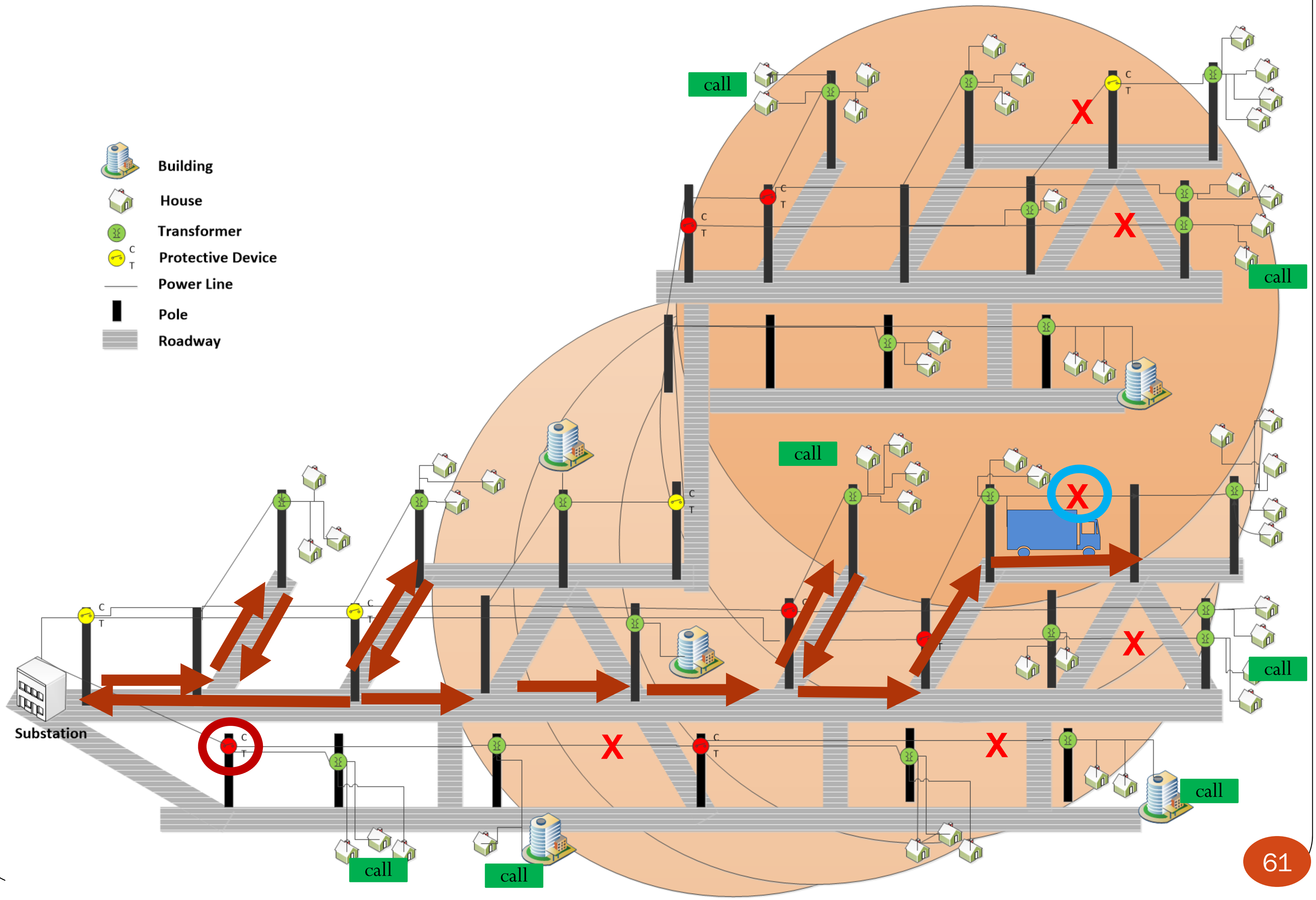
Escalation Algorithm



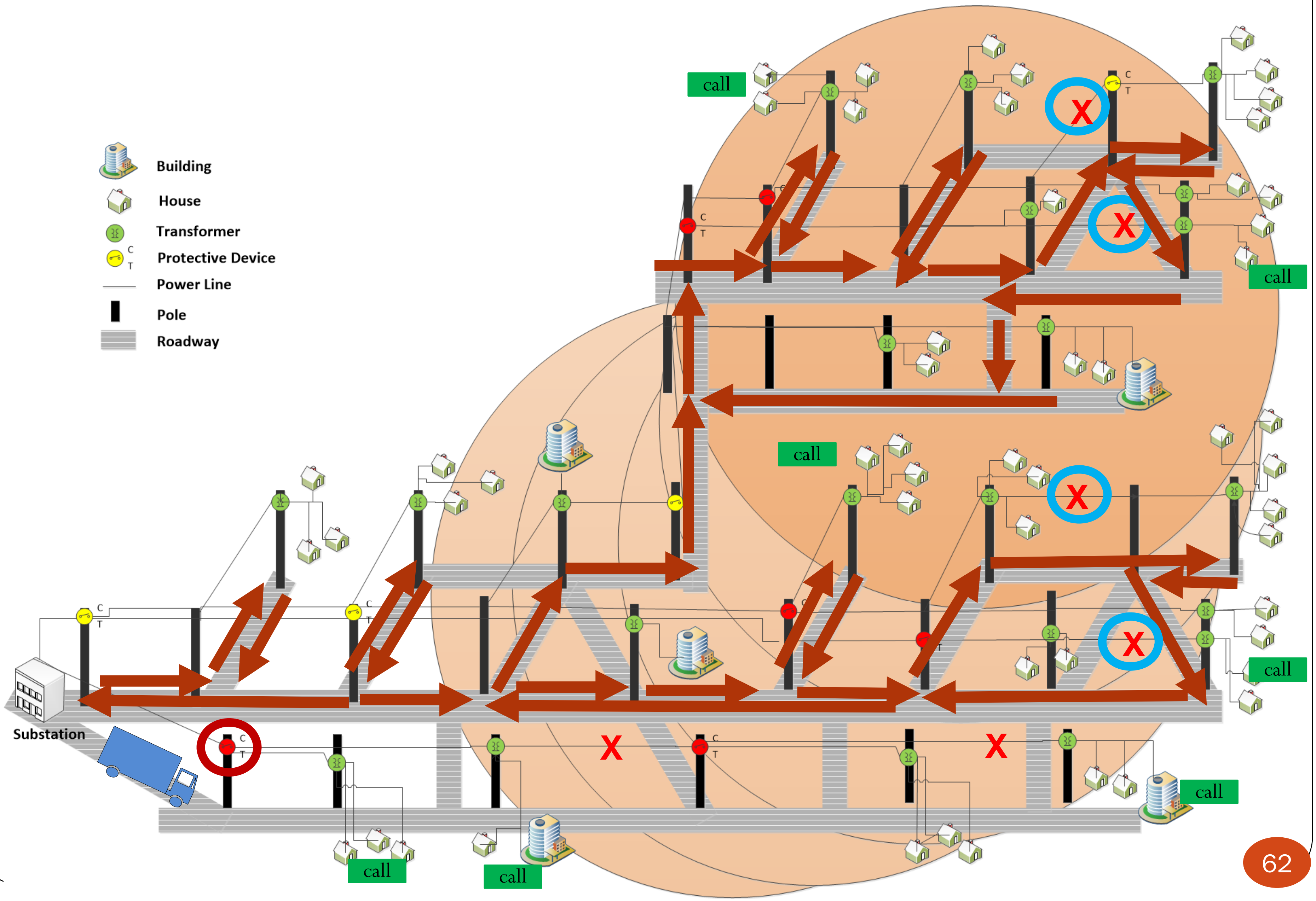
Escalation Algorithm



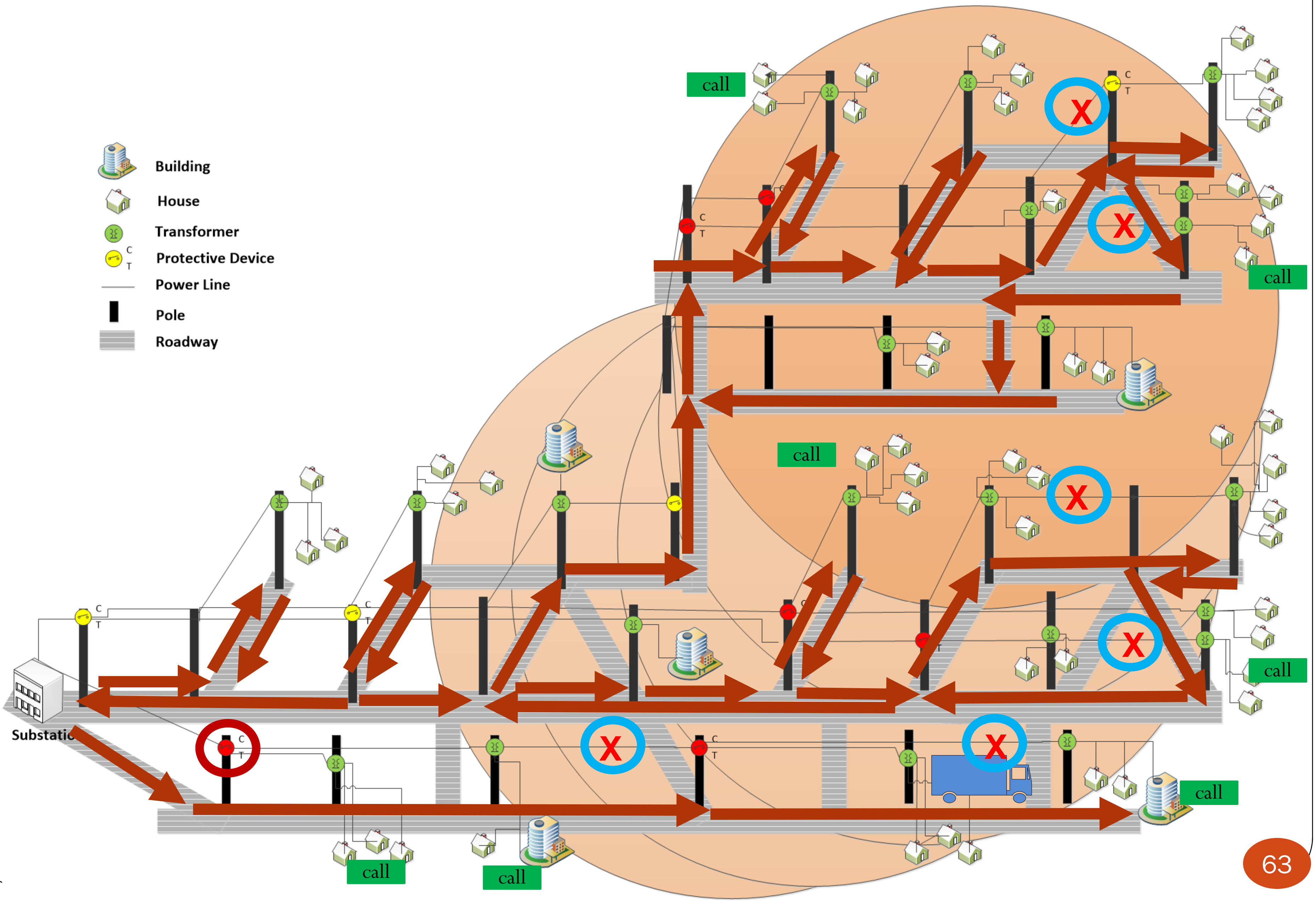
Escalation Algorithm



Escalation Algorithm

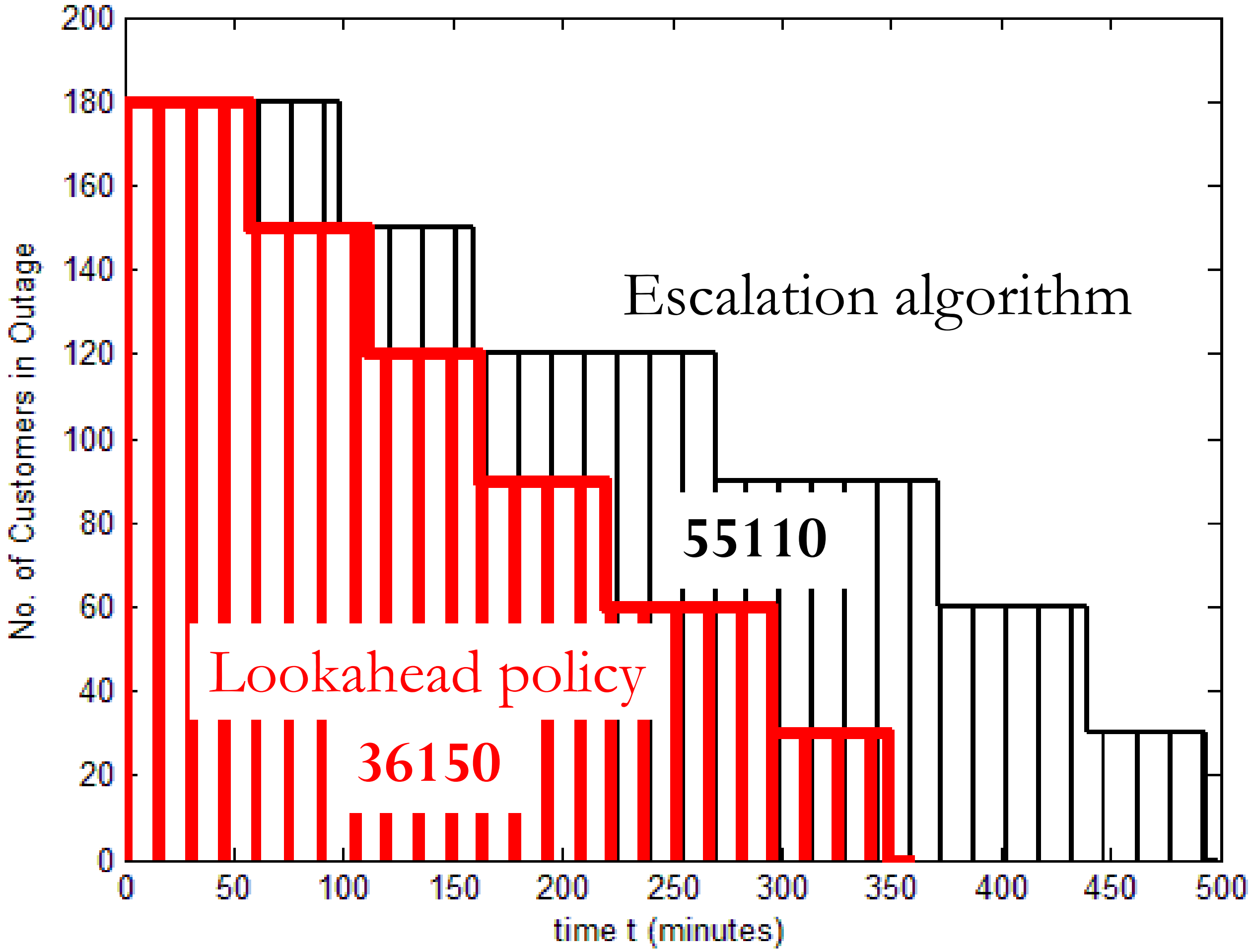


Escalation Algorithm



Numerical Results – Customer Outage-Minutes

- Performance metrics
  - Total outage minutes
  - SAIDI, CAIDI, SAIFI, …



Studies

- Possible questions:
  - What is the effect of higher call-in rates?
  - What is the best place to locate a sensor/protective device?
  - What is the value of a sensor vs. protective device?

Numerical Results

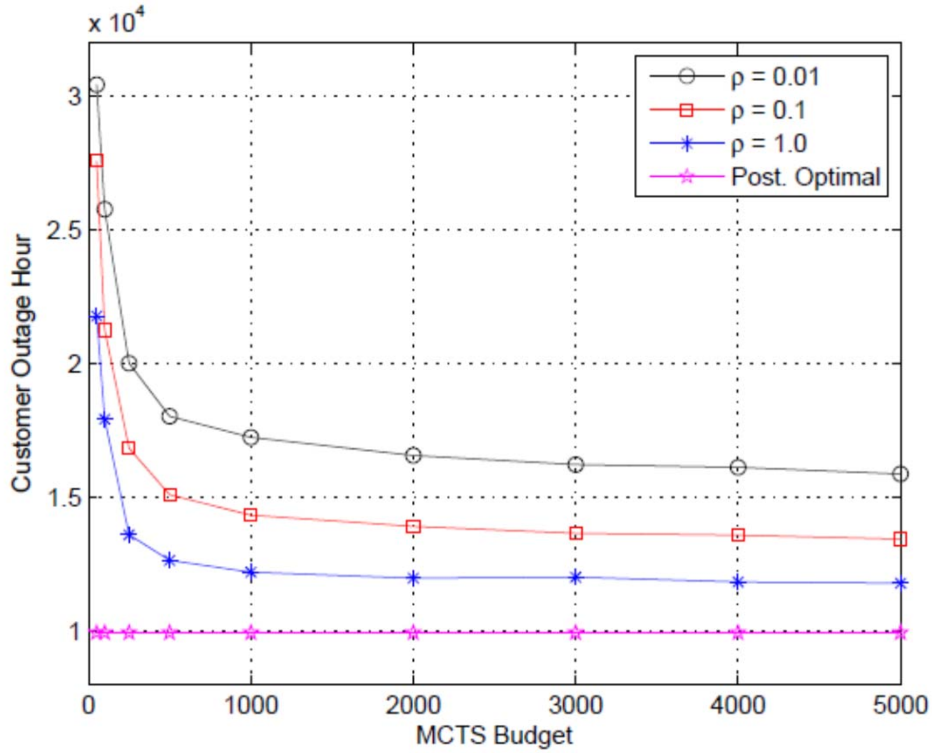
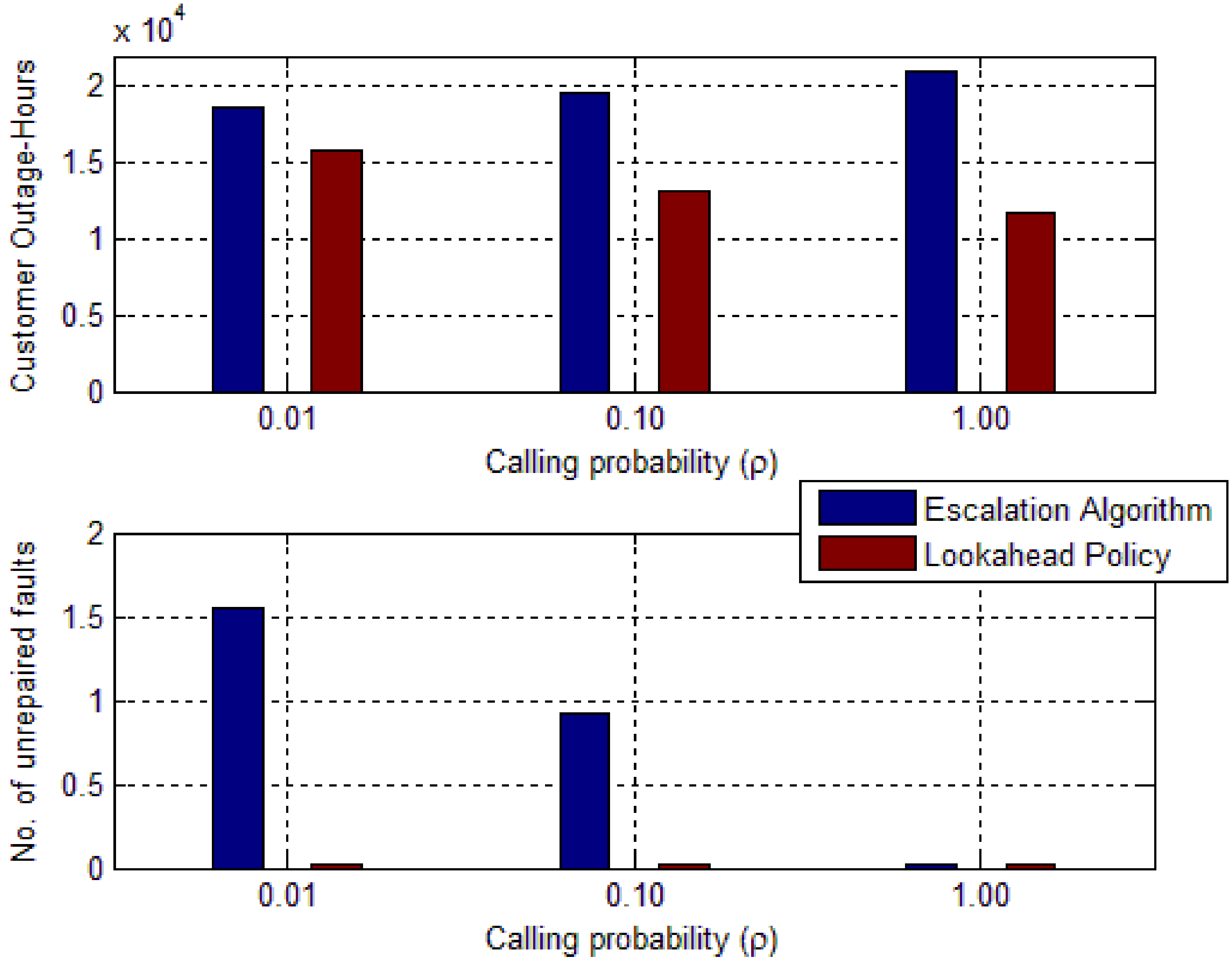


Figure 10: Average customer outage-hours vs. MCTS budget for ten networks.

Comparison to Industrial Heuristics



Outline

- Problem Description
- Probability Model for Grid Faults
- Sequential Stochastic Optimization Model for Utility Crew Routing
  - Multistage Lookahead Policy (Monte Carlo Tree Search)
  - Monte Carlo Tree Search with Information Relaxation Dual Bounds
- Stochastic Optimization Model for Sensor and Protective Device Placements

Optimal Protective Device and Sensor Placement

- Protective Device vs Sensor Placement to minimize Customer-Outage Minute:
  - A protective device prevents power flow to the downstream segments when a fault is detected.
  - A sensor feeds back information to the utility center whether power is on or off.

- Optimization problem for optimal protective device and sensor placements:

$$\min_{a} \mathbb{E}^{\pi}\left[\sum_{t=0}^{T} C(S_t(a), X^{\pi}(S_t(a)))|S_0(a)\right]$$

where

$$\pi = MCTS$$

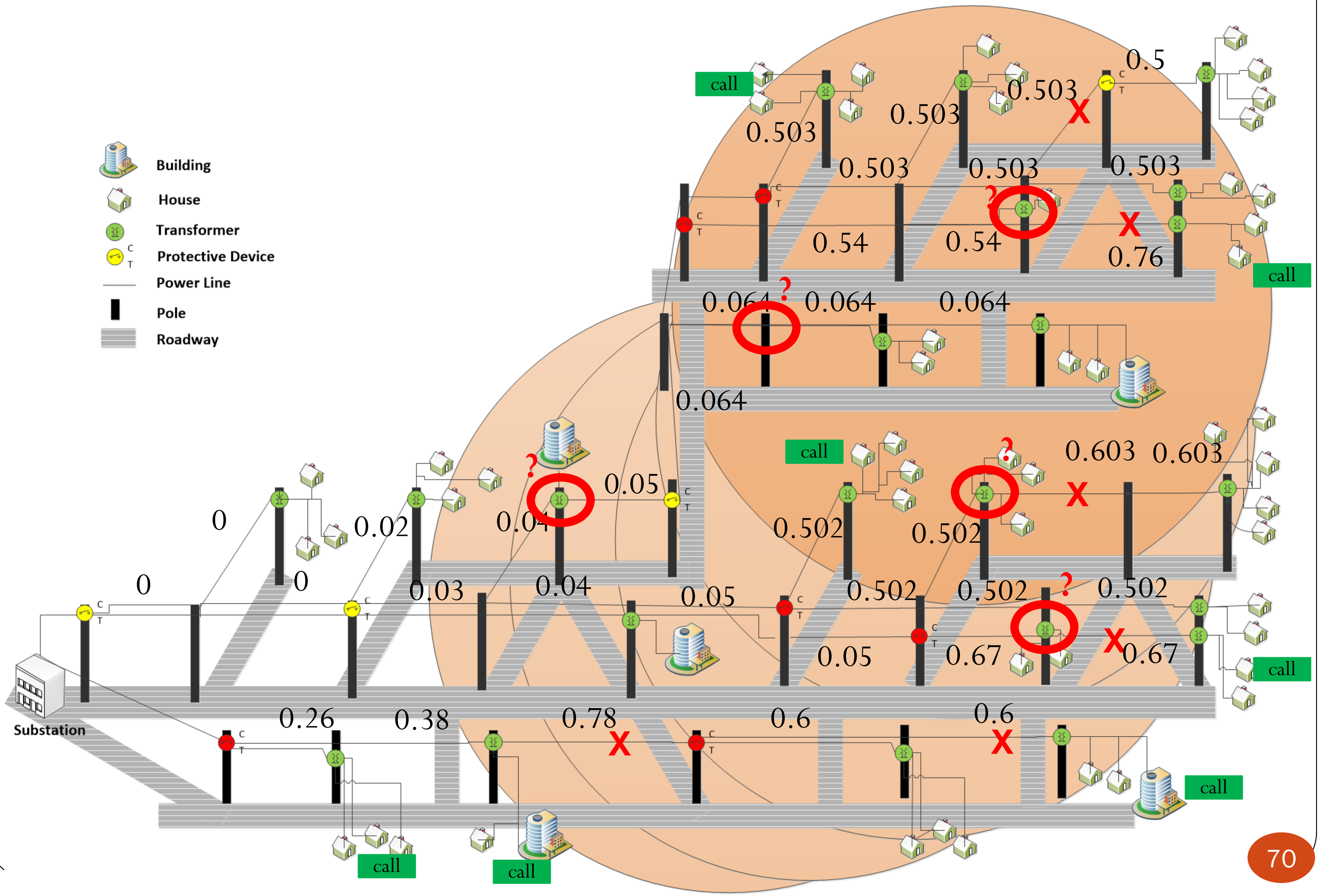
$$S_{t+1}(a) = S^M\left(S_t(a), X^{\pi}(S_t(a)), W_{t+1}(a)\right)$$

$$||a||_1 \leq M$$

a: a binary vector; $a_i = 1$ if a protective device is placed at node i and 0 otherwise.

- $\binom{N}{M}$ combinations of protective devices which results in high computation complexity.
- Solved sequentially by placing one protective device at a time.

Optimal Protective Device and Sensor Placement

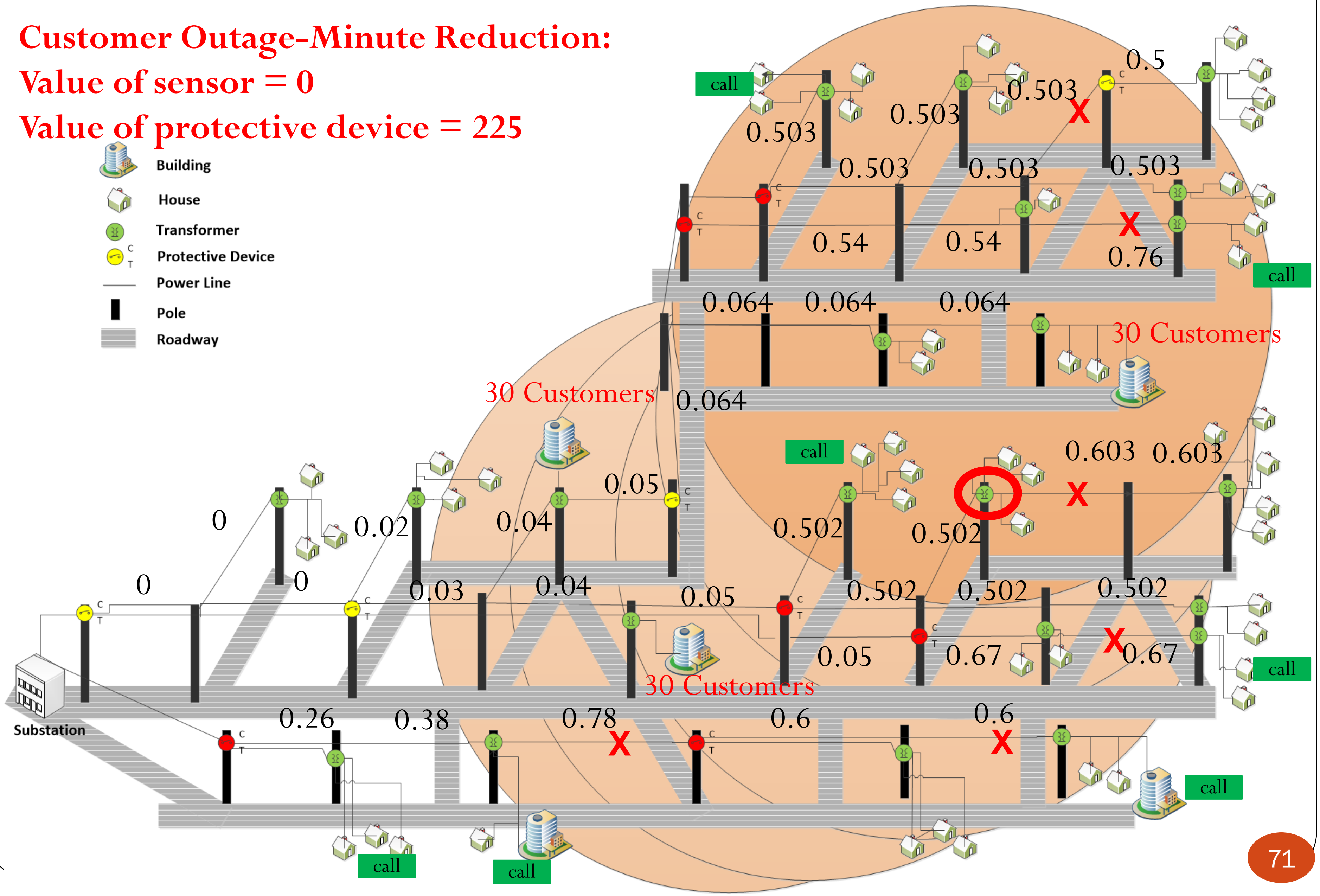


Case study A: sensor vs. protective device

Customer Outage-Minute Reduction:

Value of sensor = 0

Value of protective device = 225

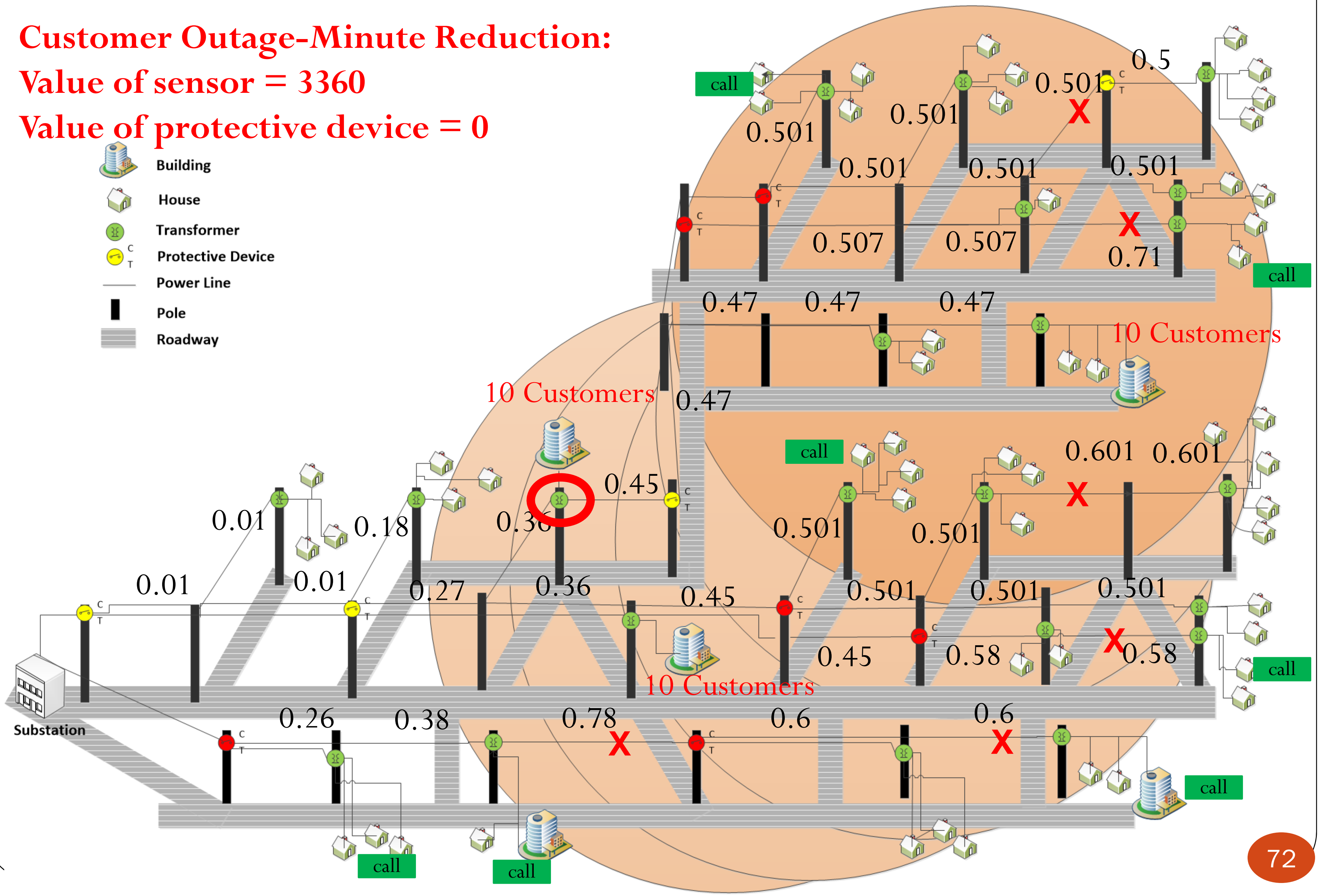


Case study B: sensor vs. protective device

Customer Outage-Minute Reduction:

Value of sensor = 3360

Value of protective device = 0



Summary

- We have developed:
  - A detailed storm and grid simulator that simulates storms, outages and lights-out calls
  - Models the behavior of protective devices and grid reconfigurations
  - An industry-standard escalation policy and a probabilistic lookahead model that uses prior knowledge and anticipates likely outages
  - Shown that the lookahead policy produces near-optimal performance when compared against optimal on a deterministic model
- Simulations
  - We have quantified the value of sensors and protective devices at different locations
  - We can find the best locations for each type of device
- Conclusion
  - We believe that our simulator can be used to develop effective rate cases for the BPU

Summary

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Thank you for your attention!

Problem Description – Grid Restoration

$\mathcal{I}$	set of poles, i.e., $\mathcal{I}=\{i, i=1,\dots,I\}$
$\mathcal{U}$	set of circuits, i.e., $\mathcal{U}=\{u,\ \ u=1,\dots,U\ \}$
$\mathcal{I}^{u}$	set of power lines on circuit u, i.e., $\mathcal{I}^{u}=\{i,\ i=1,\dots,I^{u}\ \}$
$\mathcal{N}^{u}$	Set of nodes on circuit u, i.e., $\mathcal{N}^{u}=\{N_i^{u},\ \forall i\in I^{u}\ \}$
n_i^{u}	Number of customers attached to node N_i^{u}
$n_{ti}^{u,c}$	Number of customers that called on node N_i^{u}
Ω_t	sample space at time t; each scenario ω represents phone calls, faults, travel & repair times
H_t	random vector representing the realizations of received calls at time t
L_t	random vector representing the realizations of power line faults at time t
T_{tij}	random variable representing the travel time from node i to node j at time t
R_j^{u}	random variable representing the repair time of power line j on circuit u
x_t	random vector representing the trajectory of the truck at time t; $x_t=\left(x_{tij}\right)_{i,j\in\mathcal{V}}$

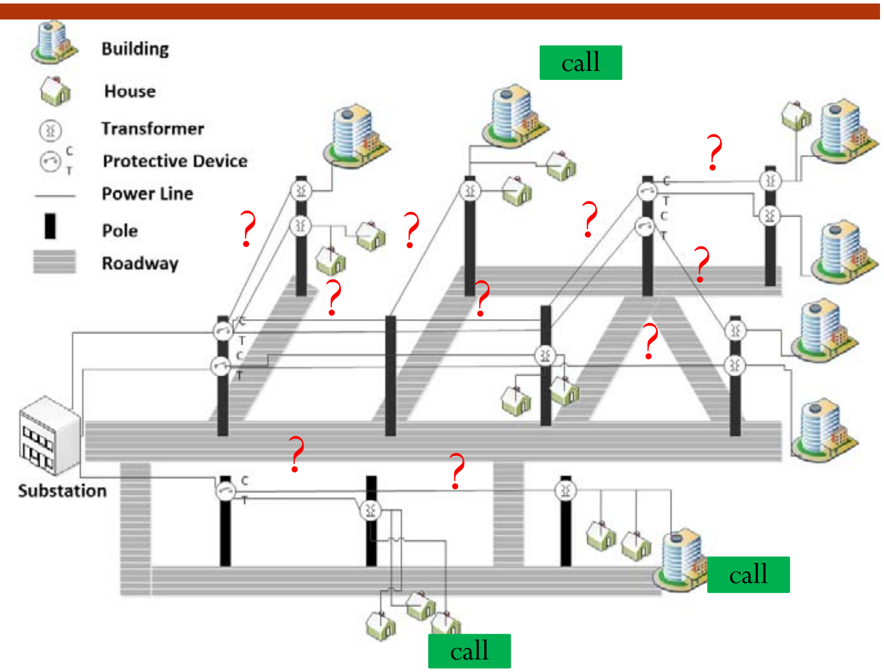


Illustration of distribution grid

- Bayes' Theorem:

Likelihood of calls given faults

Prior

Posterior

$$p(L_t|H_t)=\frac{p(H_t|L_t)p(L_t)}{p(H_t)}=\frac{p(H_t|L_t)p(L_t)}{\sum_{L_t\in\mathcal{L}}p\ (H_t|L_t)p(L_t)}$$

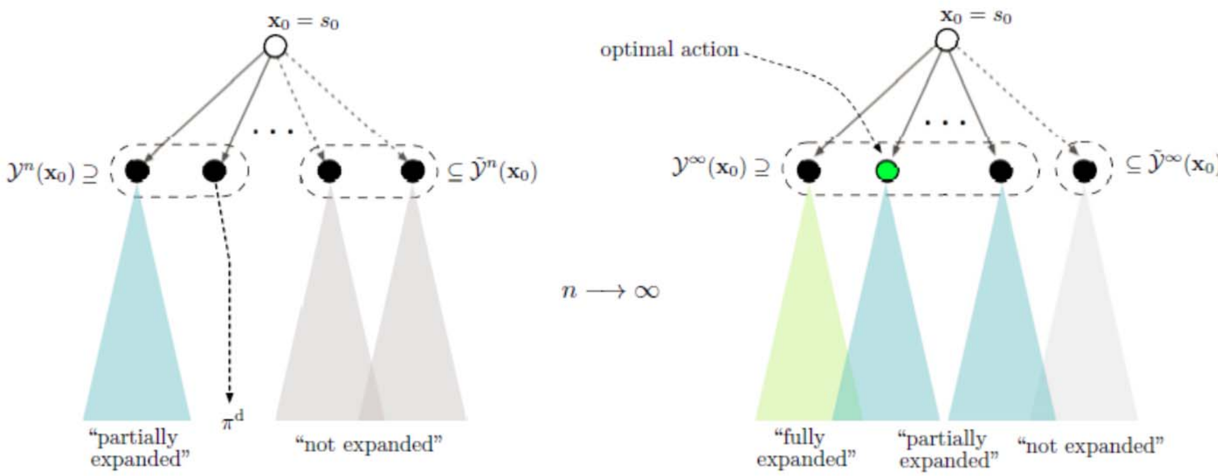
Probability of calls

[1] L. Al-Kanj, B. Bouzaeini-Ayari and W. Powell, "A Probability Model for Grid Faults Using Incomplete Information", *IEEE Transactions on Smart Grids*, July 2015.

Problem Description – Simulator

- Features of the simulator:
  - Complete model of the distribution grid:
    - Substations
    - Sensors, protective devices, reconfiguration rules, …
  - Able to simulate:
    - Many storm trajectories with varying sizes
    - Process of outages occurring as storm progresses
    - Model of who loses power (from grid configuration)
    - Process of lights-out calls (with variable call-in rates)
  - Probabilistic model of outages
    - Captures prior knowledge and probabilistic model of lights-out calls
  - Dispatch policies:
    - Industry-standard escalation policy – Uses lights-out calls and grid configuration
    - Stochastic lookahead policy – Exploits uncertainty model of outage probabilities

Monte Carlo Tree Search



Primal-Dual MCTS Algorithm

- Propose a Primal-Dual MCTS, that takes advantage of the *information relaxation bound* idea that asymptotically converges [8] to the optimal solution while ignoring suboptimal parts of the tree
- Explore a set of actions:

$$\hat{u}^n(\mathbf{x}^n_{\tau_e}, a) =$$
$$c_{\tau_e}(s, a, W^n_{\tau_e+1}) + \max_{\mathbf{a}}\left[h_{\tau_e+1}(S_{\tau_e+1}, \mathbf{a}, \mathbf{W}^n_{\tau_e+1,T}) - z^{\nu}_{\tau_e+1}(S_{\tau_e+1}, \mathbf{a}, \mathbf{W}^n_{\tau_e+1,T})\right]$$

- Smooth with previous estimates:

$$\bar{u}^n(\mathbf{x}^n_{\tau_e}, a) = (1 - \alpha^n(\mathbf{x}^n_{\tau_e}, a))\, \bar{u}^{n-1}(\mathbf{x}^n_{\tau_e}, a) + \alpha^n(\mathbf{x}^n_{\tau_e}, a)\, \hat{u}^n(\mathbf{x}^n_{\tau_e}, a).$$

- Expand it if the value is greater than $\bar{V}^{n-1}(\mathbf{x}^n_{\tau_e})$

[8] D. Jiang, L. Al-Kanj, W. B. Powell, "Monte Carlo Tree Search with Information Relaxation Dual Bounds", In preparation for Operations Research, 2017.

Simulation Policy – Integer programming

- The problem can be formulated as a non-linear integer program as follows:

$$\min_{\tilde{x}_{t''}} \sum_{\hat{t}=t'}^{t'+H} \left(N - \sum_{t''=t'}^{\hat{t}} \sum_{u\in\mathcal{U}} \sum_{j=1}^{N} \tilde{C}^u_{t''j} \right)$$

Minimizes the customer-outage minute

subject to

$$\tilde{C}^u_{t''j} = \sum_i \left(\prod_{k\in\mathcal{Q}^u_j\setminus j} 1 - \mathbb{1}_{\tilde{L}^u_{t''k}} \right) \mathbb{1}_{\tilde{L}^u_{t''j}} \left(\sum_{k\in\mathcal{S}^u_j} n^u_k + \sum_{s\in\mathcal{W}^u_j} \left(\prod_{w=\min\{\mathcal{W}^u_j\}}^{s} \prod_{k\in w} 1 - \mathbb{1}_{\tilde{L}^u_{t''k}} \right) \sum_{k\in s} n^u_k \right) \tilde{x}_{t''ij}, \quad \forall j\in\mathcal{V}, \forall t''$$

Computes number of customers with restored power

$$\mathbb{1}_{\tilde{L}^u_{t''j}} = 1 - \sum_i \sum_{\hat{t}=t'}^{t''-1} \tilde{x}_{\hat{t}ij}, \text{ such that } \tilde{L}^u_{t'j}(\tilde{\omega}) = 1, \forall j\in\mathcal{I}^u, \forall u\in\mathcal{U}, \forall t''$$

Indicates whether power line j is still in outage

$$\tilde{\Delta}_{t''ij} \geq T_{ij}(\tilde{\omega})\tilde{x}_{t''ij} + \sum_u R^u_j(\tilde{\omega}) \left(\tilde{x}_{t''ij} - \sum_{\hat{t}=t'}^{t''-1} \tilde{x}_{\hat{t}ji} \right), \forall (i,j)\in\mathcal{A}, \forall t''$$

Computes required travel and repair time by going from node i to node j

$$\tilde{\xi}_{t''j} \geq \tilde{\xi}_{t''-1i} + \sum_i \tilde{\Delta}_{t''ij}, \forall (i,j)\in\mathcal{A}, \forall t''$$

$$\tilde{\xi}_{t''j} \leq t'' \sum_i \tilde{x}_{t''ij} + \zeta \left(1 - \sum_i \tilde{x}_{t''ij} \right), \forall j\in\mathcal{V}, \forall t''$$

Guarantee that the required travel and repair times are met

$$\sum_{t''=t'}^{t'+H} \tilde{x}_{t''ij} \leq 1, \forall (i,j)\in\mathcal{A}$$

Guarantees that each arc is visited at most once in one direction

$$\sum_k \tilde{x}_{(t''+T_{jk}(\tilde{\omega}))jk} + \sum_k \tilde{x}_{(t''+T_{jk}(\tilde{\omega})+\sum_u R^u_k(\tilde{\omega}))jk} \leq \sum_i \tilde{x}_{t''ij} \leq 1, \forall j\in\mathcal{V}, \forall t''$$

Indicates whether the truck can move to the next node

$$\tilde{C}^u_{t''j} \geq 0, \tilde{\xi}_{t''j} \geq 0, \tilde{\Delta}_{t''ij} \geq 0, \tilde{x}_{t''ij} \in \{0,1\}$$

Domains of the variables