Multi-Period Dual Pricing Algorithm for Cost Allocation in Non-Convex Electricity Markets

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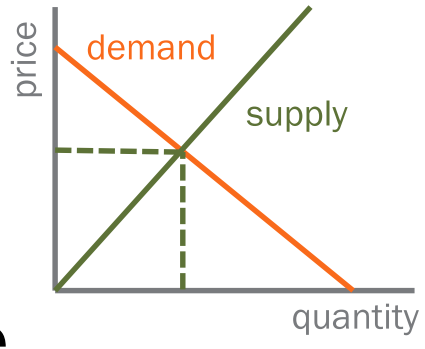
Outline

- What makes a good price?
  - Price Formation NOPR
- Dual Pricing Algorithm
  - Basic unit commitment formulation
  - Alternative pricing formulations
  - Multi-period DPA model
- Comparison of pricing methods
  - Examples

What makes a good price?

- In convex cases, market clearing prices are
  - Revenue neutral
  - Non-confiscatory
  - Incent investment (signals for entry)
- Electricity markets are non-convex due to lumpy costs



Literature

- Many proposals for non-convex pricing
  - LMP with uplift payments (O'Neill, Sotkiewicz, Hobbs, Rothkopf, Stewart)
  - Convex hull (Hogan & Ring ; Gribik, Hogan & Pope)
  - Extended LMP (Wang, Luh, Gribik, Zhang & Peng)
  - Modified LMP (Bjørndal & Jörnsten)
  - General uplift with zero-sum transfers (Motto & Galiana)
  - Semi-Lagrangean approach (Araoz & Jörnsten)
  - Primal-dual approach (Ruiz, Conejo, & Gabriel)
  - Review and internal zero-sum uplifts (Liberopoulos & Andrianesis)

	Two-Part Pricing							Single Price		
	Schweppe [25]	O'Neill [9]	Gribik [10], [11]	ELMP [14]	Bjørndal [6]	Galiana [15], [16]	DPA [4]	AIC	Araoz [18]	Ruiz [19]
Maximize market surplus	Y	Y	Y	Y	Y	Y	Y	Y	Y	N
Revenue neutral	Y	N	N	N	N	Y	Y	Y	Y	Y
Includes demand side	Y	N	N	N	N	Y	Y	Y	Y	Y
Maintain optimal dispatch	Y	Y	Y	Y	Y	Y	Y	Y	Y	N
Transparency	Y	N	N	N	N	N	Y/N	Y	Y	Y
Uplifts	Ex-post	Ex-post	Ex-post	Ex-post	Ex-post	Internal	Internal	None	None	None
Pricing problem type	LP	LP	CH	LP	LP+	NLP	LP	LP	LP+	MIP*

FERC Price Formation

- What are the goals of price formation?*
  - Maximize market surplus
  - Provide correct incentives for market participants to follow commitment and dispatch instructions and make efficient investments
  - More transparently reflect the marginal cost of serving load and operational constraints
  - Ensure suppliers can recover costs

*Adapted from Order Directing Reports on Price Formation AD14-14-000

Pricing & Cost Allocation Principles

Maximize surplus

- Assumes demand can bid their value

Non-confiscation

- Incent participants to stay in the market
- Generator profits ≥ 0
- Net demand value ≥ 0

Revenue neutrality

- For each market payments equal receipts
- Money out = money in

Incentivize efficient investments

- Prices signal entry into the market
- Transparency

Pricing Principles

Current pricing system:		
LMP	+	Side payment
Public		Private & discriminatory

- New pricing system?
  - Begin with economic principles
  - Address any deficiencies

Ramsey-Boiteux Pricing

- Ramsey: allocate fixed costs based on willingness to pay
  - Inverse elasticity pricing rule
  - Discriminatory (but not unduly discriminatory)
- Boiteux: differentiated a public and private price
  - Demand that is more elastic pays less
- Necessary for efficiency in non-convex markets

$$\lambda_r = \frac{c'(p)}{1 - \alpha/e} \quad \rightarrow \quad \lambda_r = c'(p) + \lambda_r(\alpha/e)$$

e	Demand elasticity
$c'(p)$	Marginal cost function
α	$-\gamma/(1-\gamma)$ dual of cost recovery constraint

Additional considerations

- Ease of implementation
  - Type of problem: linear, mixed-integer
- Incentive for following dispatch
  - Penalties administered or opportunity costs paid
  - Market power mitigated through regulation
- Demand side participation
  - Demand is price responsive (some markets seeing higher participation)

Post-Unit Commitment Pricing Model

$\max \; \sum_{t\in T}\left(\sum_{i\in D} b_{it} d_{it} - \sum_{i\in G}\left(c_{it} p_{it} + c_{it}^{OC} u_{it} + c_i^{SU} z_{it}\right)\right)$		Market surplus
$\sum_{i\in D} d_{it} - \sum_{i\in G} p_{it} = 0$	$\forall t \in T$	Market clearing (λ_t)
$p_i^{\min} u_{it} \le p_{it} \le p_i^{\max} u_{it}$	$\forall i \in G, t \in T$	Generation bounds
$u_{it} - u_{i,t-1} \le z_{it}$	$\forall i \in G, t \in T$	Commitment def.
$0 \le d_i \le d_i^{\max}$	$\forall i \in D, t \in T$	Demand bounds
$u_{it} = u_{it}^*$	$\forall i \in G, t \in T$	Fix optimal schedule (δ_{it})
$z_{it} = z_{it}^*$	$\forall i \in G, t \in T$	Fix optimal schedule

$c_{it}, c_{it}^{OC}, c_i^{SU}$	Generator marginal and operating costs
$p_i^{\min}, p_i^{\max}$	Generator min and max capacity
b_{it}	Demand offer
$d_i^{\max}$	Demand max capacity
d_{it}	Demand
p_{it}	Generator production variable
u_{it}	Commitment variable (*=optimal)

ELMP Pricing Model

$\max\ \sum_{t\in T}\left(\sum_{i\in D} b_{it}d_{it} - \sum_{i\in G}\left(c_{it}p_{it} + c_{it}^{OC}u_{it} + c_i^{SU}z_{it}\right)\right)$		Market surplus
$\sum_{i\in D} d_{it} - \sum_{i\in G} p_{it} = 0$	$\forall t \in T$	Market clearing (λ_t)
$p_i^{\min}u_{it} \le p_{it} \le p_i^{\max}u_{it}$	$\forall i \in G, t \in T$	Generation bounds
$u_{it} - u_{i,t-1} \le z_{it}$	$\forall i \in G, t \in T$	Commitment def.
$0 \le d_i \le d_i^{\max}$	$\forall i \in D, t \in T$	Demand bounds
$0 \le u_{it} \le 1$	$\forall i \in G, t \in T$	Relax commitment
$0 \le z_{it} \le 1$	$\forall i \in G, t \in T$	Relax startup

$c_{it}, c_{it}^{OC}, c_i^{SU}$	Generator marginal and operating costs	d_{it}	Demand
$p_i^{\min}, p_i^{\max}$	Generator min and max capacity	p_{it}	Generator production variable
b_{it}	Demand offer	u_{it}	Commitment variable (*=optimal)
$d_i^{\max}$	Demand max capacity		

Average Incremental Cost Model

$\max \; \sum_{t\in T}\left(\sum_{i\in D} b_{it}d_{it} - \sum_{i\in G^{NP}} c_{it}p_{it} - \sum_{i\in G^{MP}} c_{it}^{AIC}p_{it}\right)$		Market surplus
$\sum_{i\in D} d_{it} - \sum_{i\in G} p_{it} = 0$	$\forall t \in T$	Market clearing (λ_t)
$0 \le p_{it} \le p_i^{\max}u_{it}^*$	$\forall i \in G^{MP}, t \in T$	Generation bounds
$p_i^{\min}u_{it}^* \le p_{it} \le p_i^{\max}u_{it}^*$	$\forall i \in G^{NP}, t \in T$	Generation bounds
$0 \le d_i \le d_i^{\max}$	$\forall i \in D, t \in T$	Demand bounds

$$c_{it}^{AIC} = c_{it} + \frac{c_{it}^{OC}u_{it}^*}{p_{it}^*} + \sum_{t\in T} \frac{c_i^{SU}u_{it}^*}{p_{it}^*}$$

$c_{it}, c_{it}^{OC}, c_i^{SU}$	Generator marginal and operating costs	d_{it}	Demand
$p_i^{\min}, p_i^{\max}$	Generator min and max capacity	p_{it}	Generator production variable
b_{it}	Demand offer	u_{it}	Commitment variable (*=optimal)
$d_i^{\max}$	Demand max capacity	MP/NP	Generators with a make whole payment / no payment

Comparative pricing methods

Name	Description	Price*
LMP Locational marginal price	Fix optimal solution, rerun to obtain prices	c
ELMP Extended LMP	Relax binary commitment variable (MISO fast start pricing)	$c + \frac{c^{OC}}{p^{max}}$
LIP Locational incremental price	Relax minimum to zero, use average incremental cost in objective	$c + \frac{c^{OC}}{p^{*}}$
DPA Dual pricing algorithm	Proposed here	λ^{DPA}

New Variables

- λ^{DPA} : new LMP
- u_i^p / u_i^{pd}: make-whole payment
- u_i^c / u_i^{cd} : make–whole charge
  - Allocated by resource

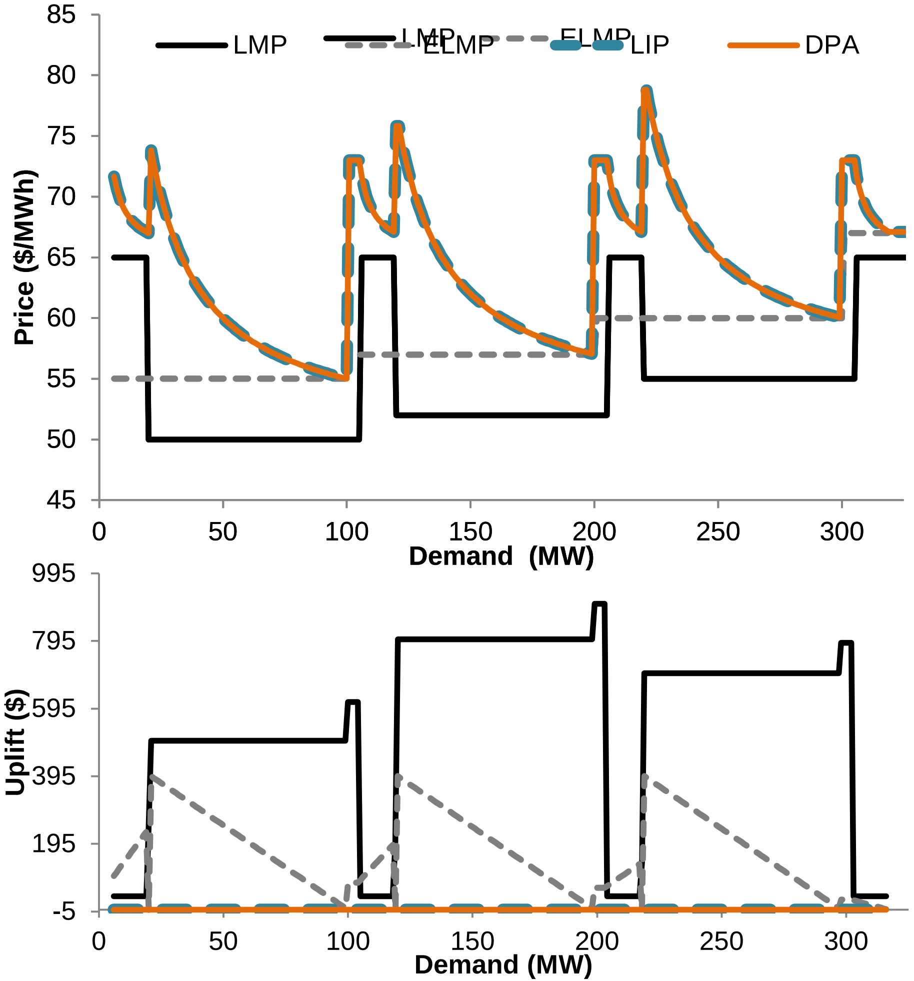
Multi-period formulation

$\min \sum_{t \in T} \left[\sum_{i \in D^+} d_{it}^* u_{it}^{pd} + \sum_{i \in G^+} p_{it}^* u_{it}^{p} + c^{up} \lambda_t^{up} + c^{dn} \lambda_t^{dn} \right]$		Uplift minimization
Subject to		
$\sum_{t \in T} \left[\sum_{i \in D^+} d_{it}^* \left(u_{it}^{pd} - u_{it}^{cd} \right) + \sum_{i \in G^+} p_{it}^* \left(u_{it}^{p} - u_{it}^{c} \right) \right] = 0$		Uplift revenue neutrality
$\Pi_i = \sum_{t \in T_r} \left(p_{it}^* \left(\lambda_t^{DPA} - c_{it} + u_{it}^{p} - u_{it}^{c} \right) - u_{it}^* c_{it}^{OC} - z_{it}^* c_i^{SU} \right)$	$\forall i \in G^+$	Profit definition
$\Psi_i = \sum_{t \in T_r} d_{it}^* \left(b_{it} - \lambda_t^{DPA} + u_{it}^{pd} - u_{it}^{cd} \right)$	$\forall i \in D^+$	Value definition
$(\lambda_t^{DPA} - \lambda_t^*)/\lambda_t^* - \lambda_t^{up} + \lambda_t^{dn} = 0$	$\forall t \in T$	Price conditioning
$\lambda_t^{DPA} \geq b_{it}$	$\forall i \in D^0, t \in T$	Non-recourse condition
$\Psi_i \geq 0$	$\forall i \in D^+$	Non-confiscation of demand
$\Pi_i \geq 0$	$\forall i \in G^+$	Non-confiscation of supply
$u_{it}^{p}, u_{it}^{c}, u_{it}^{pd}, u_{it}^{cd} \geq 0$	$\forall i \in D \cup G, t \in T$	Non-negativity

MISO example
DPA reflects average incremental costs

Gen	Min Cap (MW)	Max Cap (MW)	Marginal Cost ($/MWh)	Operating cost ($/h)
A	20	100	50	500
B	20	100	52	500
C	20	100	55	500
D	5	20	65	40

- DPA and LIP prices are the same and have no uplift
- ELMP is increasing wrt demand



Example Source: Gribik, Zhang “Extended Locational marginal Pricing,” FERC Software Conference 2010

Simple Multiperiod Example:

Conditioning impacts prices across time

Gen	Min Cap (MW)	Max Cap (MW)	Marginal Cost ($/MWh)	Operating cost ($/h)	Startup cost ($/start)
A	200	1200	30	100	900
B	50	80	50	100	600

Hour	1	2	3	4	5	6	7	8	Uplift ($)
Dispatched generator	A	A	A	A	A	A+B	A+B	A	
LMP λ_t^*	30	30	30	30	30	30	30	30	4500
DPA, multi λ_t^{DPA}	30.22	30.22	30.22	30.22	30.22	30.22	30.22	30.22	Net 0*
DPA, single $\lambda_t^{\mathrm{DPA}\prime}$	30	30	30	30	30	86	30	30	0

DPA, multi:

$$(\lambda_t^{DPA} - \lambda_t^*)/\lambda_t^* - \lambda^{up} + \lambda^{dn} = 0$$

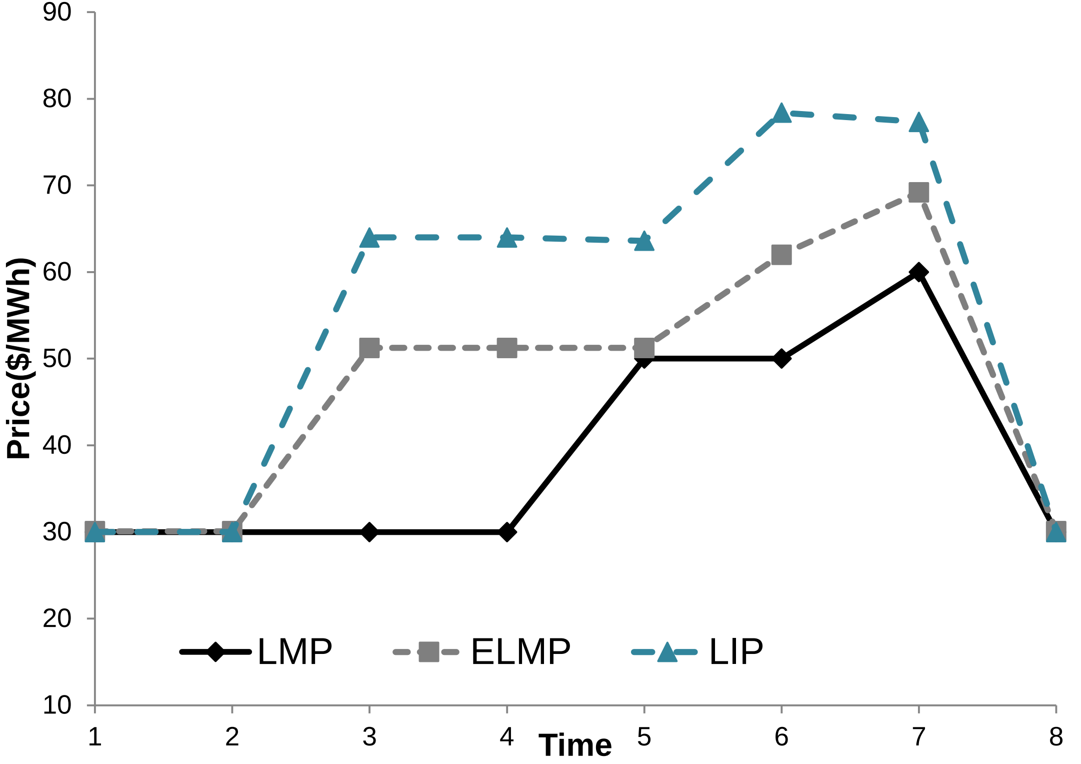
DPA, single:

$$(\lambda_t^{DPA} - \lambda_t^*)/\lambda_t^* - \lambda_t^{up} + \lambda_t^{dn} = 0$$

*Demand pays Gen B $2779

Multiperiod Comparison

Gen	Min Cap (MW)	Max Cap (MW)	Marginal Cost ($/MWh)	Operating cost ($/h)	Startup cost ($/start)
A	200	1200	30	100	900
B	50	80	50	100	600
C	25	50	60	100	360



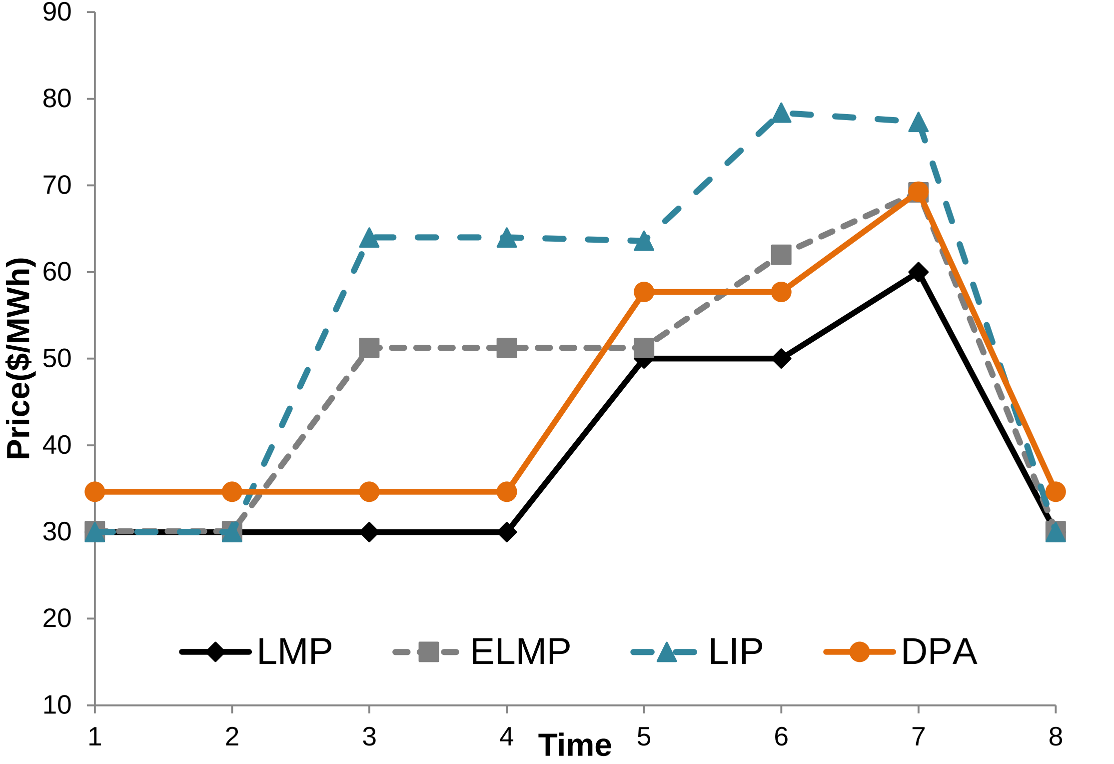
Dispatch	A	A	A+$\underline{\text{B}}$	$\overline{A}$+$\underline{\text{B}}$	$\overline{A}$+B	$\overline{A}$+$\overline{B}$+$\underline{C}$	$\overline{A}$+$\overline{B}$+C	A

Multiperiod Comparison:
DPA prices follow LMP allocating uplift in peak period

Gen	Min Cap (MW)	Max Cap (MW)	Marginal Cost ($/MWh)	Operating cost ($/h)	Startup cost ($/start)
A	200	1200	30	100	900
B	50	80	50	100	600
C	25	50	60	100	360

- Uplift:
  - LMP $3110
  - ELMP $197
  - LIP $0
  - DPA $302
    - Dem 2 pays $0.472/MWh to Gen C in period 7

$$(\lambda_t^{DPA} - \lambda_t^*)/\lambda_t^* - \lambda^{up} + \lambda^{dn} = 0$$



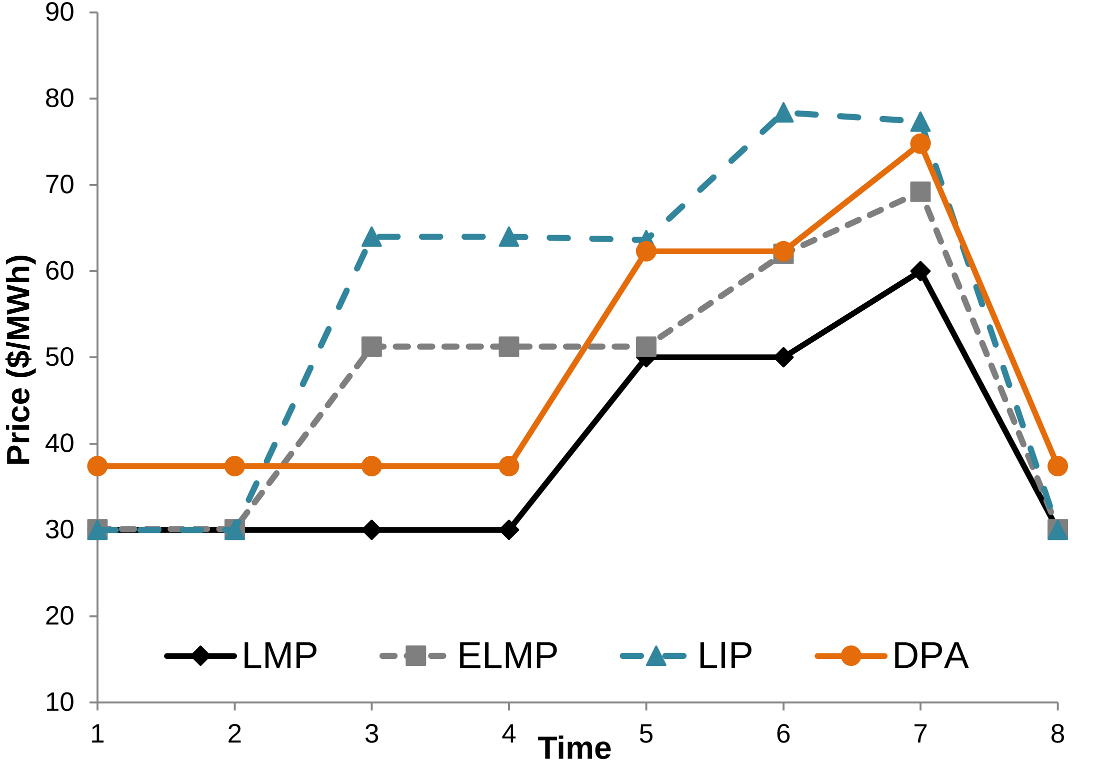
Dispatch							
A	A	$A+\underline{B}$	$\overline{A}+\underline{B}$	$\overline{A}+B$	$\overline{A}+\overline{B}+\underline{C}$	$\overline{A}+\overline{B}+C$	A

Multiperiod Comparison:

DPA prices slightly higher with no side payment

Gen	Min Cap (MW)	Max Cap (MW)	Marginal Cost ($/MWh)	Operating cost ($/h)	Startup cost ($/start)
A	200	1200	30	100	900
B	50	80	50	100	600
C	25	50	60	100	360

- Uplift:
  - LMP $3110
  - ELMP $197
  - LIP $0
  - DPA $0



$$(\lambda_t^{DPA} - \lambda_t^*)/\lambda_t^* - \lambda_t^{up} + \lambda_t^{dn} = 0$$

Dispatch	A	A	$A+\underline{B}$	$\overline{A}+\underline{B}$	$\overline{A}+B$	$\overline{A}+\overline{B}+\underline{C}$	$\overline{A}+\overline{B}+C$	A

Properties of the DPA

- Non-confiscation
- Revenue neutral (and adequate)
- Feasible solution with optimal feasible UC
- Does not change optimal dispatch solution
- Easy to implement in present ISO software
- Problem is linear – computationally efficient
- Price is non-unique
  - Can be conditioned depending on operator preference

Thank you!

Questions?

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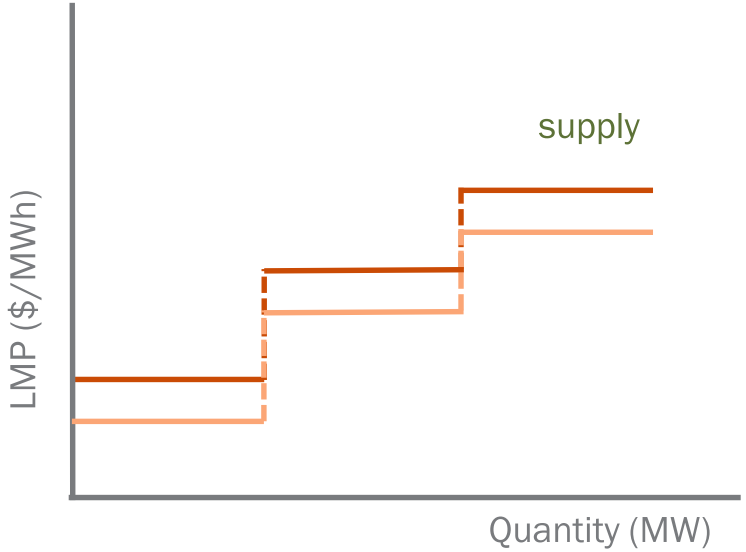
What makes a good price?

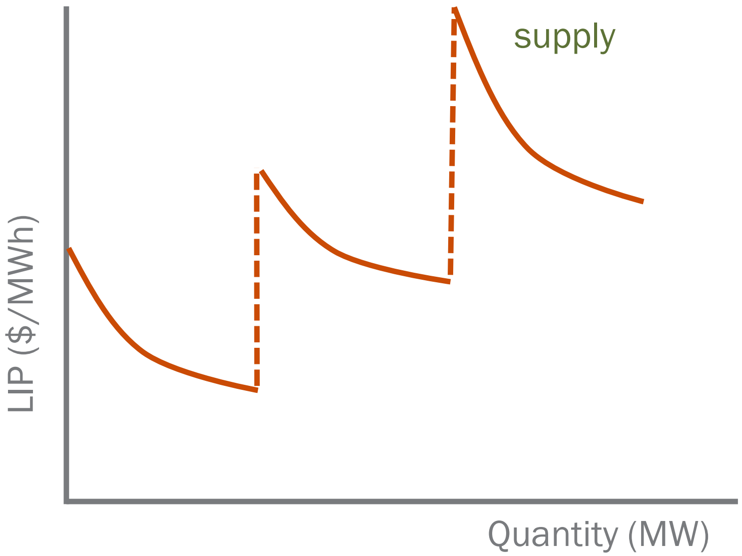
- In a non-convex market, the answer is not straightforward
  - "Nomads in an intellectual desert" – Matt White
  - "An important objective of electricity market design is to provide efficient prices with the associated incentives for operation and investment." – Bill Hogan
    - Incentives for operation: stay on dispatch
    - Incentives for investment: new resources entering the market

Comparison of methods:
Trends in prices

- LMP (lighter)
- ELMP

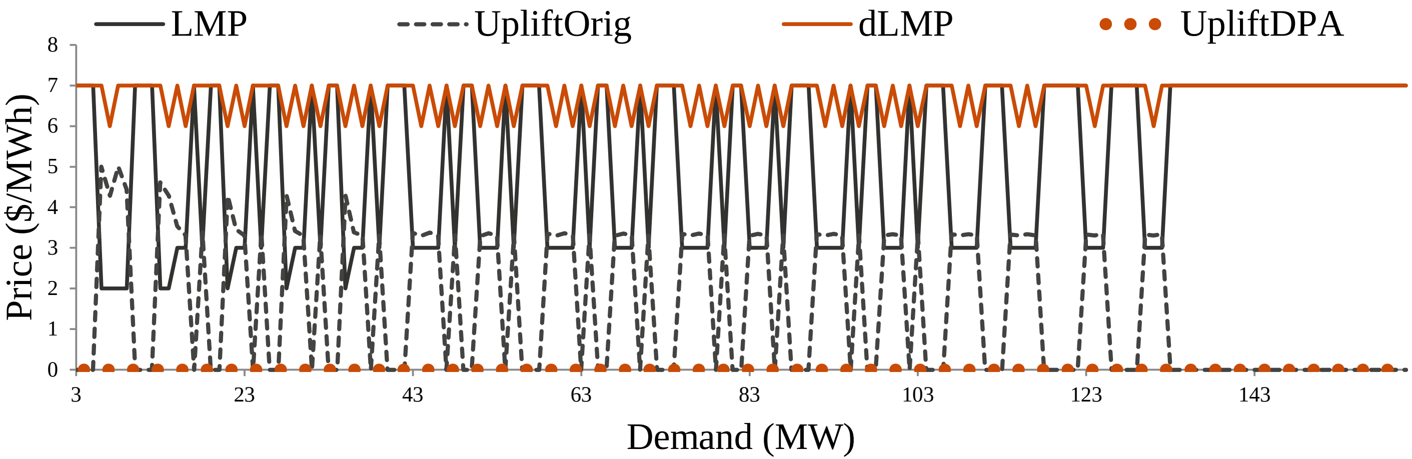


- LIP
- (DPA)



Example: Scarf

- Modified Scarf example



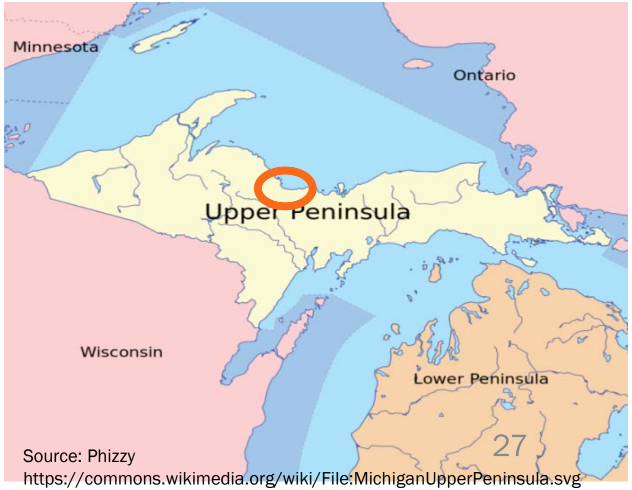
Historical Example: Canal Units

- Canal Units on Cape Cod run daily due to long startup times and regional specifications
- Units support customers on Cape Cod
  - Without that demand, they would not be needed
- Uplift broadly allocated including Lower Southeastern Massachusetts (SEMA)
  - SEMA does not benefit
  - Costs should have been allocated primarily to Cape Cod to find a cheaper alternative much sooner

Source: http://www.capecodtimes.com/article/20150716/NEWS/150719565

Historical Example: Upper Peninsula

- Presque Isle Power Plant mainly powers the Upper Peninsula (UP)
  - Generates 90% of power in UP, 12% in Wisconsin Energy system
  - Sells 50% to Empire and Tilden mines
- Used for reliability in UP
  - Costs allocated to all LSEs in Wisconsin and UP on a pro rata basis
  - FERC found this unjust and unreasonable



Source: Phizzy
https://commons.wikimedia.org/wiki/File:MichiganUpperPeninsula.svg

Post-UC Pricing Model

$\max \; \sum_{i\in D} b_i d_i - \sum_{i\in G}\left(c_i p_i + c_i^{\mathrm{SU}} z_i\right)$			Market surplus
$\sum_{i\in D} d_i - \sum_{i\in G} p_i = 0$		λ	Market clearing
$p_i^{\min} z_i \le p_i \le p_i^{\max} z_i$	$\forall i \in G$	$\beta_i^{\max}, \beta_i^{\min}$	Generation bounds
$0 \le d_i \le d_i^{\max}$	$\forall i \in D$	$\alpha_i^{\max}$	Demand bounds
$z_i = z_i^*$	$\forall i \in G$	δ_i	Fix optimal schedule

Decision variables

p_i Cleared energy

d_i Cleared demand

z_i Startup commitment

Dual Model

$\min \; \sum_{i\in D} d_i^{\max}\alpha_i^{\max} + \sum_{i\in G} z_i^{*}\delta_i$			Resource valuation
$\lambda + \alpha_i^{\max} \geq b_i$	$\forall i \in D$	d_i	Value condition
$-\lambda + \beta_i^{\max} - \beta_i^{\min} \geq -c_i$	$\forall i \in G$	p_i	Profit condition
$\delta_i - p_i^{\max}\beta_i^{\max} + p_i^{\min}\beta_i^{\min} = -c_i^{\mathrm{SU}}$	$\forall i \in G$	z_i	Startup economics
$\alpha_i^{\max}, \beta_i^{\max}, \beta_i^{\min} \geq 0$	$\forall i \in D \cup G$		Non-negativity

Objective

- Minimize uplift payments
  - $\min \sum_{i \in D^+} d_i^* u_i^{\mathrm{pd}} + \sum_{i \in G^+} p_i^* \, u_i^{\mathrm{p}}$
  - Uplift payments from demand and generation

Market Surplus

- Maintain optimal market surplus
  - $\sum_{i \in D} \Psi_i + \sum_{i \in G} \Pi_i = MS^*$
  - Use optimal dispatch, making it a redundant constraint

Maximize market surplus

Profit Definition

- From complementary slackness of the generation bounds and the profit condition, combining with the startup economics, we calculate the linear surplus of generator *i*
  - $\delta_i = p_i^*(\lambda - c_i) - c_i^{SU}$
  - dispatch*(LMP – marginal cost) – startup cost
- To ensure non-confiscation, the linear surplus and uplift payments must be non-negative
  - $\Pi_i = \delta_i + p_i^*(u_i^p - u_i^c) \geq 0$

Non-confiscation

Value Definition

- From complementary slackness of the value condition, and non-negativity of variables, demand *i*
  - $d_i^*(b_i - \lambda) = d_i^* \alpha_i^{max*} \geq 0$
- To ensure non-confiscation, the value and uplift payments must be non-negative
  - $\Psi_i = d_i^* \alpha_i^{max*} + d_i^* \left(u_i^p - u_i^c\right) \geq 0$

Non-confiscation

Additional constraints

- Revenue neutrality
  - $\sum_{i\in D^+} d_i^* \left(u_i^{\mathrm{pd}} - u_i^{\mathrm{cd}}\right) + \sum_{i\in G^+} p_i^* \left(u_i^{\mathrm{p}} - u_i^{\mathrm{c}}\right) = 0$
- Non-recourse of demand not selected
  - $\lambda^{\mathrm{DPA}} \geq b_i$
  - Value of new LMP not entice out-of-market demand to consume

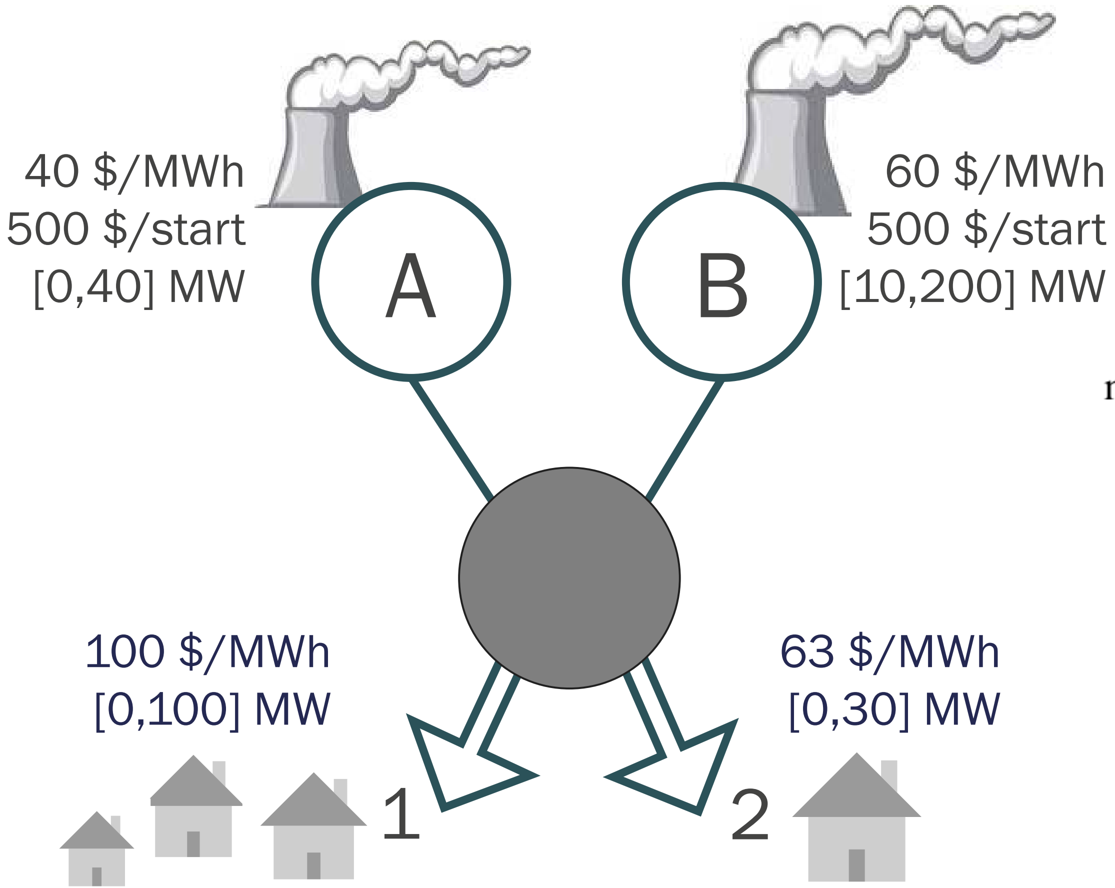
Revenue neutrality

Non-Unique Prices

- Conditioning
  - Allows the market operator to adjust LMP based on regional policies
- Example: tie new LMP to LMP from dispatch run
  - New constraint: $\frac{(\lambda^{\mathrm{DPA}}-\lambda^{*})}{\lambda^{*}} - \lambda^{\mathrm{up}} + \lambda^{\mathrm{dn}} = 0$
  - New Objective:

$$\min \sum\nolimits_{i \in D} d_i^* u_i^{\mathrm{pd}} + \sum\nolimits_{i \in G} p_i^* u_i^{\mathrm{p}} + c^{\mathrm{up}} \lambda^{\mathrm{up}} + c^{\mathrm{dn}} \lambda^{\mathrm{dn}}$$

Example: Single node, single period



$$\max MS = \sum_{i \in D} b_i d_i - \sum_{i \in G} (c_i p_i + c_i^{su} z_i)$$

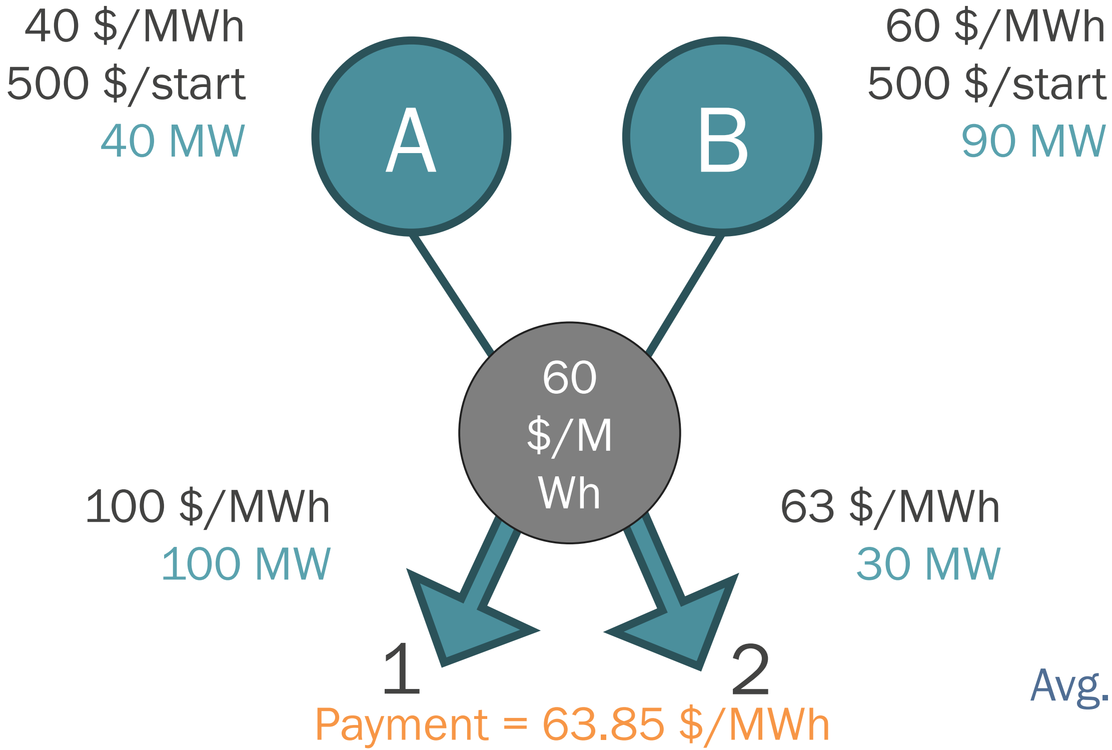
$$\sum_{i \in D} d_i - \sum_{i \in G} p_i = 0$$

$$p_i^{\min} z_i \le p_i \le p_i^{\max} z_i \quad \forall i \in G$$

$$0 \le d_i \le d_i^{\max} \quad \forall i \in D$$

$$z_i = z_i^* \quad \forall i \in G$$

Resulting UC Solution



Market surplus = $3830		
Gen	Margin ($/MWh)	Profit ($)
A	20	300
B	0	-500
Buyer	Margin ($/MWh)	Net Value ($)
1	40	4000
2	3	90

Price = $60/MWh

Uplift = $500

Avg. socialized uplift = $3.85/MWh

Results of DPA

λ^{DPA}	Make whole payment	Unallocated make whole payment
65.56	76.67	0

Gen	Marg. Cost	u^p	u^c
A	40	0	0
B	60	0	0
Buyer	**Value**	**u^p**	**u^c**
1	100	0	0.767
2	63	2.556	0

u_i^p Make whole payment

u_i^c Make whole charge

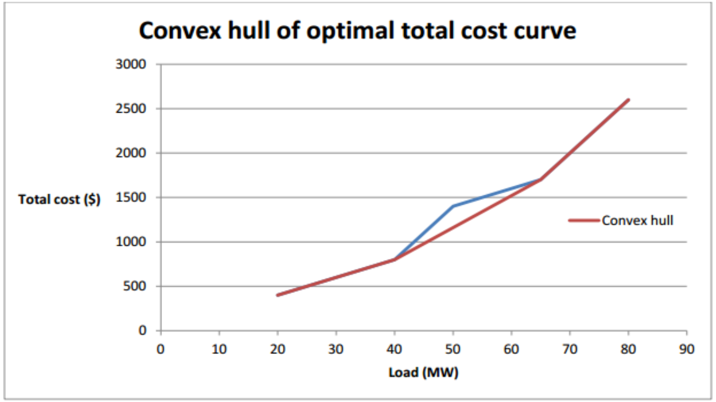
λ^{DPA} New LMP

Results of DPA

		Post-UC Value ($)		Value under DPA ($)	
LMP (λ)		60		65.56	
		Unit ($/MWh)	Total	Unit ($/MWh)	Total
Profit	Gen A	20	300	25.56 (+28%)	522.22 (+74%)
	Gen B	0	-500	5.56	0

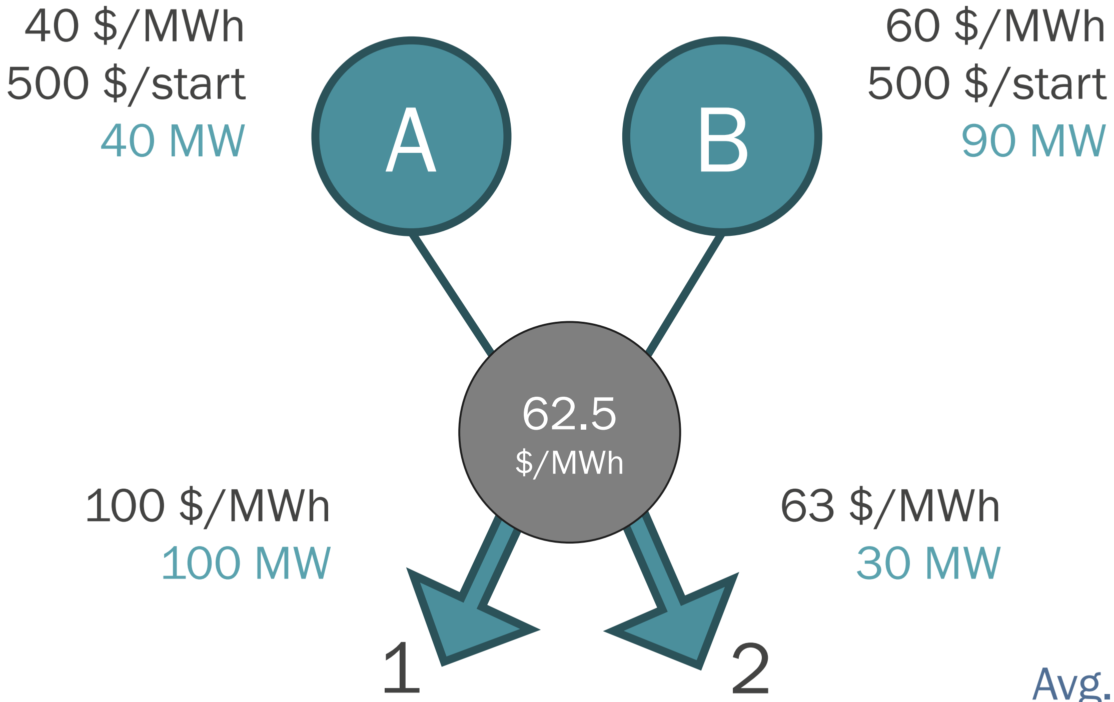
Comparison to Convex Hull

- Convex hull formulation finds a uniform price that minimizes side payments
  - Not all side payments minimized
  - Not well understood
- Formulation based on [1]



[1] D.A. Schiro, T. Zheng, F. Zhao, and E. Litvinov, "Convex Hull Pricing in Electricity Markets: Formulation, Analysis, and Implementation Challenges," ISO-NE. [Online] Available: http://www.optimization-online.org/DB_FILE/2015/03/4830.pdf

Resulting CH Solution



Payment = 64.62 $/MWh

Market surplus = $4165		
Gen	Margin ($/MWh)	Profit ($)
A	20	400
B	0	-275
Buyer	Margin ($/MWh)	Net Value ($)
1	40	3750
2	3	15

Price = $62.50/MWh

Uplift = $275

Avg. socialized uplift = $2.12/MWh

Results Comparison

		Original Value		Value under DPA		Value under Convex Hull	
LMP λ ($/MWh)		60		65.56		62.50	
		Unit ($/MWh)	Total	Unit ($/MWh)	Total	Unit ($/MWh)	Total
Profit	Gen A ($40/MWh)	20 (-)	300 (-)	25.56 (+28%)	522.22 (+74%)	22.50 (+13%)	400 (+33%)
	Gen B ($60/MWh)	0	-500	5.56	0	2.50	-275
Value	Buyer 1 ($100/MWh)	40 (-)	4000 (-)	33.678 (-19%)	3367 (-19%)	37.50 (-6%)	3750 (-6%)
	Buyer 2 ($63/MWh)	3	90	0	0	0.50	15

Revenue Adequacy and LOCs

Market surplus = $200						
Gen	**Marginal Cost ($/MWh)**	**Start Up Cost**	**Linear Profit ($)**	**Dispatch (MWh)**	**Max Capacity (MW)**	**Total Cost ($)**
A	30	900	1100	0	200	0
B	40	100	-100	60	200	2500
LMP = **$40/MWh** Uplift = **-$100** Avg. socialized uplift = **-$1.67/MWh**						
Buyer	**Value ($/MWh)**	**Load (MWh)**	**Max demand (MW)**	**Marginal Value ($/MWh)**	**Total Value ($)**	**Gross Value ($)**
1	45	60	60	5	300	2700

200 MWh($40/MWh-$30/MWh)-$900 = $1100 = LOC > MS = $200