Identifying and Controlling Risky Contingencies of Transmission Systems

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N - K criterion revisited

Previous work: Salmeron and Wood, Donde et al, Turitsyin, Hines

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- **N - 1** criterion widely used.

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- How about **N - K**, for **K** "larger"? Everybody knows that:
  - It is *too* slow. A very difficult combinatorial problem.

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Table 1: $\binom{N}{K}$

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Example: August 14 2003

U.S. - Canada report on blackout:

"Because it had been hot for several days in the Cleveland-Akron area, more air conditioners were running to overcome the persistent heat, and consuming relatively high levels of reactive power – further straining the area's limited reactive generation capabilities."

→ A **system-wide** condition that impedes the system

→ Not a cause, but a contributor

→ Look for combined events ?

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  - Perhaps **N - K** does not necessarily capture all interesting events?
- How can we deal with both types of problems?

A continuous interdiction model

- A fictitious adversary is trying to interdict the transmission system.
- This adversary negatively alters the physical parameters of equipment, e.g. transmission lines, so as to impede transmission.
- The adversary has a budget available (both system-wide and per-line).
  - On line km, reactance $\boldsymbol{x_{km}}$ increased to $\boldsymbol{(1+\lambda_{km})x_{km}}$

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  - $\boldsymbol{\sum_{km} \lambda_{km} \leq \Lambda}$ (global limit)

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- This adversary negatively alters the physical parameters of equipment, e.g. transmission lines, so as to impede transmission.
- The adversary has a budget available (both system-wide and per-line).
- Adversary maximizes the impact (e.g. voltage loss) over the available budget.
- A continuous, non-convex optimization problem with **simple** constraints.
  No emumeration!

A blast from the past: Bienstock and Verma, 2007

- **DC approximation** to power flows.
- Adversary **increases reactances** of lines.
- **Limit** on total percentage-increase of reactances, and on per-line increase.
- Adversary maximizes the maximum **line overload**:

$$\max_{\boldsymbol{x},\boldsymbol{\theta}} \ \max_{km} \left\{ \frac{|\theta_k - \theta_m|}{u_{km}\ \boldsymbol{x_{km}}} \right\}$$

$$\text{s.t.} \quad \boldsymbol{B_x}\theta = d$$

$$\boldsymbol{x} \text{ within budget}$$

  – Variables: reactances $\boldsymbol{x}$, phase angles $\boldsymbol{\pi}$

  – $\boldsymbol{x_{km}}$ = reactance of $\boldsymbol{km}$, $\boldsymbol{u_{km}}$ = limit of $\boldsymbol{km}$, $\boldsymbol{B_x}$ = bus susceptance matrix, $\boldsymbol{d}$ = net injections (given)

- Continuous, but non-smooth problem.

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- Continuous, smooth, **nonconvex**.

Technical point

$$\max_{\boldsymbol{x},\boldsymbol{\theta},\boldsymbol{\alpha}} \quad \sum_{km} (\alpha^+_{km} - \alpha^-_{km}) \frac{(\theta_k - \theta_m)}{u_{km} \; \boldsymbol{x_{km}}}$$

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Function to maximize: $\boldsymbol{F(x, \alpha)} \doteq \sum_{km} (\alpha^+_{km} - \alpha^-_{km}) \frac{(\theta_k - \theta_m)}{u_{km} x_{km}}$

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Function to maximize: $\boldsymbol{F(x, \alpha)} \doteq \sum_{km} (\alpha^+_{km} - \alpha^-_{km}) \frac{(\theta_k - \theta_m)}{u_{km}\, x_{km}}$

- **Fact:** The gradient and the Hessian of $F(x, \alpha)$ can be efficiently computed
- Optimization problem solved using **LOQO** (**IPOPT** an option)

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- Algorithm scales well (2007): CPU times of ~ 1 hour for studying systems with thousands of lines.
- Optimal * attack concentrated on a handful of lines
- Significant part of the budget expended on many lines, with visible impact

Table 6: **Attack patterns**

single = 20	total = 60	single = 10	total = 30	single = 10	total = 40
Range	**Count**	**Range**	**Count**	**Range**	**Count**
$[1, 1]$	8	$[1, 1]$	1	$[1, 1]$	14
$(1, 2]$	72	$(1, 2]$	405	$(1, 2]$	970
$(2, 3]$	4	$(2, 9]$	0	$(2, 5]$	3
$(5, 6]$	1	$(9, 10]$	**3**	$(5, 6]$	0
$(6, 7]$	1			$(6, 7]$	1
$(7, 8]$	**4**			$(7, 9]$	0
$(8, 20]$	0			$(9, 10]$	**2**

"single" = max multiplicative increase of a line's reactance

"total" = max total multiplicative increase of line reactances

Today: the AC power flows setting

As before, adversary increases impedances, subject to budgets

Adversary wants to **maximize:**

- Phase angle differences across ends of a lines
- Voltage deviations (loss)

Alternative version:

- There is a **recourse** action: shed load so as to maintain feasibility of all power flow constraints (limits)
- Adversary wants to maximize the amount of lost load

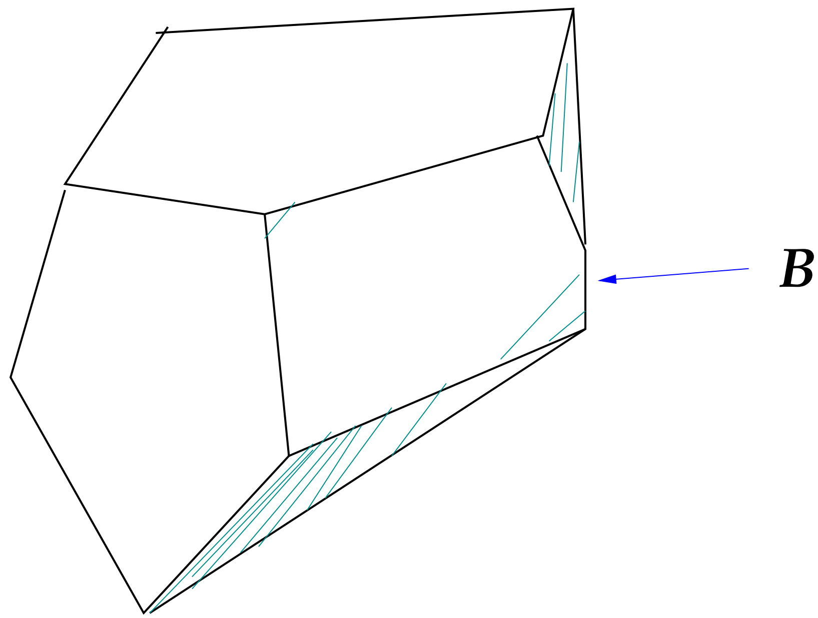
Generically:

$$\begin{aligned} \mathbf{max} \quad & \boldsymbol{\mathcal{F}(x)} \\ \text{s.t.} \quad & \boldsymbol{x} \in \mathcal{B} \end{aligned}$$

- $\boldsymbol{x}$ = impedances, $\boldsymbol{\mathcal{B}}$ = budget constraints
- $\boldsymbol{\mathcal{F}(x)}$= meausure of phase angle differences, voltage loss, load loss
- Challenge 1: $\boldsymbol{\mathcal{F}(x)}$ is implicitly defined

Basic methodology: **Frank-Wolfe**

$$\begin{array}{ll} \textbf{max} & \mathcal{F}(\boldsymbol{x}) \\ \text{s.t.} & \boldsymbol{x} \in \mathcal{B} \quad \text{(within budget)} \end{array}$$



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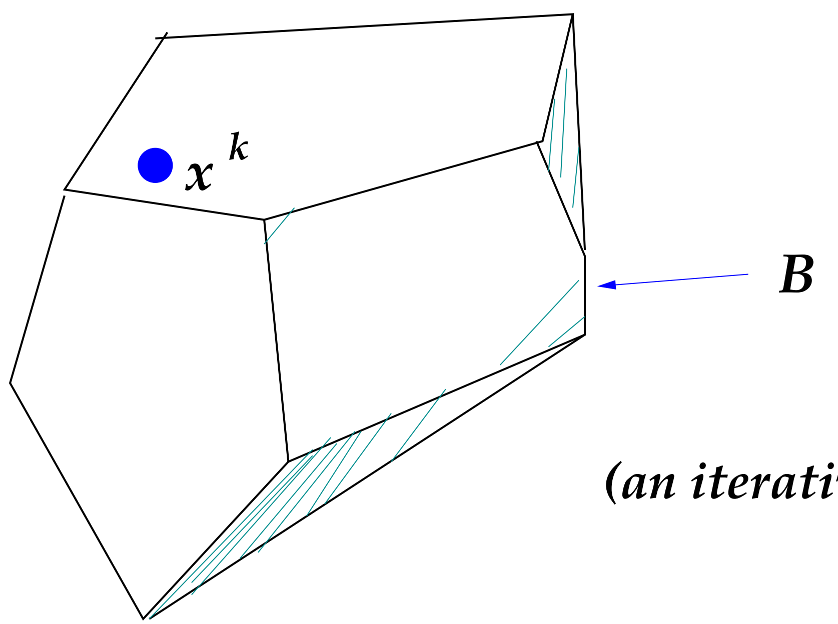

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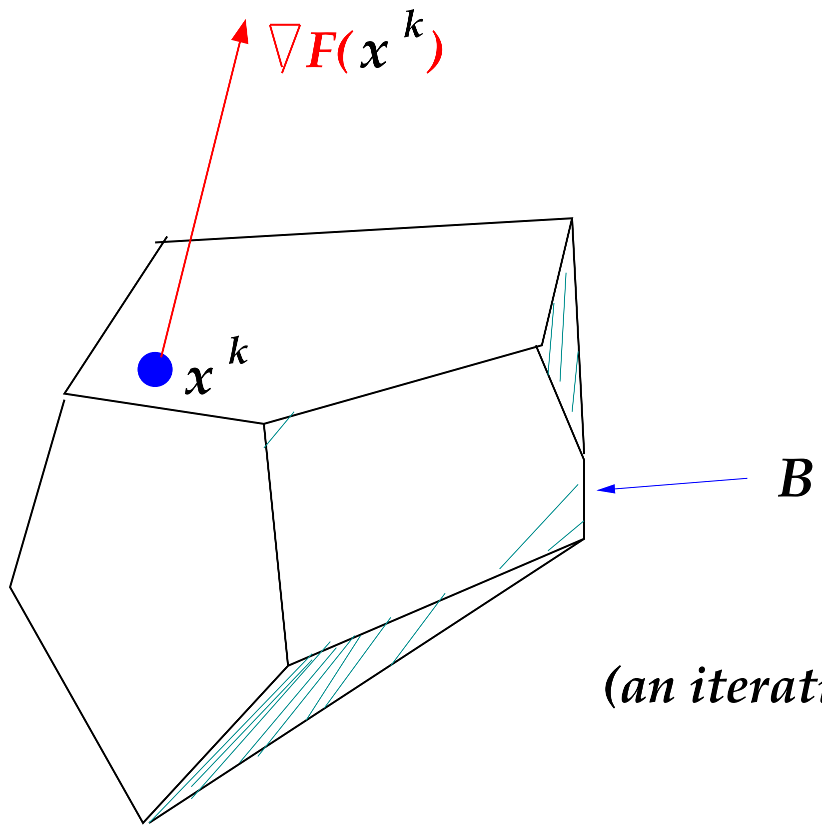

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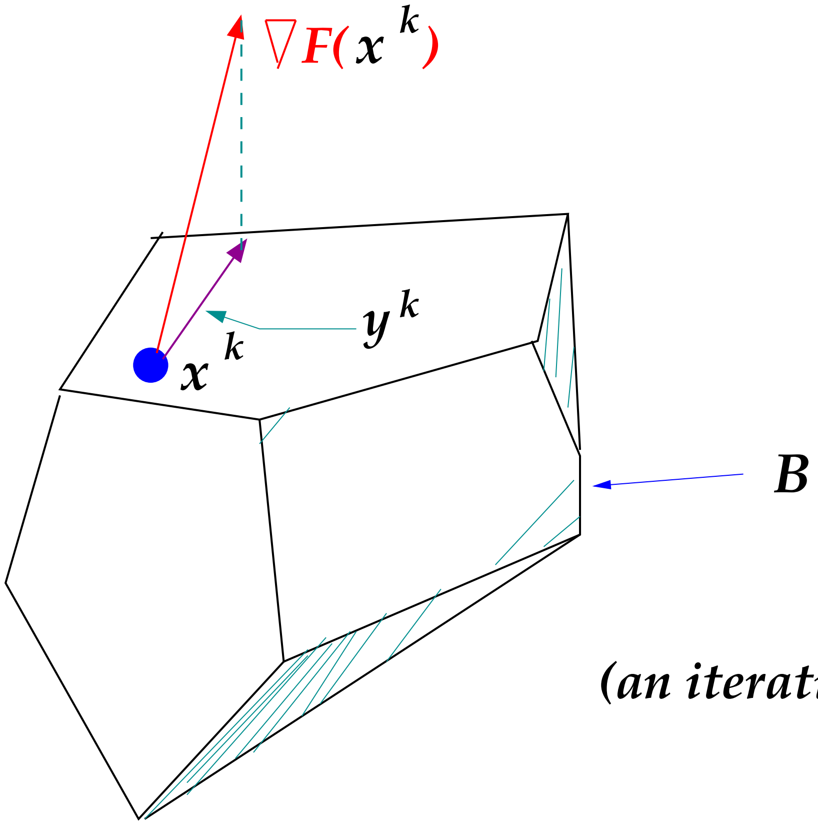

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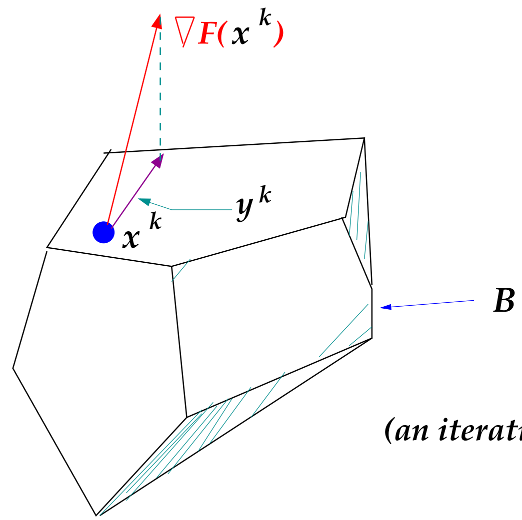

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$$\boldsymbol{y}^{\boldsymbol{k}} \text{ solves} \quad \begin{array}{ll} \textbf{max} & [\boldsymbol{\nabla}\mathcal{F}(\boldsymbol{x}^{\boldsymbol{k}})]^{\boldsymbol{T}}\boldsymbol{y} \\ \text{s.t.} & \boldsymbol{x}^{\boldsymbol{k}} + \boldsymbol{y} \ \in \ \mathcal{B} \quad \text{(within budget)} \end{array}$$

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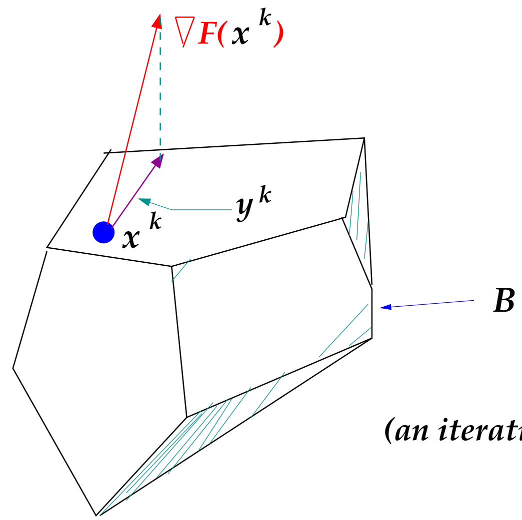
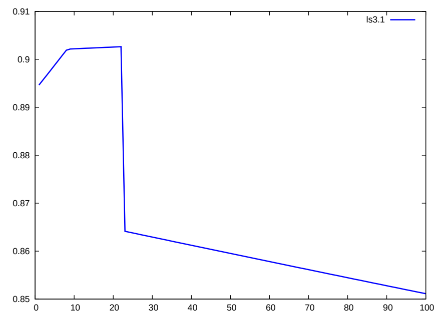


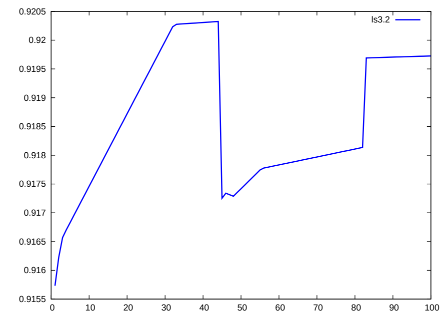
(an iterative algorithm)

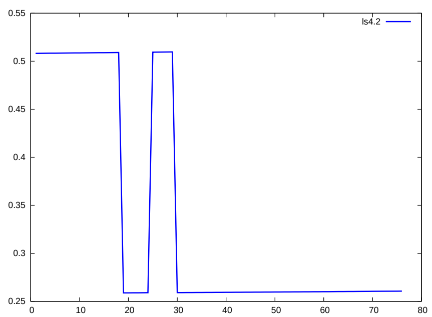
$$y^k \text{ solves} \quad \begin{array}{ll} \textbf{max} & [\nabla \mathcal{F}(x^k)]^T y \\ \text{s.t.} & x^k + y \in \mathcal{B} \quad \text{(within budget)} \end{array}$$

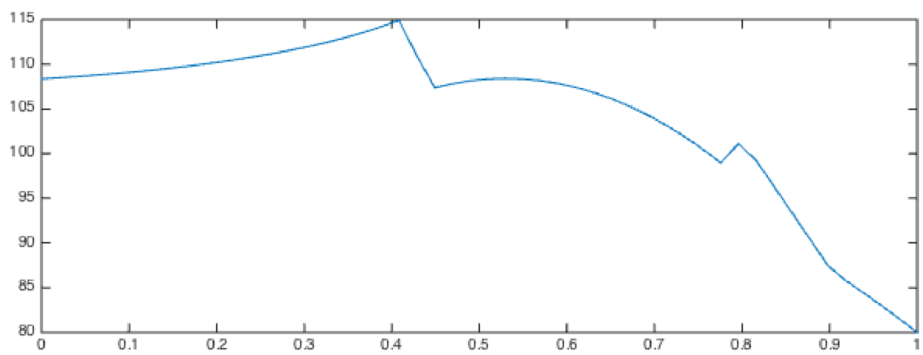
Final step is a **line search**: $x^{k+1} = x^k + \alpha y^k$, where $0 \le \alpha \le 1$ is the stepsize.

Line searches



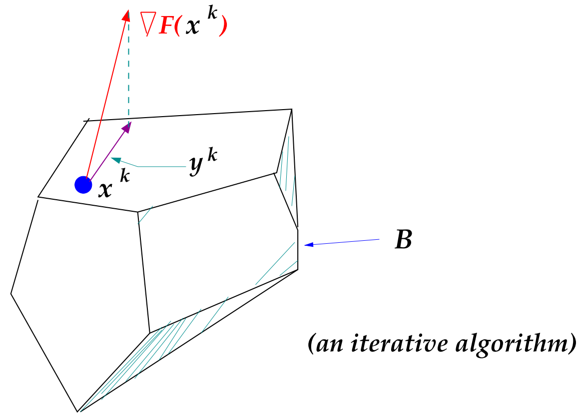






Challenge 2

$$\begin{array}{ll} \max & \mathcal{F}(\boldsymbol{x}) \\ \text{s.t.} & \boldsymbol{x} \in \mathcal{B} \quad \text{(within budget)} \end{array}$$



- Recall: $\mathcal{F}(\boldsymbol{x})$ measures e.g. the largest phase angle difference using reactances $\boldsymbol{x}$
- **Q:** exactly how do we get $\nabla\mathcal{F}(\boldsymbol{x})$?

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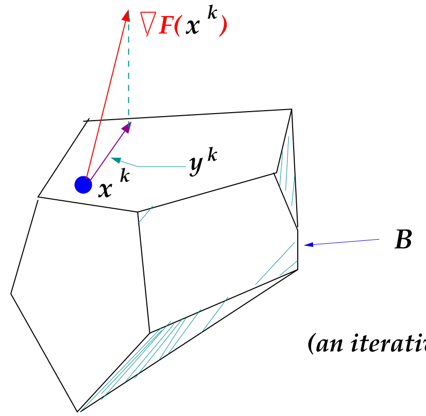

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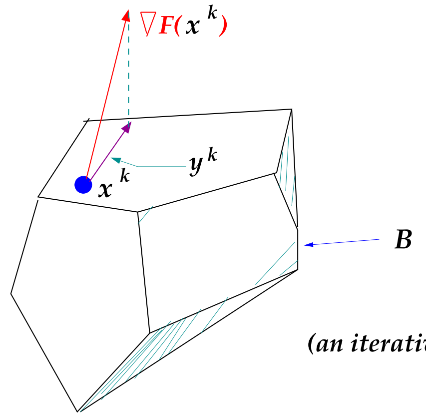

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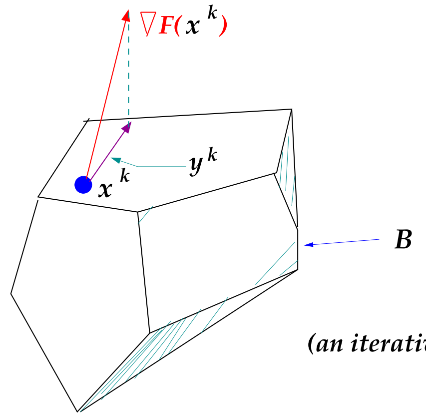

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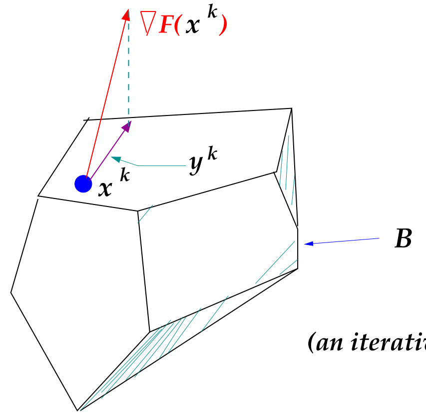

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- **"Solution"**: Estimate $\nabla\mathcal{F}(\boldsymbol{x})$ in parallel over several cores
- **Alternative:** only estimate some of the components of $\nabla\mathcal{F}(\boldsymbol{x})$:
  - **Random** subset of small size
  - **Most promising** subset

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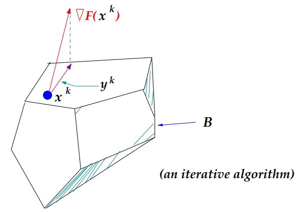

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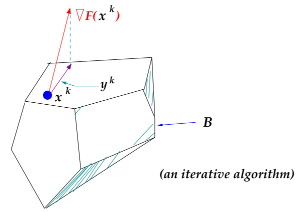

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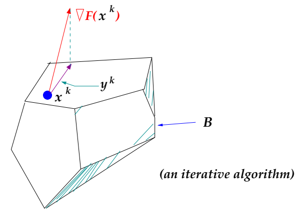

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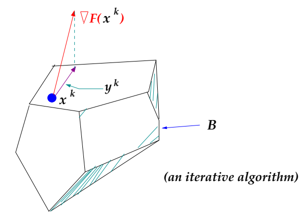

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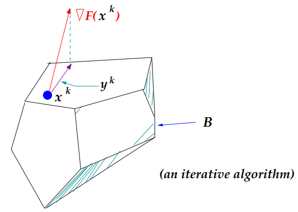

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- **solution?** violations still observed
- **solution?** Add to definition of $\mathcal{F}(x)$ sum of weighted square violations

$\longrightarrow$ Currently using **IPOPT** within Matpower (fastest for **our** purposes)
$\longrightarrow$ Infeasible cases verified using SDP relaxation

Example: phase angle attack on Polish grid (from Matpower)

1 obj=2620.72 step=1.00 [**263** 8.00; **300** 8.00; **728** 8.00;]

2 obj=2641.52 step=1.00 [**305** 8.00; **306** 8.00; **309** 8.00;]

3 obj=2649.34 step=1.00 [**168** 8.00; **263** 8.00; **321** 8.00;]

5 obj=2765.47 step=0.50 [**51** 4.00; **261** 4.00; **263** 4.00; **300** 4.00; **321** 4.00; **322** 4.00;]

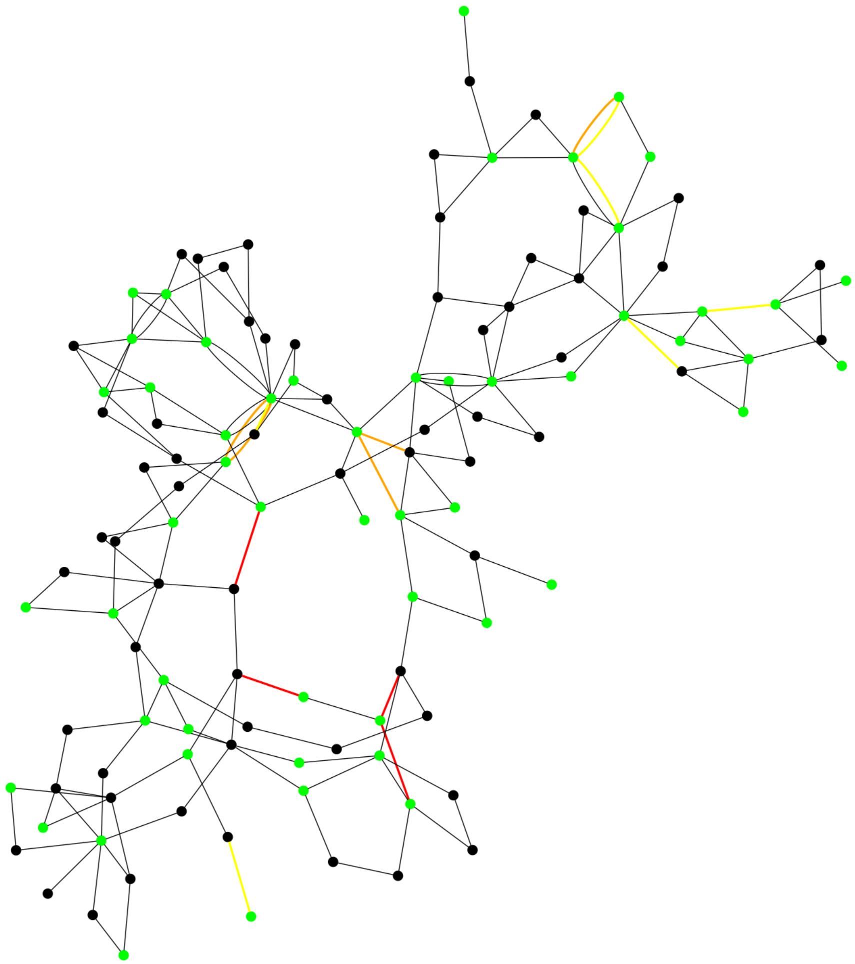
13 obj=2944.01 step=0.12 [**305** 2.60; **168** 2.32; **322** 2.17; **169** 1.90; **321** 1.85; **263** 1.57; **309** 1.50; **32** 1.15; **51** 1.08; **261** 1.08; **170** 1.00; **171** 1.00; **306** 0.85; **39** 0.75; **281** 0.75; **166** 0.57; **310** 0.57; **8** 0.43; **264** 0.43; **300** 0.42;]

20 obj=2950.54 step=0.03 [**169** 2.53; **305** 2.38; **168** 1.88; **322** 1.77; **321** 1.76; **309** 1.74; **166** 1.44; **170** 1.28; **263** 1.28; **261** 1.14; **32** 0.93; **51** 0.88; **171** 0.81; **306** 0.69; **39** 0.61; **281** 0.61; **264** 0.59; **260** 0.51; **310** 0.46; **8** 0.35; **300** 0.34;]

27 obj=2958.08 **step=0.00** [**169** 2.80; **305** 2.53; **321** 2.00; **309** 1.97; **168** 1.63; **263** 1.58; **322** 1.53; **166** 1.38; **261** 1.11; **170** 1.11; **32** 0.81; **51** 0.76; **264** 0.76; **281** 0.75; **171** 0.71; **306** 0.60; **39** 0.53; **260** 0.44; **310** 0.40; **8** 0.30; **300** 0.30;]

Example: phase angle attack on 118-bus

Three top-attacked lines in red:



Fact: phase angle attack cannot be isolated to a few lines

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Experiment on 118-bus case:

(1) Take line most heavily interdicted: line **38**

(2) Let the reactance of this line increase to infinity

(3) What happens? Phase angle difference $\rightarrow \boldsymbol{\pi/2}$?

Fact: phase angle attack cannot be isolated to a few lines

Experiment on 118-bus case:

(1) Take line most heavily interdicted: line **38**

(2) Let the reactance of this line increase to infinity

(3) What happens? Phase angle difference $\rightarrow \boldsymbol{\pi/2}$? **No.**

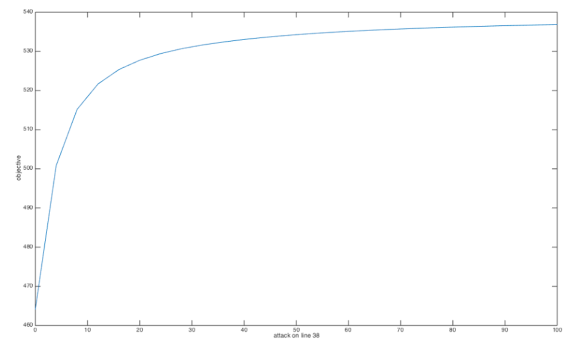
Fact: phase angle attack cannot be isolated to a few lines

Experiment on 118-bus case:

(1) Take line most heavily interdicted: line **38**

(2) Let the reactance of this line increase to infinity

(3) What happens? Phase angle difference $\rightarrow \pi/2$? **No.**
From $\approx$ **10** to $\approx$ **40**.



Fact: phase angle attack cannot be isolated to a few lines

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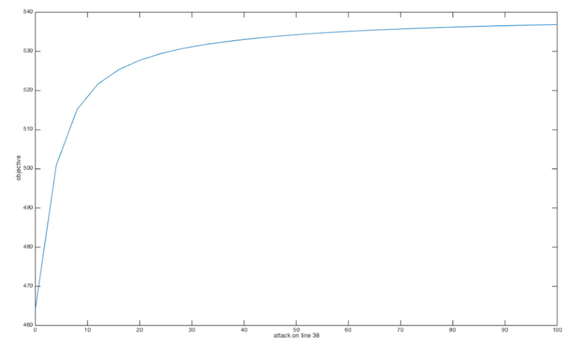
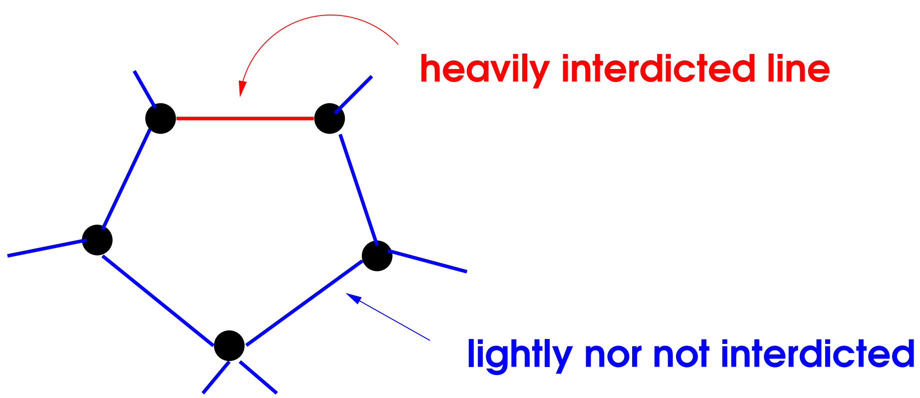
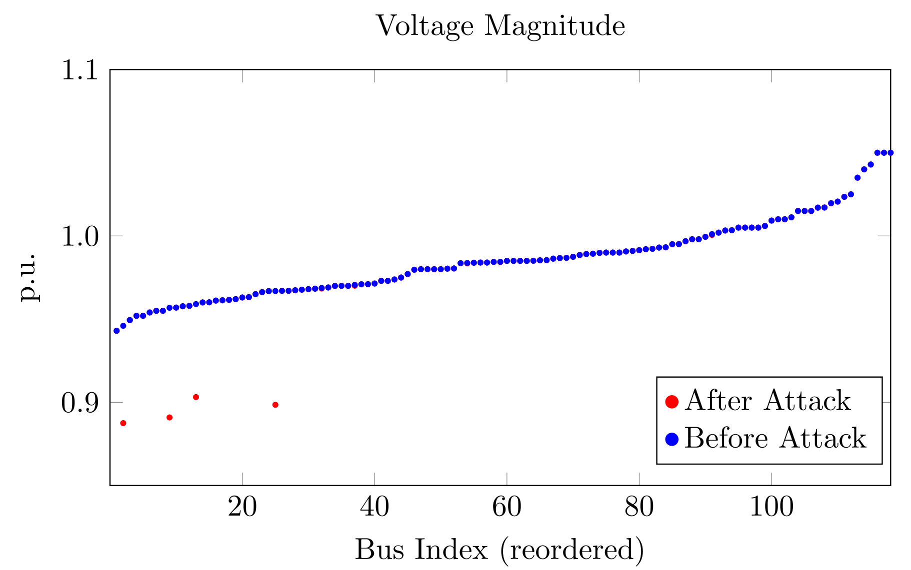
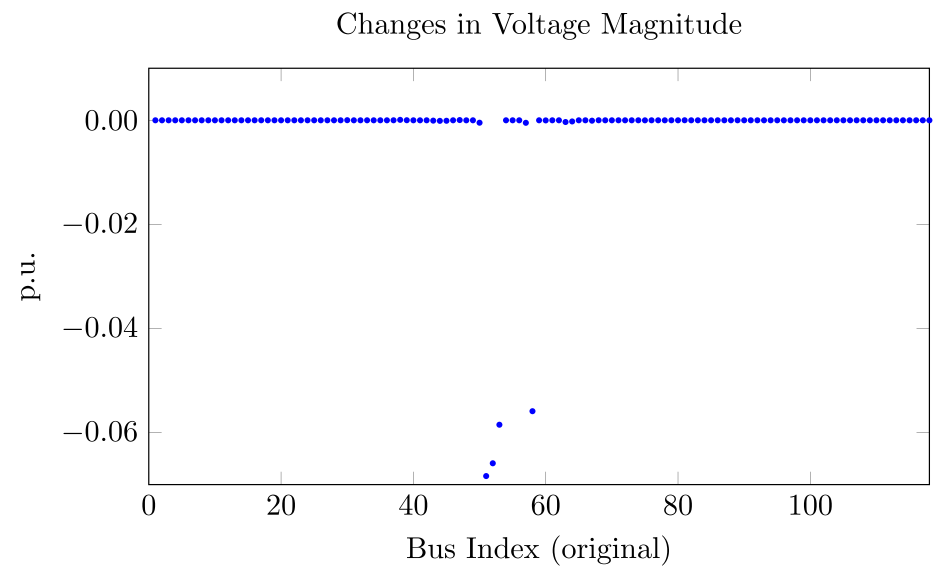


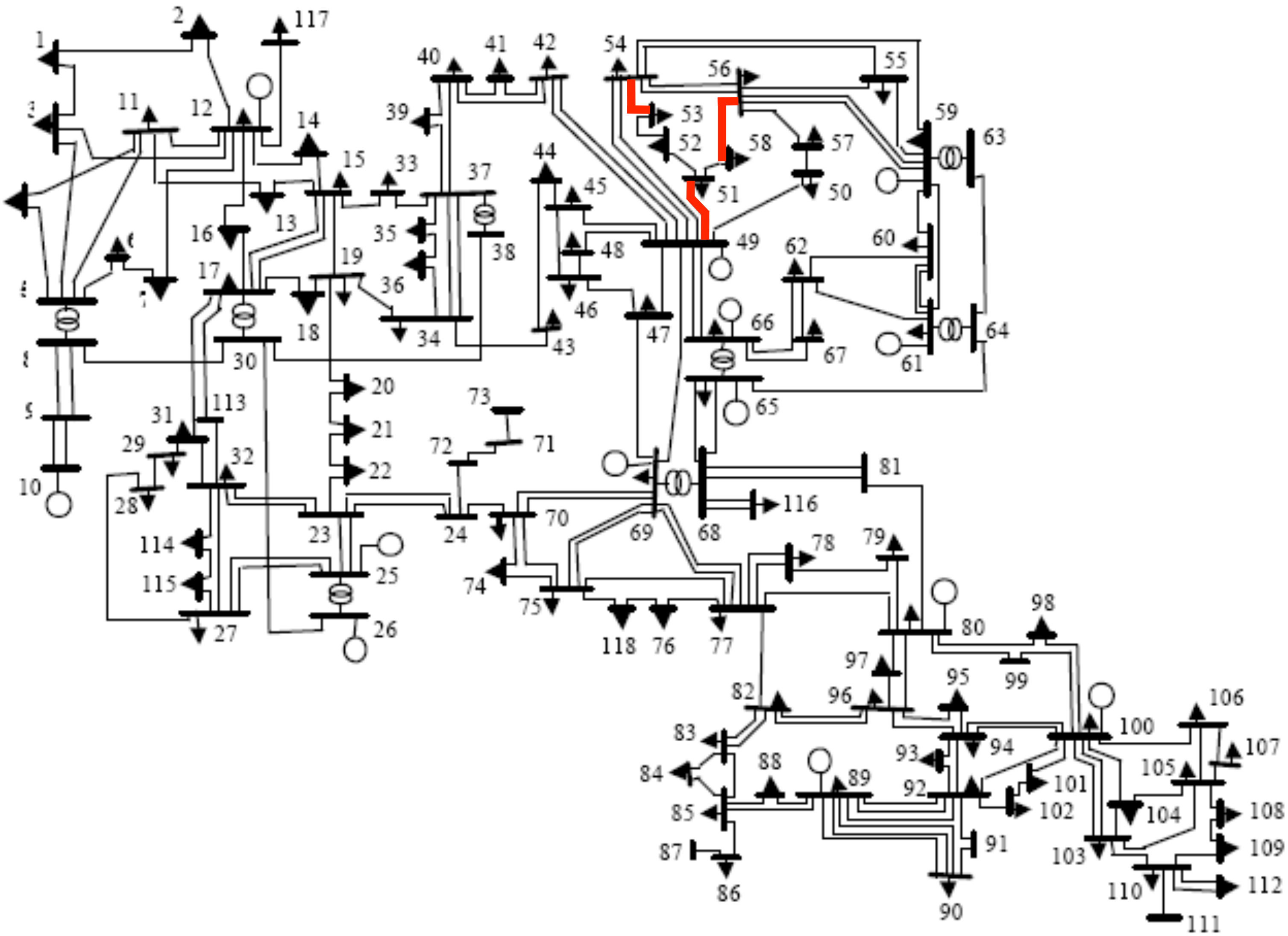


Voltage attack on 118-bus

“Triple the reactance of at most three lines”







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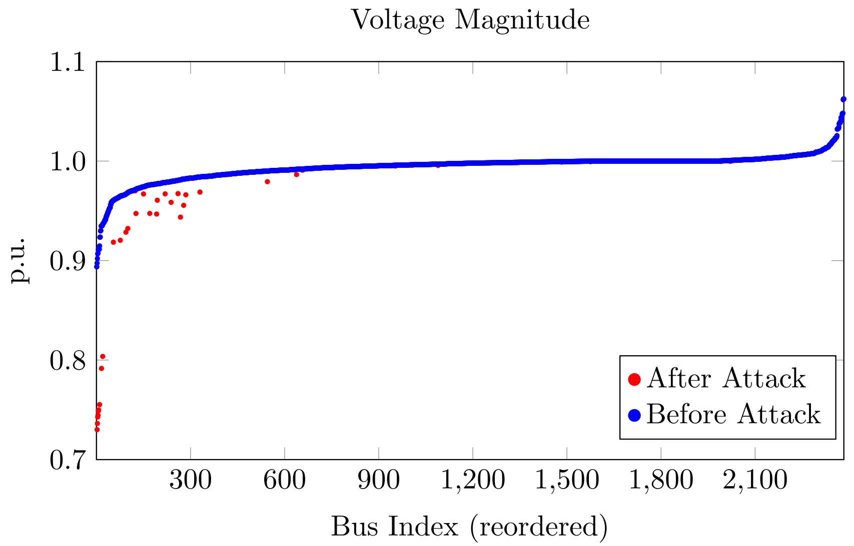
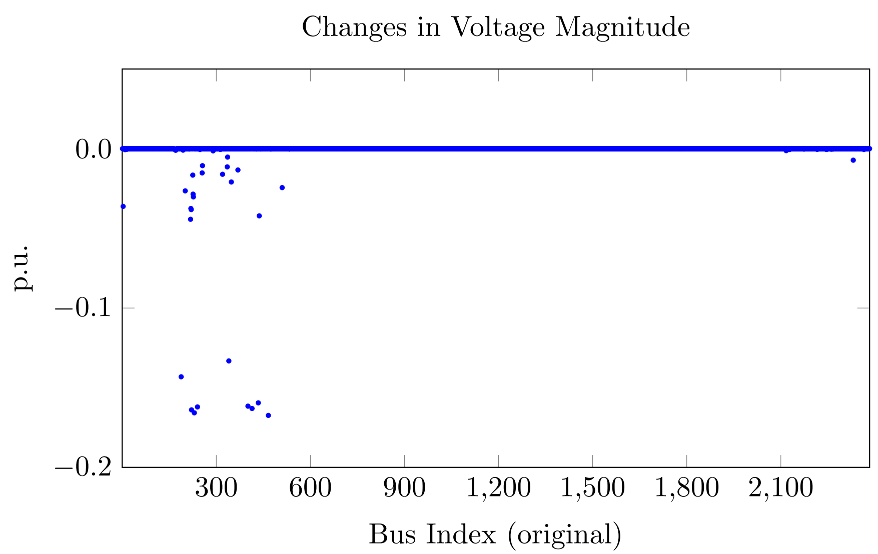
Voltage attack on 2383-bus Polish

"Double the reactance of at most three lines"





→ Primarily 4 lines interdicted