Decentralized Robust Optimization Algorithms for Tie-Line Scheduling of Multi-Area Grid with Variable Wind Energy

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Joint work with

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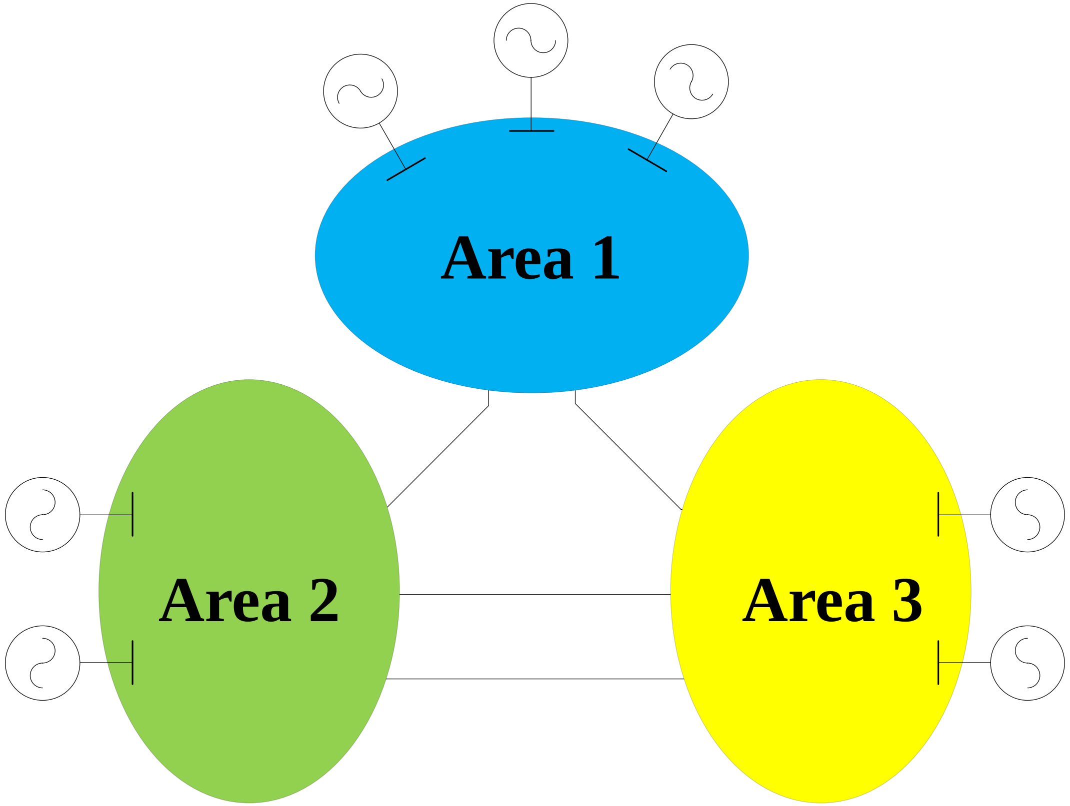
Outline

- ❑ Background and Motivation
- ❑ Robust Optimization Formulations and Properties
- ❑ Distributed Computing Methods through Alternating Direction Method of Multipliers (ADMM)
- ❑ Numerical Study and Demonstrations
- ❑ Conclusions

Part I: Background and Motivation

- Multi-area power system
  - Physically separated regions (China)
  - Different electricity market
- Intermittent renewable energy
- Random contingencies
- Aligned LMPs for boundary nodes



Challenges in Multi-Area Power Grids

- Systems are cooperative but operated independently
  - Privacy
  - Commercial information
- Protocols between systems to support cooperation:
  - Coordinated Transaction Scheduling: NYISO and PJM
  - Interchange Optimization: PJM and MISO
  - Inter-Regional Interchange Scheduling: ISONE and NYISO
- Interfaces between multiple systems: **tie-lines**
  - Power flow decisions (day-ahead and intraday)
  - Unit commitment decisions (day-ahead)

Decision Models for Multi-Area Power Grids

- Lagrangian relaxation
  - Conejo and Aguado (1998), Multi-area coordinated decentralized DC optimal power flow
  - Aguado, Quintana, and Conejo (1999), Optimal power flows of interconnected power systems
- Augmented Lagrangian decomposition
  - Kim and Baldick (1997)
  - Ahmadi-Khatir, Conejo, and Cherkaoui (2014)
- Alternative Direction Method of Multiplers (ADMM)
  - Kim and R. Baldick (2000)
  - Zheng, Wu, Zhang, Sun, and Liu (2015)

Uncertainty Consideration in Power Grids

- Scenario based probabilistic models
  - Stochastic programming (SP) and chance constrained formulation
  - Explicit large-scale models but many existing computing methods: e.g., Ahmadi-Khatir, Conejo, and Cherkaoui (2013, 2014):
    - *SP+ augmented Lagrangian relaxation*
  - Issue: Inaccurate prediction of scenarios and probabilities -> infeasible solutions

- Robust optimization models
  - Uncertainty set based compact formulation
  - Produce highly feasible solutions by considering all possibilities inside uncertainty set
  - Two types of algorithms: Benders decomposition and *column-and-constraint generation*
  - **Our aim: integration of decentralized computing scheme + robust optimization**

Part II: Robust Optimization Formulations and Properties

Multi-Area Robust Tie-Line Scheduling

- Two-stage decision making framework

$$\min_{\xi^f \in \Omega^f} \sum_{a \in \mathrm{A}} \left\{ \max_{\tilde{\mathbf{P}}_a^w \in \mathrm{U}_a^w} \left[\min_{\xi_a^s \in \Omega_a^s(\xi_a^f, \tilde{\mathbf{P}}_a^w)} C_a^{ED}(\xi_a^s, \tilde{\mathbf{P}}_a^w) \right] \right\}$$

- Tie-line interchanges: the first-stage decisions before the availability of wind energy is known
  - Phase angles at boundary buses:

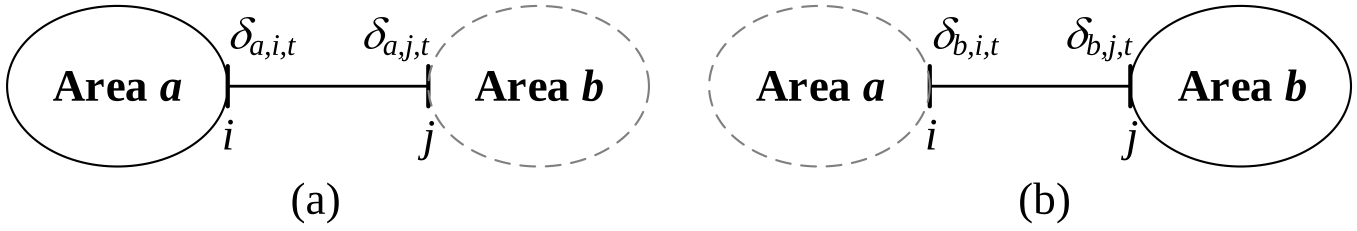
$$\xi^f = \{\xi_a^f, \forall a \in \mathrm{A}\}, \xi_a^f = \left\{\delta_{a,i,t}, \forall g \in \mathrm{G}_a, i \in \mathrm{N}_a^{BB} \cup \mathrm{N}_a'^{BB}, t \in \mathrm{T}\right\}$$

$$\Omega^f = \left\{\xi^f \,\middle|\, \delta_{\phi(i),i,t} = \delta_{\phi(j),i,t},\ \delta_{\phi(i),j,t} = \delta_{\phi(j),j,t}, \left|\delta_{\phi(i),i,t} - \delta_{\phi(i),j,t}\right| / X_{i,j} \le \bar{F}_{i,j}, \forall (i,j) \in \mathrm{E}^{tie}, i > j, t \in \mathrm{T},\right.$$
$$\left. \delta_{a',ref,t} = 0, \forall a' \in \Phi(ref), t \in \mathrm{T} \right\}$$

- Economic dispatch: the second-stage decision after wind energy is revealed.
  - Continuous model

Inter-regional constraints

- Region-coupling constraints - perceived phase angles are same at two ends of a tie-line



$$\delta_{a,i,t} = \delta_{b,i,t}, \delta_{a,j,t} = \delta_{b,j,t}, \forall t \in \mathrm{T}$$

Economic Dispatch of Each Area

- Dispatch of conventional generation units, wind farms, and phase angles of internal buses

$$\Omega_a^s(\boldsymbol{\xi}_a^f, \tilde{\mathbf{P}}_a^w) = \Big\{ \boldsymbol{\xi}_a^s \Big|$$

$$\sum_{j \in \Psi_a^{IB}(i)} (\theta_{i,t} - \theta_{j,t}) \big/ X_{i,j} + \sum_{j \in \Psi^{BB}(i)} (\theta_{i,t} - \delta_{a,j,t}) \big/ X_{i,j}$$

$$= \sum_{g \in \Psi_a^G(i)} p_{g,t}^G + \sum_{k \in \Psi_a^W(i)} p_{k,t}^w - \sum_{d \in \Psi_a^D(i)} p_{d,t}^D, \forall i \in \mathrm{N}_a^{IB}, t,$$

$$\sum_{j \in \Psi_a^{IB}(i)} (\delta_{a,i,t} - \theta_{j,t}) \big/ X_{i,j} + \sum_{j \in \Psi^{BB}(i)} (\delta_{a,i,t} - \delta_{a,j,t}) \big/ X_{i,j}$$

$$= \sum_{g \in \Psi_a^G(i)} p_{g,t}^G + \sum_{k \in \Psi_a^W(i)} p_{k,t}^w - \sum_{d \in \Psi_a^D(i)} p_{d,t}^D, \forall i \in \mathrm{N}_a^{BB}, t,$$

$$-\overline{F}_{i,j} \le (\theta_{i,t} - \theta_{j,t}) \big/ X_{i,j} \le \overline{F}_{i,j}, \forall i \in \mathrm{N}_a^{IB}, \; j \in \Psi_a^{IB}(i), \; j > i,$$

$$-\overline{F}_{i,j} \le (\delta_{a,i,t} - \theta_{j,t}) \big/ X_{i,j} \le \overline{F}_{i,j}, \forall i \in \mathrm{N}_a^{BB}, \; j \in \Psi_a^{IB}(i),$$

$$p_{g,t}^G + r_{g,t}^{G+} \le P_g^G, \forall g \in \mathrm{G}_a, t \in \mathrm{T},$$

$$r_{g,t}^{G-} \ge p_{g,t}^G - \underline{P}_g, \forall g \in \mathrm{G}_a, t \in \mathrm{T},$$

$$\sum_{g \in \mathrm{G}_a} r_{g,t}^{G+} \ge SR_{a,t}^+, \sum_{g \in \mathrm{G}_a} r_{g,t}^{G-} \ge SR_{a,t}^-, \forall t \in \mathrm{T},$$

$$0 \le r_{g,t}^{G+} \le RU_g^G, 0 \le r_{g,t}^{G-} \le RD_g^G, \forall g \in \mathrm{G}_a, t \in \mathrm{T},$$

$$0 \le p_{k,t}^w \le \tilde{P}_{k,t}^w, \forall k \in \mathrm{W}_a, t \in \mathrm{T} \Big\}$$

Observation:

- If the uncertainty sets U_a^w are polyhedra, the robust multi-area tie-line schedule problem is a convex optimization (an extremely large-scale linear program)

- Idea: enumerating all extreme points of U_a^w to construct the equivalent formulation, which is an LP

Multi-Area Generation Unit and Tie-line Scheduling

- Two-stage decision making framework

$$\min_{\xi^f \in \Omega^f} \sum_{a \in \mathrm{A}} \left\{ C_a^{UD}(\xi_a^f) + \max_{\tilde{\mathbf{P}}_a^w \in \mathrm{U}_a^w} \left[\min_{\xi_a^s \in \Omega_a^s(\xi_a^f, \tilde{\mathbf{P}}_a^w)} C_a^{ED}(\xi_a^s, \tilde{\mathbf{P}}_a^w) \right] \right\}$$

- First-stage decisions: unit commitments and tie-line interchanges

$$\xi^f = \{\xi_a^f, \forall a \in \mathrm{A}\}, \xi_a^f = \{u_{g,t}^G, x_{g,t}^G, y_{g,t}^G, \delta_{a,i,t}, \forall g \in \mathrm{G}_a, i \in \mathrm{N}_a^{BB} \cup \mathrm{N}_a'^{BB}, t \in \mathrm{T}\}$$

$$\Omega^f = \Big\{ \xi^f \Big| \forall a \in \mathrm{A}, g \in \mathrm{G}_a, t \in \mathrm{T}, \delta_{\phi(i),i,t} = \delta_{\phi(j),i,t}, \ \delta_{\phi(i),j,t} = \delta_{\phi(j),j,t}, \left|\delta_{\phi(i),i,t} - \delta_{\phi(i),j,t}\right| / X_{i,j} \le \bar{F}_{i,j}, \forall (i,j) \in \mathrm{E}^{tie}, i > j,$$

$$\delta_{a',ref,t} = 0, \forall a' \in \Phi(ref), t \in \mathrm{T},$$

$$u_{g,t}^G - u_{g,t-1}^G = x_{g,t}^G - y_{g,t}^G, \quad \sum_{\tau = \max\{1, t - MU_g^G + 1\}}^{t} x_{g,\tau}^G \le u_{g,t}^G, \quad \sum_{\tau = \max\{1, t - MD_g^G + 1\}}^{t} y_{g,\tau}^G \le 1 - u_{g,t}^G, \forall g, t$$

$$u_{g,t}^G \in \{0,1\}, 0 \le x_{g,t}^G \le 1, 0 \le y_{g,t}^G \le 1 \Big\}$$

Multi-Area Generation Unit and Tie-line Scheduling (Cont'd)

- Second-stage decisions: economic dispatch after the available wind power is revealed and unit status are determined
- Observation:
  - Due to binary variables for unit scheduling, the robust formulation is equivalent to a non-convex and discrete mixed integer program
- Challenge: augmented Lagrangian methods typically do not converge

Part III: Distributed Computing Methods through ADMM

Augmented Lagrangian Relaxation

- Relaxing $\delta_{a,i,t} = \delta_{b,i,t}, \delta_{a,j,t} = \delta_{b,j,t}, \forall t \in \mathrm{T}$

- Averaging $\bar{\delta}_{i,t} = \sum_{a \in \Phi(i)} \delta_{a,i,t} / |\Phi(i)|$

- Augmented model

$$\min_{\boldsymbol{\xi}_a^f} L_a(\boldsymbol{\xi}_a^f, \boldsymbol{\lambda}_a, \bar{\boldsymbol{\delta}})$$

$$\begin{aligned} L_a(\boldsymbol{\xi}_a^f, \boldsymbol{\lambda}_a, \bar{\boldsymbol{\delta}}) &= \sum_{i \in \mathrm{N}_a^{BB} \cup \mathrm{N}_a^{\prime BB}} \sum_{t \in \mathrm{T}} \left[\lambda_{a,i,t} (\delta_{a,i,t} - \bar{\delta}_{i,t}) + \frac{\rho}{2} (\delta_{a,i,t} - \bar{\delta}_{i,t})^2 \right] \\ &+ \max_{\tilde{\mathbf{P}}_a^w \in \mathrm{U}_a^w} \left[\min_{\boldsymbol{\xi}_a^s \in \Omega_a^s(\boldsymbol{\xi}_a^f, \tilde{\mathbf{P}}_a^w)} C_a^{ED}(\boldsymbol{\xi}_a^s, \tilde{\mathbf{P}}_a^w) \right] \\ &= C_a'(\boldsymbol{\xi}_a^f, \boldsymbol{\lambda}_a, \bar{\boldsymbol{\delta}}) + \max_{\tilde{\mathbf{P}}_a^w \in \mathrm{U}_a^w} \left[\min_{\boldsymbol{\xi}_a^s \in \Omega_a^s(\boldsymbol{\xi}_a^f, \tilde{\mathbf{P}}_a^w)} C_a^{ED}(\boldsymbol{\xi}_a^s, \tilde{\mathbf{P}}_a^w) \right] \end{aligned}$$

Overall Algorithm Scheme

- Using ADMM, each area can be computed independently
  - distributed computing and privacy protection

- For a single area problem: two-stage robust optimization
  - column and constraint generation method
  - finitely convergent for a polyhedron uncertainty set

- **Integrated ADMM+CCG (IAC)** Solution Method
  - Multi-area robust tie-line scheduling: ADMM+CCG converges to optimal value
  - Multi-area robust generation unit and tie-line scheduling: convergence is NOT guaranteed
  - Computational enhancements? Speed and convergence

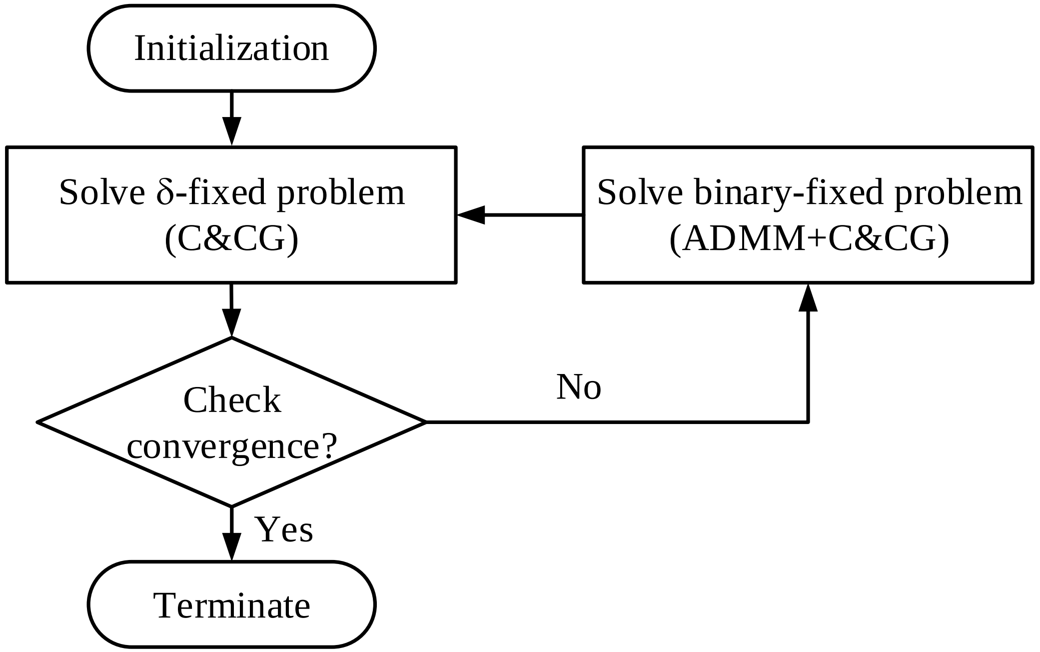
Fast Computing

- *Warm Start (WS)*:
  - select initial values of the first-stage variables and dual variables using the deterministic version
- *Scenario Retaining (SR)*:
  - CCG is repeatedly called within ADMM framework
  - Keep and re-use existing scenarios generated in previous CCG calls
- *Scenario Discard (SD):*
  - Remove non-critical scenarios to maintain a small pool
  - Dynamically manage a scenario pool through a changing threshold
- SR and SD are key steps in distributed computation of Robust Optimization

Convergence Issue from UC

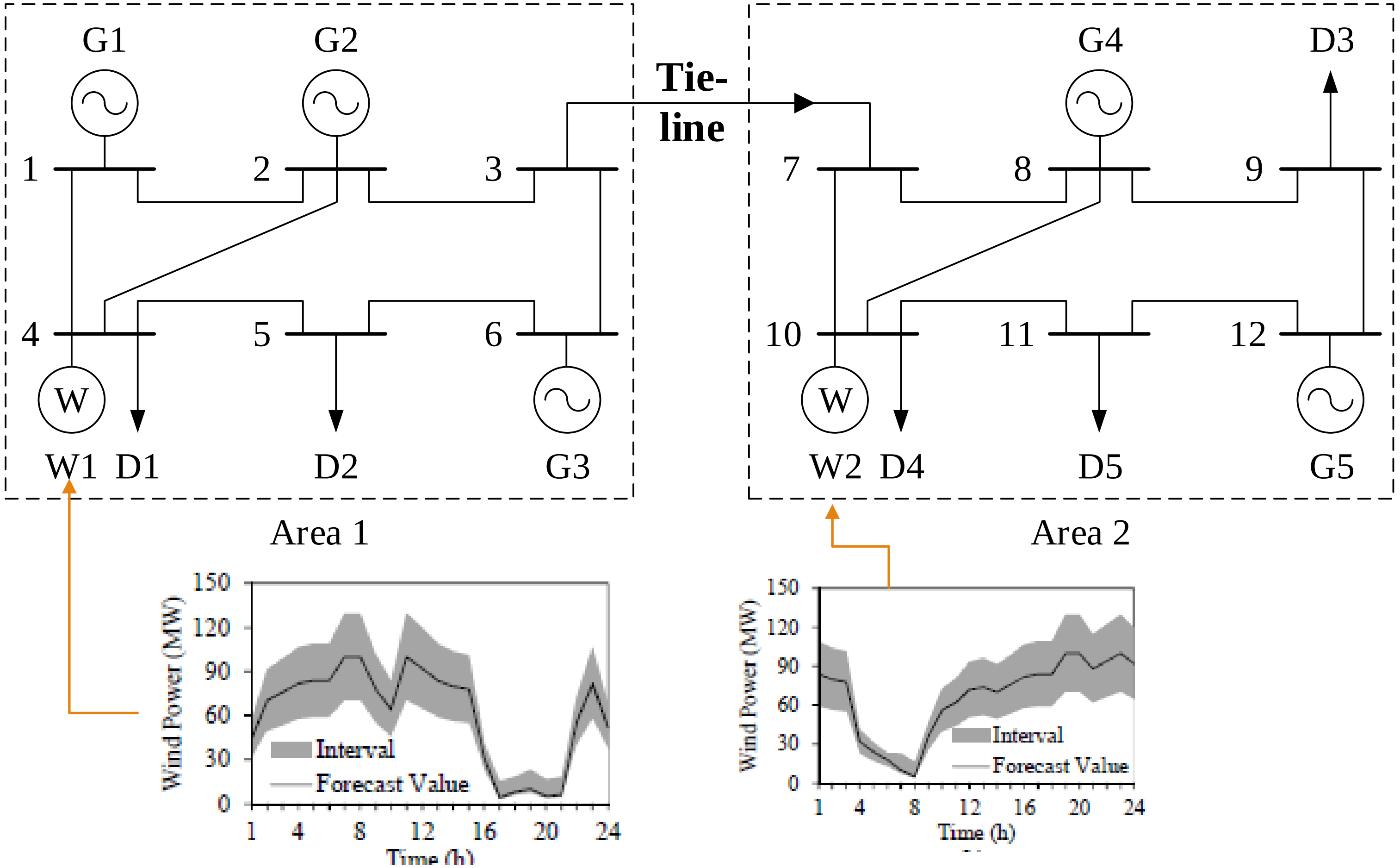
- Non-convergence due to non-convex structure from binary commitment decisions

- Alternating optimization procedure to ensure convergence (heuristically)
  - Alternatively computing with boundary phase angles or commitment status are fixed
  - A repeated commitment status indicates termination
  - **Finitely converged**

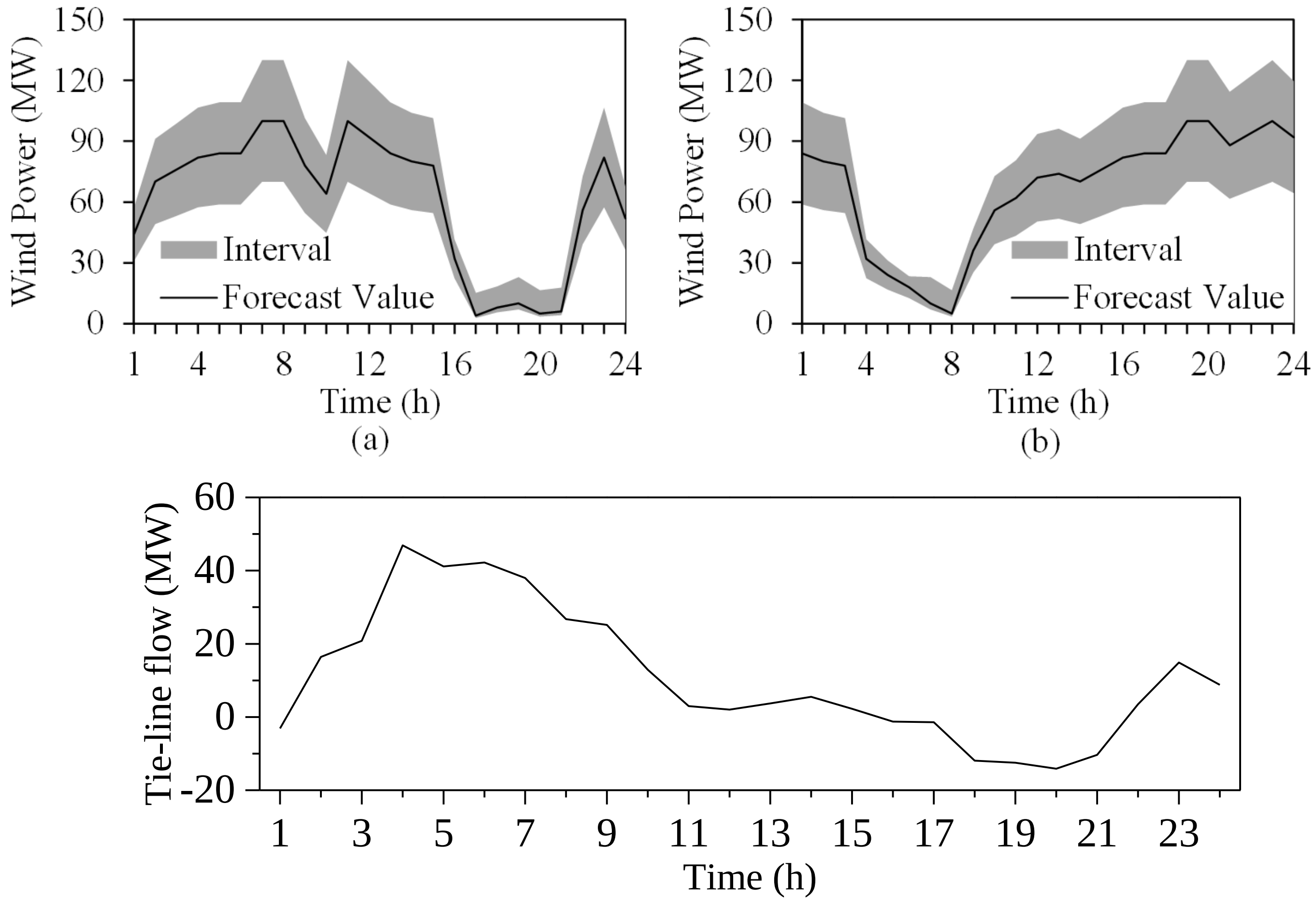


Part IV: Numerical Study and Demonstrations

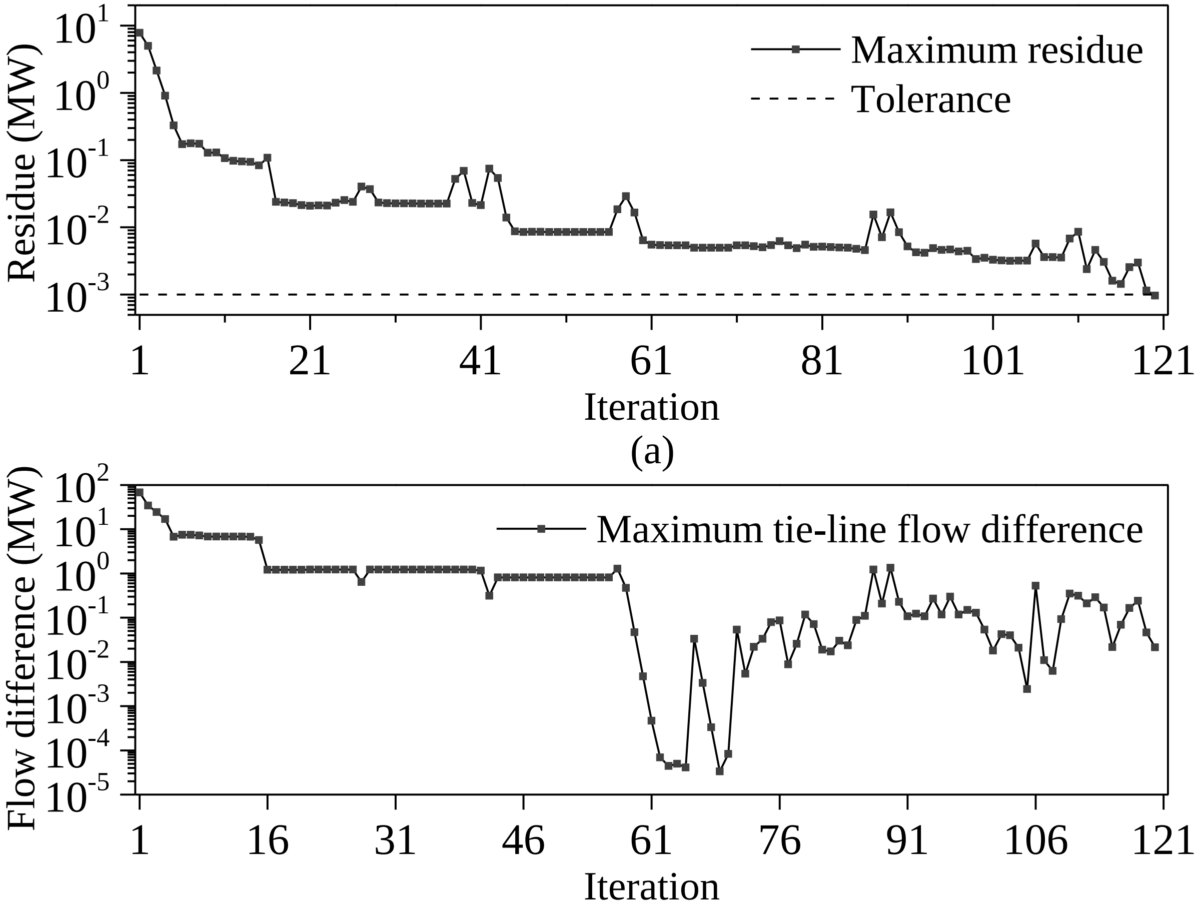
Two-Area 12-Bus Interconnected System



Tie-line Flows



IAC Performance for Tie-line Scheduling



(a)

(b)

Computational Enhancement Strategies

Case	Strategy			Initialization		ADMM							Total time (s)
							C&CG						
							# solution		Time (s)				
	WS	SR	SD	# iter.	Time (s)	# iter.	MP	SP	MP	SP	SD		
M0	-	-	-	-	-	120	516	469	20.1	33.3	-		53.4
M1	-	√	-	-	-	120	151	152	223.2	11.0	-		234.2
M2	-	√	√	-	-	120	179	179	16.6	13.7	5.8		36.1
M3	√	-	-	21	5.1	41	82	82	0.9	4.3	-		10.3
M4	√	√	√	21	5.2	41	41	41	0.5	1.5	0.5		7.7

*M0: IAC without enhancement

Coordination Effect with Unit Commitment

slightly higher than the centralized solution by 0.11%

SCHEDULING RESULTS ON 12-BUS SYSTEM

Hour	Proposed IAC						Centralized C&CG						Non-coordinated					
	Area 1			Area 2		Tie	Area 1			Area 2		Tie	Area 1			Area 2		Tie
	G1	G2	G3	G4	G5	(MW)	G1	G2	G3	G4	G5	(MW)	G1	G2	G3	G4	G5	(MW)
1	1	0	0	1	0	-10.4	1	0	0	1	0	-7.9	1	0	0	1	0	0.0
2	1	0	0	1	0	-1.9	1	0	0	1	0	16.1	1	0	0	1	0	0.0
3	1	0	0	1	0	5.4	1	0	0	1	0	18.1	1	0	0	1	0	0.0
4	1	0	0	1	0	11.8	1	0	0	1	0	24.4	1	0	0	1	0	0.0
5	1	0	0	1	0	11.6	1	0	0	1	0	25.3	1	0	0	1	0	0.0
6	1	0	0	1	0	12.6	1	0	0	1	0	21.4	1	0	0	1	0	0.0
7	1	0	0	1	0	25.9	1	0	0	1	0	25.9	1	0	0	1	1	0.0
8	1	0	0	1	0	16.7	1	0	0	1	0	16.5	1	0	0	1	1	0.0
9	1	0	0	1	0	-0.3	1	0	0	1	0	19.9	1	0	0	1	1	0.0
10	1	0	0	1	0	-3.7	1	0	0	1	0	12.7	1	0	0	1	1	0.0
11	1	0	1	1	1	3.0	1	0	1	1	0	11.0	1	0	1	1	1	0.0
12	1	0	1	1	1	2.0	1	0	1	1	0	7.6	1	0	1	1	1	0.0
13	1	0	1	1	1	2.7	1	0	1	1	0	6.7	1	0	0	1	1	0.0
14	1	0	1	1	1	3.4	1	0	1	1	0	7.6	1	0	0	1	1	0.0
15	1	0	1	1	1	1.7	1	0	1	1	0	6.1	1	0	1	1	1	0.0
16	1	0	0	1	0	-8.4	1	0	0	1	0	-0.6	1	0	0	1	0	0.0
17	1	0	0	1	0	-11.4	1	0	0	1	0	-8.4	1	0	0	1	0	0.0
18	1	0	0	1	0	-2.7	1	0	0	1	0	-2.0	1	0	0	1	0	0.0
19	1	0	0	1	0	-5.2	1	0	0	1	0	-9.9	1	0	0	1	0	0.0
20	1	0	0	1	0	-2.5	1	0	0	1	0	-34.3	1	0	0	1	0	0.0
21	1	0	0	1	0	-6.8	1	0	0	1	0	-9.0	1	0	0	1	0	0.0
22	1	0	0	1	0	9.3	1	0	0	1	0	4.6	1	0	0	1	0	0.0
23	1	0	0	1	0	7.1	1	0	0	1	0	0.4	1	0	0	1	0	0.0
24	1	0	0	1	0	-2.0	1	0	0	1	0	4.4	1	0	0	1	0	0.0
Obj.	$150,419						$150,243						$153,645					

Performance in Large Systems

System	Areas	Units	Buses	Int. Lines	Tie-lines
2A-RTS	2	66	48	76	3
3A-RTS	3	99	73	115	5
118-Bus	3	79	118	174	12

IAC for Tie-line Scheduling

SIMULATION RESULTS ON LARGE-SCALE TEST SYSTEMS

Uncertainty Budget			Γ = 0	Γ = 6	Γ = 12	Γ = 24
2A-RTS	Centr.	Obj. ($)	5,982,249	6,104,215	6,182,390	6,203,326
		Time (s)	2.4	15.2	24.0	9.8
	IAC	Obj. ($)	5,982,497	6,118,620	6,198,730	6,226,089
		# iter.	101	103	103	107
		Time (s)	17.9	92.1	79.4	98.7
		Error (%)	0.00	0.24	0.26	0.37
3A-RTS	Centr.	Obj. ($)	9,151,653	9,333,234	9,459,759	9,510,397
		Time (s)	3.6	33.6	106.7	9.9
	IAC	Obj. ($)	9,151,694	9,348,872	9,468,978	9,512,639
		# iter.	226	320	316	339
		Time (s)	34.4	830.4	501.9	676.5
		Error (%)	0.00	0.17	0.10	0.02
118-Bus	Centr.	Obj. ($)	2,249,048	2,250,762	2,252,142	2,255,044
		Time (s)	4.4	4.6	4.7	4.2
	IAC	Obj. ($)	2,249,120	2,251,392	2,253,339	2,257,712
		# iter.	491	535	573	600
		Time (s)	197.1	285.1	325.2	348.3
		Error (%)	0.00	0.03	0.05	0.12

IAC for Generation Unit and Tie-line Scheduling

SIMULATION RESULTS ON LARGE-SCALE TEST SYSTEMS

Uncertainty Budget			$\Gamma = 0$	$\Gamma = 6$	$\Gamma = 12$	$\Gamma = 24$
2A-RTS	Centr.	Obj. ($)	1,014,164	1,018,237	1,039,095	1,044,114
		Time (s)	597	3,005	1,617	1,451
	IAC	Obj. ($)	1,019,117	1,034,468	1,055,462	1,061,706
		Time (s)	127	3,642	4,186	788
		Gap (%)	0.49	1.59	1.58	1.66
3A-RTS	Centr.	Obj. ($)	1,529,977	1,535,347	1,565,097	1,576,184
		Time (s)	25	4,196	5,876	4,821
	IAC	Obj. ($)	1,541,769	1,562,666	1,583,698	1,591,784
		Time (s)	125	2,360	1,757	3,486
		Gap (%)	0.77	1.78	1.19	0.99
118-Bus	Centr.	Obj. ($)	1,101,662	1,111,969	1,112,181	1,112,431
		Time (s)	19	922	316	867
	IAC	Obj. ($)	1,111,116	1,113,739	1,117,006	1,119,490
		Time (s)	172	4,775	918	437
		Gap (%)	0.85	1.06	1.34	1.54

Part V: Conclusions

Observations and Conclusions

- An integrated ADMM+CCG computing method
  - Supporting information and privacy protection in handling uncertainties
  - Advanced enhancement strategies for fast computation
  - New strategies to ensure convergence in non-convex structures
- Coordination plays a critical role in multi-area grid performance
  - For tie-line scheduling, IAC performs (almost) the same as centralized method
  - For commitment and tie-line scheduling, IAC significantly outperforms non-coordinated control
- Future Improvement
  - Economic implications from IAC computation
  - Novel algorithmic improvement to support fast computing