Parallel Solution of a Nonlinear Stochastic Programming Formulation for the N-1 Contingency-Constrained ACOPF Problem

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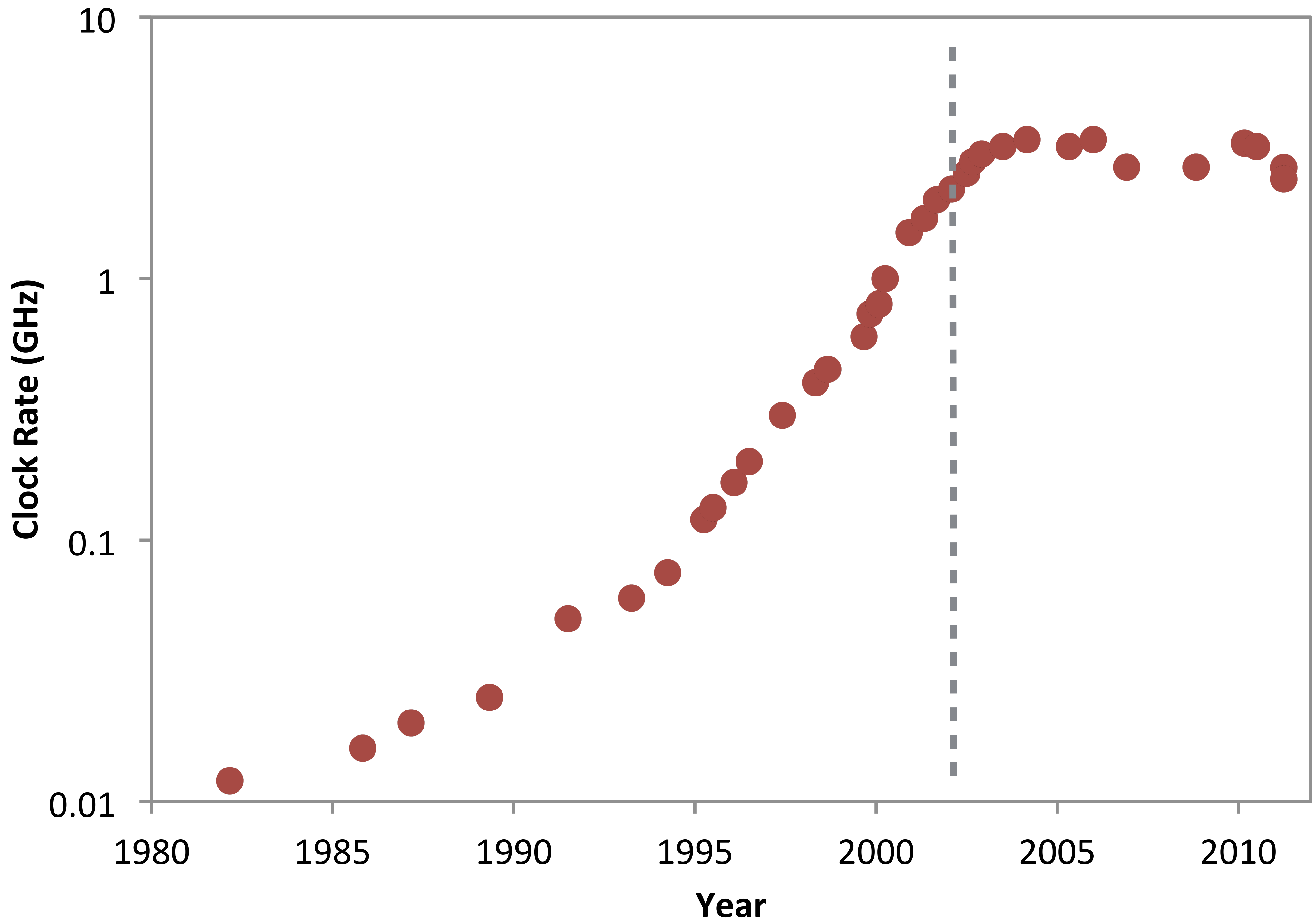
Laird Research Group: http://allthingsoptimal.com

PURDUE CHEMICAL ENGINEERING

Landscape of Scientific Computing



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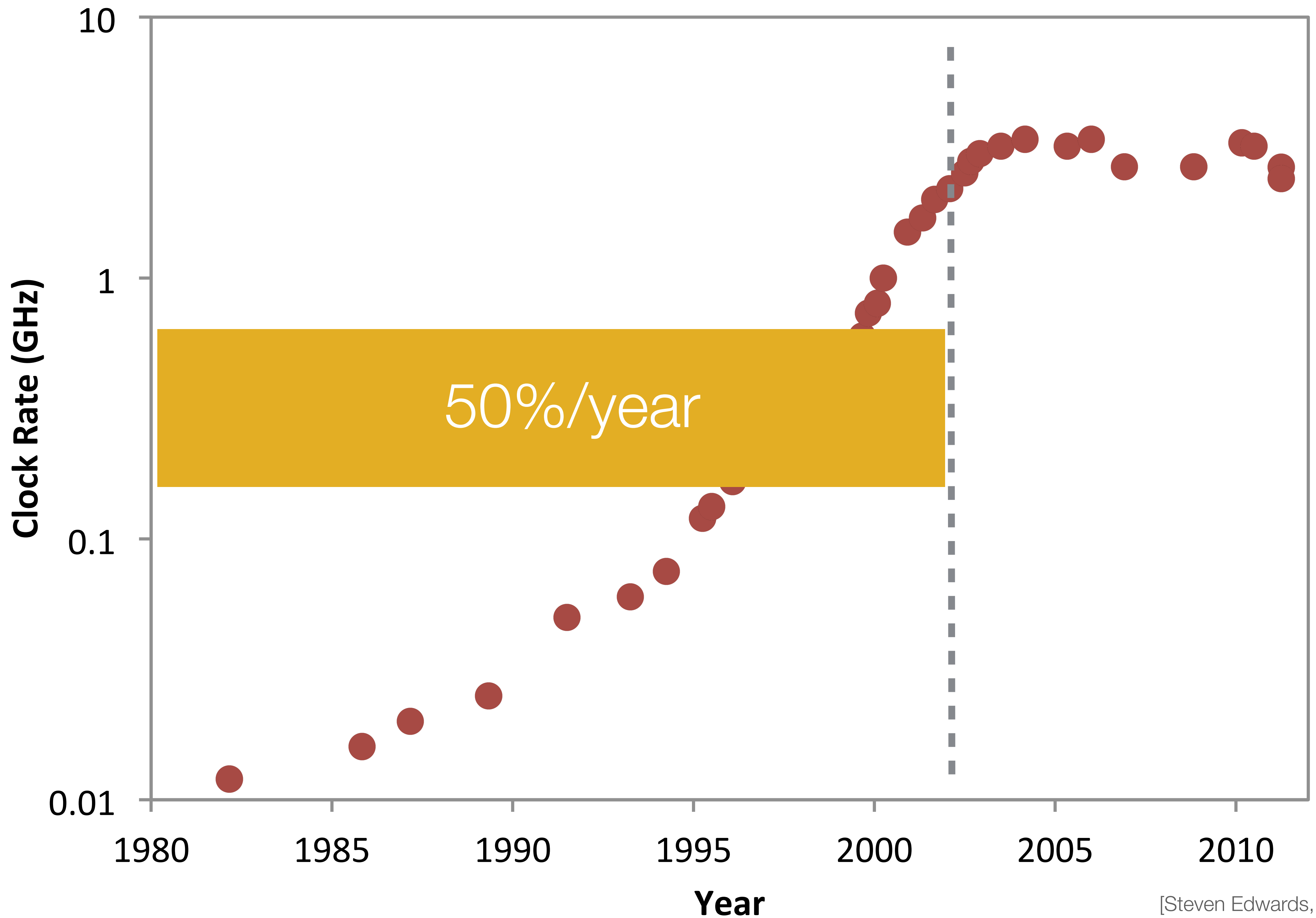

[Steven Edwards, Columbia University]

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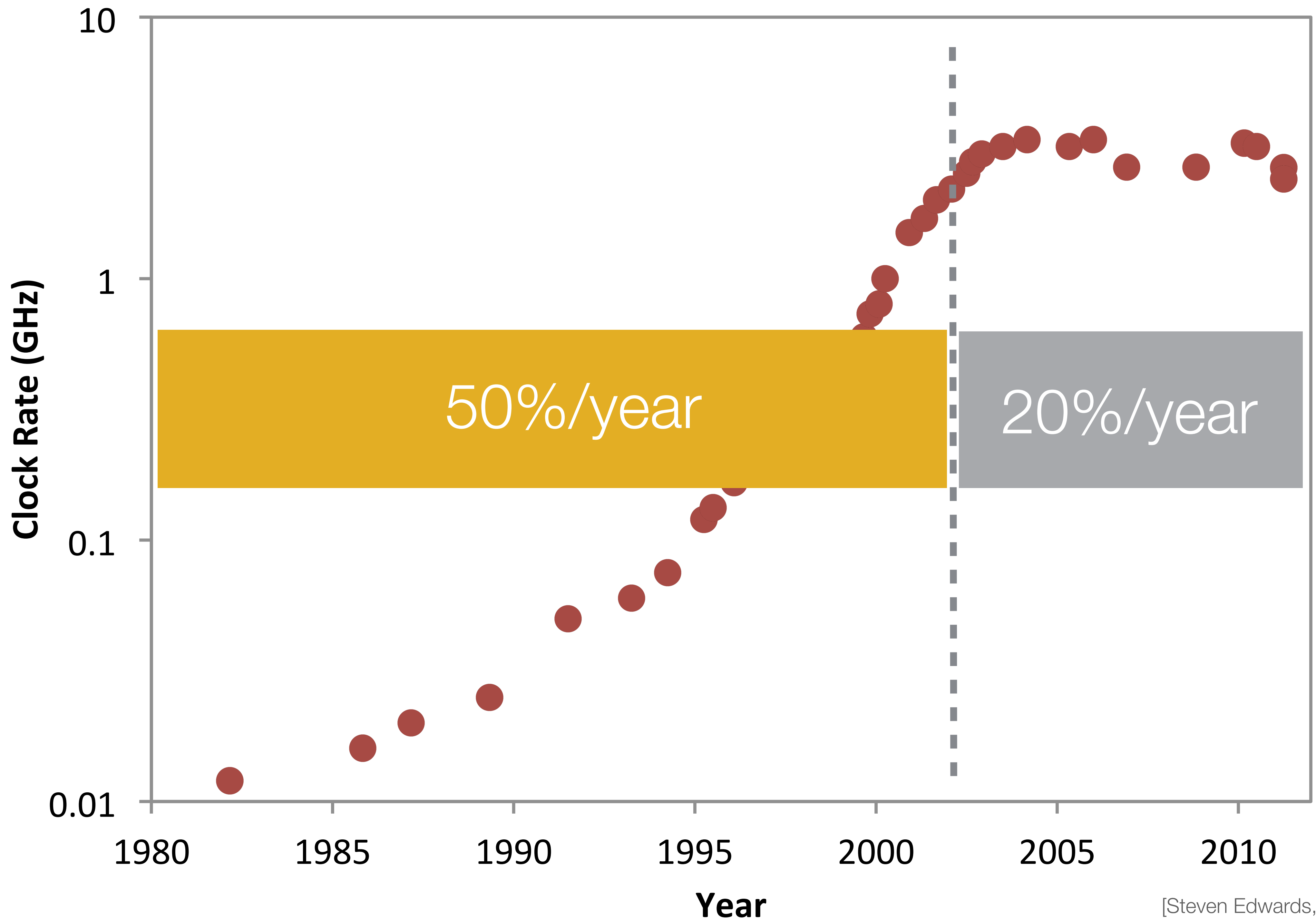

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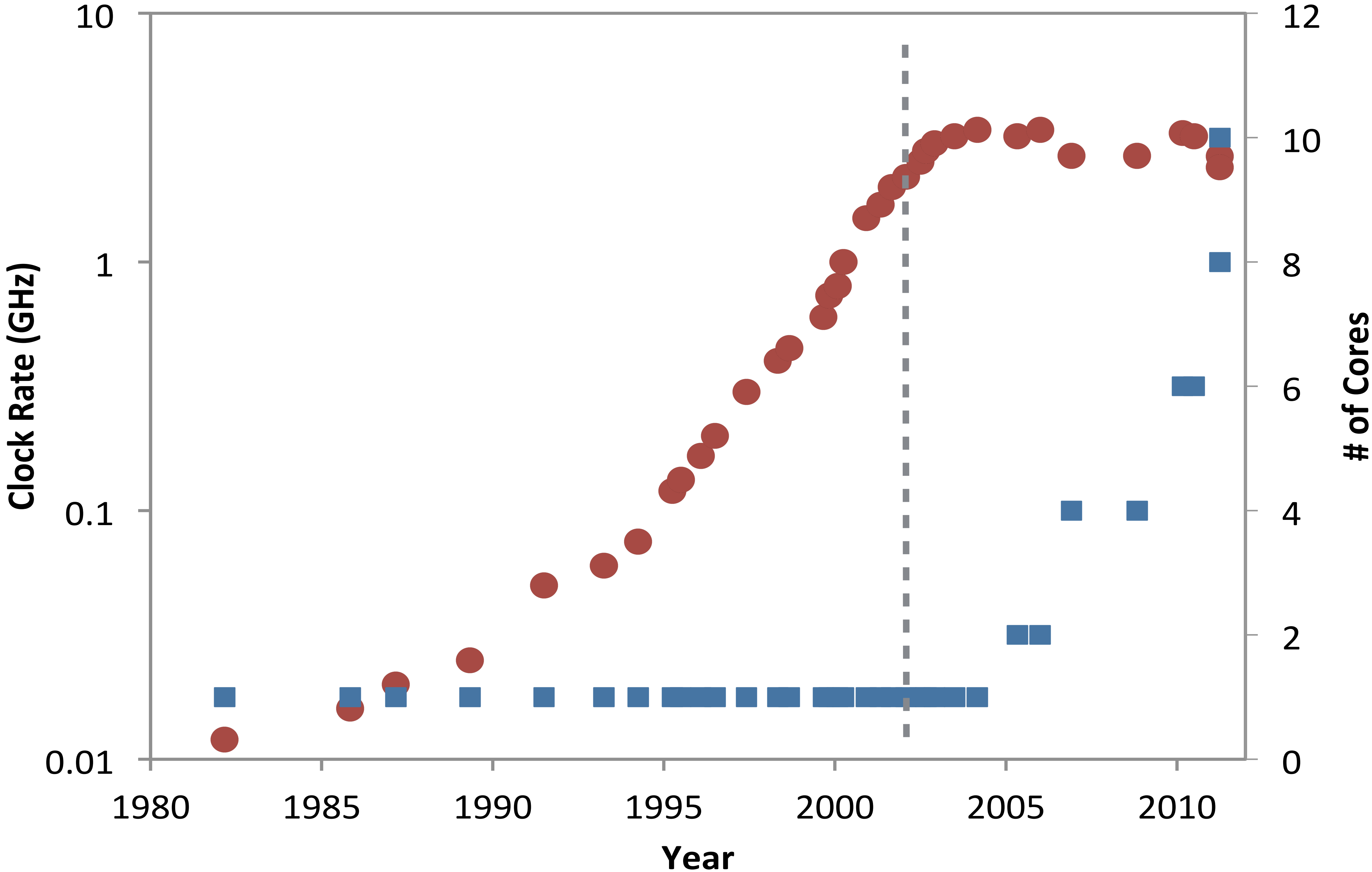
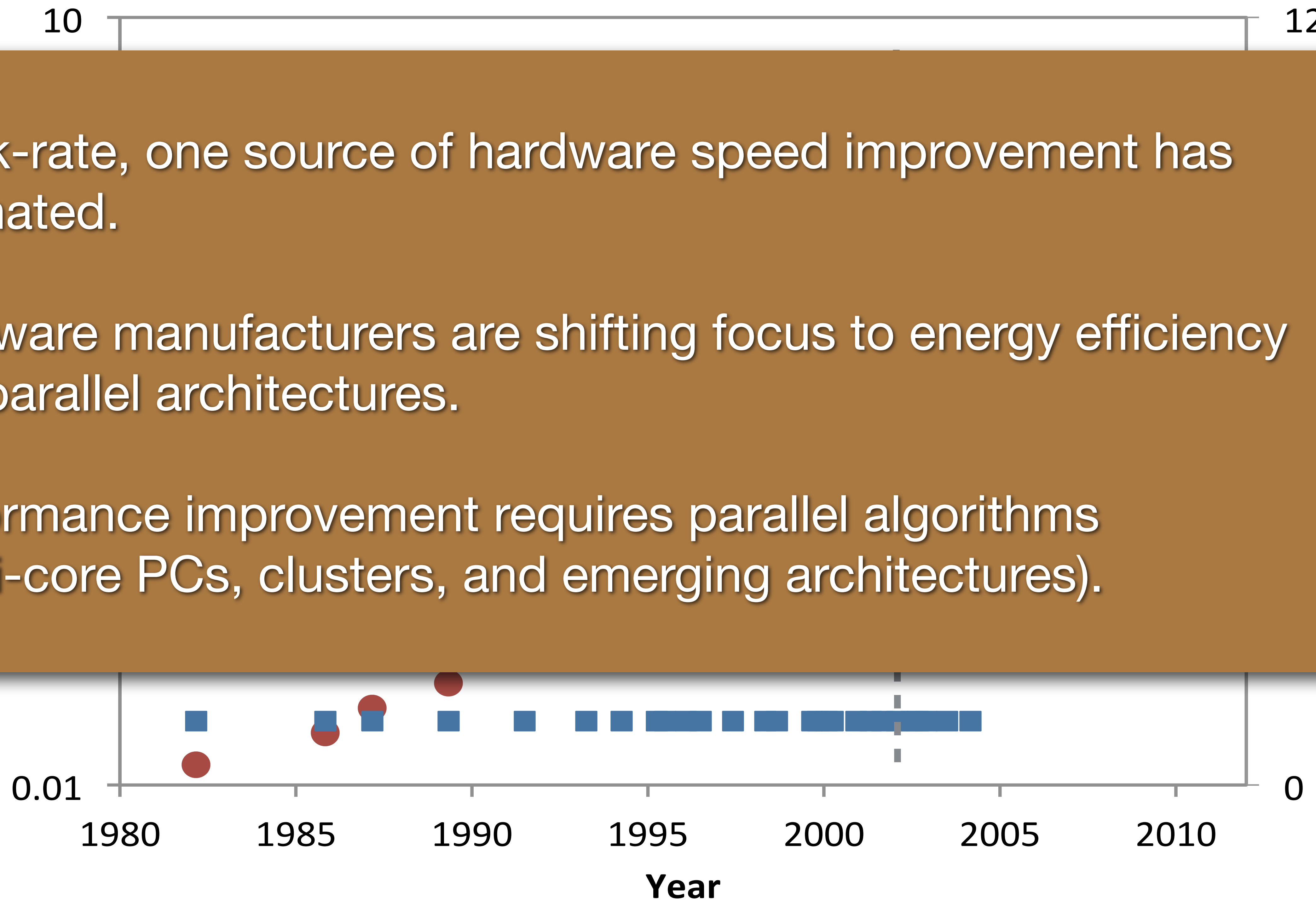


Landscape of Scientific Computing

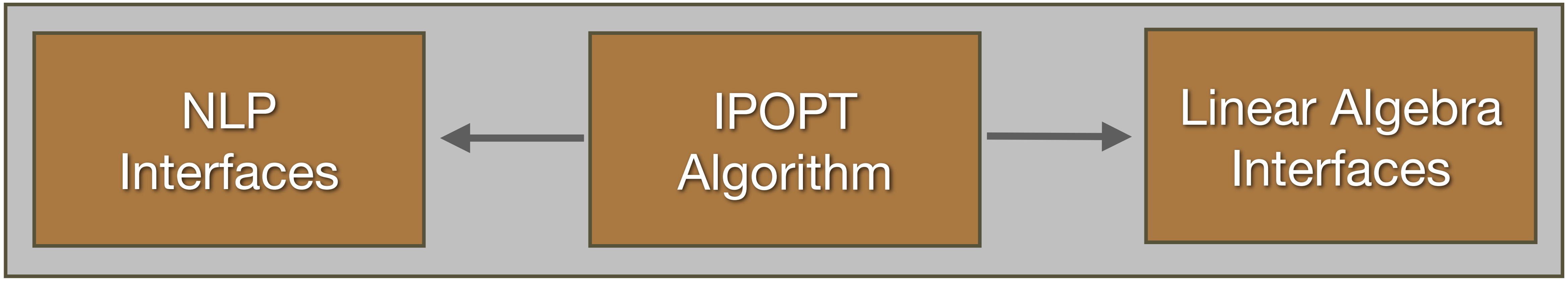
Clock-rate, one source of hardware speed improvement has stagnated.

Hardware manufacturers are shifting focus to energy efficiency and parallel architectures.

Performance improvement requires parallel algorithms (multi-core PCs, clusters, and emerging architectures).



Parallel Interior-Point Methods



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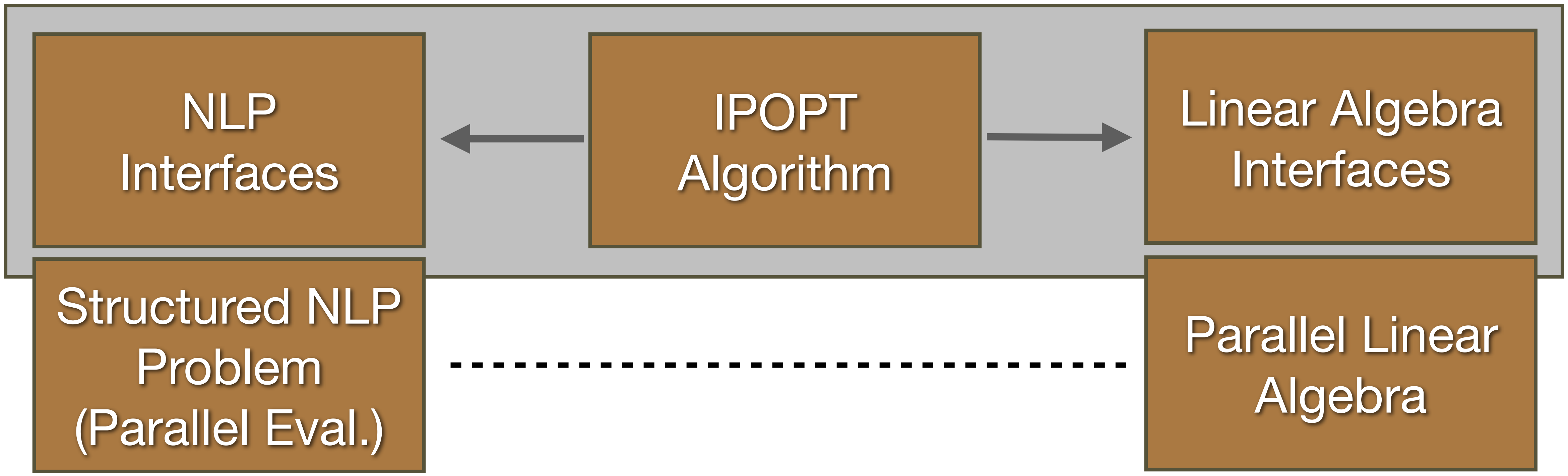
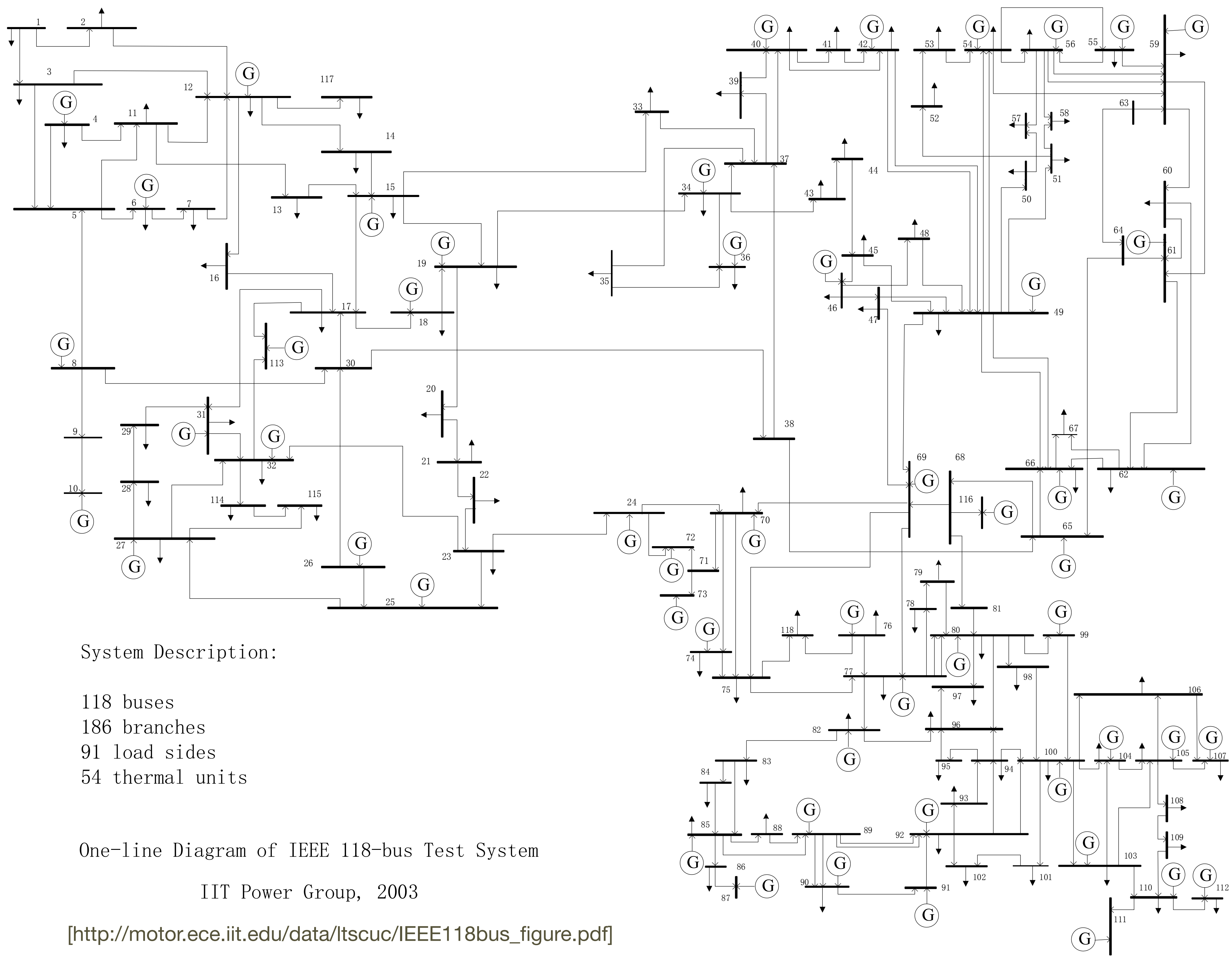


ACOPF Implementation (Pyomo & IPOPT)

Contingency-Constrained ACOPF (PySP)

Parallel Interior-point Methods (Schur-IPOPT)

AC Optimal Power Flow (ACOPF)



System Description:

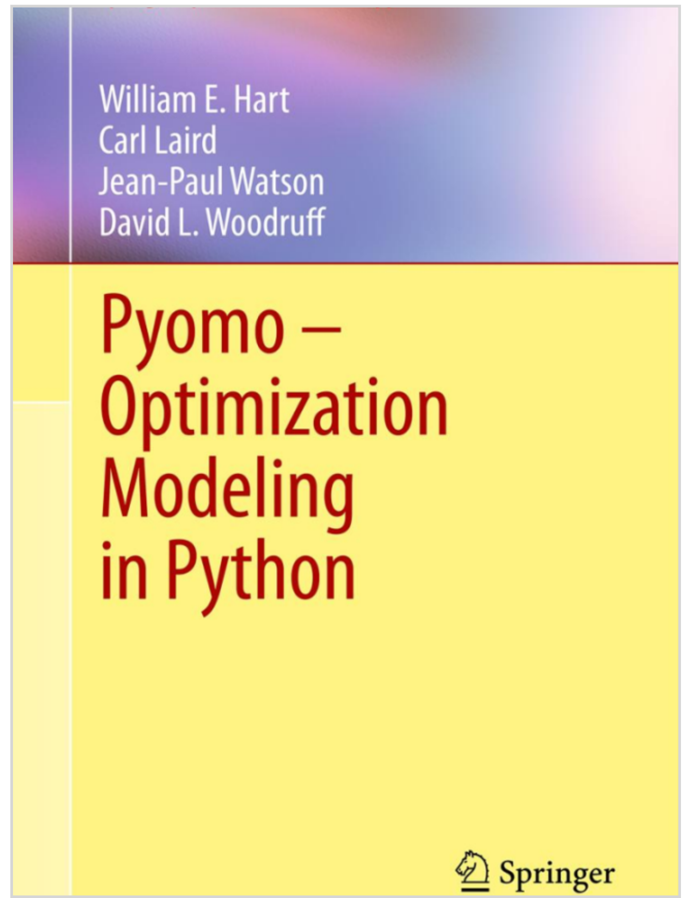
118 buses
186 branches
91 load sides
54 thermal units

One-line Diagram of IEEE 118-bus Test System

IIT Power Group, 2003

[http://motor.ece.iit.edu/data/ltscuc/IEEE118bus_figure.pdf]

Building the model with Pyomo



Hackebeil, Hart, Laird,
Siirola, Watson, Woodruff,
and many others…

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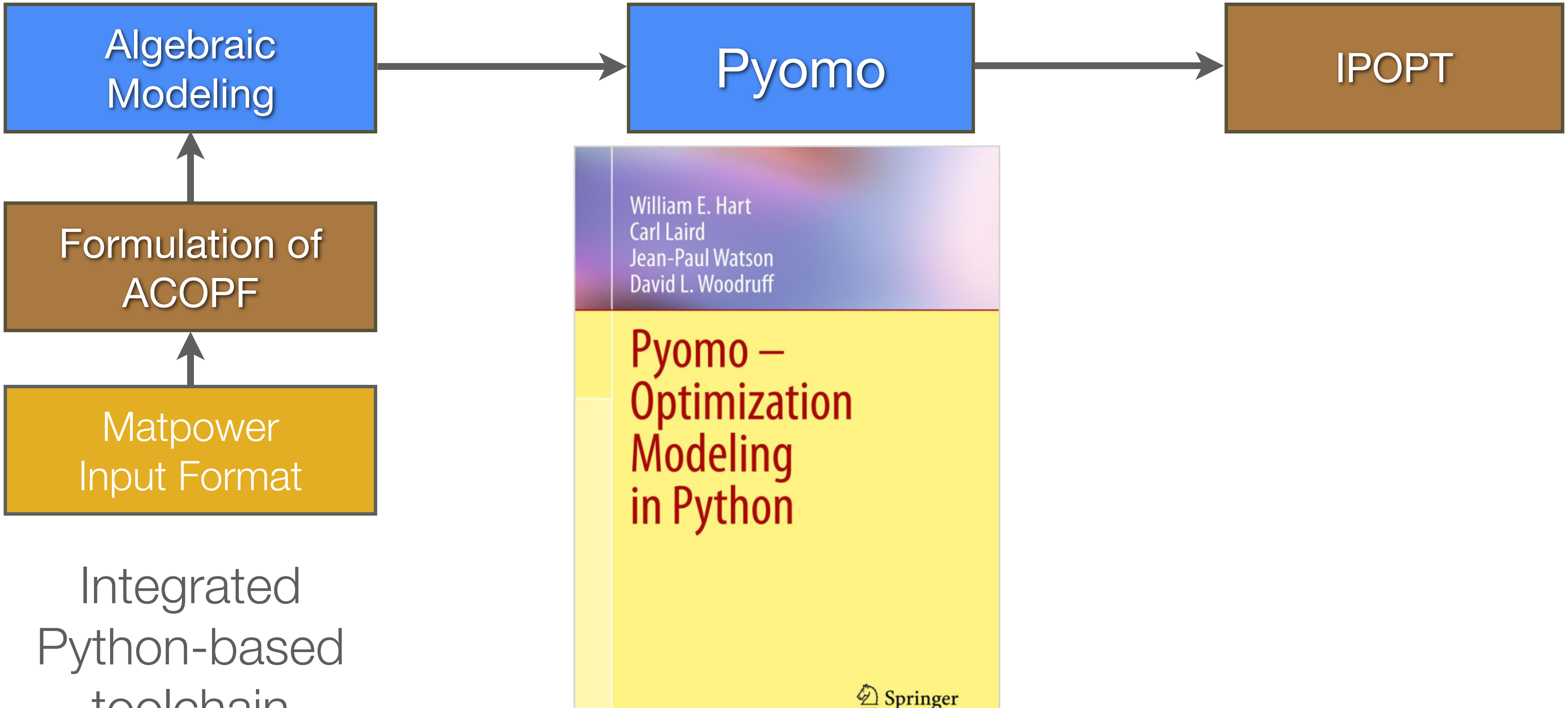


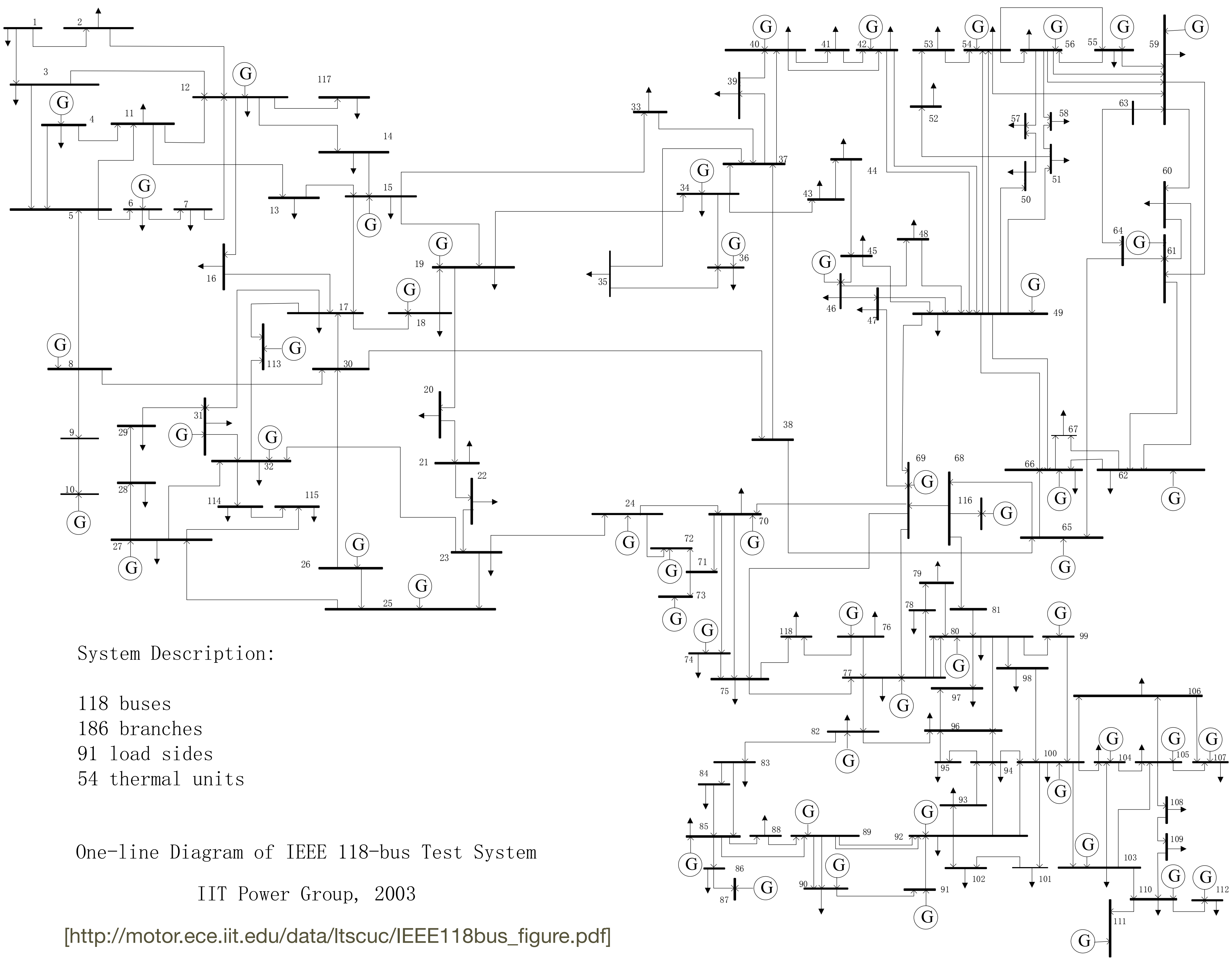
Integrated Python-based toolchain

Hackebeil, Hart, Laird, Siirola, Watson, Woodruff, and many others…

ACOPF Solution with Pyomo and Ipopt

Case Name	Number of Variables	Δ Objective Function ($) compared with Matpower
case30	46	2.410E-05
case9	95	-2.931E-04
case9Q	95	-4.806E-04
case6ww	105	-7.390E-05
case14	197	-9.430E-05
case30Q	399	-3.940E-05
case_ieee30	399	-8.082E-04
case24_ieee_rts	416	-5.421E-03
case39	465	2.282E-04
case57	767	8.225E-04
case118_laird	1,831	-1.020E-03
case2383wp	28,456	-2.939E-02
case2737sop	33,742	1.139E-03
case2736sp	33,807	-5.044E-03
case2746wop	34,013	-1.544E-04
case2746wp	34,063	-6.135E-03
case3012wp	35,242	-1.976E-02
case3120sp	36,247	-1.149E-02

N-1 Contingency-Constrained ACOPF Problem



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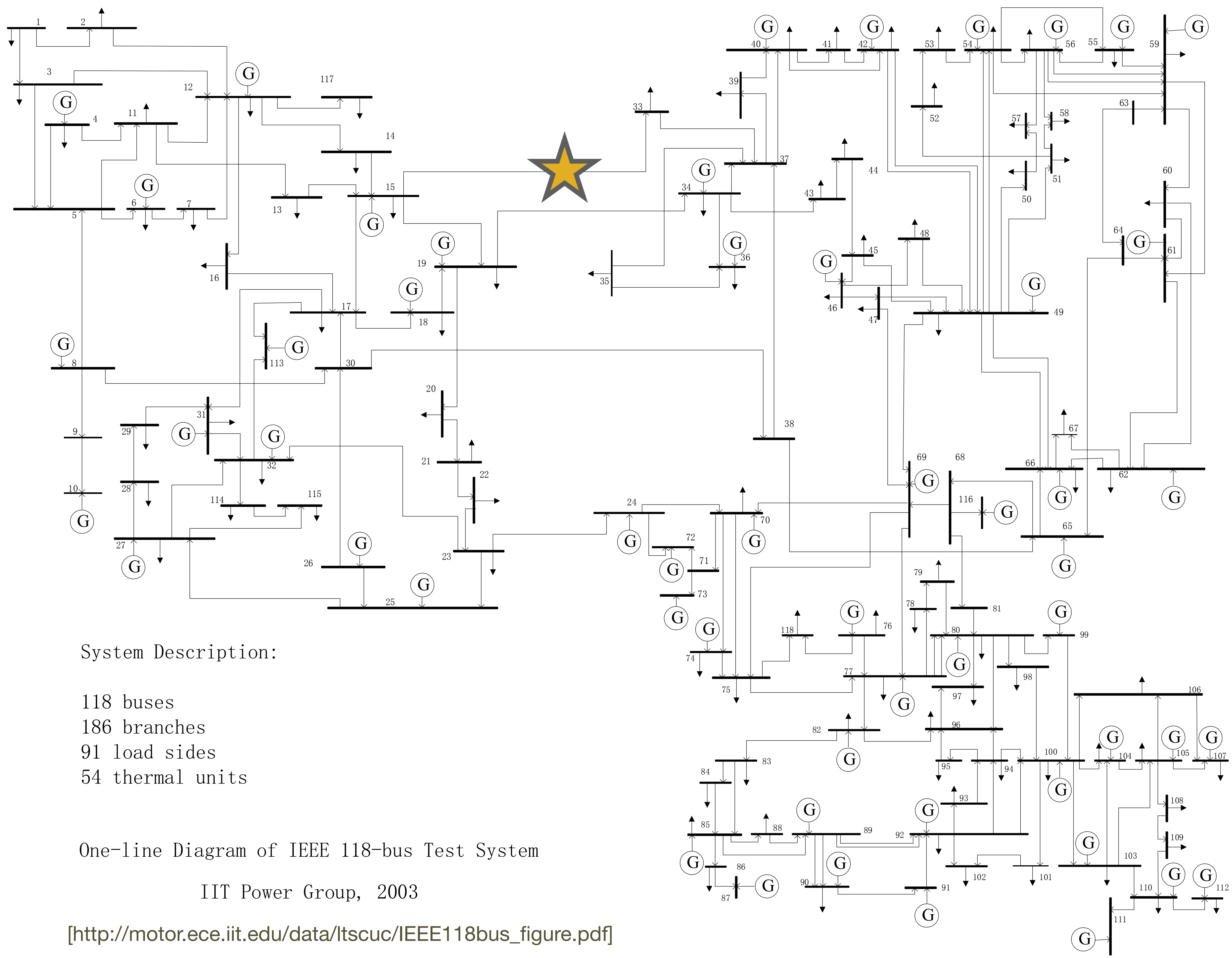

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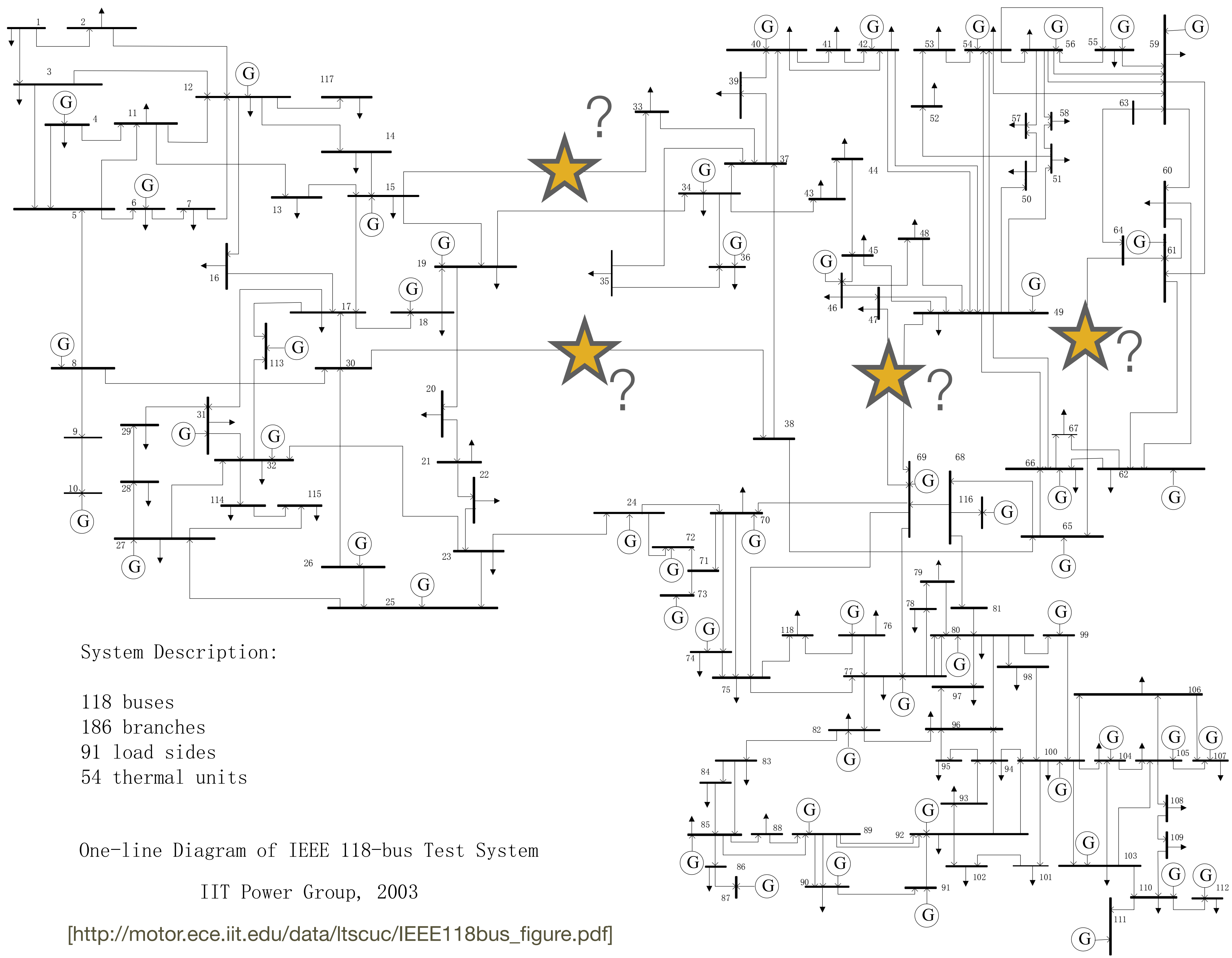

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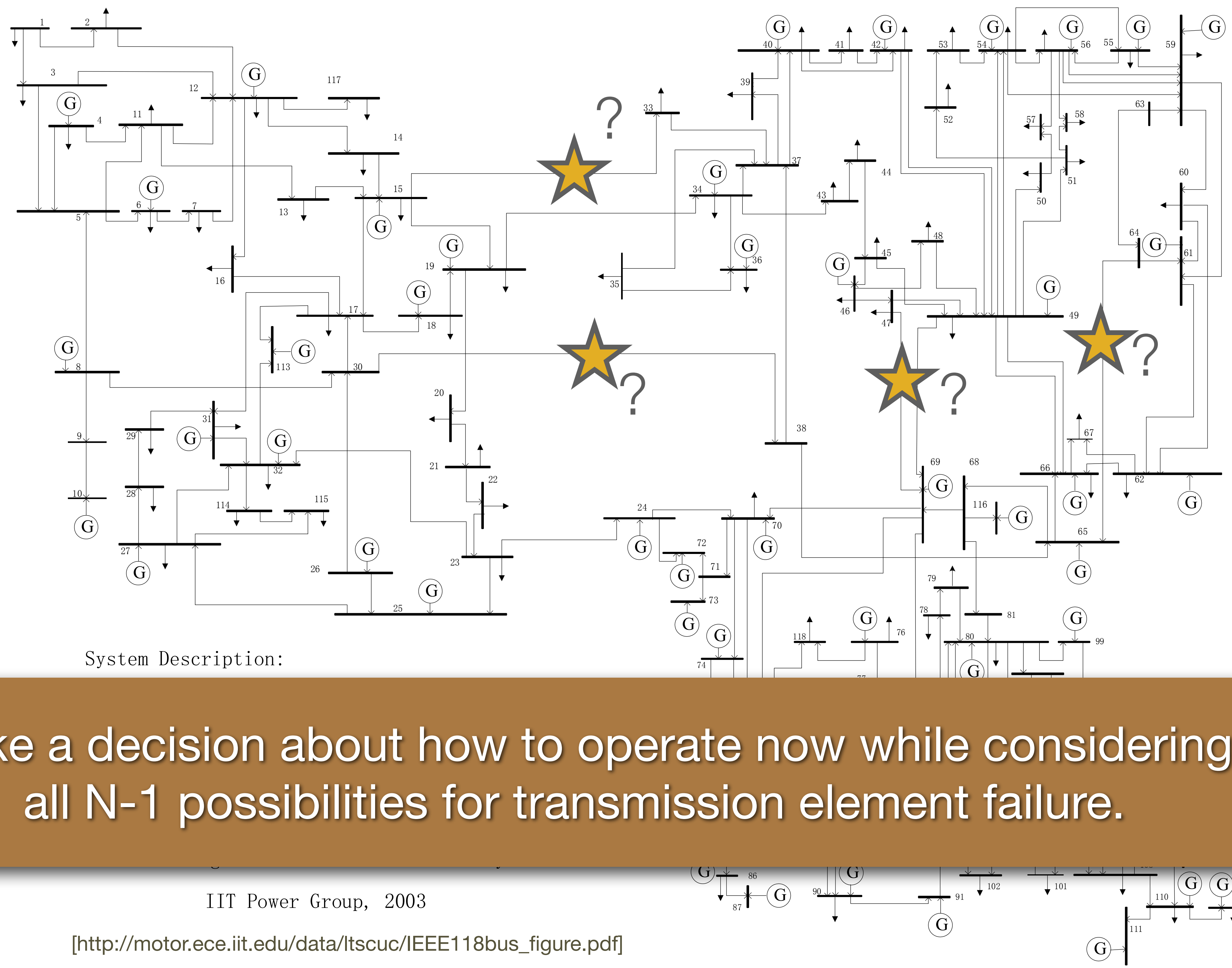
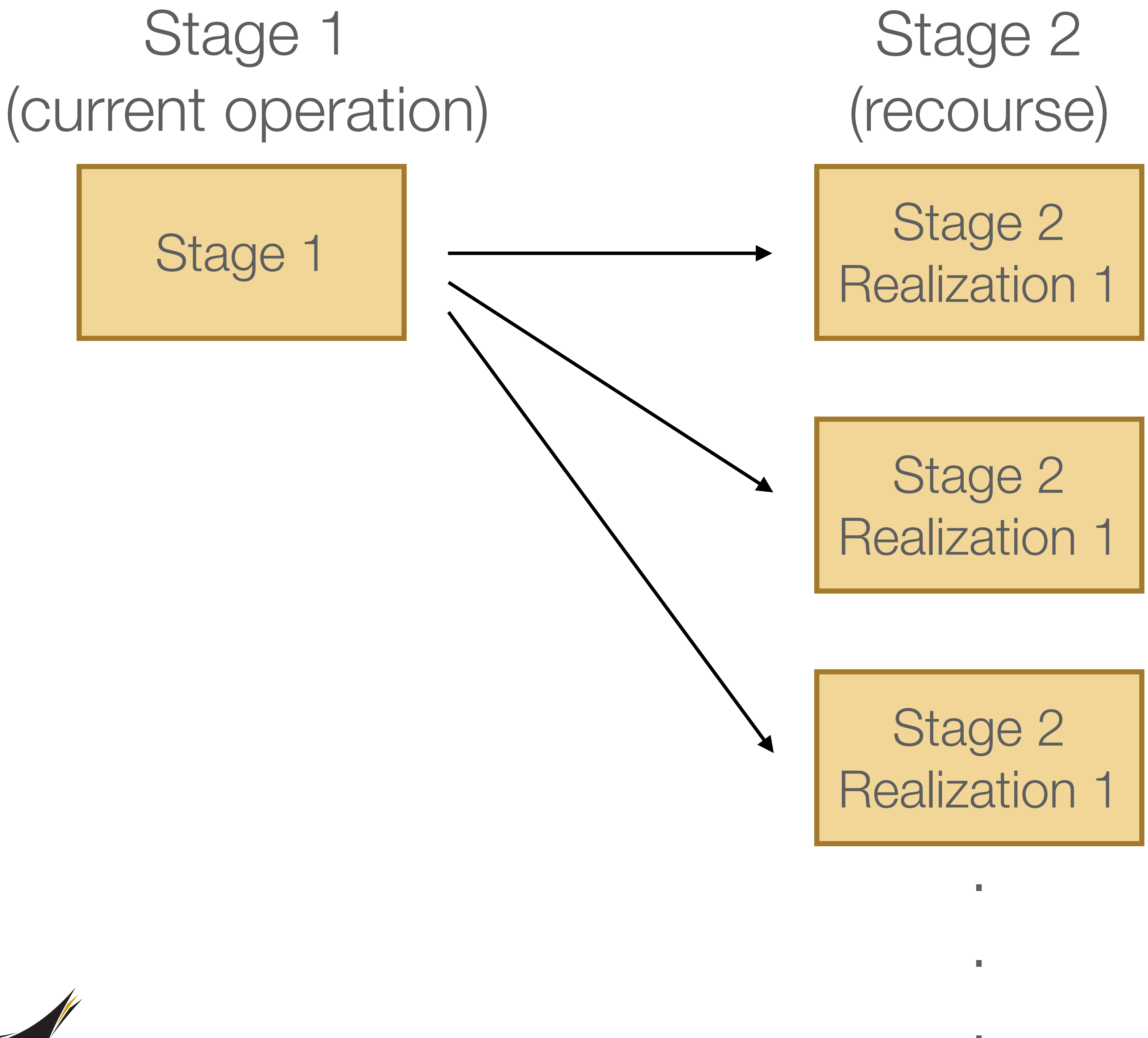


Make a decision about how to operate now while considering all N-1 possibilities for transmission element failure.

[http://motor.ece.iit.edu/data/ltscuc/IEEE118bus_figure.pdf]

Two-Stage Stochastic Programming



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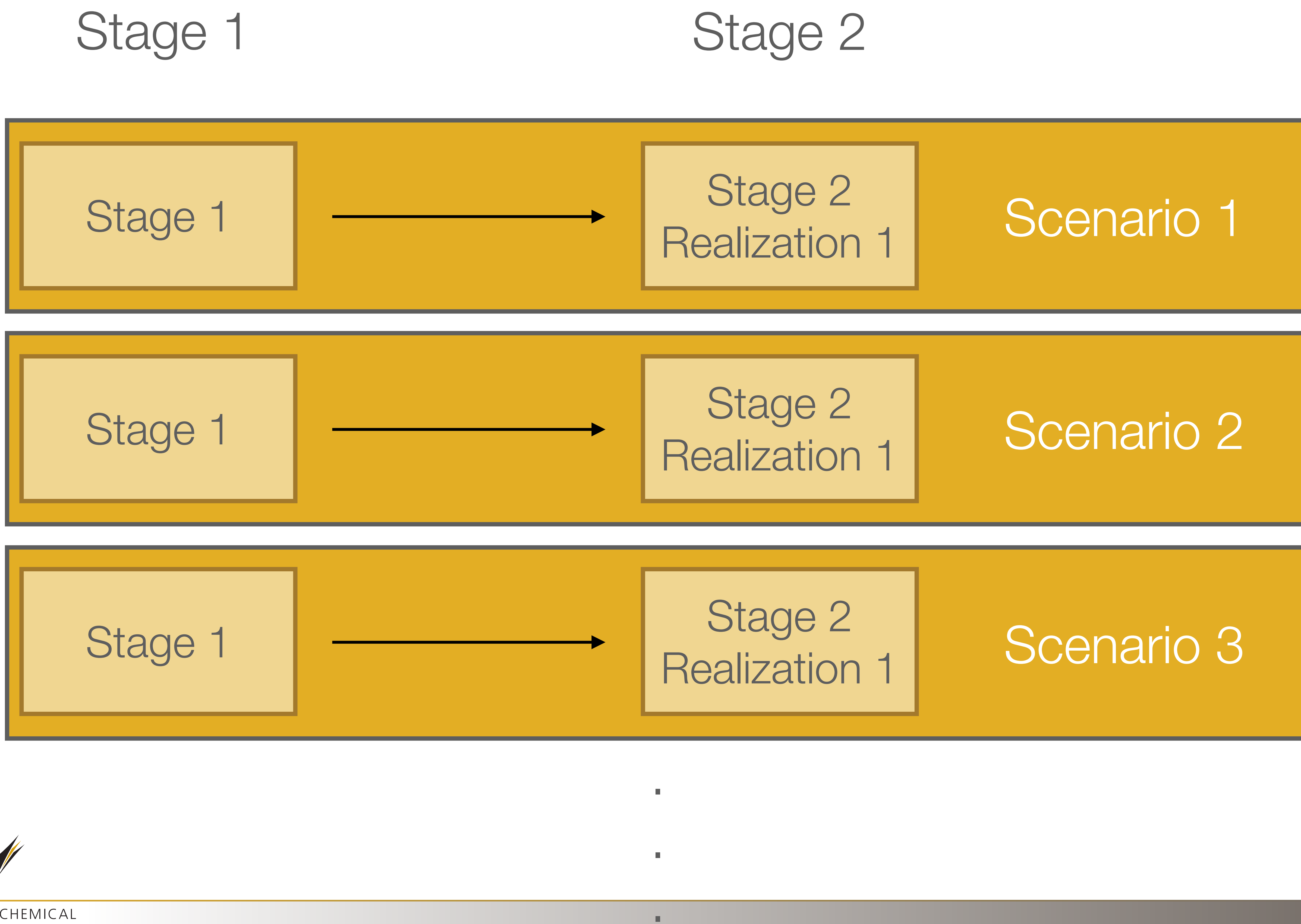

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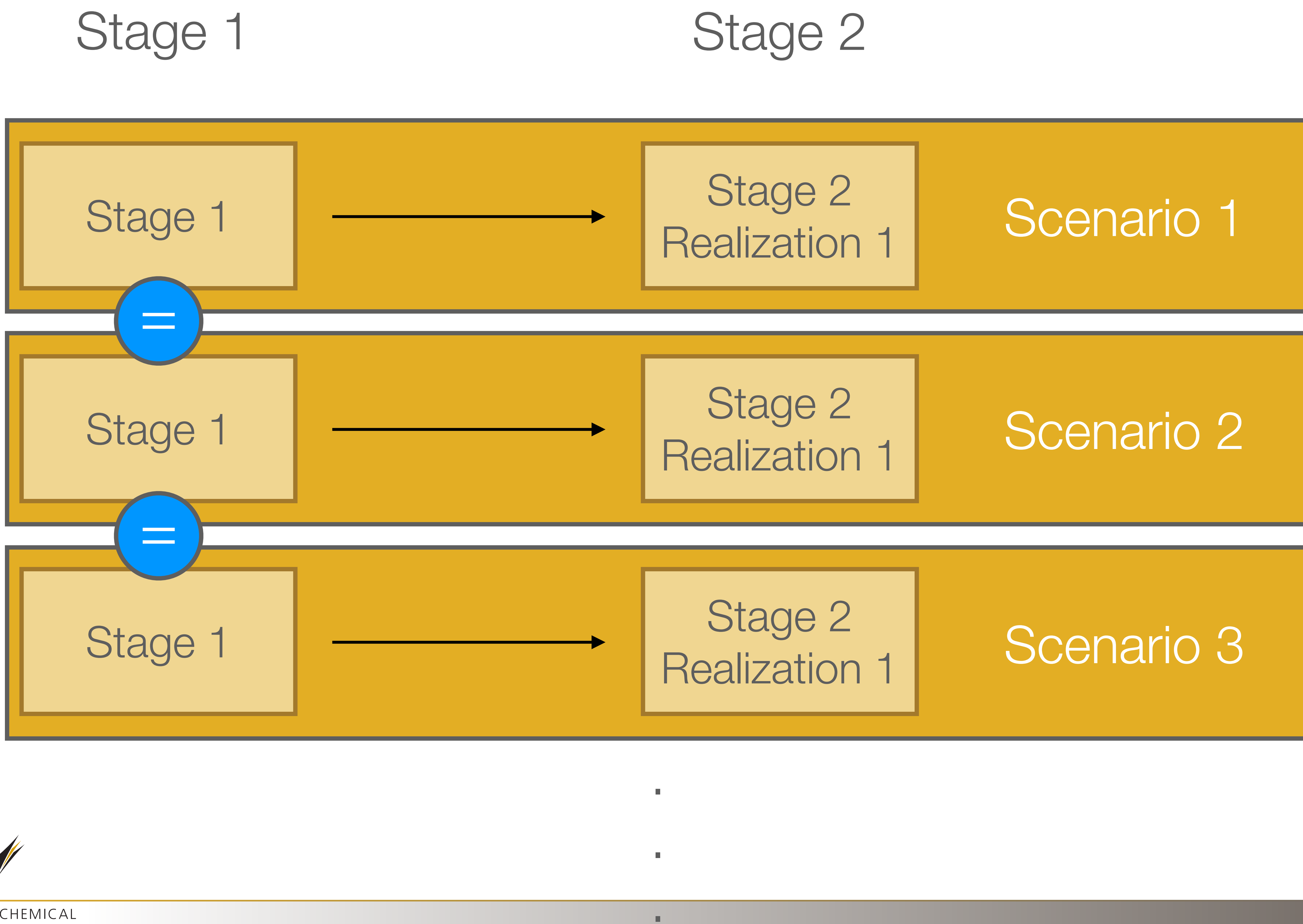
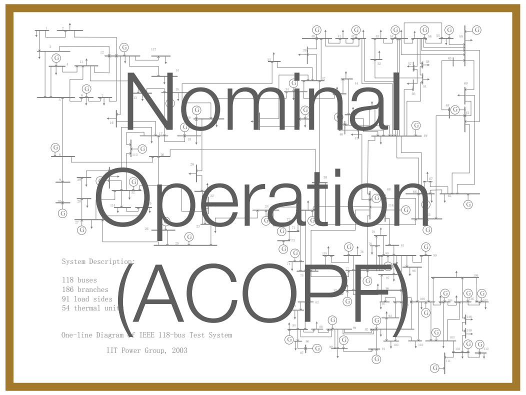


Contingency-Constrained ACOPF Problem



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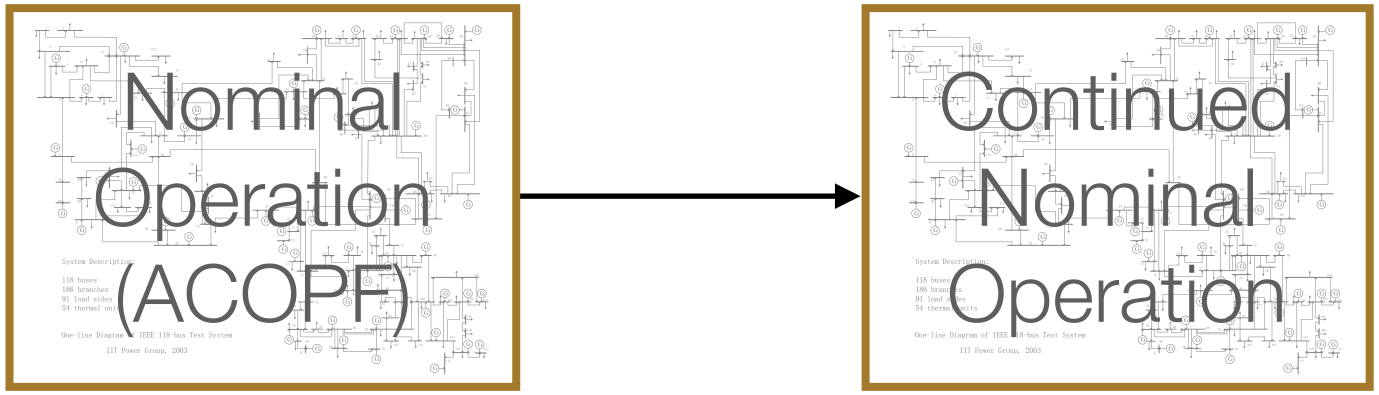

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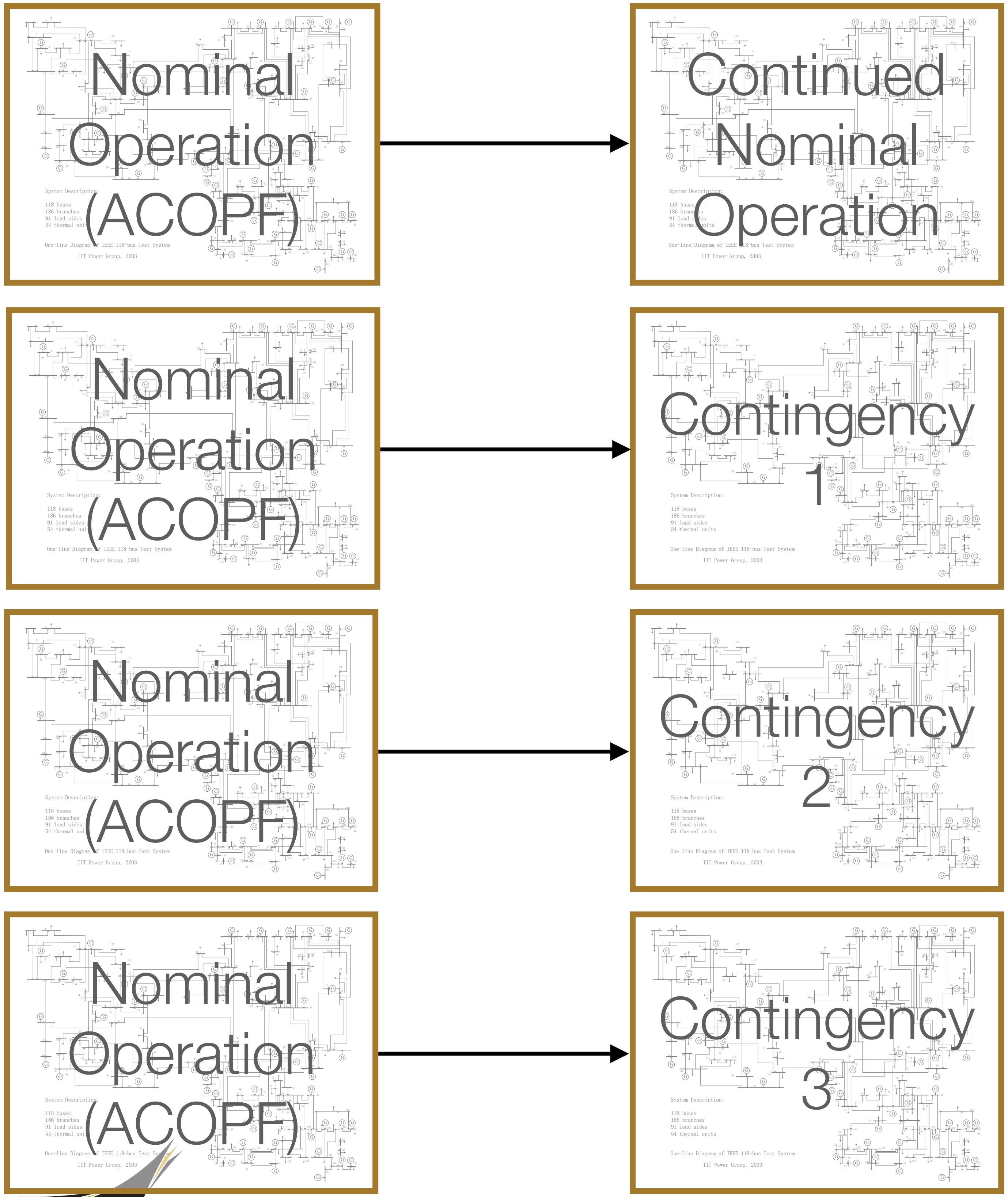

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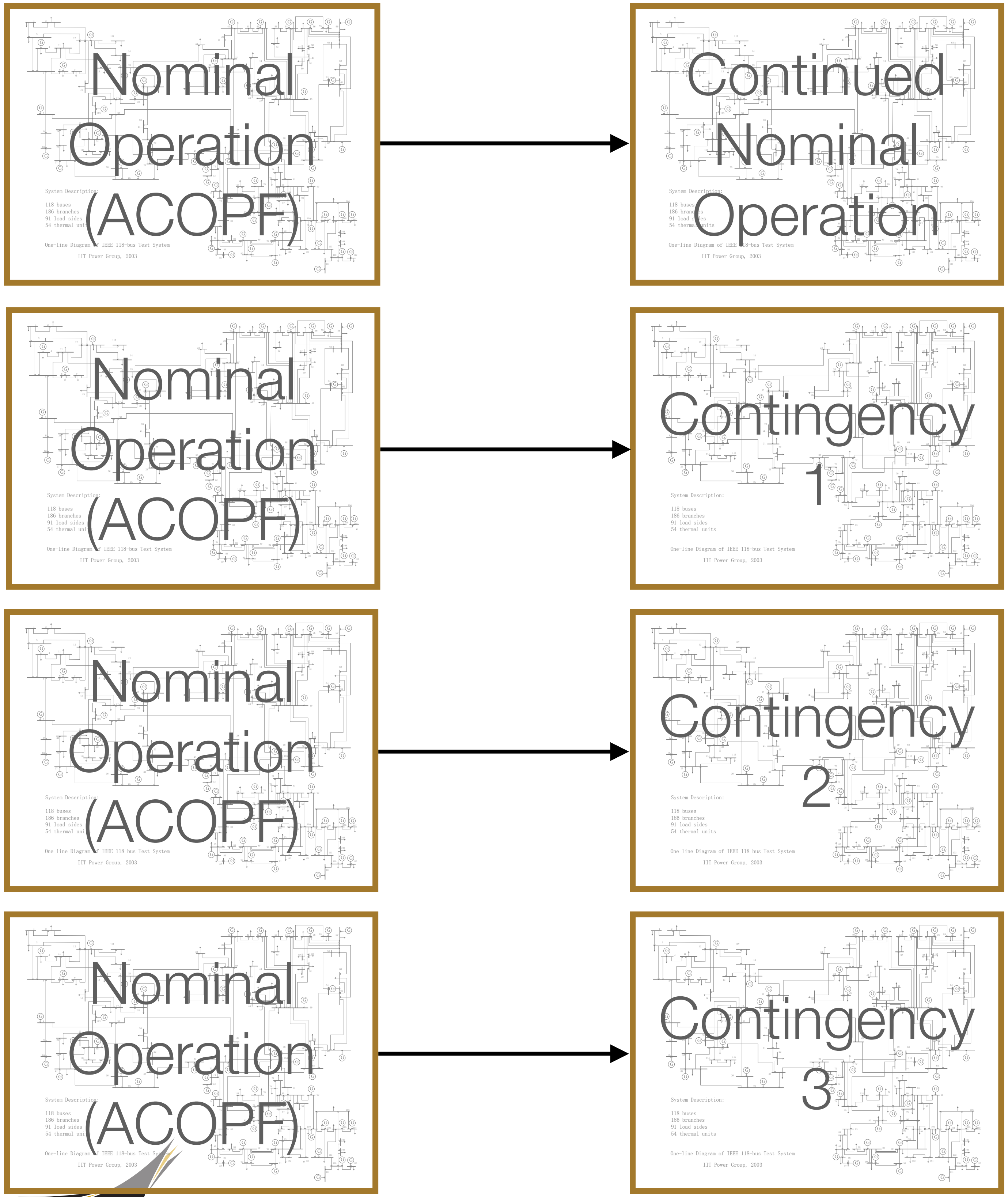

Nonlinear two-stage stochastic programming problem

AC power flow model for each scenario and stage

Contingency scenario for each (almost) transmission line

Penalty for inability to meet demands

Ramping constraints for changes in generator set points

Very large-scale nonlinear programming problem (Millions of variables)

Contingency-Constrained Stochastic Programming ACOPF Formulation

$\mathcal{L}$	Lines	$\mathcal{G}$	Generators
$\mathcal{B}$	All Buses	$\mathcal{D}$	Load Buses
$\mathcal{S}$	Scenarios	$\mathcal{T}$	Time periods

$$\min \sum_{s\in S} p_s \sum_{t\in T} \Bigg[\sum_{g\in\mathcal{G}} C_g^G(P_{g,t,s}^G, Q_{g,t,s}^G) + \rho_1 \sum_{g\in\mathcal{G}} \left[\left(P_{g,t,s}^G - P_{g,t,s}^{\star G}\right)^2 + \left(Q_{g,t,s}^G - Q_{g,t,s}^{\star G}\right)^2 \right]$$

$$+\rho_2 \sum_{b\in\mathcal{D}} \left[\left(P_{b,t,s}^L - P_b^{\star L}\right)^2 + \left(Q_{b,t,s}^L - Q_b^{\star L}\right)^2 \right] \Bigg]$$

$$\text{s.t.} \quad \begin{bmatrix} i_{l,t,s}^{fr} \\ i_{l,t,s}^{fj} \\ i_{l,t,s}^{tr} \\ i_{l,t,s}^{tj} \end{bmatrix} = Y_{l,t,s} \begin{bmatrix} v_{bf(l),t,s}^r \\ v_{bf(l),t,s}^j \\ v_{bt(l),t,s}^r \\ v_{bt(l),t,s}^j \end{bmatrix} \qquad \forall\, l \in \mathcal{L},\ t \in \mathcal{T},\ s \in \mathcal{S}$$

$$P_{b,t,s}^S = Y_b^S \left[(v_{b,t,s}^r)^2 + (v_{b,t,s}^j)^2 \right] \qquad \forall\, b \in \mathcal{H},\ t \in \mathcal{T},\ s \in \mathcal{S}$$

$$Q_{b,t,s}^S = -Y_b^S \left[(v_{b,t,s}^r)^2 + (v_{b,t,s}^j)^2 \right] \qquad \forall\, b \in \mathcal{H},\ t \in \mathcal{T},\ s \in \mathcal{S}$$

$$0 = \sum_{l\in\mathcal{I}_b} P_{l,t,s}^t + \sum_{l\in\mathcal{O}_b} P_{l,t,s}^f + P_{b,t,s}^S + P_{b,t,s}^L - P_{b,t,s}^G \qquad \forall\, b \in \mathcal{B},\ t \in \mathcal{T},\ s \in \mathcal{S}$$

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$$+\rho_2 \sum_{b\in\mathcal{D}}\left[\left(P^L_{b,t,s} - P^{\star L}_b\right)^2 + \left(Q^L_{b,t,s} - Q^{\star L}_b\right)^2\right]\Bigg]$$

Objective: Expected Value of Generator Op. Costs + Penalty

$$\text{s.t.} \quad \begin{bmatrix} i^{fr}_{l,t,s} \\ i^{fj}_{l,t,s} \\ i^{tr}_{l,t,s} \\ i^{tj}_{l,t,s} \end{bmatrix} = Y_{l,t,s} \begin{bmatrix} v^r_{bf(l),t,s} \\ v^j_{bf(l),t,s} \\ v^r_{bt(l),t,s} \\ v^j_{bt(l),t,s} \end{bmatrix} \qquad \forall\, l\in\mathcal{L},\ t\in\mathcal{T},\ s\in\mathcal{S}$$

$$P^S_{b,t,s} = Y^S_b\left[(v^r_{b,t,s})^2 + (v^j_{b,t,s})^2\right] \qquad \forall\, b\in\mathcal{H},\ t\in\mathcal{T},\ s\in\mathcal{S}$$

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IV Relationships (to/from) Every Line (all S and T)

$$P_{b,t,s}^S = Y_b^S \left[(v_{b,t,s}^r)^2 + (v_{b,t,s}^j)^2 \right] \qquad \forall\, b \in \mathcal{H},\ t \in \mathcal{T},\ s \in \mathcal{S}$$

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PI Transmission Model

$$0 = \sum_{l\in\mathcal{I}_b} P_{l,t,s}^t + \sum_{l\in\mathcal{O}_b} P_{l,t,s}^f + P_{b,t,s}^S + P_{b,t,s}^L - P_{b,t,s}^G \qquad \forall\, b \in \mathcal{B},\ t \in \mathcal{T},\ s \in \mathcal{S}$$

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Power Balance (to/from) Every Bus (all S and T)

Contingency-Constrained Stochastic Programming ACOPF Formulation

$$P^f_{l,t,s} = v^r_{bf(l),t,s} \cdot i^{fr}_{l,t,s} + v^j_{bf(l),t,s} \cdot i^{fj}_{l,t,s} \quad \forall\, l \in \mathcal{L},\ t \in \mathcal{T},\ s \in \mathcal{S}$$
$$Q^f_{l,t,s} = v^j_{bf(l),t,s} \cdot i^{fr}_{l,t,s} - v^r_{bf(l),t,s} \cdot i^{fj}_{l,t,s} \quad \forall\, l \in \mathcal{L},\ t \in \mathcal{T},\ s \in \mathcal{S}$$
$$P^t_{l,t,s} = v^r_{bt(l),t,s} \cdot i^{tr}_{l,t,s} + v^j_{bt(l),t,s} \cdot i^{tj}_{l,t,s} \quad \forall\, l \in \mathcal{L},\ t \in \mathcal{T},\ s \in \mathcal{S}$$
$$Q^t_{l,t,s} = v^j_{bt(l),t,s} \cdot i^{tr}_{l,t,s} - v^r_{bt(l),t,s} \cdot i^{tj}_{l,t,s} \quad \forall\, l \in \mathcal{L},\ t \in \mathcal{T},\ s \in \mathcal{S}$$

Power (to/from)
Every Line
(all S and T)

$$(P^f_{l,t,s})^2 + (Q^f_{l,t,s})^2 \leq (S^U)^2 \quad \forall\, l \in \mathcal{L},\ t \in \mathcal{T},\ s \in \mathcal{S}$$
$$(P^t_{l,t,s})^2 + (Q^t_{l,t,s})^2 \leq (S^U)^2 \quad \forall\, l \in \mathcal{L},\ t \in \mathcal{T},\ s \in \mathcal{S}$$
$$(V^L_b)^2 \leq (v^r_{b,t,s})^2 + (v^j_{b,t,s})^2 \leq (V^U_b)^2 \quad \forall\, b \in \mathcal{B},\ t \in \mathcal{T},\ s \in \mathcal{S}$$
$$P^{GL}_g \leq P^G_{g,t,s} \leq P^{GU}_g \quad \forall\, g \in \mathcal{G},\ t \in \mathcal{T},\ s \in \mathcal{S}$$
$$Q^{GL}_g \leq Q^G_{g,t,s} \leq Q^{GU}_g \quad \forall\, g \in \mathcal{G},\ t \in \mathcal{T},\ s \in \mathcal{S}$$
$$v^j_{rb,t,s} = 0 \quad \forall\, t \in \mathcal{T},\ s \in \mathcal{S}$$
$$P^L_{b,1,s} = P^{\star L}_b \quad \forall\, b \in \mathcal{D},\ s \in \mathcal{S}$$
$$Q^L_{b,1,s} = Q^{\star L}_b \quad \forall\, b \in \mathcal{D},\ s \in \mathcal{S}$$
$$-P^{GR}_g \leq P^G_{g,1,s} - P^G_{g,2,s} \leq P^{GR}_g \quad \forall\, g \in \mathcal{G},\ s \in \mathcal{S}$$
$$P^G_{g,1,0} = P^G_{g,2,0} \quad \forall\, g \in \mathcal{G}$$
$$Q^G_{g,1,0} = Q^G_{g,2,0} \quad \forall\, g \in \mathcal{G}$$
$$P^G_{g,1,0} = P^G_{g,1,s} \quad \forall\, g \in \mathcal{G},\ s \in \mathcal{S}/\{0\}$$
$$Q^G_{g,1,0} = Q^G_{g,1,s} \quad \forall\, g \in \mathcal{G},\ s \in \mathcal{S}/\{0\}$$

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$$Q^f_{l,t,s} = v^j_{bf(l),t,s} \cdot i^{fr}_{l,t,s} - v^r_{bf(l),t,s} \cdot i^{fj}_{l,t,s} \qquad \forall\, l \in \mathcal{L},\ t \in \mathcal{T},\ s \in \mathcal{S}$$

$$P^t_{l,t,s} = v^r_{bt(l),t,s} \cdot i^{tr}_{l,t,s} + v^j_{bt(l),t,s} \cdot i^{tj}_{l,t,s} \qquad \forall\, l \in \mathcal{L},\ t \in \mathcal{T},\ s \in \mathcal{S}$$

$$Q^t_{l,t,s} = v^j_{bt(l),t,s} \cdot i^{tr}_{l,t,s} - v^r_{bt(l),t,s} \cdot i^{tj}_{l,t,s} \qquad \forall\, l \in \mathcal{L},\ t \in \mathcal{T},\ s \in \mathcal{S}$$

$$(P^f_{l,t,s})^2 + (Q^f_{l,t,s})^2 \le (S^U)^2 \qquad \forall\, l \in \mathcal{L},\ t \in \mathcal{T},\ s \in \mathcal{S}$$

$$(P^t_{l,t,s})^2 + (Q^t_{l,t,s})^2 \le (S^U)^2 \qquad \forall\, l \in \mathcal{L},\ t \in \mathcal{T},\ s \in \mathcal{S}$$

$$(V^L_b)^2 \le (v^r_{b,t,s})^2 + (v^j_{b,t,s})^2 \le (V^U_b)^2 \qquad \forall\, b \in \mathcal{B},\ t \in \mathcal{T},\ s \in \mathcal{S}$$

$$P^{GL}_g \le P^G_{g,t,s} \le P^{GU}_g \qquad \forall\, g \in \mathcal{G},\ t \in \mathcal{T},\ s \in \mathcal{S}$$

$$Q^{GL}_g \le Q^G_{g,t,s} \le Q^{GU}_g \qquad \forall\, g \in \mathcal{G},\ t \in \mathcal{T},\ s \in \mathcal{S}$$

$$v^j_{rb,t,s} = 0 \qquad \forall\, t \in \mathcal{T},\ s \in \mathcal{S}$$

$$P^L_{b,1,s} = P^{\star L}_b \qquad \forall\, b \in \mathcal{D},\ s \in \mathcal{S}$$

$$Q^L_{b,1,s} = Q^{\star L}_b \qquad \forall\, b \in \mathcal{D},\ s \in \mathcal{S}$$

$$-P^{GR}_g \le P^G_{g,1,s} - P^G_{g,2,s} \le P^{GR}_g \qquad \forall\, g \in \mathcal{G},\ s \in \mathcal{S}$$

$$P^G_{g,1,0} = P^G_{g,2,0} \qquad \forall\, g \in \mathcal{G}$$

$$Q^G_{g,1,0} = Q^G_{g,2,0} \qquad \forall\, g \in \mathcal{G}$$

$$P^G_{g,1,0} = P^G_{g,1,s} \qquad \forall\, g \in \mathcal{G},\ s \in \mathcal{S}/\{0\}$$

$$Q^G_{g,1,0} = Q^G_{g,1,s} \qquad \forall\, g \in \mathcal{G},\ s \in \mathcal{S}/\{0\}$$

Line limits on (to/from) of every line (all S and T)

Contingency-Constrained Stochastic Programming ACOPF Formulation

$$P^f_{l,t,s} = v^r_{bf(l),t,s} \cdot i^{fr}_{l,t,s} + v^j_{bf(l),t,s} \cdot i^{fj}_{l,t,s} \qquad \forall\, l \in \mathcal{L},\ t \in \mathcal{T},\ s \in \mathcal{S}$$

$$Q^f_{l,t,s} = v^j_{bf(l),t,s} \cdot i^{fr}_{l,t,s} - v^r_{bf(l),t,s} \cdot i^{fj}_{l,t,s} \qquad \forall\, l \in \mathcal{L},\ t \in \mathcal{T},\ s \in \mathcal{S}$$

$$P^t_{l,t,s} = v^r_{bt(l),t,s} \cdot i^{tr}_{l,t,s} + v^j_{bt(l),t,s} \cdot i^{tj}_{l,t,s} \qquad \forall\, l \in \mathcal{L},\ t \in \mathcal{T},\ s \in \mathcal{S}$$

$$Q^t_{l,t,s} = v^j_{bt(l),t,s} \cdot i^{tr}_{l,t,s} - v^r_{bt(l),t,s} \cdot i^{tj}_{l,t,s} \qquad \forall\, l \in \mathcal{L},\ t \in \mathcal{T},\ s \in \mathcal{S}$$

$$(P^f_{l,t,s})^2 + (Q^f_{l,t,s})^2 \leq (S^U)^2 \qquad \forall\, l \in \mathcal{L},\ t \in \mathcal{T},\ s \in \mathcal{S}$$

$$(P^t_{l,t,s})^2 + (Q^t_{l,t,s})^2 \leq (S^U)^2 \qquad \forall\, l \in \mathcal{L},\ t \in \mathcal{T},\ s \in \mathcal{S}$$

$$(V^L_b)^2 \leq (v^r_{b,t,s})^2 + (v^j_{b,t,s})^2 \leq (V^U_b)^2 \qquad \forall\, b \in \mathcal{B},\ t \in \mathcal{T},\ s \in \mathcal{S}$$

$$P^{GL}_g \leq P^G_{g,t,s} \leq P^{GU}_g \qquad \forall\, g \in \mathcal{G},\ t \in \mathcal{T},\ s \in \mathcal{S}$$

$$Q^{GL}_g \leq Q^G_{g,t,s} \leq Q^{GU}_g \qquad \forall\, g \in \mathcal{G},\ t \in \mathcal{T},\ s \in \mathcal{S}$$

$$v^j_{rb,t,s} = 0 \qquad \forall\, t \in \mathcal{T},\ s \in \mathcal{S}$$

$$P^L_{b,1,s} = P^{\star L}_b \qquad \forall\, b \in \mathcal{D},\ s \in \mathcal{S}$$

$$Q^L_{b,1,s} = Q^{\star L}_b \qquad \forall\, b \in \mathcal{D},\ s \in \mathcal{S}$$

$$-P^{GR}_g \leq P^G_{g,1,s} - P^G_{g,2,s} \leq P^{GR}_g \qquad \forall\, g \in \mathcal{G},\ s \in \mathcal{S}$$

$$P^G_{g,1,0} = P^G_{g,2,0} \qquad \forall\, g \in \mathcal{G}$$

$$Q^G_{g,1,0} = Q^G_{g,2,0} \qquad \forall\, g \in \mathcal{G}$$

$$P^G_{g,1,0} = P^G_{g,1,s} \qquad \forall\, g \in \mathcal{G},\ s \in \mathcal{S}/\{0\}$$

$$Q^G_{g,1,0} = Q^G_{g,1,s} \qquad \forall\, g \in \mathcal{G},\ s \in \mathcal{S}/\{0\}$$

Voltage limits on every bus (all S and T)

Contingency-Constrained Stochastic Programming ACOPF Formulation

$$P^f_{l,t,s} = v^r_{bf(l),t,s} \cdot i^{fr}_{l,t,s} + v^j_{bf(l),t,s} \cdot i^{fj}_{l,t,s} \qquad \forall\, l \in \mathcal{L},\ t \in \mathcal{T},\ s \in \mathcal{S}$$

$$Q^f_{l,t,s} = v^j_{bf(l),t,s} \cdot i^{fr}_{l,t,s} - v^r_{bf(l),t,s} \cdot i^{fj}_{l,t,s} \qquad \forall\, l \in \mathcal{L},\ t \in \mathcal{T},\ s \in \mathcal{S}$$

$$P^t_{l,t,s} = v^r_{bt(l),t,s} \cdot i^{tr}_{l,t,s} + v^j_{bt(l),t,s} \cdot i^{tj}_{l,t,s} \qquad \forall\, l \in \mathcal{L},\ t \in \mathcal{T},\ s \in \mathcal{S}$$

$$Q^t_{l,t,s} = v^j_{bt(l),t,s} \cdot i^{tr}_{l,t,s} - v^r_{bt(l),t,s} \cdot i^{tj}_{l,t,s} \qquad \forall\, l \in \mathcal{L},\ t \in \mathcal{T},\ s \in \mathcal{S}$$

$$(P^f_{l,t,s})^2 + (Q^f_{l,t,s})^2 \leq (S^U)^2 \qquad \forall\, l \in \mathcal{L},\ t \in \mathcal{T},\ s \in \mathcal{S}$$

$$(P^t_{l,t,s})^2 + (Q^t_{l,t,s})^2 \leq (S^U)^2 \qquad \forall\, l \in \mathcal{L},\ t \in \mathcal{T},\ s \in \mathcal{S}$$

$$(V^L_b)^2 \leq (v^r_{b,t,s})^2 + (v^j_{b,t,s})^2 \leq (V^U_b)^2 \qquad \forall\, b \in \mathcal{B},\ t \in \mathcal{T},\ s \in \mathcal{S}$$

$$P^{GL}_g \leq P^G_{g,t,s} \leq P^{GU}_g \qquad \forall\, g \in \mathcal{G},\ t \in \mathcal{T},\ s \in \mathcal{S}$$

$$Q^{GL}_g \leq Q^G_{g,t,s} \leq Q^{GU}_g \qquad \forall\, g \in \mathcal{G},\ t \in \mathcal{T},\ s \in \mathcal{S}$$

$$v^j_{rb,t,s} = 0 \qquad \forall\, t \in \mathcal{T},\ s \in \mathcal{S}$$

$$P^L_{b,1,s} = P^{\star L}_b \qquad \forall\, b \in \mathcal{D},\ s \in \mathcal{S}$$

$$Q^L_{b,1,s} = Q^{\star L}_b \qquad \forall\, b \in \mathcal{D},\ s \in \mathcal{S}$$

$$-P^{GR}_g \leq P^G_{g,1,s} - P^G_{g,2,s} \leq P^{GR}_g \qquad \forall\, g \in \mathcal{G},\ s \in \mathcal{S}$$

$$P^G_{g,1,0} = P^G_{g,2,0} \qquad \forall\, g \in \mathcal{G}$$

$$Q^G_{g,1,0} = Q^G_{g,2,0} \qquad \forall\, g \in \mathcal{G}$$

$$P^G_{g,1,0} = P^G_{g,1,s} \qquad \forall\, g \in \mathcal{G},\ s \in \mathcal{S}/\{0\}$$

$$Q^G_{g,1,0} = Q^G_{g,1,s} \qquad \forall\, g \in \mathcal{G},\ s \in \mathcal{S}/\{0\}$$

Generator bounds on every generator (all S and T)

Contingency-Constrained Stochastic Programming ACOPF Formulation

$$P^f_{l,t,s} = v^r_{bf(l),t,s} \cdot i^{fr}_{l,t,s} + v^j_{bf(l),t,s} \cdot i^{fj}_{l,t,s} \qquad \forall\, l \in \mathcal{L},\; t \in \mathcal{T},\; s \in \mathcal{S}$$

$$Q^f_{l,t,s} = v^j_{bf(l),t,s} \cdot i^{fr}_{l,t,s} - v^r_{bf(l),t,s} \cdot i^{fj}_{l,t,s} \qquad \forall\, l \in \mathcal{L},\; t \in \mathcal{T},\; s \in \mathcal{S}$$

$$P^t_{l,t,s} = v^r_{bt(l),t,s} \cdot i^{tr}_{l,t,s} + v^j_{bt(l),t,s} \cdot i^{tj}_{l,t,s} \qquad \forall\, l \in \mathcal{L},\; t \in \mathcal{T},\; s \in \mathcal{S}$$

$$Q^t_{l,t,s} = v^j_{bt(l),t,s} \cdot i^{tr}_{l,t,s} - v^r_{bt(l),t,s} \cdot i^{tj}_{l,t,s} \qquad \forall\, l \in \mathcal{L},\; t \in \mathcal{T},\; s \in \mathcal{S}$$

$$(P^f_{l,t,s})^2 + (Q^f_{l,t,s})^2 \le (S^U)^2 \qquad \forall\, l \in \mathcal{L},\; t \in \mathcal{T},\; s \in \mathcal{S}$$

$$(P^t_{l,t,s})^2 + (Q^t_{l,t,s})^2 \le (S^U)^2 \qquad \forall\, l \in \mathcal{L},\; t \in \mathcal{T},\; s \in \mathcal{S}$$

$$(V^L_b)^2 \le (v^r_{b,t,s})^2 + (v^j_{b,t,s})^2 \le (V^U_b)^2 \qquad \forall\, b \in \mathcal{B},\; t \in \mathcal{T},\; s \in \mathcal{S}$$

$$P^{GL}_g \le P^G_{g,t,s} \le P^{GU}_g \qquad \forall\, g \in \mathcal{G},\; t \in \mathcal{T},\; s \in \mathcal{S}$$

$$Q^{GL}_g \le Q^G_{g,t,s} \le Q^{GU}_g \qquad \forall\, g \in \mathcal{G},\; t \in \mathcal{T},\; s \in \mathcal{S}$$

$$v^j_{rb,t,s} = 0 \qquad \forall\, t \in \mathcal{T},\; s \in \mathcal{S}$$

$$P^L_{b,1,s} = P^{\star L}_b \qquad \forall\, b \in \mathcal{D},\; s \in \mathcal{S}$$

$$Q^L_{b,1,s} = Q^{\star L}_b \qquad \forall\, b \in \mathcal{D},\; s \in \mathcal{S}$$

$$-P^{GR}_g \le P^G_{g,1,s} - P^G_{g,2,s} \le P^{GR}_g \qquad \forall\, g \in \mathcal{G},\; s \in \mathcal{S}$$

$$P^G_{g,1,0} = P^G_{g,2,0} \qquad \forall\, g \in \mathcal{G}$$

$$Q^G_{g,1,0} = Q^G_{g,2,0} \qquad \forall\, g \in \mathcal{G}$$

$$P^G_{g,1,0} = P^G_{g,1,s} \qquad \forall\, g \in \mathcal{G},\; s \in \mathcal{S}/\{0\}$$

$$Q^G_{g,1,0} = Q^G_{g,1,s} \qquad \forall\, g \in \mathcal{G},\; s \in \mathcal{S}/\{0\}$$

Reference Bus Constraint (all S and T)

Contingency-Constrained Stochastic Programming ACOPF Formulation

$$P^f_{l,t,s} = v^r_{bf(l),t,s} \cdot i^{fr}_{l,t,s} + v^j_{bf(l),t,s} \cdot i^{fj}_{l,t,s} \quad \forall\, l \in \mathcal{L},\ t \in \mathcal{T},\ s \in \mathcal{S}$$

$$Q^f_{l,t,s} = v^j_{bf(l),t,s} \cdot i^{fr}_{l,t,s} - v^r_{bf(l),t,s} \cdot i^{fj}_{l,t,s} \quad \forall\, l \in \mathcal{L},\ t \in \mathcal{T},\ s \in \mathcal{S}$$

$$P^t_{l,t,s} = v^r_{bt(l),t,s} \cdot i^{tr}_{l,t,s} + v^j_{bt(l),t,s} \cdot i^{tj}_{l,t,s} \quad \forall\, l \in \mathcal{L},\ t \in \mathcal{T},\ s \in \mathcal{S}$$

$$Q^t_{l,t,s} = v^j_{bt(l),t,s} \cdot i^{tr}_{l,t,s} - v^r_{bt(l),t,s} \cdot i^{tj}_{l,t,s} \quad \forall\, l \in \mathcal{L},\ t \in \mathcal{T},\ s \in \mathcal{S}$$

$$(P^f_{l,t,s})^2 + (Q^f_{l,t,s})^2 \leq (S^U)^2 \quad \forall\, l \in \mathcal{L},\ t \in \mathcal{T},\ s \in \mathcal{S}$$

$$(P^t_{l,t,s})^2 + (Q^t_{l,t,s})^2 \leq (S^U)^2 \quad \forall\, l \in \mathcal{L},\ t \in \mathcal{T},\ s \in \mathcal{S}$$

$$(V^L_b)^2 \leq (v^r_{b,t,s})^2 + (v^j_{b,t,s})^2 \leq (V^U_b)^2 \quad \forall\, b \in \mathcal{B},\ t \in \mathcal{T},\ s \in \mathcal{S}$$

$$P^{GL}_g \leq P^G_{g,t,s} \leq P^{GU}_g \quad \forall\, g \in \mathcal{G},\ t \in \mathcal{T},\ s \in \mathcal{S}$$

$$Q^{GL}_g \leq Q^G_{g,t,s} \leq Q^{GU}_g \quad \forall\, g \in \mathcal{G},\ t \in \mathcal{T},\ s \in \mathcal{S}$$

$$v^j_{rb,t,s} = 0 \quad \forall\, t \in \mathcal{T},\ s \in \mathcal{S}$$

$$P^L_{b,1,s} = P^{\star L}_b \quad \forall\, b \in \mathcal{D},\ s \in \mathcal{S}$$

$$Q^L_{b,1,s} = Q^{\star L}_b \quad \forall\, b \in \mathcal{D},\ s \in \mathcal{S}$$

$$-P^{GR}_g \leq P^G_{g,1,s} - P^G_{g,2,s} \leq P^{GR}_g \quad \forall\, g \in \mathcal{G},\ s \in \mathcal{S}$$

$$P^G_{g,1,0} = P^G_{g,2,0} \quad \forall\, g \in \mathcal{G}$$

$$Q^G_{g,1,0} = Q^G_{g,2,0} \quad \forall\, g \in \mathcal{G}$$

$$P^G_{g,1,0} = P^G_{g,1,s} \quad \forall\, g \in \mathcal{G},\ s \in \mathcal{S}/\{0\}$$

$$Q^G_{g,1,0} = Q^G_{g,1,s} \quad \forall\, g \in \mathcal{G},\ s \in \mathcal{S}/\{0\}$$

Fix load values
for Nominal stage
(all S)

Contingency-Constrained Stochastic Programming ACOPF Formulation

$$P^f_{l,t,s} = v^r_{bf(l),t,s} \cdot i^{fr}_{l,t,s} + v^j_{bf(l),t,s} \cdot i^{fj}_{l,t,s} \qquad \forall\, l \in \mathcal{L},\ t \in \mathcal{T},\ s \in \mathcal{S}$$

$$Q^f_{l,t,s} = v^j_{bf(l),t,s} \cdot i^{fr}_{l,t,s} - v^r_{bf(l),t,s} \cdot i^{fj}_{l,t,s} \qquad \forall\, l \in \mathcal{L},\ t \in \mathcal{T},\ s \in \mathcal{S}$$

$$P^t_{l,t,s} = v^r_{bt(l),t,s} \cdot i^{tr}_{l,t,s} + v^j_{bt(l),t,s} \cdot i^{tj}_{l,t,s} \qquad \forall\, l \in \mathcal{L},\ t \in \mathcal{T},\ s \in \mathcal{S}$$

$$Q^t_{l,t,s} = v^j_{bt(l),t,s} \cdot i^{tr}_{l,t,s} - v^r_{bt(l),t,s} \cdot i^{tj}_{l,t,s} \qquad \forall\, l \in \mathcal{L},\ t \in \mathcal{T},\ s \in \mathcal{S}$$

$$(P^f_{l,t,s})^2 + (Q^f_{l,t,s})^2 \leq (S^U)^2 \qquad \forall\, l \in \mathcal{L},\ t \in \mathcal{T},\ s \in \mathcal{S}$$

$$(P^t_{l,t,s})^2 + (Q^t_{l,t,s})^2 \leq (S^U)^2 \qquad \forall\, l \in \mathcal{L},\ t \in \mathcal{T},\ s \in \mathcal{S}$$

$$(V^L_b)^2 \leq (v^r_{b,t,s})^2 + (v^j_{b,t,s})^2 \leq (V^U_b)^2 \qquad \forall\, b \in \mathcal{B},\ t \in \mathcal{T},\ s \in \mathcal{S}$$

$$P^{GL}_g \leq P^G_{g,t,s} \leq P^{GU}_g \qquad \forall\, g \in \mathcal{G},\ t \in \mathcal{T},\ s \in \mathcal{S}$$

$$Q^{GL}_g \leq Q^G_{g,t,s} \leq Q^{GU}_g \qquad \forall\, g \in \mathcal{G},\ t \in \mathcal{T},\ s \in \mathcal{S}$$

$$v^j_{rb,t,s} = 0 \qquad \forall\, t \in \mathcal{T},\ s \in \mathcal{S}$$

$$P^L_{b,1,s} = P^{\star L}_b \qquad \forall\, b \in \mathcal{D},\ s \in \mathcal{S}$$

$$Q^L_{b,1,s} = Q^{\star L}_b \qquad \forall\, b \in \mathcal{D},\ s \in \mathcal{S}$$

$$-P^{GR}_g \leq P^G_{g,1,s} - P^G_{g,2,s} \leq P^{GR}_g \qquad \forall\, g \in \mathcal{G},\ s \in \mathcal{S}$$

$$P^G_{g,1,0} = P^G_{g,2,0} \qquad \forall\, g \in \mathcal{G}$$

$$Q^G_{g,1,0} = Q^G_{g,2,0} \qquad \forall\, g \in \mathcal{G}$$

$$P^G_{g,1,0} = P^G_{g,1,s} \qquad \forall\, g \in \mathcal{G},\ s \in \mathcal{S}/\{0\}$$

$$Q^G_{g,1,0} = Q^G_{g,1,s} \qquad \forall\, g \in \mathcal{G},\ s \in \mathcal{S}/\{0\}$$

Ramping constraint on all generators from stage 1 to stage 2 (all S)

Contingency-Constrained Stochastic Programming ACOPF Formulation

$$
\begin{aligned}
P^f_{l,t,s} &= v^r_{bf(l),t,s} \cdot i^{fr}_{l,t,s} + v^j_{bf(l),t,s} \cdot i^{fj}_{l,t,s} && \forall\, l \in \mathcal{L},\ t \in \mathcal{T},\ s \in \mathcal{S} \\
Q^f_{l,t,s} &= v^j_{bf(l),t,s} \cdot i^{fr}_{l,t,s} - v^r_{bf(l),t,s} \cdot i^{fj}_{l,t,s} && \forall\, l \in \mathcal{L},\ t \in \mathcal{T},\ s \in \mathcal{S} \\
P^t_{l,t,s} &= v^r_{bt(l),t,s} \cdot i^{tr}_{l,t,s} + v^j_{bt(l),t,s} \cdot i^{tj}_{l,t,s} && \forall\, l \in \mathcal{L},\ t \in \mathcal{T},\ s \in \mathcal{S} \\
Q^t_{l,t,s} &= v^j_{bt(l),t,s} \cdot i^{tr}_{l,t,s} - v^r_{bt(l),t,s} \cdot i^{tj}_{l,t,s} && \forall\, l \in \mathcal{L},\ t \in \mathcal{T},\ s \in \mathcal{S} \\
(P^f_{l,t,s})^2 + (Q^f_{l,t,s})^2 &\le (S^U)^2 && \forall\, l \in \mathcal{L},\ t \in \mathcal{T},\ s \in \mathcal{S} \\
(P^t_{l,t,s})^2 + (Q^t_{l,t,s})^2 &\le (S^U)^2 && \forall\, l \in \mathcal{L},\ t \in \mathcal{T},\ s \in \mathcal{S} \\
(V^L_b)^2 \le (v^r_{b,t,s})^2 + (v^j_{b,t,s})^2 &\le (V^U_b)^2 && \forall\, b \in \mathcal{B},\ t \in \mathcal{T},\ s \in \mathcal{S} \\
P^{GL}_g \le P^G_{g,t,s} &\le P^{GU}_g && \forall\, g \in \mathcal{G},\ t \in \mathcal{T},\ s \in \mathcal{S} \\
Q^{GL}_g \le Q^G_{g,t,s} &\le Q^{GU}_g && \forall\, g \in \mathcal{G},\ t \in \mathcal{T},\ s \in \mathcal{S} \\
v^j_{rb,t,s} &= 0 && \forall\, t \in \mathcal{T},\ s \in \mathcal{S} \\
P^L_{b,1,s} &= P^{\star L}_b && \forall\, b \in \mathcal{D},\ s \in \mathcal{S} \\
Q^L_{b,1,s} &= Q^{\star L}_b && \forall\, b \in \mathcal{D},\ s \in \mathcal{S} \\
-P^{GR}_g \le P^G_{g,1,s} - P^G_{g,2,s} &\le P^{GR}_g && \forall\, g \in \mathcal{G},\ s \in \mathcal{S} \\
P^G_{g,1,0} &= P^G_{g,2,0} && \forall\, g \in \mathcal{G} \\
Q^G_{g,1,0} &= Q^G_{g,2,0} && \forall\, g \in \mathcal{G} \\
P^G_{g,1,0} &= P^G_{g,1,s} && \forall\, g \in \mathcal{G},\ s \in \mathcal{S}/\{0\} \\
Q^G_{g,1,0} &= Q^G_{g,1,s} && \forall\, g \in \mathcal{G},\ s \in \mathcal{S}/\{0\}
\end{aligned}
$$

Enforce generator power solution equal in stage 1 and 2 for Nominal to Nominal

Contingency-Constrained Stochastic Programming ACOPF Formulation

$$P^f_{l,t,s} = v^r_{bf(l),t,s} \cdot i^{fr}_{l,t,s} + v^j_{bf(l),t,s} \cdot i^{fj}_{l,t,s} \qquad \forall\, l \in \mathcal{L},\ t \in \mathcal{T},\ s \in \mathcal{S}$$

$$Q^f_{l,t,s} = v^j_{bf(l),t,s} \cdot i^{fr}_{l,t,s} - v^r_{bf(l),t,s} \cdot i^{fj}_{l,t,s} \qquad \forall\, l \in \mathcal{L},\ t \in \mathcal{T},\ s \in \mathcal{S}$$

$$P^t_{l,t,s} = v^r_{bt(l),t,s} \cdot i^{tr}_{l,t,s} + v^j_{bt(l),t,s} \cdot i^{tj}_{l,t,s} \qquad \forall\, l \in \mathcal{L},\ t \in \mathcal{T},\ s \in \mathcal{S}$$

$$Q^t_{l,t,s} = v^j_{bt(l),t,s} \cdot i^{tr}_{l,t,s} - v^r_{bt(l),t,s} \cdot i^{tj}_{l,t,s} \qquad \forall\, l \in \mathcal{L},\ t \in \mathcal{T},\ s \in \mathcal{S}$$

$$(P^f_{l,t,s})^2 + (Q^f_{l,t,s})^2 \le (S^U)^2 \qquad \forall\, l \in \mathcal{L},\ t \in \mathcal{T},\ s \in \mathcal{S}$$

$$(P^t_{l,t,s})^2 + (Q^t_{l,t,s})^2 \le (S^U)^2 \qquad \forall\, l \in \mathcal{L},\ t \in \mathcal{T},\ s \in \mathcal{S}$$

$$(V^L_b)^2 \le (v^r_{b,t,s})^2 + (v^j_{b,t,s})^2 \le (V^U_b)^2 \qquad \forall\, b \in \mathcal{B},\ t \in \mathcal{T},\ s \in \mathcal{S}$$

$$P^{GL}_g \le P^G_{g,t,s} \le P^{GU}_g \qquad \forall\, g \in \mathcal{G},\ t \in \mathcal{T},\ s \in \mathcal{S}$$

$$Q^{GL}_g \le Q^G_{g,t,s} \le Q^{GU}_g \qquad \forall\, g \in \mathcal{G},\ t \in \mathcal{T},\ s \in \mathcal{S}$$

$$v^j_{rb,t,s} = 0 \qquad \forall\, t \in \mathcal{T},\ s \in \mathcal{S}$$

$$P^L_{b,1,s} = P^{\star L}_b \qquad \forall\, b \in \mathcal{D},\ s \in \mathcal{S}$$

$$Q^L_{b,1,s} = Q^{\star L}_b \qquad \forall\, b \in \mathcal{D},\ s \in \mathcal{S}$$

$$-P^{GR}_g \le P^G_{g,1,s} - P^G_{g,2,s} \le P^{GR}_g \qquad \forall\, g \in \mathcal{G},\ s \in \mathcal{S}$$

$$P^G_{g,1,0} = P^G_{g,2,0} \qquad \forall\, g \in \mathcal{G}$$

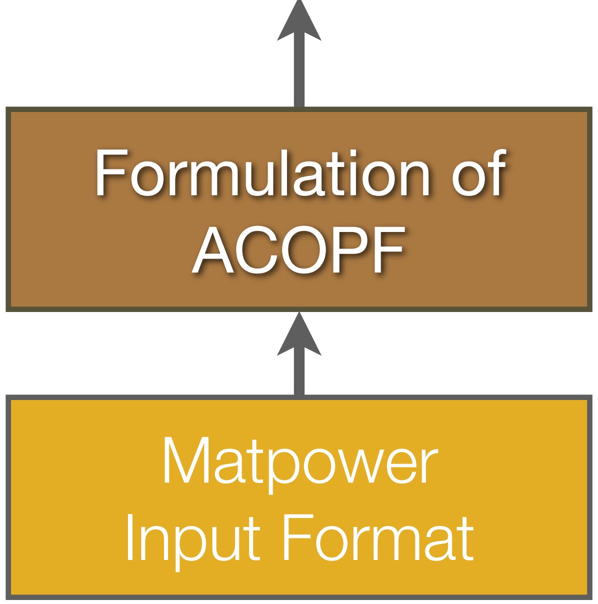
$$Q^G_{g,1,0} = Q^G_{g,2,0} \qquad \forall\, g \in \mathcal{G}$$

$$P^G_{g,1,0} = P^G_{g,1,s} \qquad \forall\, g \in \mathcal{G},\ s \in \mathcal{S}/\{0\}$$

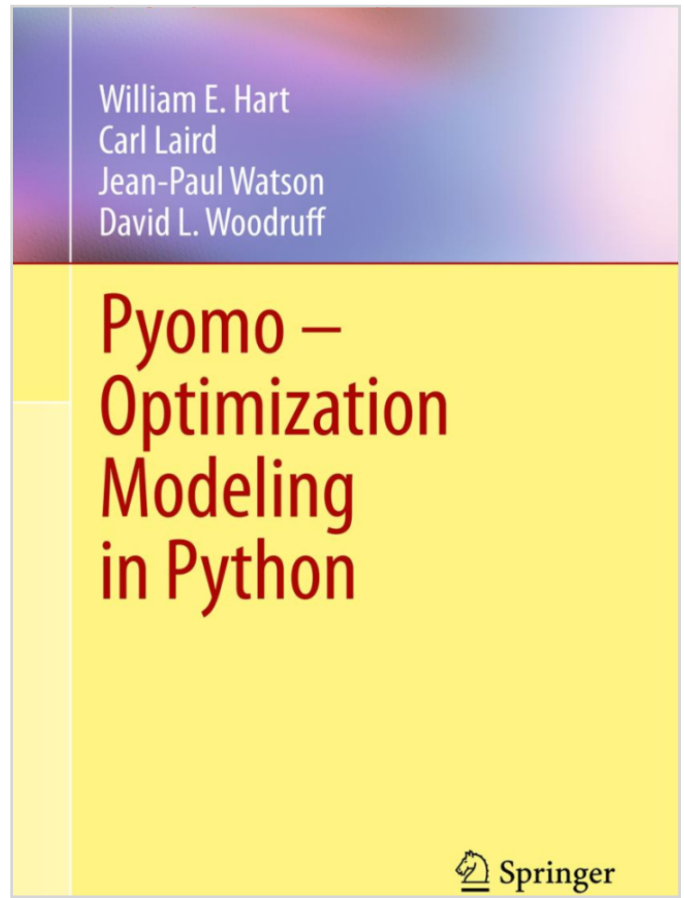
$$Q^G_{g,1,0} = Q^G_{g,1,s} \qquad \forall\, g \in \mathcal{G},\ s \in \mathcal{S}/\{0\}$$

Non-anticipativity constraints

Building the model with Pyomo and PySP



Integrated Python-based toolchain



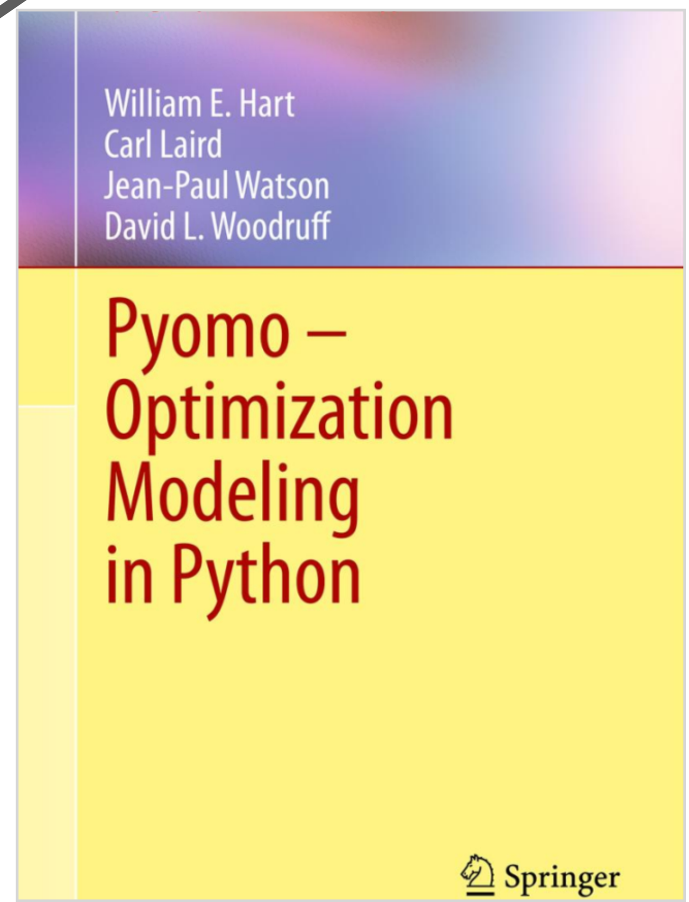
Hackebeil, Hart, Laird, Siirola, Watson, Woodruff, and many others…

Building the model with Pyomo and PySP

Algebraic Modeling → Pyomo → IPOPT

PySP - Stochastic Programming (Watson, Woodruff) → Pyomo

Matpower Input Format → Formulation of ACOPF → PySP - Stochastic Programming (Watson, Woodruff)



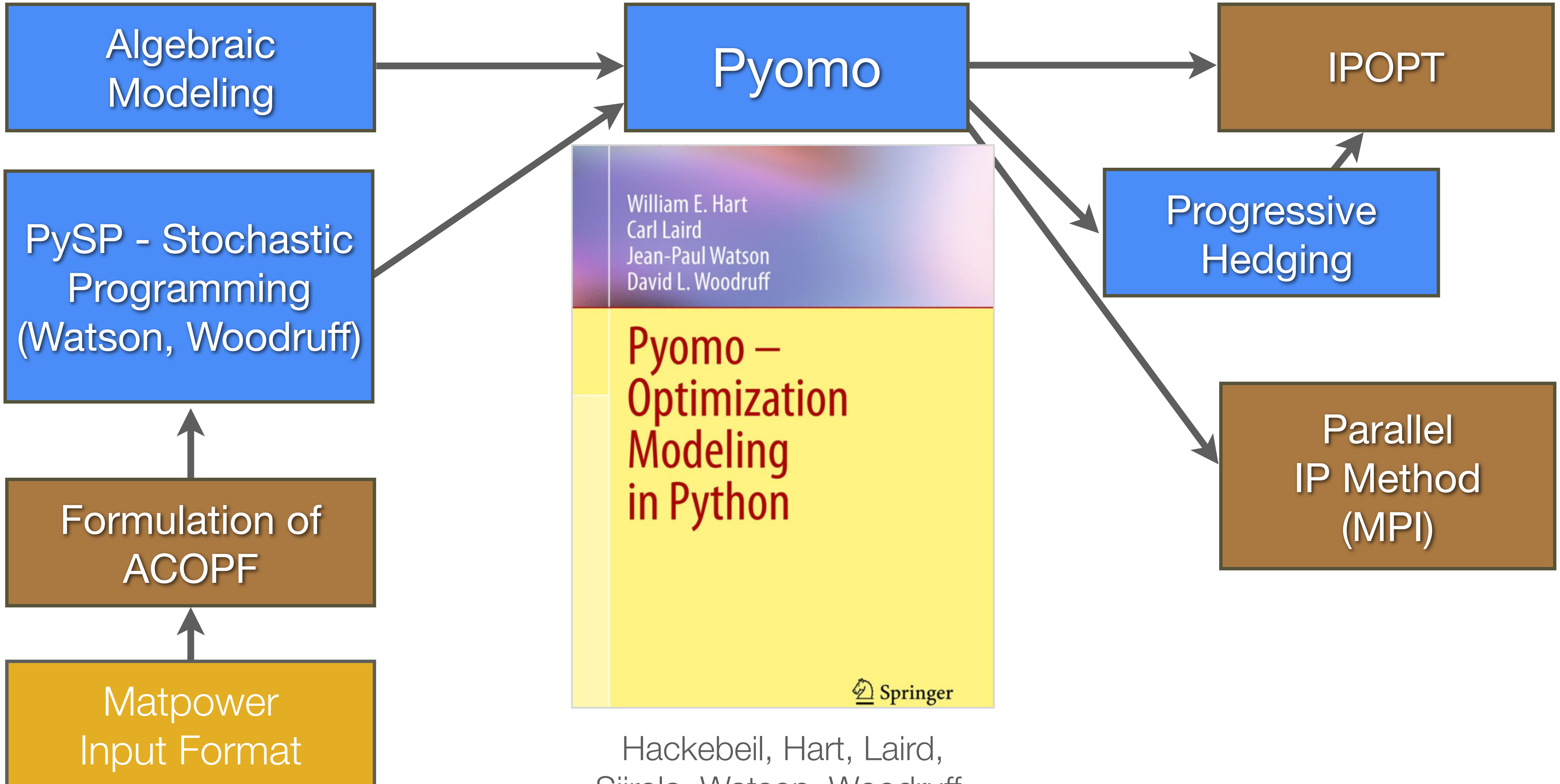
Hackebeil, Hart, Laird, Siirola, Watson, Woodruff, and many others…

Integrated Python-based toolchain

Parallel Algorithms for Large-Scale Nonlinear Optimization

External Partitioning (Problem Level)	Internal Partitioning (Linear Algebra)
Bender’s Decomposition Lagrangian Decomposition Progressive Hedging	Host Algorithm: parallelize all scale-dependent operations
Less intrusive Easier to implement	More intrusive Difficult to implement
Less favorable convergence rates	Retains convergence rates of host algorithm

Building the model with Pyomo and PySP



Hackebeil, Hart, Laird, Siirola, Watson, Woodruff, and many others…

Integrated Python-based toolchain

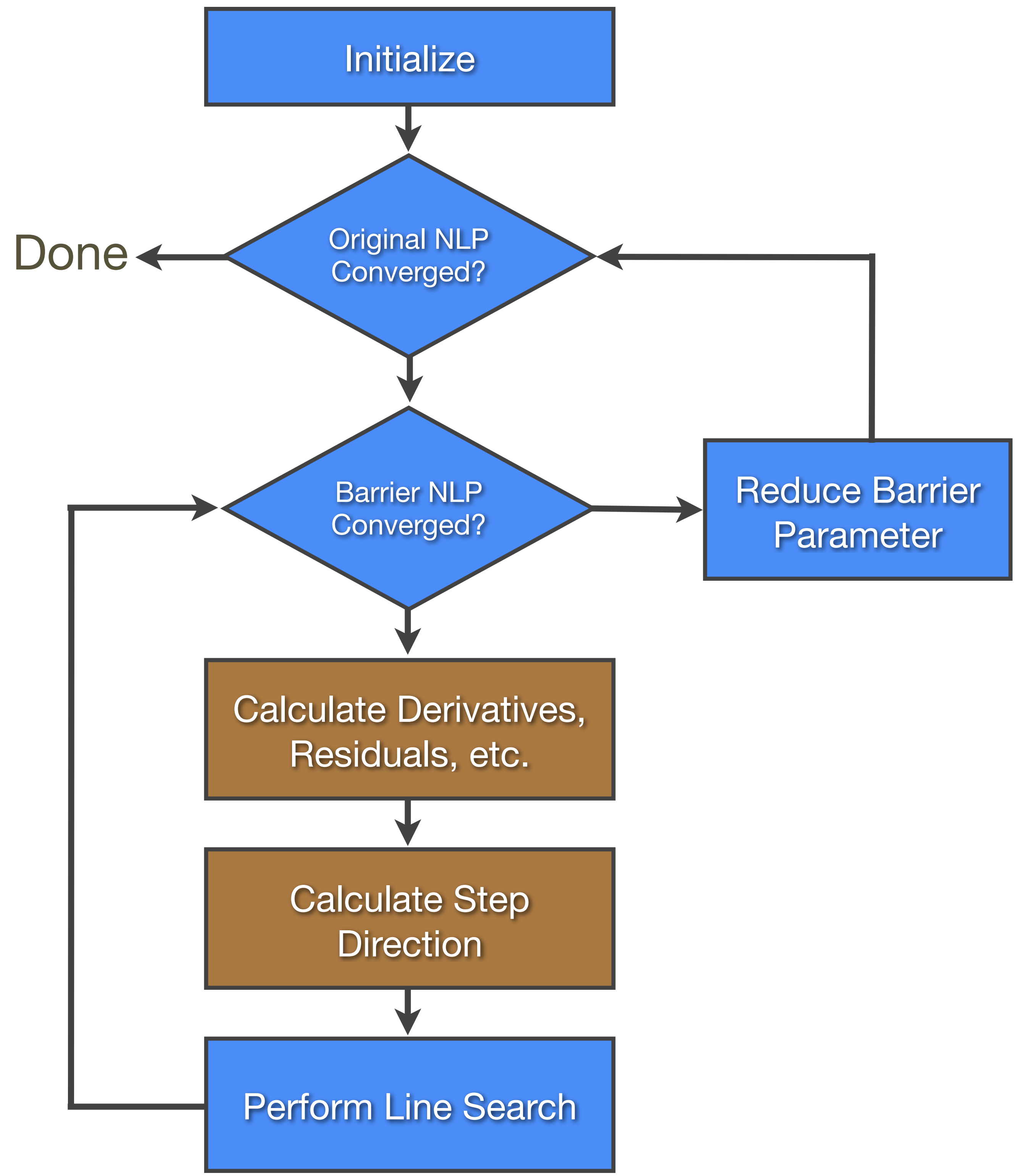
Nonlinear Interior-Point Methods

Original NLP

$$\min_x \quad f(x)$$
$$\text{s.t.} \quad c(x) = 0$$
$$x \geq 0$$

Barrier NLP

$$\min_x \quad f(x) - \mu \cdot \sum_i ln(x_i)$$
$$\text{s.t.} \quad c(x) = 0$$



KNITRO (Byrd, Nocedal, Hribar, Waltz)
LOQO (Benson, Vanderbei, Shanno)
IPOPT (Wachter, Laird, Biegler)

Nonlinear Interior-Point Methods

$$\min_x \quad f(x) \qquad \text{s.t} \quad c(x)=0, \quad x \geq 0$$

$\longrightarrow$

$$\min_x \quad f(x) - \mu \sum_i \ln(x_i) \qquad \text{s.t} \quad c(x)=0$$

$$\begin{aligned} \nabla f(x) + \nabla c(x)^T \lambda - z &= 0 \\ c(x) &= 0 \\ Xz &= \mu e \\ (x > 0, z > 0) \end{aligned}$$

$\longleftarrow \; z=\mu X^{-1} e$

$$\begin{aligned} \nabla f(x) + \nabla c(x)^T \lambda - \mu X^{-1} e &= 0 \\ c(x) &= 0 \\ (x > 0) \end{aligned}$$

$$\left[\begin{array}{cc} W_k + \Sigma_k + \delta_w I & \nabla c(x_k) \\ \nabla c(x_k)^T & -\delta_c I \end{array} \right] \left[\begin{array}{c} \Delta x \\ \Delta \lambda \end{array} \right] = - \left[\begin{array}{c} \nabla \psi_\mu(x_k) + \nabla c(x_k)^T \lambda_k \\ c(x_k) \end{array} \right]$$

$$\left(W_k = \nabla^2_{xx} \mathcal{L}, \; \Sigma_k = Z_k X_k^{-1} \right)$$

Nonlinear Interior-Point Methods

$$\min_x \quad f(x)$$
$$\text{s.t} \quad c(x)=0$$
$$x \geq 0$$

$\longrightarrow$

$$\min_x \quad f(x) - \mu \sum_i \ln(x_i)$$
$$\text{s.t} \quad c(x)=0$$

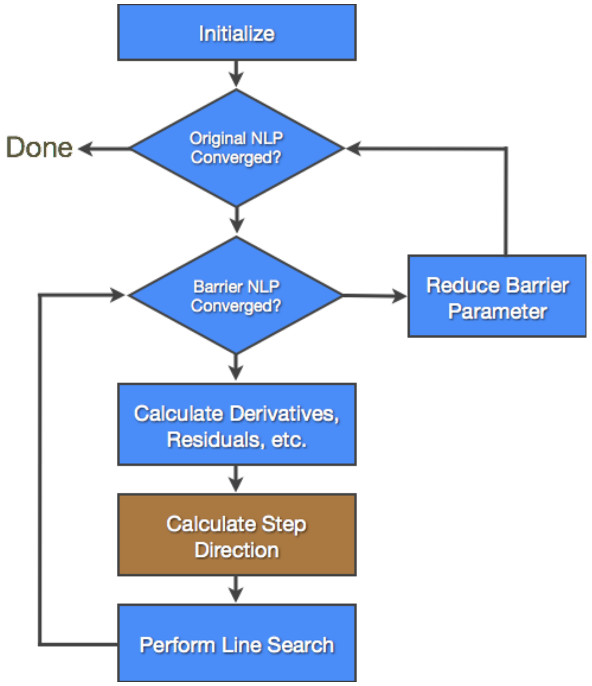
∇f

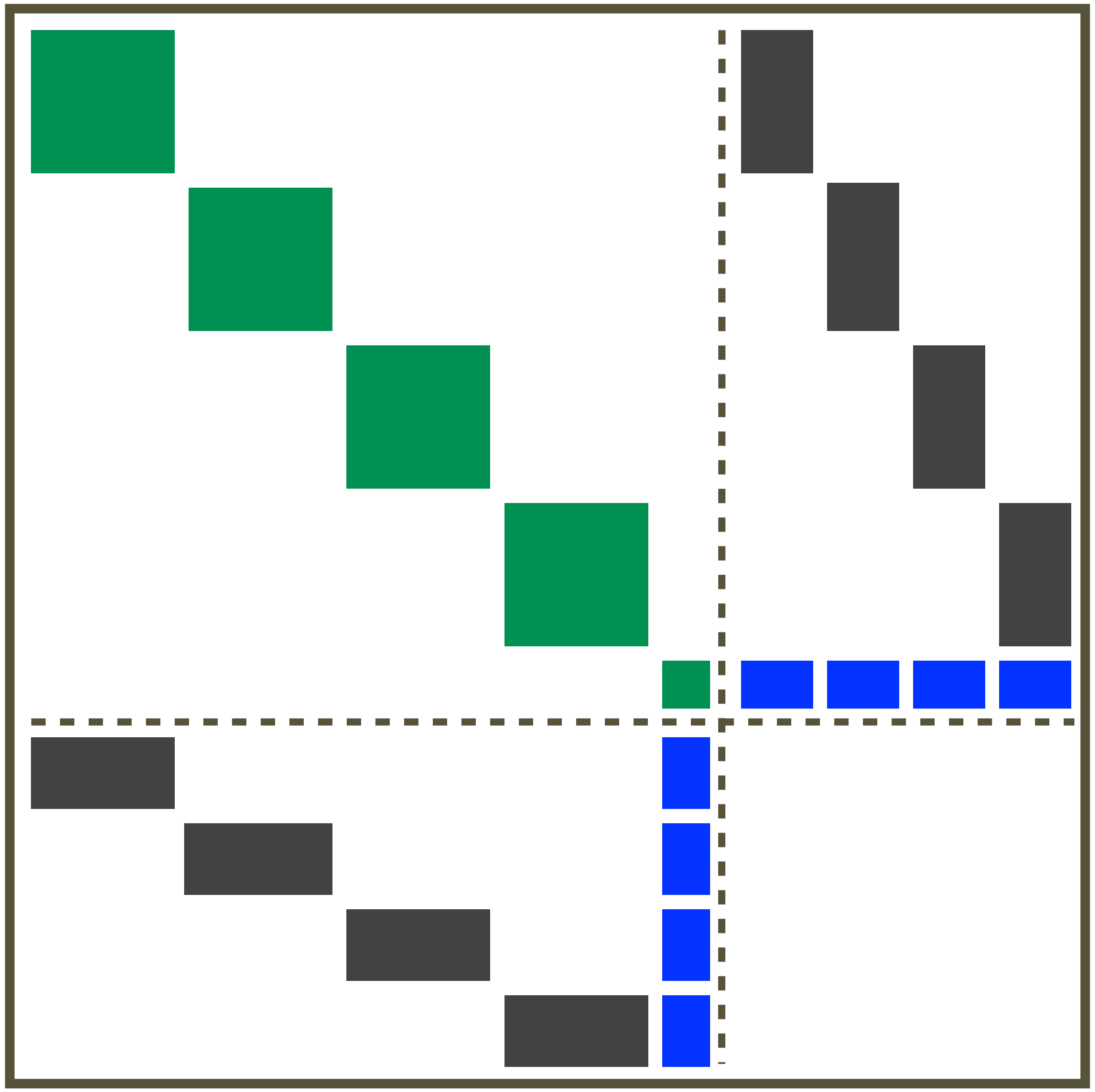
Structure in the optimization problem
Structure in the linear system

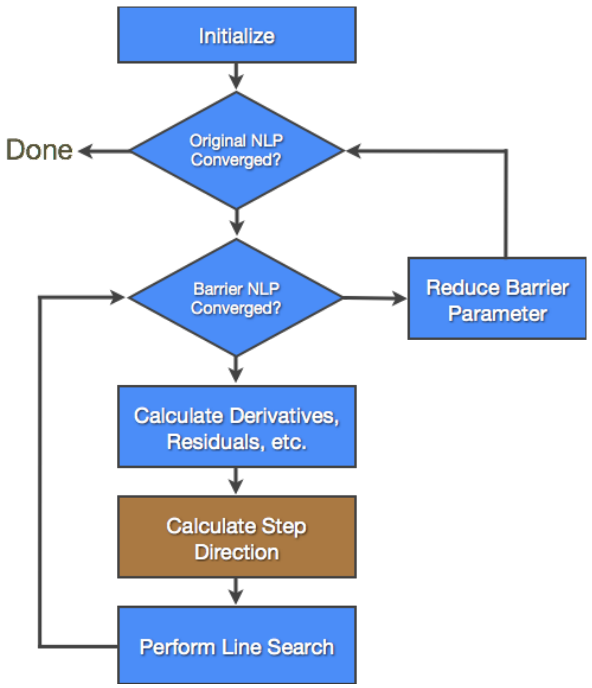
$(x > ($ $> 0)$

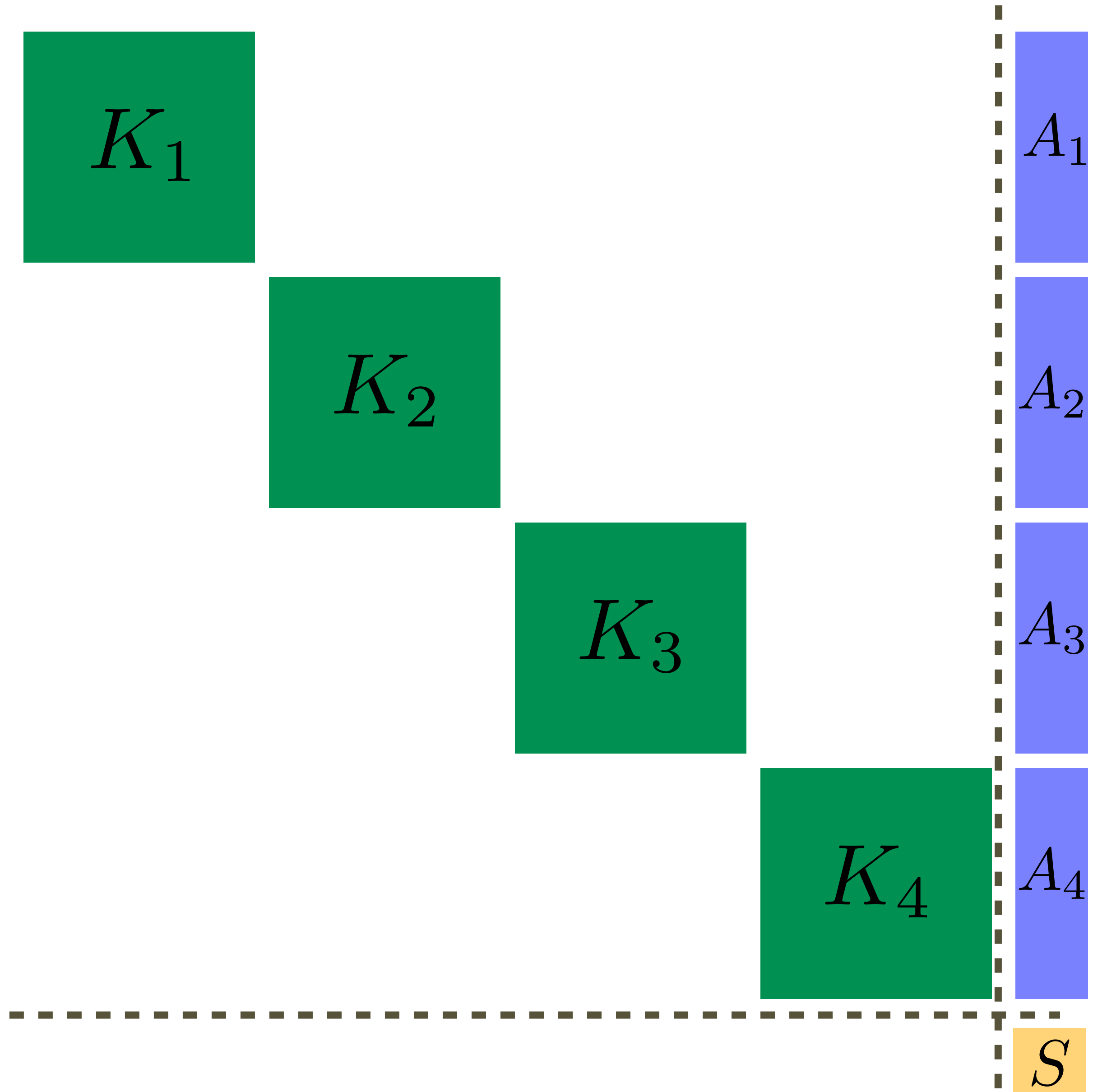
$$\begin{bmatrix} W_k + \Sigma_k + \delta_w I & \nabla c(x_k) \\ \nabla c(x_k)^T & -\delta_c I \end{bmatrix} \begin{bmatrix} \Delta x \\ \Delta \lambda \end{bmatrix} = - \begin{bmatrix} \nabla \psi_\mu(x_k) + \nabla c(x_k)^T \lambda_k \\ c(x_k) \end{bmatrix}$$

$$(W_k = \nabla^2_{xx}\mathcal{L}, \; \Sigma_k = Z_k X_k^{-1})$$


Initialize
Original NLP Converged?
Done
Barrier NLP Converged?
Reduce Barrier Parameter
Calculate Derivatives, Residuals, etc.
Calculate Step Direction
Perform Line Search







$$\left[D_y - \sum_{q\in Q} A_q^T K_q^{-1} A_q\right]\Delta y = r_y - \sum_{q\in Q} A_q^T K_q^{-1} r_q$$

$$\left[D_y - \sum_{q \in Q} A_q^T K_q^{-1} A_q\right] \Delta y = r_y - \sum_{q \in Q} A_q^T K_q^{-1} r_q$$

1: for each i in $1, ..., n_e$ **1.1:** factor K_i	Factor K Blocks

2: let $S = [-\delta_c I]$

3: let $r_{sc} = r_s$

4: for each i in $1, ..., n_e$ **4.1:** for each column j in A_i^T **4.1.1:** solve the system $K_i q_i^{<j>} = [A_i^T]^{<j>}$ **4.1.2:** let $S^{<j>} = S^{<j>} + A_i q_i^{<j>}$ **4.2:** solve the system $K_i p_i = r_i$ **4.3:** let $r_{sc} = r_{sc} - A_i p_i$	Form Schur-Complement
5: solve $S \Delta \nu_s = r_{sc}$ for $\Delta \nu_s$	Solve Schur-Complement
6: for each i in $1, ..., n_e$ **6.1:** solve $K_i \Delta \nu_i = r_i - A_i^T \Delta \nu_s$ for $\Delta \nu_i$	Solve Remaining Vars

$$\left[D_y - \sum_{q \in Q} A_q^T K_q^{-1} A_q\right] \Delta y = r_y - \sum_{q \in Q} A_q^T K_q^{-1} r_q$$

1: ~~for each i in 1, ..., n_e~~

1.1: factor K_i

Factor K Blocks

2: let $S = [-\delta_c I]$

3: let $r_{sc} = r_s$

4: ~~for each i in 1, ..., n_e~~

4.1: for each column j in A_i^T

4.1.1: solve the system $K_i q_i^{<j>} = [A_i^T]^{<j>}$

4.1.2: let $S^{<j>} = S^{<j>} + A_i q_i^{<j>}$

4.2: solve the system $K_i p_i = r_i$

4.3: let $r_{sc} = r_{sc} - A_i p_i$

Form Schur-Complement

5: solve $S \Delta\nu_s = r_{sc}$ for $\Delta\nu_s$

Solve Schur-Complement

6: ~~for each i in 1, ..., n_e~~

6.1: solve $K_i \Delta\nu_i = r_i - A_i^T \Delta\nu_s$ for $\Delta\nu_i$

Solve Remaining Vars

Parallel Nonlinear Programming with Schur-complement Decomposition

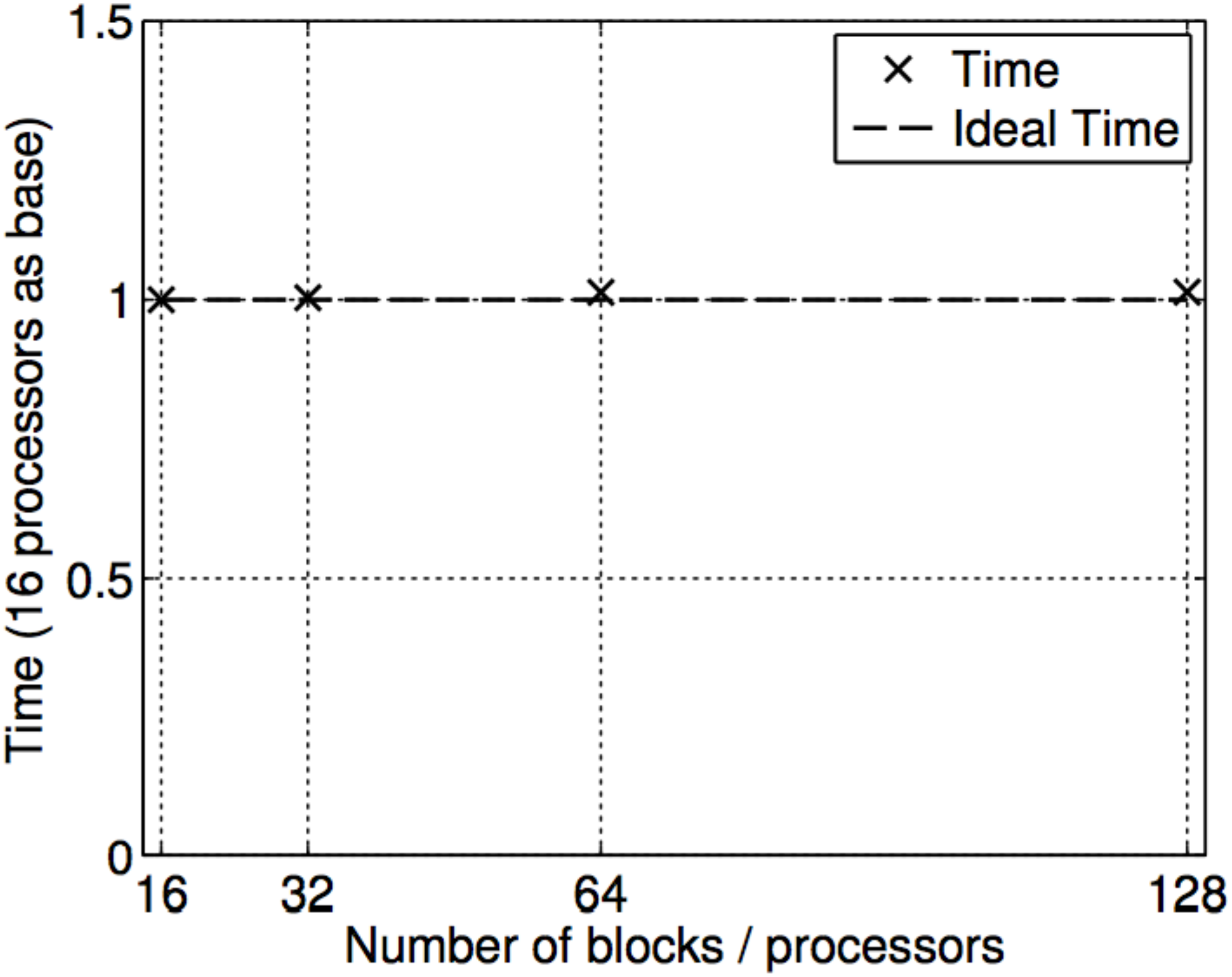
OOPS [Gondzio & Gothrey, 2007, 2009]
PIPS-NLP [Chiang et al. 2014, Lubin et. al, 2011]
PRBLOCK_IP [Castro 2007]
Schur-IPOPT [Kang et al. 2014, Zavala et al., 2008]

- Parameter estimation in LDPE process [Zavala et al. 2008]
- Spatial decomposition of water network [Zhu, 2011]
- Optimal operation of air separation plants with uncertain demand and energy pricing [Zhu, Legg, Laird, 2011a,b]
- Temporal decomposition for dynamic optimization [Word et al. 2014]

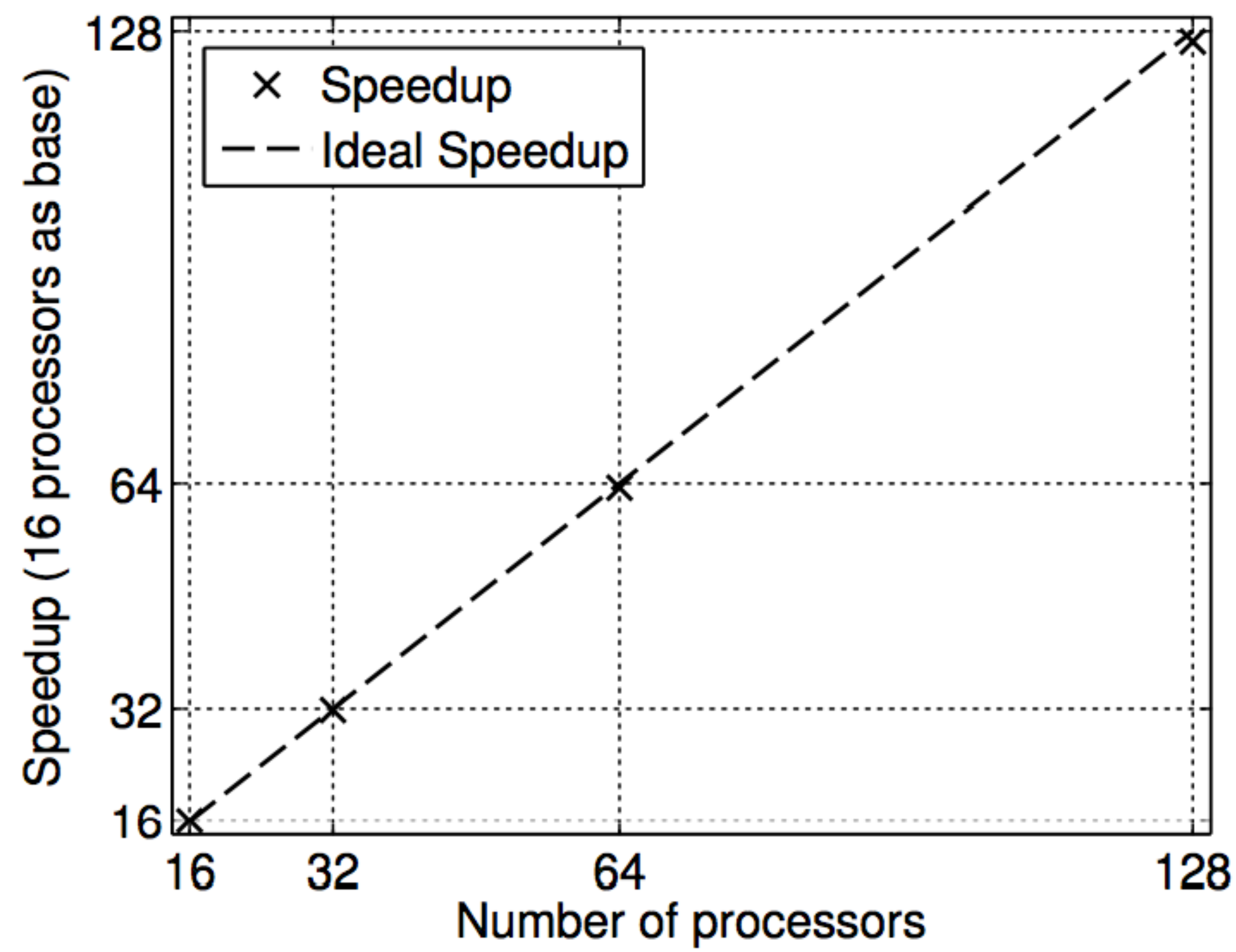
- Implicit Schur-complement approach [Kang et al. 2014]
    - Never explicitly form the Schur-complement
    - Solve it iteratively with PCG using preconditioner from Morales & Nocedal (2000)

Parallel Performance: Dynamic Optimization Under Uncertainty

Dynamic optimization under uncertainty problem
1200 coupling variables - over 15 million total variables



(a) Weak Scaling



(b) Strong Scaling

Kang, J., Word, D.P., and Laird, C.D., "An interior-point method for efficient solution of block-structured NLP problems using an implicit Schur-complement decomposition", to appear in Computers and Chemical Engineering, 2014.

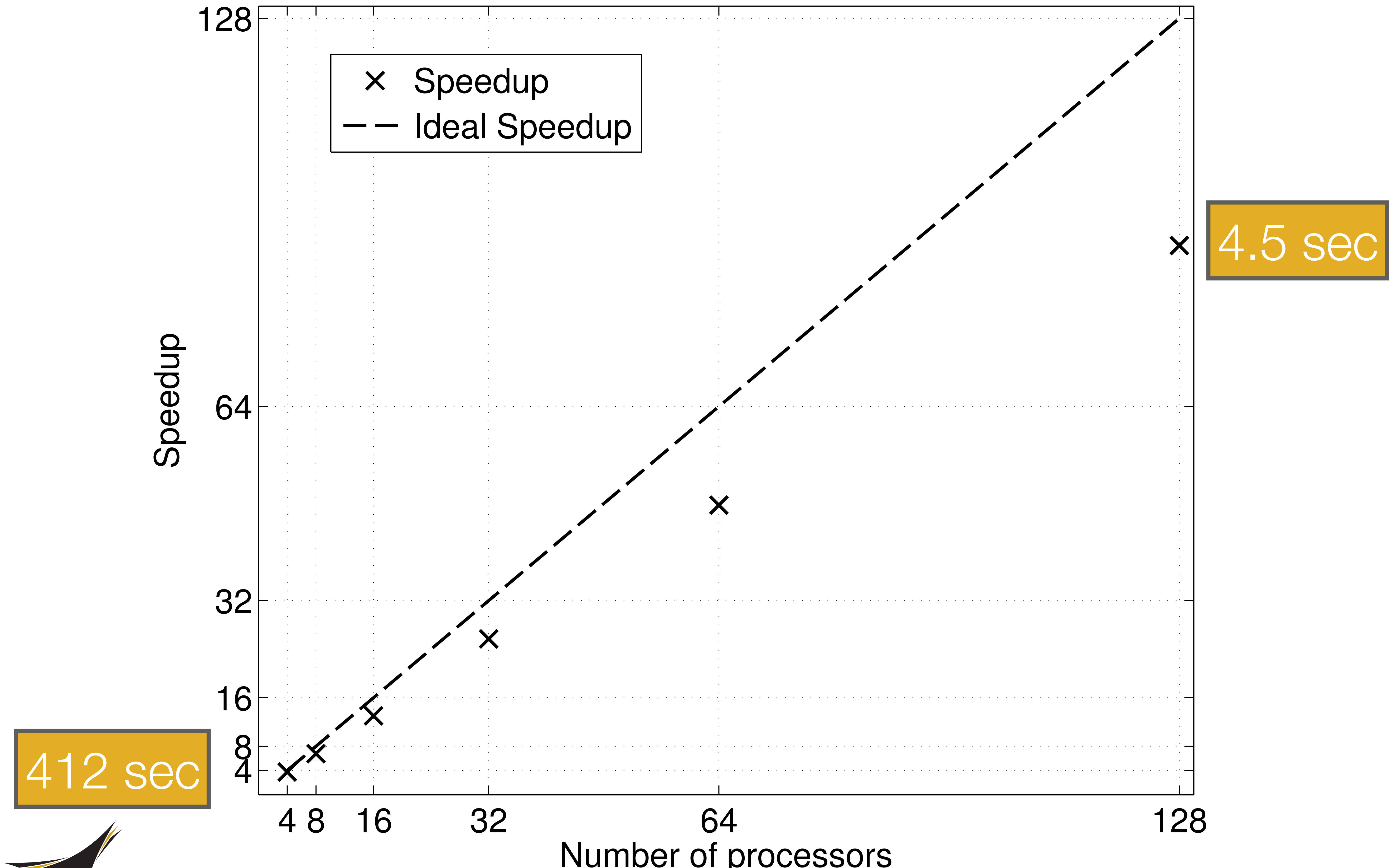
Contingency-constrained ACOPF results

Problem data: case118 distributed with Matpower 4.1
- 118 buses, 54 active generators, and 186 branches

Multi-scenario problem with 128 scenarios in total
- Normal operating scenario and 127 contingencies
- Problem size: ~400,000 variables and ~385,000 constraints

*Wall-clock time obtained from the Red Mesa supercomputing cluster at Sandia National Laboratories. Each node has 12 GB RAM and two, 2.93 GHz quad-core, Nehalem X5570 processors.

Contingency-constrained ACOPF results



Summary and Conclusions

Contingency-constrained ACOPF
- two-stage nonlinear stochastic programming problem
- straightforward implementation within Pyomo/PySP
- parallel IP methods show significant speedup

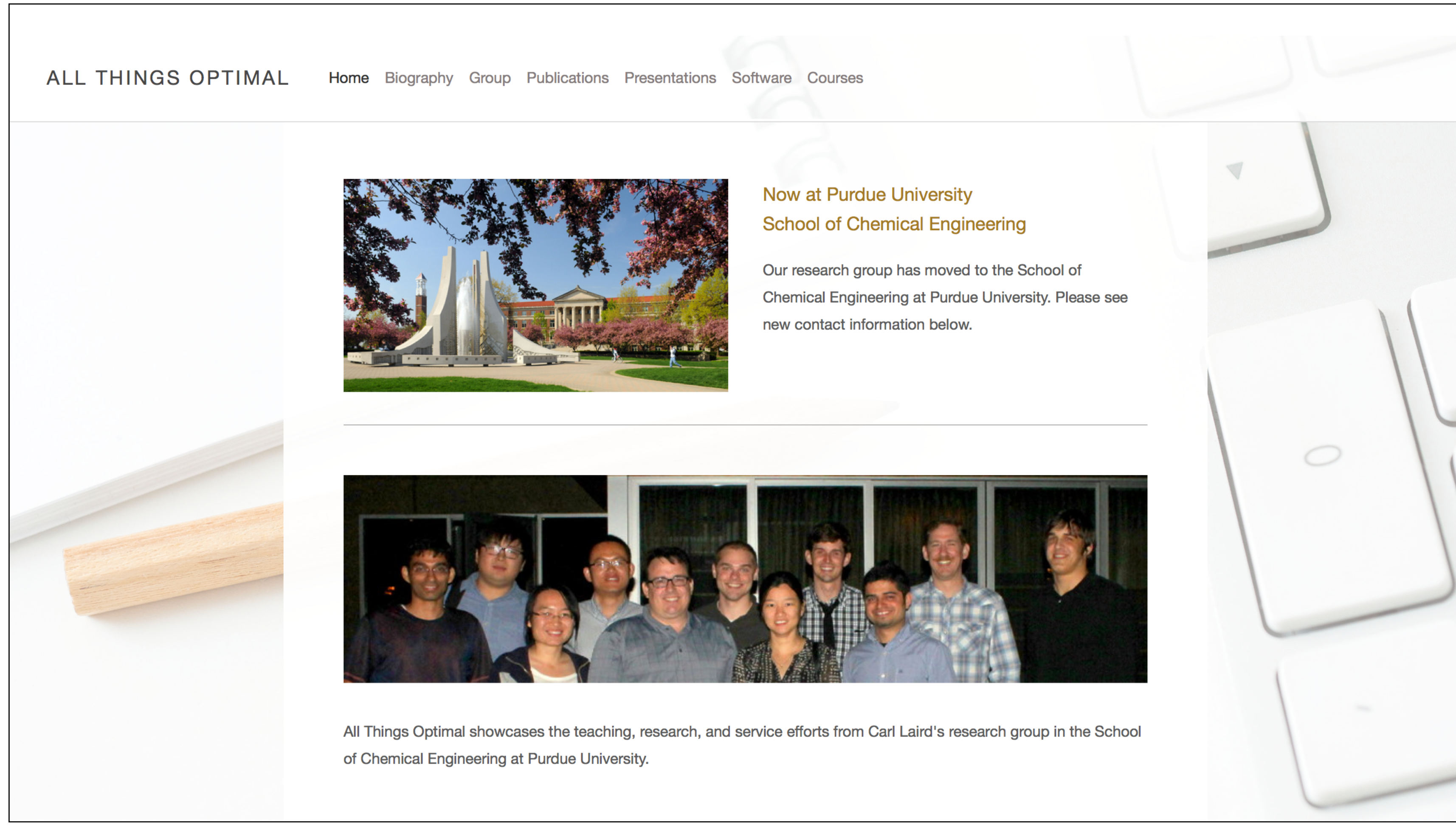
PySP provides access to other parallel solvers.

Progressive Hedging (PySP, Watson & Woodruff)
- IEEE-57, Parallel solution time: 9 seconds (under 10 iterations).
- case118, Parallel solution time: 20 seconds (under 10 iterations).
- case2383wp, (current work)

Increasing demand for solution of challenging large-scale nonlinear programming problems
- need for parallel algorithms
- opportunities for improving design and operation

Acknowledgements

- Current Students/Researchers
  - **Jia Kang**
  - Arpan Seth
  - Yankai Cao
  - Jianfeng Liu
  - Michael Bynum
- Former Students/Postdocs
  - Yu Zhu
  - Ahmed Rabie
  - George Abbott III
  - Chen Wang
  - Sean Legg
  - Daniel Word
  - Angelica Wong
  - Xiaorui Yu
  - Gabriel Hackebeil
  - Shawn McGee
  - Alberto Benavides-Serrano
- Collaborators
  - **Anya Castillo - Johns Hopkins U. / FERC**
  - D. Cummings - JHSPH
  - S. Iamsirithaworn - MPHT
  - W. Hart, S. McKenna, K. Klise, **J.P. Watson** **C. Silva-Monroy - Sandia**
  - T. Haxton, R. Murray - EPA
  - Johan Akesson, Lund University
  - S. Davis, A. Bratteteig - GexCon
  - M. S. Mannan - MKOPSC



Support

- National Science Foundation Cyber-Enabled Discovery and Innovation (CDI)-Type II
- National Science Foundation (CAREER Grant CBET# 0955205).
- Sandia National Laboratories
- MKO Process Safety Center
- P2SAC