



UNIVERSITY OF WASHINGTON  
ELECTRICAL ENGINEERING

# Optimizing Primary Frequency Response

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# Primary frequency response

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- Background:
  - Compulsory provision
  - Fixed droop for all conventional generators
  - Response characteristic has drastically decreased over the past decades:
    - 37.5 MW/mHz → 30.7 MW/mHz
  - Increasing penetration of renewable generation will further decrease this response

# Objectives

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- Improve reliability by accurately assessing the needs for primary frequency response
- Reduce the cost of providing primary reserve
  - Significant cost savings even for moderate wind penetration levels

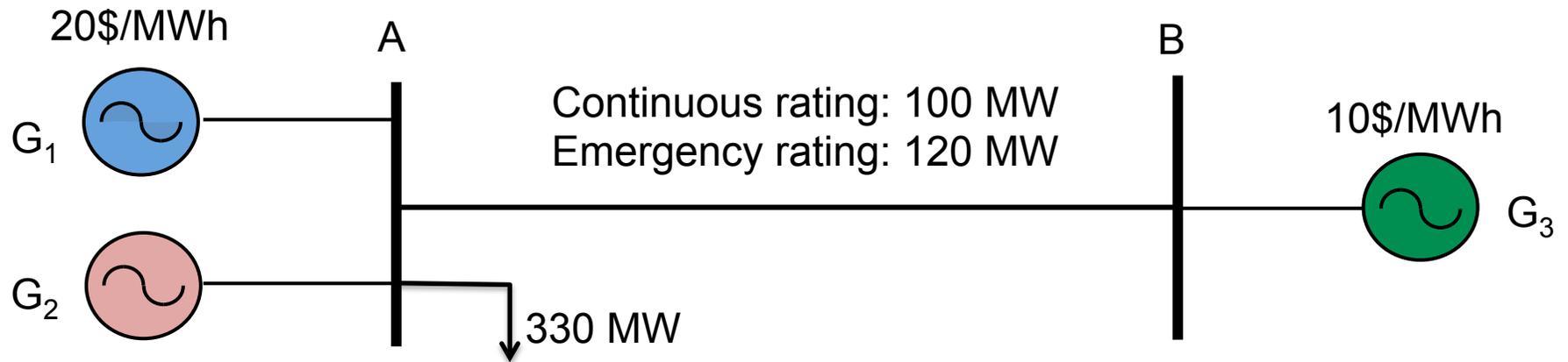
# Approach

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- Optimally allocate primary reserve between all generating units
  - Optimize individual droop coefficients
  - Co-optimize each generator's contribution to primary reserve with its energy production
  - No generator contingency should:
    - Cause an excessive frequency deviation
    - Overload a line or transformer
- Preventive Security Constrained OPF (PSCOPF)
  - Consider generator contingencies

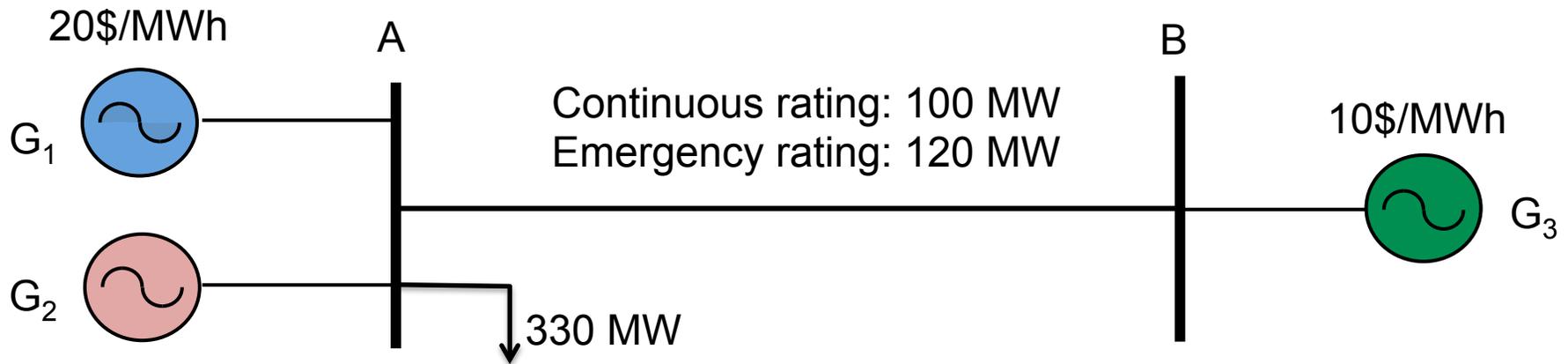
# Example

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All generators have the same rating

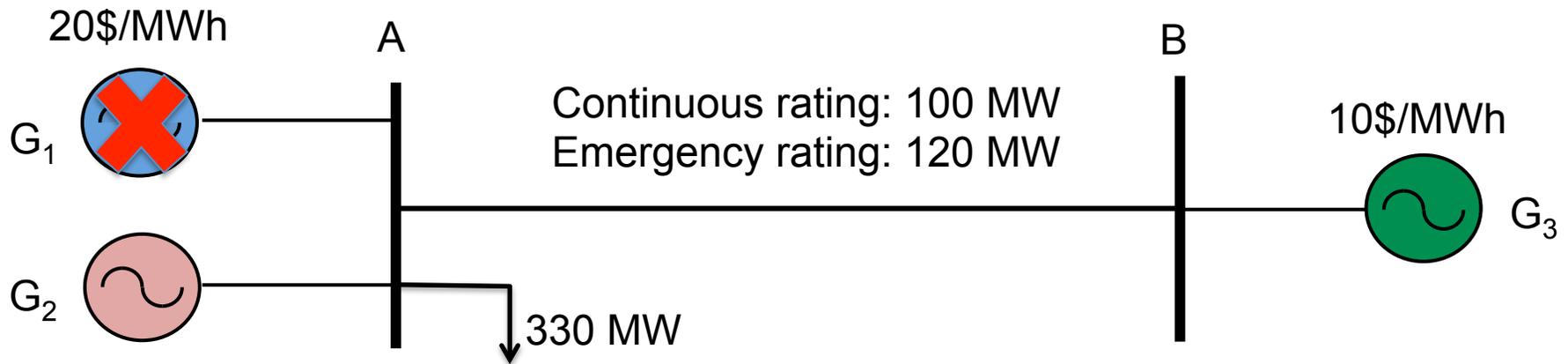
# OPF **without** generator contingency constraints



Output of  $G_3$  is limited by continuous rating of line A-B:

State	$G_1$	$G_2$	$G_3$
Pre-contingency dispatch (MW)	115	115	100

# Generator contingency with identical droops



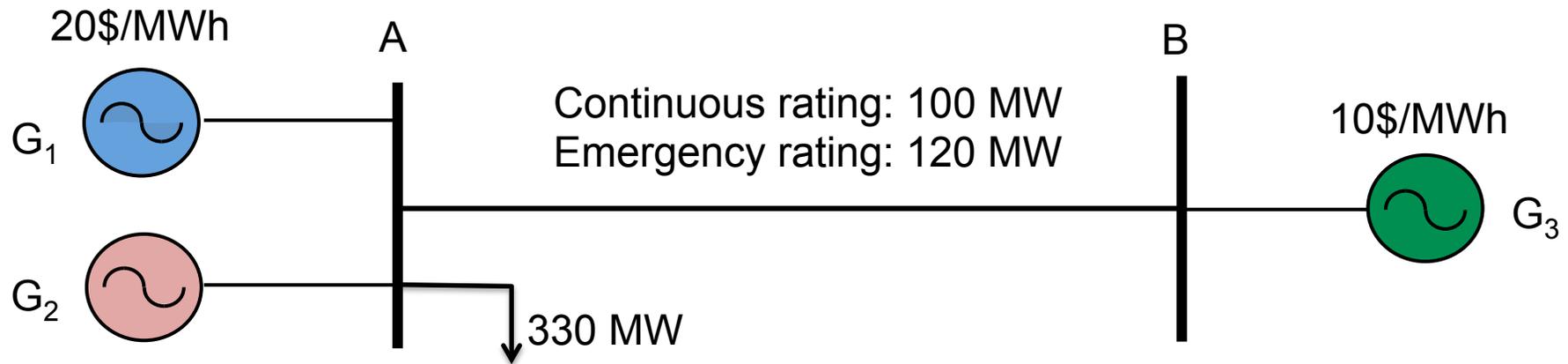
If  $G_2$  and  $G_3$  have the same droop and the same size:

State	$G_1$	$G_2$	$G_3$
Pre-contingency dispatch (MW)	115	115	100
Response (MW)	--	57.5	57.5
Post-contingency dispatch (MW)	--	172.5	157.5



Exceeds the emergency rating of line A-B

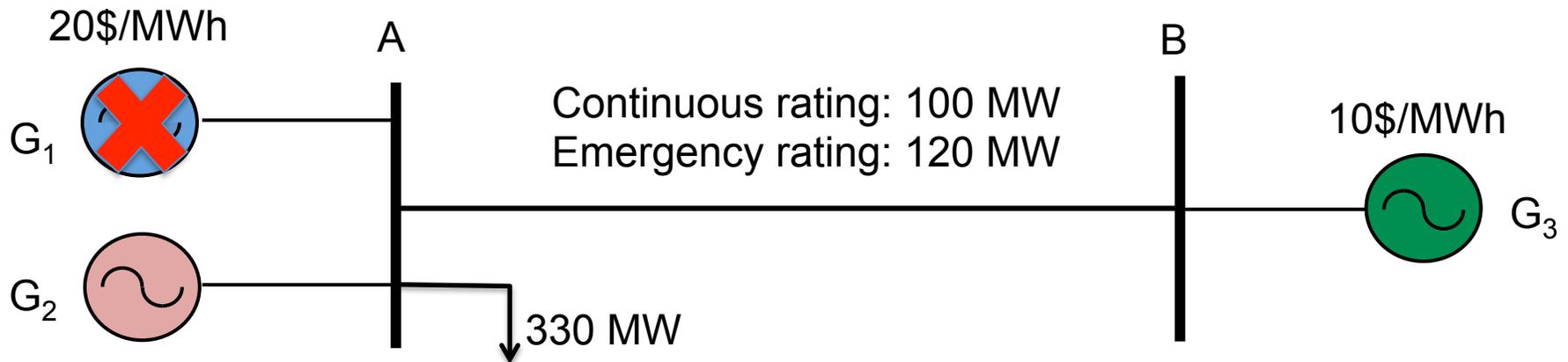
# PSCOPF **with** generator contingency constraints



The dispatch takes into account the emergency rating of line A-B:

State	$G_1$	$G_2$	$G_3$
Pre-contingency dispatch (MW)	140	140	50

# Generator contingency with identical droops



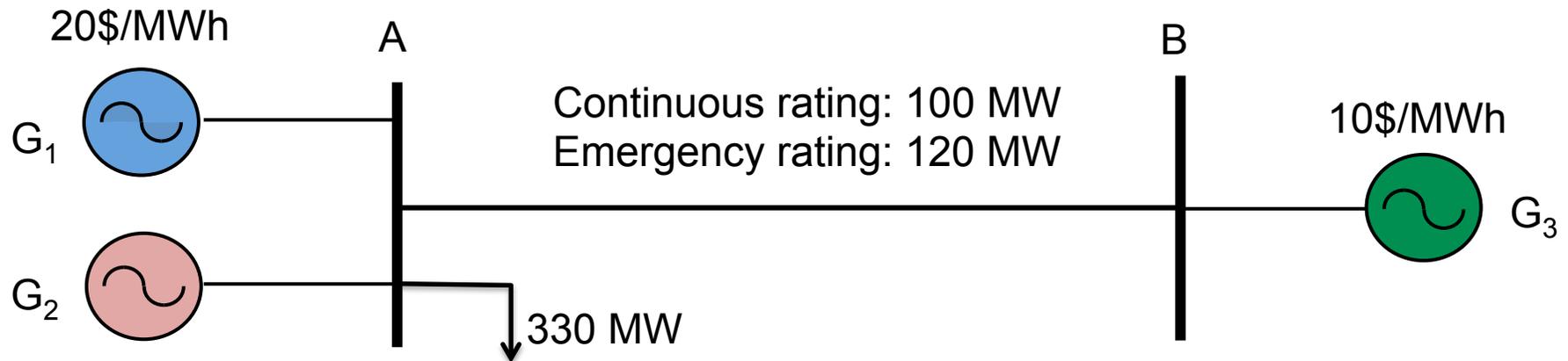
Security constrained dispatch with identical droops:

State	$G_1$	$G_2$	$G_3$
Pre-contingency dispatch (MW)	140	140	50
Response (MW)	--	70	70
Post-contingency dispatch (MW)	--	210	120



Within the emergency rating of line A-B

# OPF vs. PSCOPF

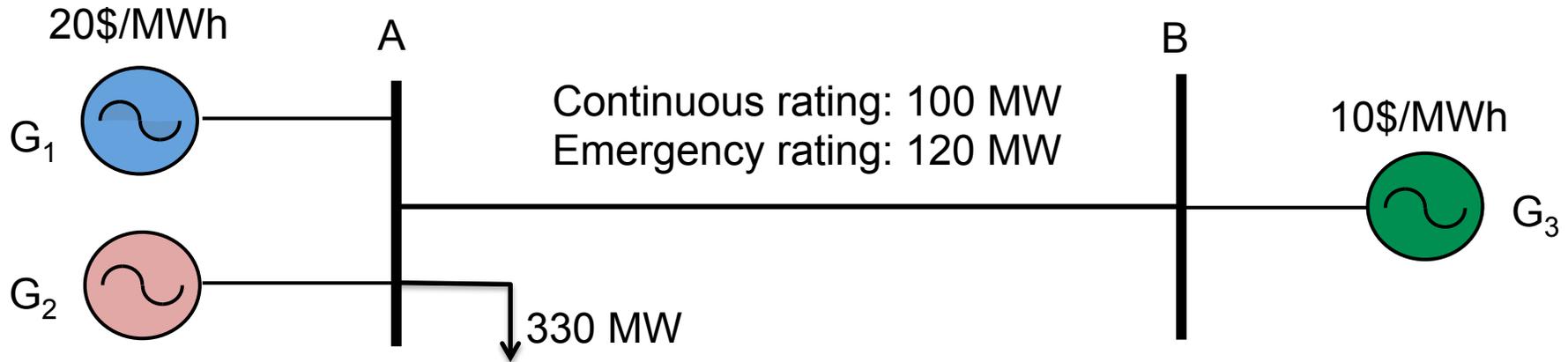


State	$G_1$	$G_2$	$G_3$	Cost (\$/h)
OPF dispatch (MW)	115	115	100	5,600
PSCOPF dispatch (MW)	140	140	50	6,100

Cost of security:  $\$6,100 - \$5,600 = \$500$

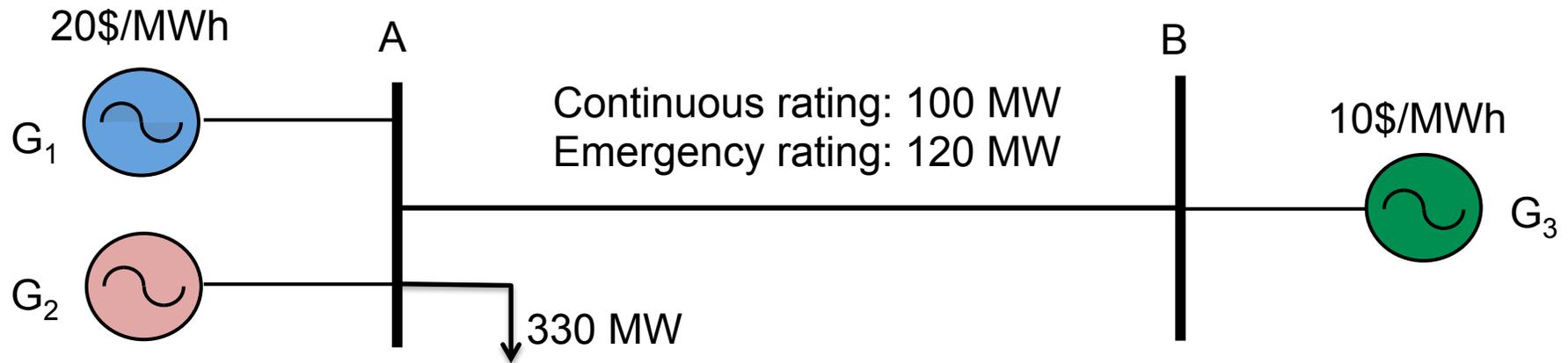
# Reducing the security cost

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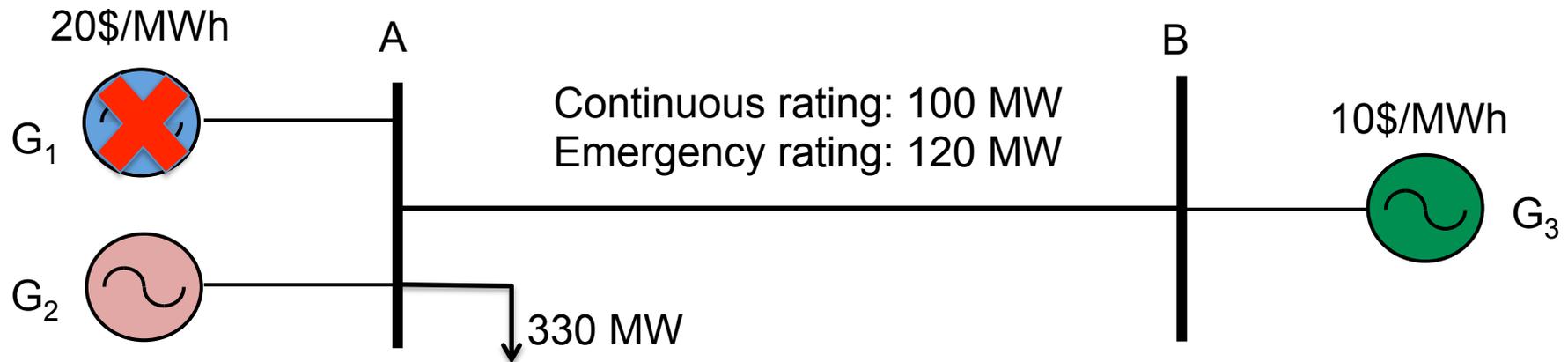
- Increase the output of  $G_3$  in the **pre**-contingency state
- Decrease the output of  $G_3$  in the **post**-contingency state
  - Avoid exceeding the emergency rating of the line
- Increase the primary response of  $G_1$  and  $G_2$ 
  - Avoid excessive frequency drop
- Optimize the droop of each generator

# PSCOPF with droop optimization



State	$G_1$	$G_2$	$G_3$
Pre-contingency dispatch (MW)	120	120	90
Droop	2%	2%	6%

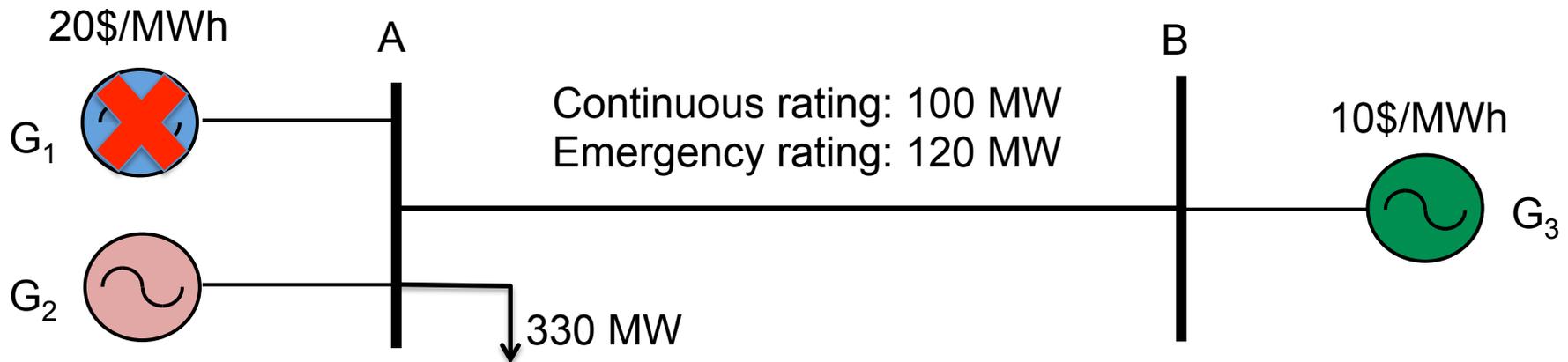
# PSCOPF with droop optimization



State	$G_1$	$G_2$	$G_3$
Pre-contingency dispatch (MW)	120	120	90
Droop	2%	2%	6%
Response (MW)	--	90	30
Post-contingency dispatch (MW)	--	210	120

Large pre-contingency output but low response from  $G_3$

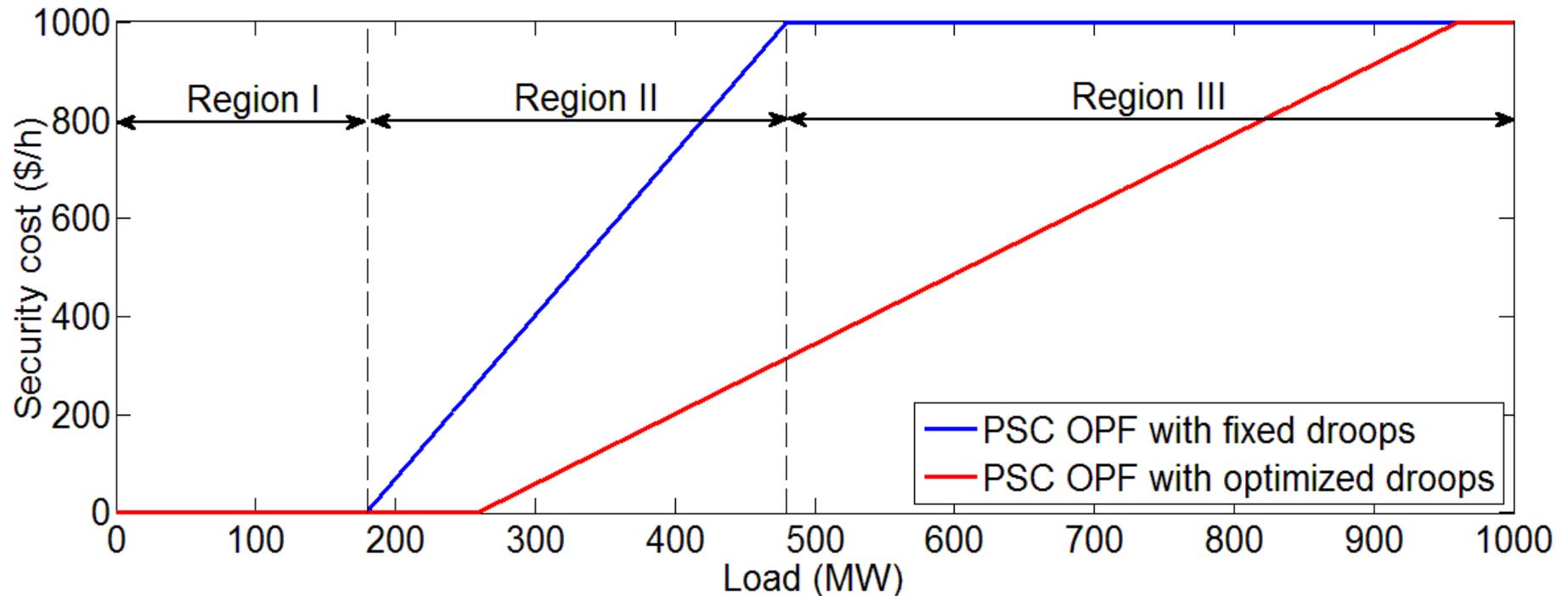
# Comparison



Case	State	$G_1$	$G_2$	$G_3$	Cost
Without droop optimization	Pre-contingency	140	140	50	\$ 6,100
	Response	--	70	70	
With droop optimization	Pre-contingency	120	120	90	\$ 5,700
	Response	--	90	30	

Droop optimization **reduces the total cost** while ensuring the **same level of security** as PSCOPF with fixed droops

# Cost savings



The savings depend on the load, but are limited by the acceptable range of droop coefficients: 2 to 6%

# PSCOPF formulation

$$\min_{P_{g0}} \sum_g C_g(P_{g0})$$

$$\sum_g \mathbb{1}_{ng} P_{gc} + P_n^w + \sum_k \mathbb{1}_{nk} P_{kc} = P_n^d, \forall n, c$$

$$\mathbb{1}_{gc} P_g^{\min} \leq P_{gc} \leq \mathbb{1}_{gc} P_g^{\max}, \forall g, c$$

$$0 \leq P_n^w \leq P_n^{\text{f-w}}, \forall n$$

$$P_{kc} = B_k \sum_n \mathbb{1}_{nk} \theta_{nc}, \forall k, c$$

$$-P_{kc}^{\max} \leq P_{kc} \leq P_{kc}^{\max}, \forall k, c$$

$$\theta^{\min} \leq \theta_{nc} \leq \theta^{\max}, \forall n > 1, c$$

$$\theta_{1c} = 0, \forall c$$

$$P_{gc_g} \geq z_{gc_g} P_g^{\max}, \forall g, c_g$$

$$P_{gc_g} \leq \mathbb{1}_{gc_g} (P_{g0} - D_g \Delta f_{c_g}), \forall g, c_g$$

$$P_{gc_g} \geq \mathbb{1}_{gc_g} (P_{g0} - D_g \Delta f_{c_g}) - M z_{gc_g}, \forall g, c_g$$

Objective function

Dispatch constraints

Primary response constraints

# PSCOPF formulation

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Droop coefficients:  
New discretized  
decision variables

Primary response  
constraints

# Solving this PSCOPF

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- Non-linear problem because the droop coefficients are decision variables
  - Product of binary and continuous variables
- Non-convex problem
  - Use a Benders-like decomposition
  - No guarantee of optimality
  - Test results show that the loss of optimality is acceptable

# Test System and Data

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- Modified versions of the one- and three-area Reliability Test System
- Yearly load and simulated wind generation profiles
- Day-ahead UC decisions are optimized using a deterministic UC model
- Optimization is performed using the CPLEX 12.1 solver in GAMS 24.0.2 on a 2.5 GHz Intel Xeon processor with 16 GB RAM

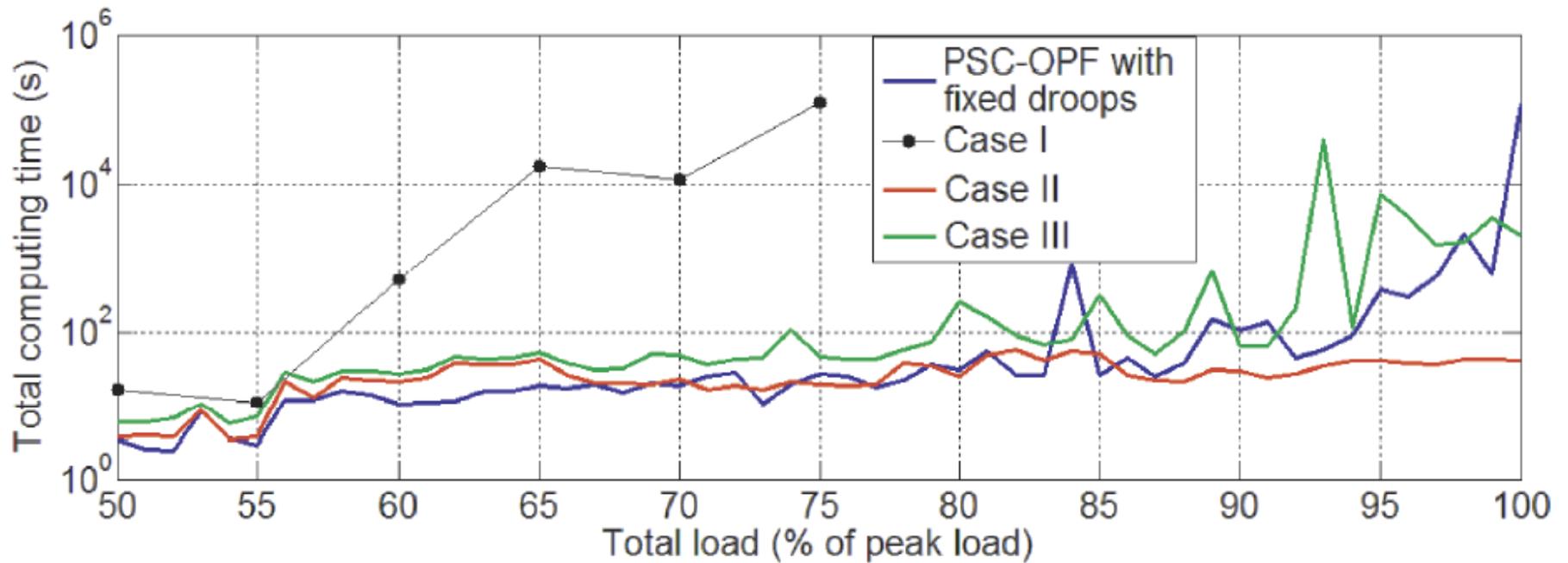
# Cases

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- Case 0: PSC OPF with fixed droops
- Case I: PSC OPF with droops optimization but **without** Benders' decomposition
- Case II: PSC OPF with droops optimization but **with** Benders' decomposition
- Case III: Determine the optimal droops using Benders decomposition, then re-solve the PSC OPF with these droops fixed.

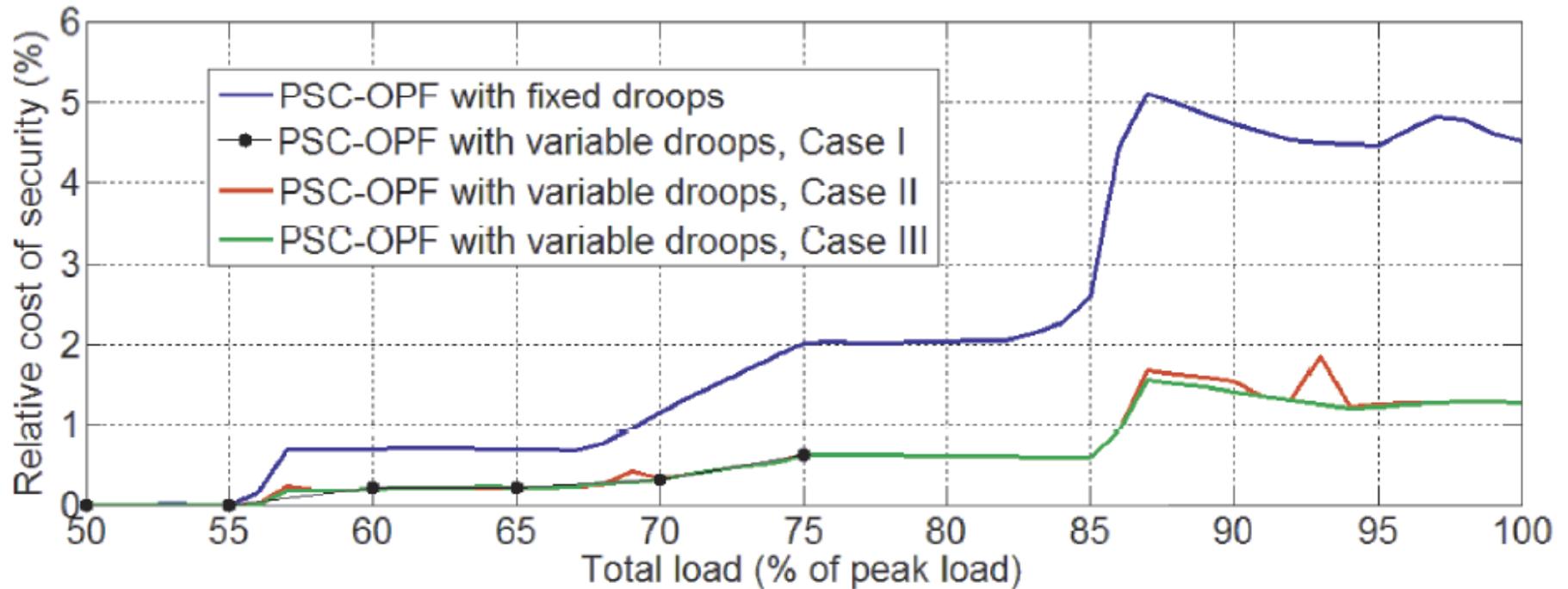
# Computational performance

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Tests performed on the one-area RTS

# Cost of security



Tests performed on the one-area RTS

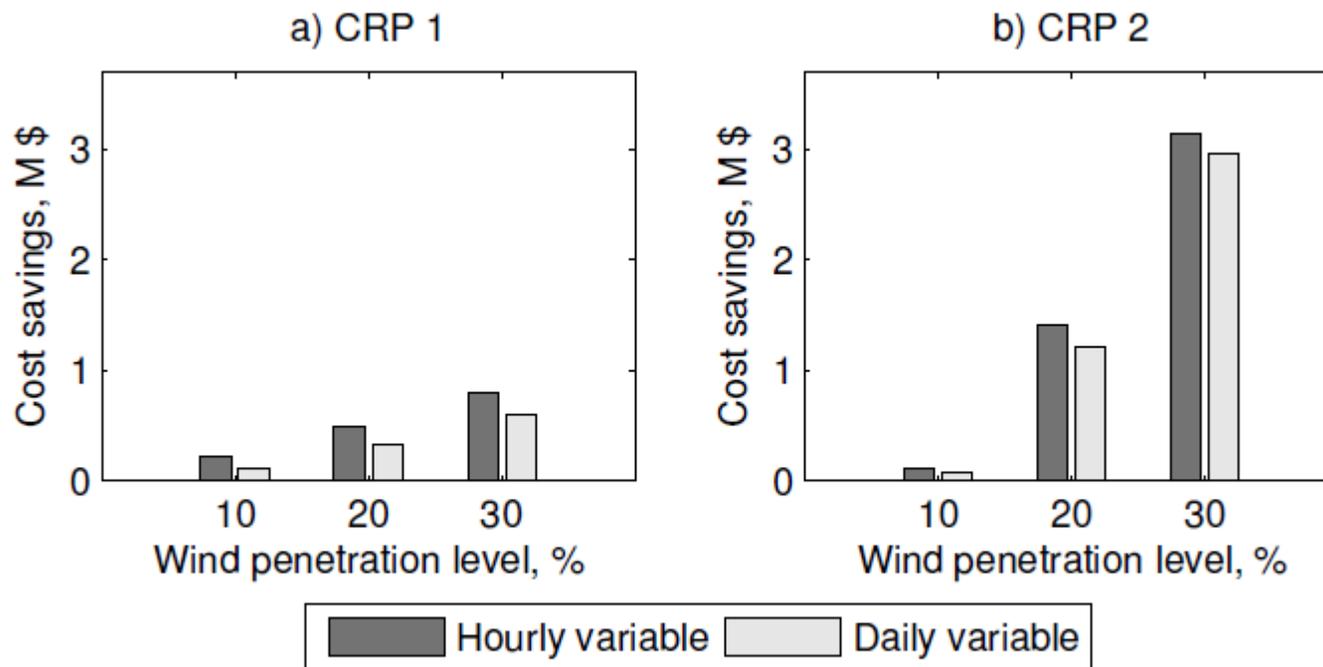
# Cost savings over a year

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- Three-area RTS
- Case III technique is used for year-long simulations
- Hourly and daily droop optimization
- Three wind penetration levels: 10, 20, 30%
- Two contingency reserve policies (CRP):
  - CRP1 (state-of-the-art): (N-1) reserve requirement is arbitrarily shared between three areas
  - CRP2 (potential policy): Each area provides a third of the (N-1) reserve requirement

# Savings over a year

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- Cost savings increase with the wind penetration
- Hourly variable droop optimization provides more flexibility for procuring primary response and achieves larger savings
- The proposed methodology is especially valuable for CRP2

# Conclusions

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- Optimizing droops enables more flexible allocation of primary response
- Cost savings achieved with the proposed methodology increase with wind penetration
- The proposed methodology is computationally tractable and compatible with PSCOPF

# References

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- **Motivation for modelling generation contingencies in PSCOPF:**

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- **Evidence of reducing primary response:**

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[6] R. Doherty, G. Lalor, and M. O'Malley, "Frequency Control in Competitive Electricity Market Dispatch," *IEEE Transactions on Power Systems*, Vol. 20, No. 3, pp. 1588-1596, 2005.