Linearized Reactive Power and Voltage Constraints for DC OPF

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Outline

Introduction

- Motivating problem
- Current practices
- Literature review

Formulation

Sensitivity

Conclusion

Market efficiency through central dispatch

- ISO markets in the US optimize generator schedules to produce power at the lowest cost

- In the US: \$1-4 billion in savings are possible from a 1% decrease in operational costs[1]

Picture credit: http://w3.siemens.com/smartgrid/global/en/projects/pages/pjm.aspx

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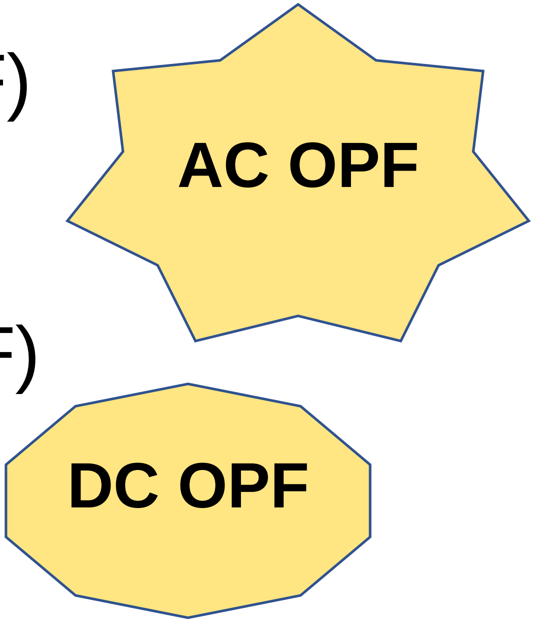
[1] M.B. Cain, R. P. O'Neill, and A. Castillo, "History of optimal power flow and formulations," Federal Energy Regulatory Commission, December 2012.

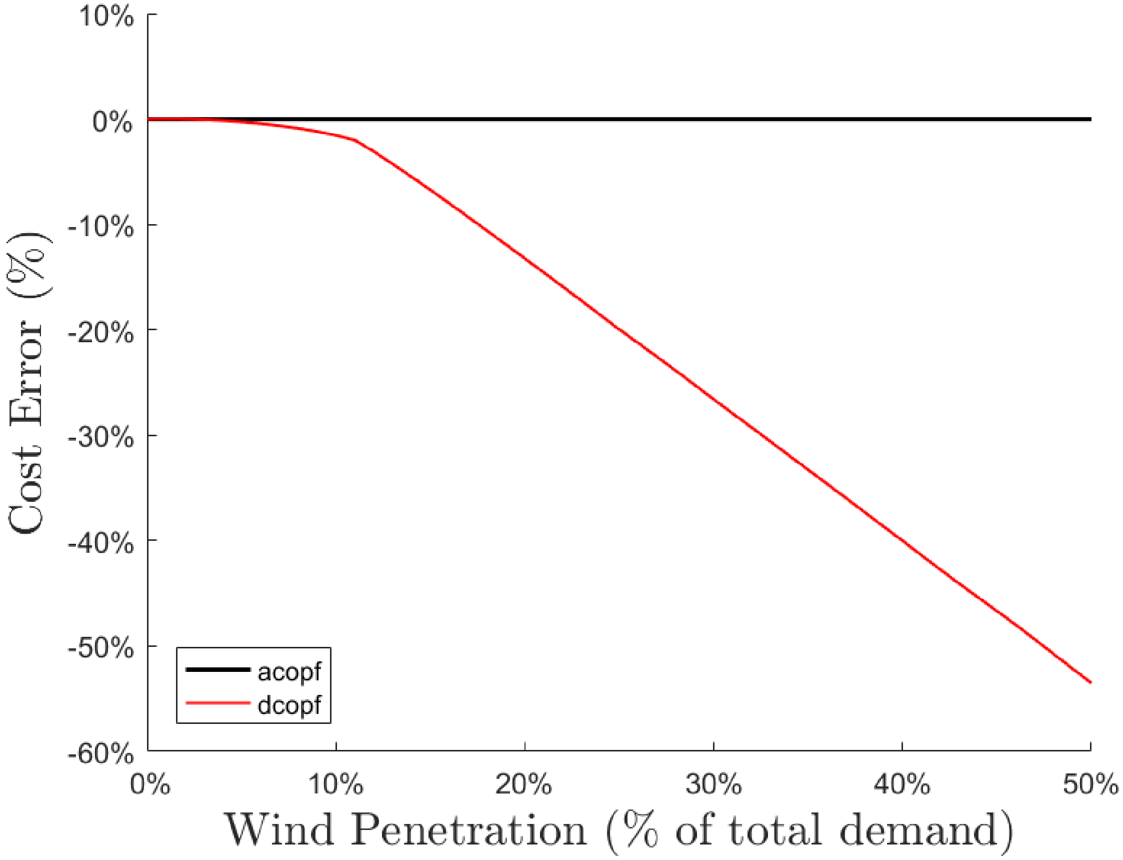
AC vs DC modeling

- AC optimal power flow (AC OPF)
  - P, Q, Θ, V
  - Nonlinear, non-convex
- DC optimal power flow (DC OPF)
  - P only in ISO software
  - Linear



- DC-based market dispatch requires additional tweaks to account for voltage

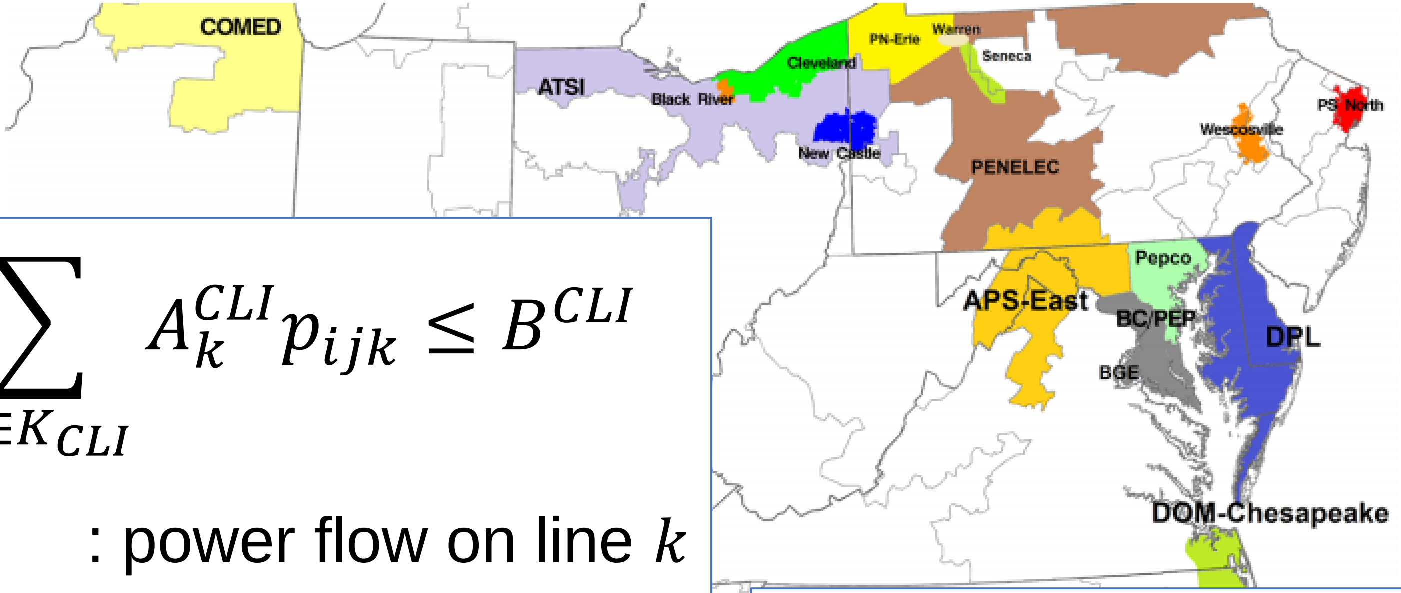
Example: IEEE 57-bus



More supply at “distant” busses
=
Binding voltage constraints
=
Poor DCOPF accuracy

In PJM: closed loop interfaces

Pre-defined borders to proxy voltage constraints



$$\sum_{k \in K_{CLI}} A_k^{CLI} p_{ijk} \leq B^{CLI}$$

p_{ijk} : power flow on line k

A_k^{CLI} : usually 1 if $k \in K_{CLI}$

B^{CLI} : import limit

i, j, n : buses or nodes

k : transmission lines

Source: http://www.monitoringanalytics.com/reports/Reports/2016/IMM_Comments_Docket_No_AD14-14-000_20160406.pdf

MISO market enhancements

- Current practice: Operating Guides and uplift
- Est. $90M/yr in uplift for MISO South load pockets
- New project: “Pricing for Voltage and Local Reliability Commitments”
- Goal: send appropriate price signals and decrease uplift
    - Directly affects 6000-9000MW
    - Production cost savings and pricing efficiency

Source: https://www.misoenergy.org/WhatWeDo/MarketEnhancements/Pages/MarketEnhancements.aspx

Literature review

- Power Flow
  - Stott, Alsac (1973) - *Fast decoupled load flow*
  - Alsac, Bright, Prais, Stott (1990) - *Further developments in LP-based power flow*
  - Overbye, Cheng, Sun (2004) - *Comparison of AC and DC power flow models*
  - Coffrin, Van Hentenryck (2014) - *LP approximation of AC power flows*
- Pricing/markets
  - Baughman and S. N. Siddiqi (1991) - *Real time pricing of reactive power*
  - Khan, Baldick (1994) - *Reactive power is a cheap constraint*
  - Hogan (1996) - *Markets in real electric networks require reactive power prices*

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Formulation

- DCOPF
- Q,V Linearization
- Linear OPF

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DCOPF formulations

- B-theta

$$p_{ijk} = \frac{-1}{x_k}(\theta_i - \theta_j)$$

- PTDF

$$p_{ijk} = \sum t_{kn} p_n = \sum \frac{-1}{x_k}\left(\frac{d\theta_i}{dp_n} - \frac{d\theta_j}{dp_n}\right) p_n$$

- PTDF is equivalent to B-theta
  - Fewer variables, dense sensitivity matrix
  - Only model flowgates with high prob. of binding
  - **Assumptions ignore reactive power and voltage!**

Nonlinear reactive power flow

$$q_k(v,\theta) = -B_k \boxed{v_i^2} - \boxed{v_i v_j}\left(G_k \boxed{\sin\theta_{ij}} - B_k \boxed{\cos\theta_{ij}}\right)$$

- Small angle approximations

$$\sin\theta \approx \theta \text{ and } \cos\theta \approx 1 - \theta^2/2$$

$$q_k(v,p_k) = -B_k\left(v_i^2 - v_i v_j\right) - X_k v_i v_j\left(-G_k p_k - p_k^2/2\right)$$

- Linearization

$$q_k(v,p_k) \approx q_k(\bar{v},\bar{p}_k) + \nabla q_k(\bar{v},\bar{p}_k)(v - \bar{v}, p_k - \bar{p}_k)$$

Linear OPF formulation

$$\min \sum_{i} c_i p_i^g$$

{standard DCOPF with losses}

$$q_i^g - q_i^d - \sum_{k \in K_i} q_k = 0$$

Nodal balance

$$q_k = q_k\boxed{(\bar{v},\bar{p}_k)} + \nabla q_k\boxed{(\bar{v},\bar{p}_k)}\boxed{(v} - \boxed{\bar{v},}\boxed{p_k} - \boxed{\bar{p}_k})$$

Q flow Taylor series

$$v^{\min} \leq v \leq v^{\max}$$

Voltage limits

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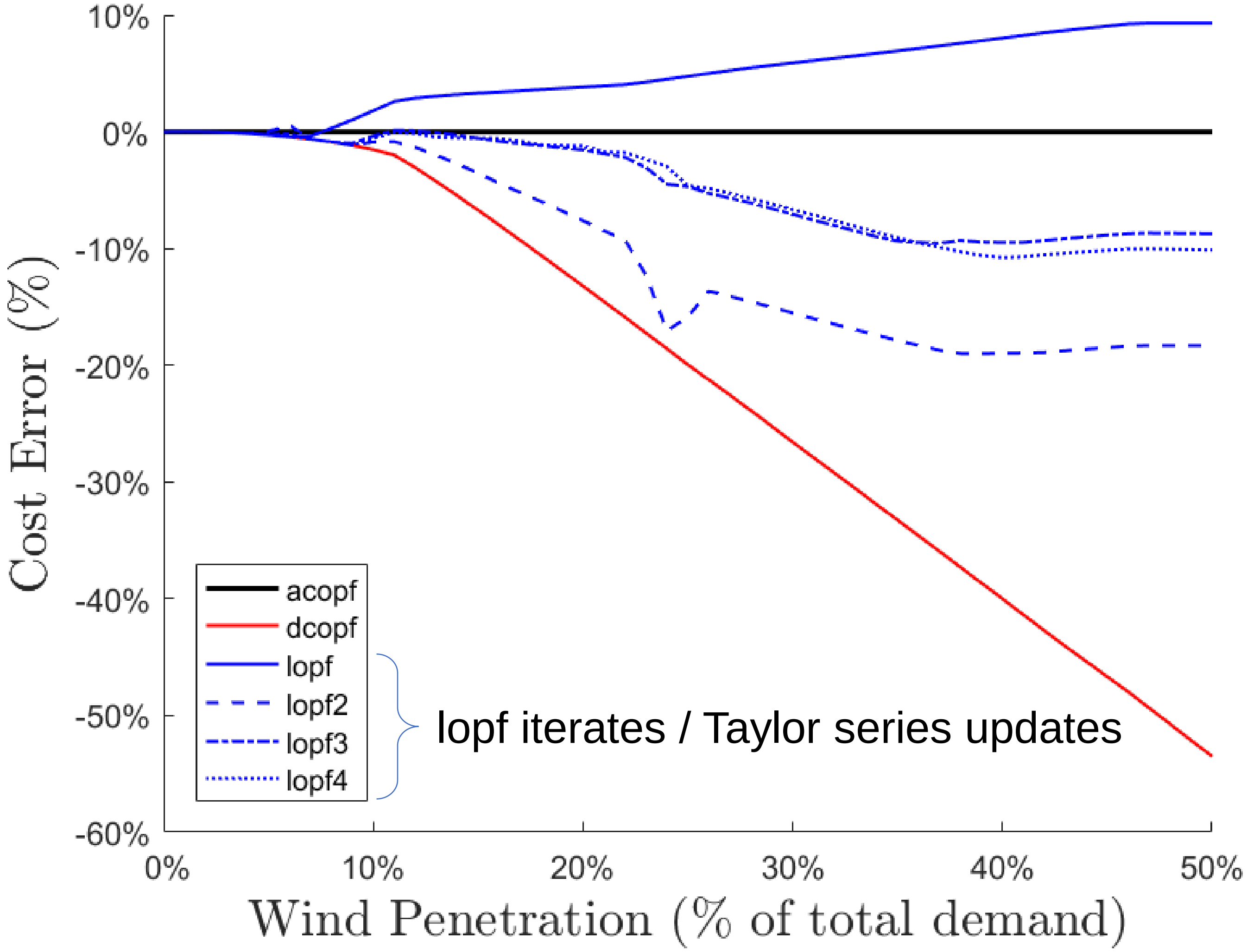
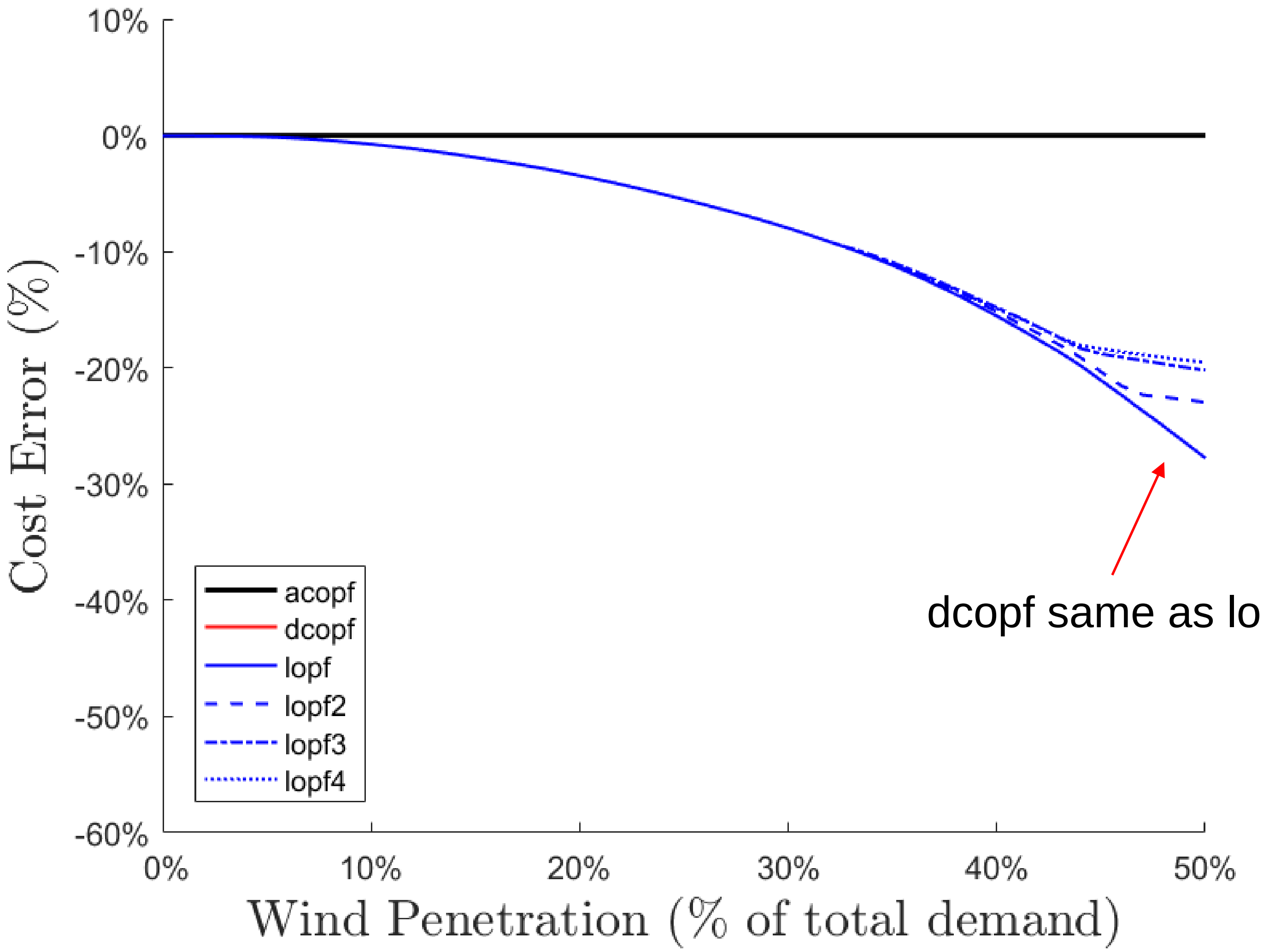
- Motivating example
- 118, 300 bus cases
- Solution times

Conclusion

Example: IEEE 57-bus

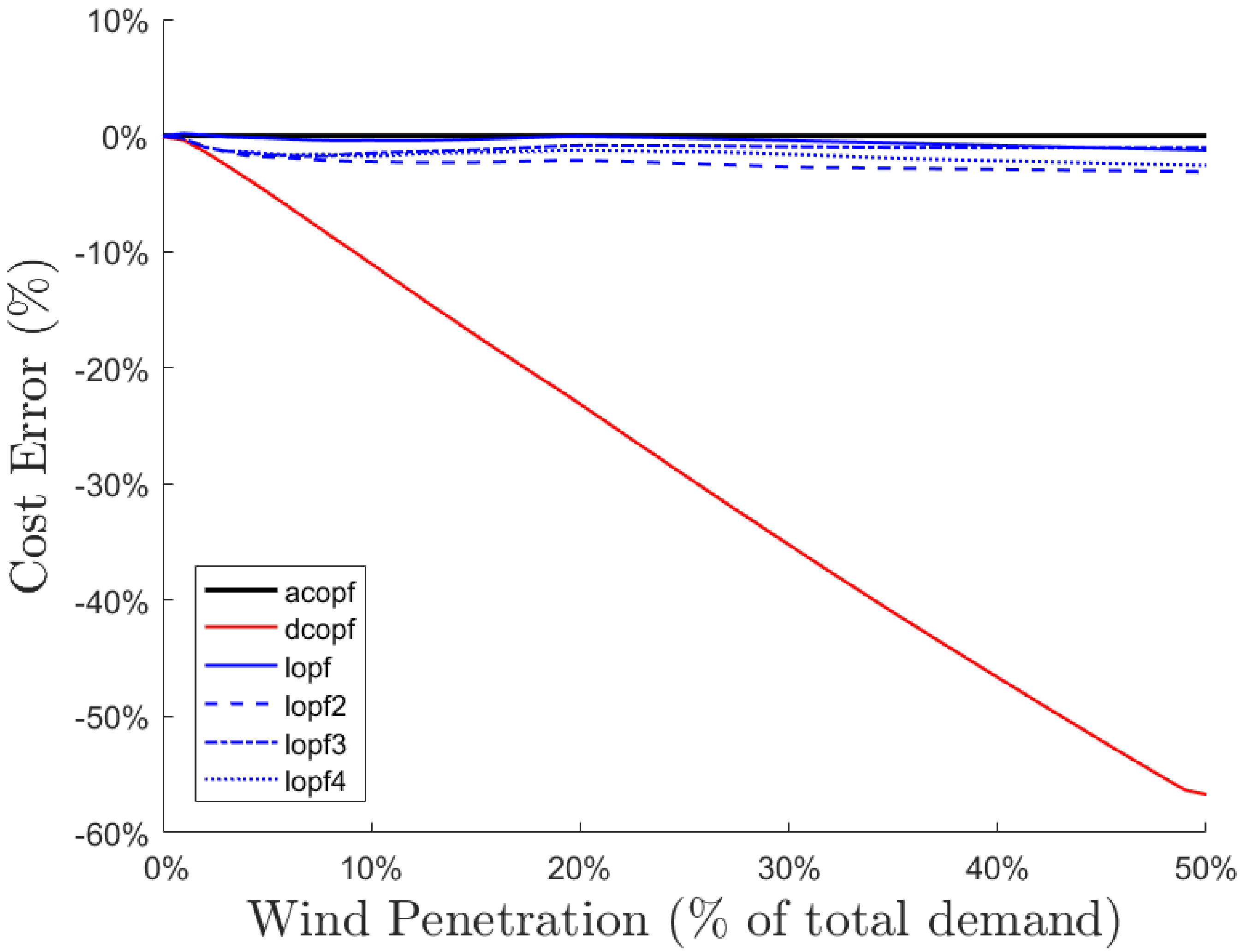


Example: IEEE 118-bus



dcopf same as lopf

Example: IEEE 300-bus



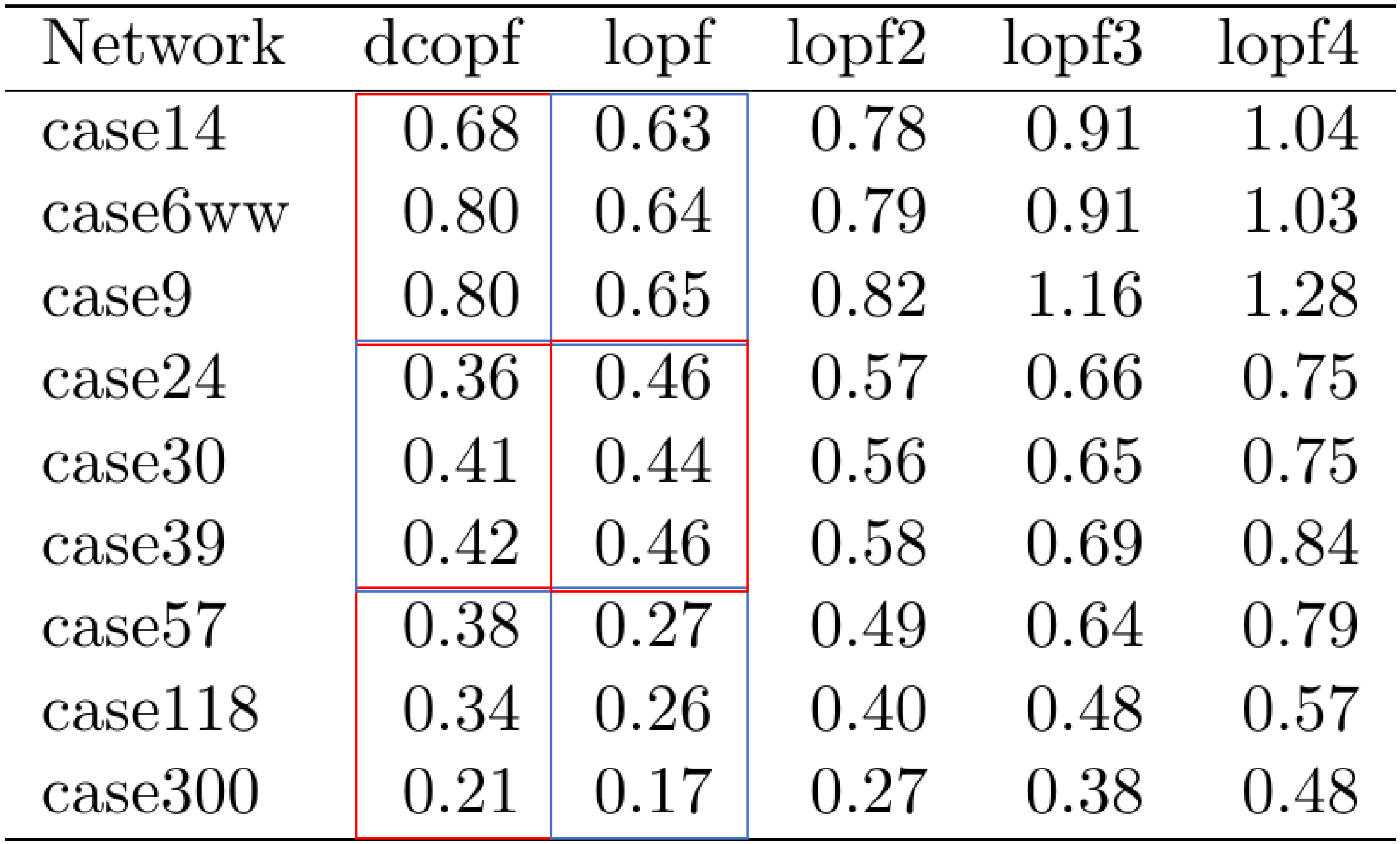
Summary: Avg Cost Error (%)

Network	dcopf	lopf	lopf2	lopf3	lopf4
case14	-2.4	-2.4	-2.4	-2.4	-2.4
case6ww	-0.7	-0.7	-0.7	-0.7	-0.7
case9	-0.5	-0.5	-0.5	-0.5	-0.5
case24	-0.5	-0.5	-0.5	-0.5	-0.5
case30	1.3	1.3	1.3	1.3	1.3
case39	-0.8	-0.8	-0.8	-0.8	-0.8
case57	-21.5	4.8	-10.7	-4.6	-4.8
case118	-8.2	-8.2	-7.9	-7.5	-7.4
case300	-28.9	-0.5	-2.4	-1.1	-1.7

No difference

* Relative to ACOPF objective function

Summary: Avg Solution Time



Network	dcopf	lopf	lopf2	lopf3	lopf4
case14	0.68	0.63	0.78	0.91	1.04
case6ww	0.80	0.64	0.79	0.91	1.03
case9	0.80	0.65	0.82	1.16	1.28
case24	0.36	0.46	0.57	0.66	0.75
case30	0.41	0.44	0.56	0.65	0.75
case39	0.42	0.46	0.58	0.69	0.84
case57	0.38	0.27	0.49	0.64	0.79
case118	0.34	0.26	0.40	0.48	0.57
case300	0.21	0.17	0.27	0.38	0.48

* Relative to ACOPF solution time

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- The DCOPF is the standard electric dispatch model
  - Linear programming solves quickly
  - MIP formulation for unit commitment
  - Ignores reactive power and voltage
- Reactive power and voltage are not optimized
- Can be modeled with additional constraints
  - First-order Taylor series approximation
  - Base point solution updated with DC approximation
  - May speed up computation time!

Future Work

Improved approximation

- Physically meaningful Q and V
- MVA transmission limits, D-curves, line losses
- Convergence to AC solution?

Computational enhancements

- Restrict modeling to specific network areas
- Base point estimation, piecewise linearization

Economics and pricing

- Cost allocation to real power or voltage
- Better incentives to install synchronous Var compensators, etc.

Thank you!