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FERC – Technical Conference on Increasing Real-Time and Day-Ahead Market Efficiency

Exact solutions to binary equilibrium problems with compensation and the power market uplift problem

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Washington, D.C., June 23, 2015

1 Introduction – The power market uplift problem

In electricity markets, marginal-cost pricing may yield incentive-incompatible dispatch and losses for generators

Generators may earn negative profits based on market-clearing prices derived from least-cost unit commitment & dispatch

⇒ Generators then have incentives to leave the market or not follow the dispatch as prescribed by the ISO ("self-scheduling")

More generally, in markets with non-convexities & indivisibilities, using duals as market-clearing (Walrasian) prices doesn't quite work

⇒ There may not even exist any Nash equilibrium in many cases

In practice, the ISO pays out compensation to "make whole" individual generators after market clearing ("no-loss rule")

⇒ Potential for gaming and issues of acceptance by consumers

There exists no scalable approach to find Nash equilibria in non-cooperative games with binary decision variables

Overview of current approaches for equilibria in binary games…

- neglect the gaming aspects altogether
- compute all permutations of the binary variables

⇒ quickly grows beyond the bounds of computational tractability

- use a two-stage approach (convex-hull/minimum uplift)
- relax/linearize the binary variables, apply equilibrium methods

  see Gabriel, Conejo, Ruiz, and Siddiqui (2013)

⇒ doesn't yield exact solution, not clear whether it's truly a NE

We propose an exact solution approach to find equilibria in games with binary variables

Outline of the proposed solution methodology:

- derive KKT conditions for *both states* of each binary variable
- add an objective function as *equilibrium selection* mechanism
- include compensation payments to ensure incentive compatibility as decision variables of the equilibrium selection problem

The reformulated problem…

⇒ is a *multi-objective bi-level* optimization program

⇒ allows to incorporate the trade-off between market efficiency and compensation payment budget

⇒ can be solved as a mixed-binary linear program

We introduce the notion of a binary quasi-equilibrium to describe incentive-compatible outcomes with compensation

Definition: *Binary game*

We have a set of players $i \in I$,

each seeking to solve a binary problem:

$$\min_{\substack{x_i \in \{0,1\} \\ y_i \in \mathbb{R}^m}} \quad f_i\left(x_i, y_i, y_{-i}\left(x_{-i}\right)\right)$$
$$\text{s.t.} \quad g_i\left(x_i, y_i\right) \leq 0 \qquad \left(\lambda_i\right)$$

Definition: *Equilibrium in a binary game*

A (Nash) equilibrium in binary variables is a feasible vector $\left(x_i^*, y_i^*\right)_{i \in I}$

such that: $f_i\left(x_i^*, y_i^*, y_{-i}^*\left(x_{-i}^*\right)\right) \leq f_i\left(x_i^\times, y_i^\times, y_{-i}^*\left(x_{-i}^*\right)\right) \quad \forall\ i \in I$

Definition: *Quasi-equilibrium in a binary game with compensation*

A (Nash) equilibrium in binary variables is a feasible vector $\left(x_i^*, y_i^*\right)_{i \in I}$

and a vector of compensation payments $\left(\zeta_i\right)_{i \in I}$

such that: $f_i\left(x_i^*, y_i^*, y_{-i}^*\left(x_{-i}^*\right)\right) - \zeta_i \leq f_i\left(x_i^\times, y_i^\times, y_{-i}^*\left(x_{-i}^*\right)\right) \quad \forall\ i \in I$

We compute the optimal value w.r.t. the continuous variables for both states of the binary variable simultaneously

Assumption:

First-order optimality (KKT) conditions are necessary and sufficient w.r.t. continuous variables y_i for fixed binary x_i and given rivals actions y_{-i}

Then, we can compute the optimal response of each player for both states of variable x_i:

$$0 = \nabla_{y_i} f_i\left(\mathbf{\overline{x}_i}, \widetilde{y}_i^{(\mathbf{\overline{x}_i})}, y_{-i}(x_{-i})\right) + \widetilde{\lambda}_i^{(\mathbf{\overline{x}_i})} \nabla_{y_i} g_i\left(\mathbf{\overline{x}_i}, \widetilde{y}_i^{(\mathbf{\overline{x}_i})}\right) \quad , \quad \widetilde{y}_i^{(\mathbf{\overline{x}_i})}$$

$$0 \geq g_i\left(\mathbf{\overline{x}_i}, \widetilde{y}_i^{(\mathbf{\overline{x}_i})}\right) \quad \perp \quad \widetilde{\lambda}_i^{(\mathbf{\overline{x}_i})} \geq 0$$

And then, we check which strategy is optimal:

$$f_i\left(\mathbf{1}, \widetilde{y}_i^{(\mathbf{1})}, y_{-i}(x_{-i})\right) < f_i\left(\mathbf{0}, \widetilde{y}_i^{(\mathbf{0})}, y_{-i}(x_{-i})\right) \quad \Rightarrow \quad x_i^* = 1$$

$$f_i\left(\mathbf{1}, \widetilde{y}_i^{(\mathbf{1})}, y_{-i}(x_{-i})\right) > f_i\left(\mathbf{0}, \widetilde{y}_i^{(\mathbf{0})}, y_{-i}(x_{-i})\right) \quad \Rightarrow \quad x_i^* = 0$$

$$f_i\left(\mathbf{1}, \widetilde{y}_i^{(\mathbf{1})}, y_{-i}(x_{-i})\right) = f_i\left(\mathbf{0}, \widetilde{y}_i^{(\mathbf{0})}, y_{-i}(x_{-i})\right) \quad \Rightarrow \quad x_i^* = \{0, 1\}$$

The core idea of our solution approach (II)

We use the switch value to the incentive-compatibility check to replace the cumbersome "if-then" conditions

The "if-then" conditions to determine the individually optimally binary decision are very painful to compute in large-scale problems:

$$
\begin{aligned}
f_i\big(\mathbf{1}, \widetilde{y}_i^{(\mathbf{1})}, y_{-i}(x_{-i})\big) &< f_i\big(\mathbf{0}, \widetilde{y}_i^{(\mathbf{0})}, y_{-i}(x_{-i})\big) \quad && \Rightarrow \quad x_i^* = 1 \\
f_i\big(\mathbf{1}, \widetilde{y}_i^{(\mathbf{1})}, y_{-i}(x_{-i})\big) &> f_i\big(\mathbf{0}, \widetilde{y}_i^{(\mathbf{0})}, y_{-i}(x_{-i})\big) \quad && \Rightarrow \quad x_i^* = 0 \\
f_i\big(\mathbf{1}, \widetilde{y}_i^{(\mathbf{1})}, y_{-i}(x_{-i})\big) &= f_i\big(\mathbf{0}, \widetilde{y}_i^{(\mathbf{0})}, y_{-i}(x_{-i})\big) \quad && \Rightarrow \quad x_i^* = \{0,1\}
\end{aligned}
$$

We use the switch value κ and introduce a compensation payment ξ:

$$
\begin{aligned}
f_i\Big(\mathbf{1}, \widetilde{y}_i^{(\mathbf{1})}, y_{-i}\Big) + \kappa_i^{(\mathbf{1})} - \zeta_i^{(\mathbf{1})} - \kappa_i^{(\mathbf{0})} + \zeta_i^{(\mathbf{0})} &= f_i\Big(\mathbf{0}, \widetilde{y}_i^{(\mathbf{0})}, y_{-i}\Big) \\
\kappa_i^{(\mathbf{1})} + \zeta_i^{(\mathbf{1})} &\leq x_i \, \widetilde{K} \\
\kappa_i^{(\mathbf{0})} + \zeta_i^{(\mathbf{0})} &\leq \big(1 - x_i\big) \, \widetilde{K} \\
\kappa_i^{(\mathbf{1})}, \kappa_i^{(\mathbf{0})}, \zeta_i^{(\mathbf{1})}, \zeta_i^{(\mathbf{0})} &\in \mathbb{R}_+
\end{aligned}
$$

3 The core idea of our solution approach (III)

We solve for a binary equilibrium using a two-stage problem by introducing an upper-level "market operator" player

We introduce an additional player called "market operator"

⇒ not exclusively related to electricity, but rather a "coordinator"

⇒ can also be interpreted as an equilibrium selection mechanism

The market operator acts as an upper-level player, optimizing:

$$\min \quad F\left(\left(x_i, y_i\right)_{i \in I}\right) + G\left(\left(\zeta_i\right)_{i \in I}\right)$$

while guaranteeing feasibility, optimality & incentive compatibility for each player.

⇒ This player can effectively consider the trade-off between market efficiency and compensation payments!

The market operator incorporates the trade-off of efficiency vs. compensation, subject to a binary (quasi-)equilibrium

$$\min_{\substack{x_i, y_i, \widetilde{y}_i^{(\overline{\mathbf{x}}_\mathbf{i})}, \widetilde{\lambda}_i^{(\overline{\mathbf{x}}_\mathbf{i})} \\ \kappa_i^{(\overline{\mathbf{x}}_\mathbf{i})}, \zeta_i^{(\overline{\mathbf{x}}_\mathbf{i})}}} \quad F\Big(\big(x_i, y_i\big)_{i \in I}\Big) + G\Big(\big(\zeta_i^{(\overline{\mathbf{x}}_\mathbf{i})}\big)_{i \in I}\Big)$$

$$\begin{aligned}
\text{s.t.} \quad \nabla_{y_i} f_i\Big(\overline{\mathbf{x}}_\mathbf{i}, \widetilde{y}_i^{(\overline{\mathbf{x}}_\mathbf{i})}, y_{-i}\Big) + \widetilde{\lambda}_i^{(\overline{\mathbf{x}}_\mathbf{i})} \nabla_{y_i} g_i\Big(\overline{\mathbf{x}}_\mathbf{i}, \widetilde{y}_i^{(\overline{\mathbf{x}}_\mathbf{i})}\Big) &= 0 \\
0 \leq -g_i\Big(\overline{\mathbf{x}}_\mathbf{i}, \widetilde{y}_i^{(\overline{\mathbf{x}}_\mathbf{i})}\Big) \perp \widetilde{\lambda}_i^{(\overline{\mathbf{x}}_\mathbf{i})} &\geq 0 \\
f_i\Big(\mathbf{1}, y_i^{(\mathbf{1})}, y_{-i}\Big) + \kappa_i^{(\mathbf{1})} - \zeta_i^{(\mathbf{1})} - \kappa_i^{(\mathbf{0})} + \zeta_i^{(\mathbf{0})} &= f_i\Big(\mathbf{0}, y_i^{(\mathbf{0})}, y_{-i}\Big) \\
\kappa_i^{(\mathbf{1})} + \zeta_i^{(\mathbf{1})} &\leq x_i \, \widetilde{K} \\
\kappa_i^{(\mathbf{0})} + \zeta_i^{(\mathbf{0})} &\leq \big(1 - x_i\big) \, \widetilde{K} \\
\widetilde{y}_i^{(\mathbf{0})} - x_i \, \widetilde{K} \leq y_i &\leq \widetilde{y}_i^{(\mathbf{0})} + x_i \, \widetilde{K} \\
\widetilde{y}_i^{(\mathbf{1})} - \big(1 - x_i\big) \, \widetilde{K} \leq y_i &\leq \widetilde{y}_i^{(\mathbf{1})} + \big(1 - x_i\big) \, \widetilde{K}
\end{aligned}$$

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The market operator’s problem is an exact solution method for binary equilibria – and in many cases, it’s a linear program!

Theorem: *Exact reformulation of a binary equilibrium program*

Any feasible solution to the market operator’s program
is a (quasi-) equilibrium in a binary equilibrium problem

⇒ Using the market operator’s objective function towards “optimality”

Theorem: *Quadratic mixed-binary program with linear constraints*

Under certain conditions, the market operator’s program can be
reformulated as a linear or quadratic mixed-binary program
with linear constraints

⇒ These conditions hold for the power market uplift problem,
fossil fuel & resource markets, agriculture, investment games, etc.

We believe that our method is at least as good in terms of social welfare as the currently applied two-stage approach

Proposition:

Our approach is always at least as good than the two-stage approach following O'Neill et al. (2005), irrespective of which market rules apply

Idea for the proof:

Our method:

$$\begin{aligned} \min \quad & F\left(\left(x_i, y_i\right)_{i \in I}\right) + G\left(\left(\zeta_i\right)_{i \in I}\right) \\ \text{s.t.} \quad & \text{feasibility, optimality,} \\ & \text{incentive compatibility} \end{aligned}$$

Two-stage approach:

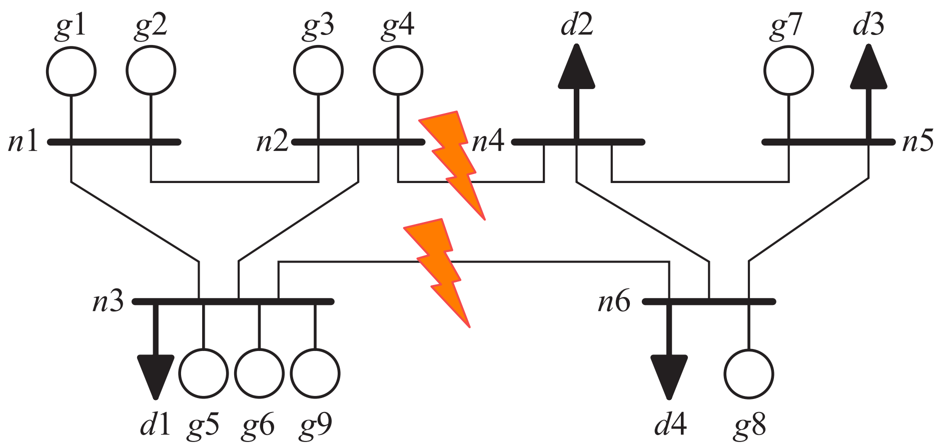
$$\begin{aligned} \min \quad & G\left(\left(\zeta_i\right)_{i \in I}\right) \\ \text{s.t.} \quad & \min F\left(\left(x_i, y_i\right)_{i \in I}\right) \ \text{s.t. feasibility} \\ & \text{incentive compatibility} \end{aligned}$$

Using an integrated model is, in principle, at least as good as solving a two-stage model

Power markets are a great field of application for our method

We use the nodal-pricing power market uplift problem example from Gabriel, Conejo, Ruiz, and Siddiqui (2013):

- 6 nodes, 9 generators, 4 load units, 2 periods (high/low demand)
- Each generator has on-off decisions, start-up/shut-down costs, and minimum generation constraints (if active)
- Two zones, with transmission bottlenecks on lines N2-N4 and N3-N6



4 An application: the power market uplift problem (II)

There are different market rules in real-world power markets and our approach is flexible to implement a variety of those

The power market uplift problem:

⇒ can be reformulated and solved as a mixed-binary linear program

⇒ a variety of market rules can be implemented as linear constraints

We compare three different market rule implementations:

Standard

- Two-step method following O'Neill et al. (2005), **standard approach**; taking into account a no-loss rule for every generator

New approach

- A **game-theoretic** solution (binary equilibrium with compensation)
- A regulatory framework that no generator should loose money, but only active generators receive compensation (**no-loss & active**)

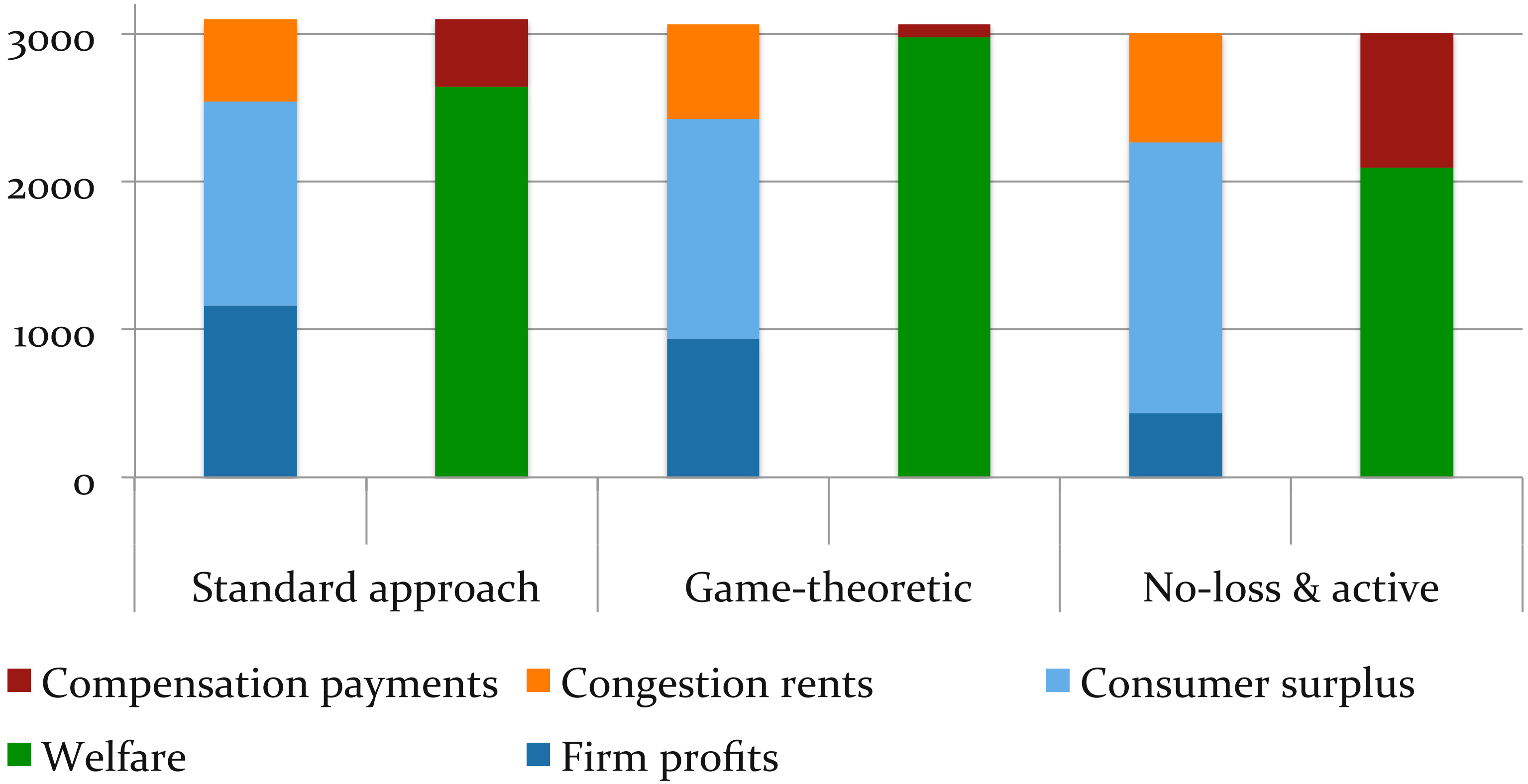
Compensation isn't necessary for incentive-compatibility here; and overly restrictive market rules reduce overall efficiency

Player		Standard approach			Game-theoretic			No-loss & Active		
	x^{init}	(x_{1i}, x_{2i})	π_i	ξ_i	(x_{1i}, x_{2i})	π_i	ξ_i	(x_{1i}, x_{2i})	π_i	ξ_i
g3	1	(0,0)	-300	300	(0,0)	-300	15	(1,1)	-525	525
g4	1	(1,1)	-160	160	(1,1)	-250	65	(1,0)	-335	335
g5	1	(1,1)	50		(1,1)	-50		(1,1)	-50	50
g6	1	(1,1)	200		(1,1)	100		(1,1)	100	
g7	0	(1,1)	690		(1,1)	715		(1,1)	690	
g8	0	(1,1)	680		(1,1)	730		(1,1)	550	
g9	0	(0,0)	0		(1,1)	-5	5	(0,0)	0	
All			1160	460		940	85		435	910
Other			1940			2120			2570	
Total			3100	460		3060	85		3005	910

Generators g1 and g2 are not active in any case; "Other": consumer & congestion rent

4 Illustrative results for the power market uplift problem (II)

In this example, the market rules to "protect" generators actually induce a substantial welfare shift towards consumers



Rents by stakeholder group in the three method/market rule cases

The number of binary variables increases only linearly in the number of generators and hours

The main challenge in computing equilibria in binary variables is the exponential increase in the number of binary variables

⇒ Size of the trial problem: 6 nodes, 9 generators, 4 load units, 2 hours

- Standard approach (welfare-optimal unit-commitment)

⇒ number of binary variables: $|T| \cdot |I| = 18$

- Brute-force enumeration of equilibria

⇒ number of equilibrium problems: $2^{|T| \cdot |I|} > 266\text{k}$

- Exact binary quasi-equilibrium solution method

⇒ number of binary variables: $|T|\left(2\left(|I|+|J|+|L|+|N|-1\right)+|I|\right)=122$

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Summary

We propose a novel and very flexible method to find Nash equilibria in binary games with compensation

We develop a method to find solutions to binary equilibrium problems such as Nash games with binary decision variables

We introduce the term "quasi-equilibrium" for market results that are incentive-compatible only if compensation is paid to some players

Under very general conditions, the problem can be solved as a Mixed-Binary Linear/Quadratic Program using standard methods

The method allows to include a multitude of market rules and regulations to replicate real-world settings and considerations

⇒ No-loss rules, compensation only for active generators, etc.

The method opens up a host of future research opportunities towards real-world applications and new algorithms

Applying the method to real-world size problems

⇒ illustrate the trade-off between market efficiency & compensation

⇒ can our method realize welfare gains similar to the switch to MIP?

Using the switch value for better numerical algorithms

⇒ stopping criteria, guidance in branch-and-bound, etc.

Yield a better understanding of gaming opportunities in power sector

⇒ our method can explicitly compare game-theoretic aspects

Extend the method to Generalized-Nash games, etc.

Thank you very much for your attention!

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Introduction – Some perspective on power market efficiency gains

Even small improvements in power market operation can have huge societal benefits due to increased efficiency

Over the past decade, power market operation was greatly improved by using Mixed-Integer Programming (MIP):

- *In 2004, PJM implemented MIP in its day-ahead market [...], with savings estimated at $100 million/year.*
- *On April 1, 2009, the California ISO (CAISO) implemented its Market Redesign and Technology Update (MRTU), [...] achieving an estimated $52 million in annual estimated savings by using MIP.*

Quoted from: "Recent ISO Software Enhancements and Future Software and Modelling Plans", FERC Staff Report, 2011, pages 3-4

2

The math – Other methods to solve binary Nash games

Recent research focuses on additional constraints or relaxations of integrality or complementarity (optimality)

- Björndal and Jornsten (2008)
- Gabriel, Conejo, Ruiz, and Siddiqui (2013)
- Todd (2015)

What are duals in integer or binary programming, and why can we use them as market-clearing prices?

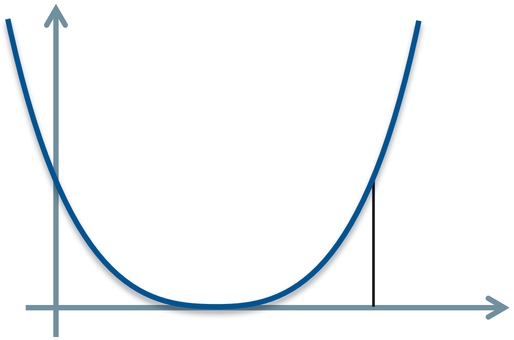
Using optimization for determining prices relies on duality

⇒ But duality in integer programs is difficult to establish

⇒ The notion of a “marginal relaxation” doesn’t quite make sense

Look at a stylized example:

$$\min_x \quad (x-0.5)^2$$
$$\text{s.t.} \quad x \in \{0,1\}$$



If you solve this program in GAMS (or most numerical solver packages), you will get some “duals” reported

⇒ Where do these values come from?

Duals in integer programs using a two-step procedure

O'Neill et al. (2005) proposed a two-step approach:

1. Solve the MIP using standard methods (Problem 1)
2. Solve the linearized LP model (Problem 2), fixing discrete/binary variables at optimal level x^* as determined by Problem (1)

$$\begin{aligned} \min_{x,y} \quad & f(x,y) && (1) \\ \text{s.t.} \quad & g(x,y) \leq 0 \\ & x \in \{0,1\}^n \\ & \mathrm{y} \in \mathbb{R}^m \end{aligned}$$

$$\begin{aligned} \min_{x,y} \quad & f(x,y) && (2) \\ \text{s.t.} \quad & g(x,y) \leq 0 && (\lambda) \\ & x = x^* && (\mu) \\ & (x,y) \in \mathbb{R}^{n+m} \end{aligned}$$

⇒ The dual variables (λ^*, μ^*) to Problem (2) can be interpreted as market-clearing, Walrasian prices!

⇒ But these are not actually the prices used in real-world markets!

3 An exact solution method – Introducing the "switch value"

Rather than focusing on relaxations of binary variables, let's look at the loss from deviating ("switch value")

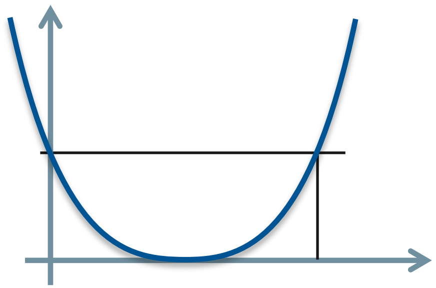
We introduce the term "switch value κ" to describe the absolute (not marginal) loss when deviating from the optimal value x^*:

$$f\left(x^*, y^*\right) = f\left(x^\times, y^\times\right) - \kappa$$

where $x^\times = 1 - x^*$ and $y^\times = \arg\min_y f\left(x^\times, y\right)$

Applying this idea to the simple example:

$$\min_x \quad (x - 0.5)^2$$
$$\text{s.t.} \quad x \in \{0,1\}$$



$$\mu(x^* = 0) = -1$$
$$\mu(x^* = 1) = 1$$
$$\kappa = 0$$

⇒ The two-stage approach suggests that marginal improvements are possible, but the switch value shows that the player is indifferent