An Extended Hybrid Markovian and Interval Unit Commitment Considering Renewable Generation Uncertainties

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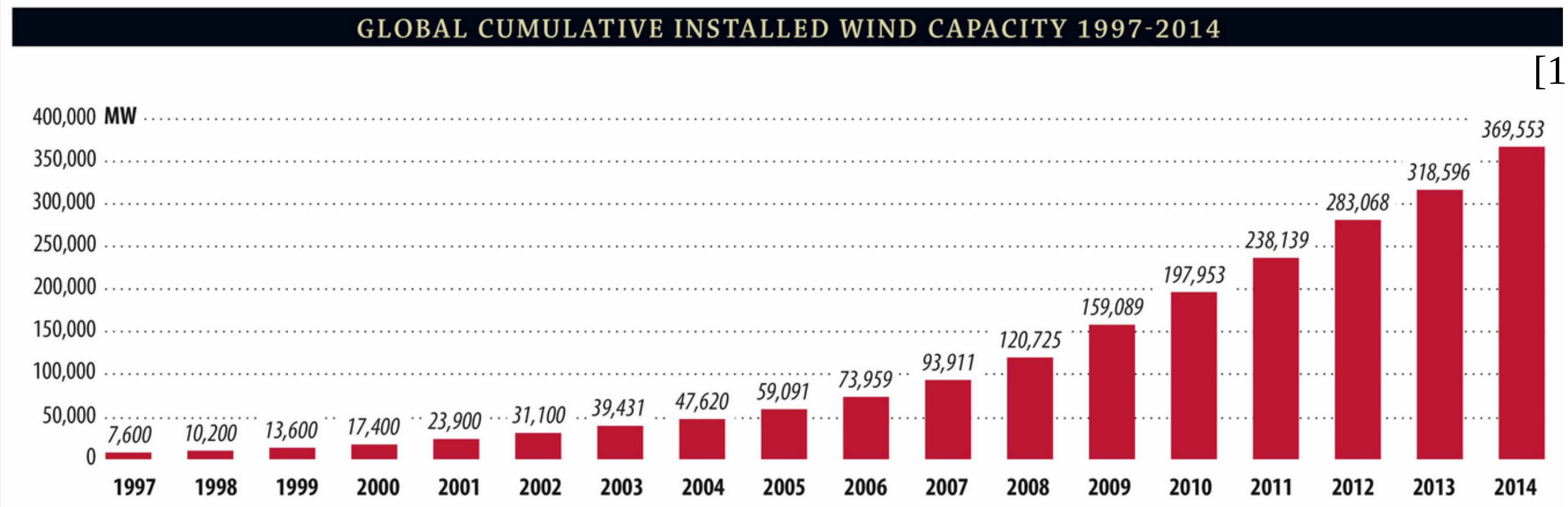
Introduction – Wind integration



[1]

- Is wind generation "free" beyond installation & maintenance?
  - Difficulties: Intermittent/uncertain nature of wind generation
    - In Spain, an unprecedented decrease in wind generation in Feb. 2012 was equivalent to the sudden down of 6 nuclear plants
    - 4 units not unusual ~ Hidden secret of intermittent renewables

1. http://breakingenergy.com/2015/03/19/wind-2000-gw-by-2030/

Existing Approaches

- **Deterministic Approach**
  - Uncertainties not explicitly considered
  - Solutions not robust against realizations of wind generation
  - Flexible ramping product being investigated
- **Stochastic Programming**
  - Modeling wind generation by representative scenarios
  - Solution methodology
    - Branch-and-cut
    - Benders' decomposition with branch-and-cut
    - Lagrangian relaxation with branch-and-cut
  - The number of scenarios: Too many or two few?

- **Robust optimization**
  - Uncertainties modeled by an uncertainty set, and optimized against the worst possible realization ~ Conservative
  - Min Max ~ Computationally challenging
  - Methodology: Benders' decomposition with outer approximation
- **Interval optimization** [2], [3], [4]
  - Wind generation modeled by closed intervals
  - Solutions to be feasible for extreme cases of system demand, transmission capacity, and ramp rate constraints ~ Conservative
  - Methodology: Benders' decomposition with branch-and-cut and interval arithmetic
- Better ways?

2. J. W. Chinneck and K. Ramadan, "Linear programming with interval coefficients," *Journal of the Operational Research Society*, Vol. 51, No. 2, pp. 209-220, 2000.
3. Y. Wang, Q. Xia, and C. Kang, "Unit commitment with volatile node injections by using interval optimization," *IEEE Transactions on Power Systems*, Vol. 26, No. 3, pp. 1705-1713, 2011.
4. L. Wu, M. Shahidehpour, and Z. Li, "Comparison of Scenario-Based and Interval Optimization Approaches to Stochastic SCUC," *IEEE Transactions on Power Systems*, Vol. 27, No. 2, pp. 913-921, 2012.

Outline

- Wind integration w/o transmission (with ISO-NE) [5]
  - Stochastic UC formulation – Generation based on wind states
- Wind integration with transmission capacities (ISO-NE) [6]
  - Markovian and interval formulation – Generation ~ local state
  - Both problems are solved by using branch-and-cut
- An extended hybrid Markovian and interval approach (with the ABB team)
  - Generation of an isolated unit can depend on a remote wind farm
  - Solved by a synergistic integration of Surrogate Lagrangian Relaxation and branch-and-cut

5. P. B. Luh, Y. Yu, B. Zhang, E. Litvinov, T. Zheng, F. Zhao, J. Zhao and C. Wang, "Grid Integration of Intermittent Wind Generation: a Markovian Approach," *IEEE Transactions on Smart Grid*, Vol. 5, No. 2, March 2014.
6. Y. Yu, P. B. Luh, E. Litvinov, T. Zheng, J. Zhao and F. Zhao, "Grid Integration of Distributed Wind Generation: Hybrid Markovian and Interval Unit Commitment," *IEEE Trans. on Smart Grid*, early access since June 2015.

Stochastic Unit Commitment Formulation

- Modeling aggregate wind generation – A Markov chain
  - The state at a time instant summarizes the information of all the past in a probabilistic sense for reduced complexity
  - Net system demand = System demand – wind generation
- Minimize the sum of expected energy and startup/no-load costs

Exp. Energy cost | Start-up cost | No-load cost

$$\min_{\{x_i(t)\}_{i,t},\{p_{i,n}(t)\}_{i,n,t}} \sum_{i=1}^{I}\sum_{t=1}^{T}\left\{\sum_{n=1}^{N}\left[\varphi_n(t)C_{i,n}(p_{i,n}(t))\right]+u_i(t)S_i+x_i(t)S_i^{NL}\right\}$$

  - s.t. system demand constraint for each state at every hour

$$\sum_{i=1}^{I} p_{i,n}(t) = P_n^D(t), \forall n, \forall t$$

– Individual unit constraints

- Generation capacity constraints for each state

$$x_i(t)p_{i\min} \le p_{i,n}(t) \le x_i(t)p_{i\max}, \forall i, \forall t, \forall n$$

- Time-coupling ramp rate constraints for any state transition whose probability is nonzero

$$p_{i,m}(t-1) - \Delta_i \le p_{i,n}(t) \le p_{i,m}(t-1) + \Delta_i,$$

$$\forall i, \forall n, \forall t, \forall m \in \{m \mid \pi_{mn} \ne 0\}$$ (Ramp-up and ramp-down)

- A linear mixed-integer optimization problem
- Solution methodology – Branch-and-cut

Difficulties when considering transmission

- Transmission capacities – A major complication
  - With congestion, wind generation cannot be aggregated
  - Global state: A combination of nodal states ~ Too many
- What can be done?
- Key ideas: Markov + interval-based optimization
  - Divide the generation of a unit into two components
    - Markovian component: Depending on the local wind state
    - Interval component: To manage extreme combinations of non-local states
  - Much simpler than the pure Markovian approach
  - Less conservative as compared to the pure interval approach

- Generation capacity constraints

The Markovian component: Depending on the local state n_i

$$x_{i,k}(t)p_{i,k}^{\min} \le p_{i,k,n_i}^{M}(t) + p_{i,k,\bar{n}_i}^{I}(t) \le x_{i,k}(t)p_{i,k}^{\max}, \forall i, \forall k, \forall t, \forall n_i, \forall \bar{n}_i$$

The interval component: Depending on the combination of non-local states $\bar{n}_i$

- Nodal injection

$$P_{i,n_i,\bar{n}_i}(t) = \sum_k p_{i,k,n_i}^{M}(t) + p_{i,n_i}^{W}(t) - p_i^{L}(t) + \sum_k p_{i,k,\bar{n}_i}^{I}(t), \forall i, \forall t, \forall n_i, \forall \bar{n}_i$$

Markovian nodal injection $\equiv P_{i,n_i}^{M}(t)$ Interval nodal injection $\equiv P_{i,\bar{n}_i}^{I}(t)$

- System demand constraints ~ Sum of nodal injections = 0
  - Sum of nodal injections = 0 for both min/max guarantee the satisfaction for in-between demand levels

$$\sum_i P_{i,n_{i,\min},\bar{n}_{i,\min}}(t) = 0, \forall t \qquad \sum_i P_{i,n_{i,\max},\bar{n}_{i,\max}}(t) = 0, \forall t$$

- Transmission: |Power flow| ≤ Transmission capacity
  - A line flow depends on injections from many nodes

    Generation Shift Factors (GSFs which can be + or -)

$$f_l(t) = \sum_i (a_l^i \cdot P_{i,n_i,\bar{n}_i}(t))$$

Where are uncertainties?

$$= \sum_i \left[a_l^i \cdot \left(\sum_k p_{i,k,n_i}^M(t) + p_{i,n_i}^W(t) - p_i^L(t) \right) \right] + \sum_i \left[a_l^i \cdot \left(\sum_k p_{i,k,\bar{n}_i}^I(t) \right) \right], \forall l, \forall t$$

Markovian nodal injection $\equiv P_{i,n_i}^M(t)$ Interval nodal injection $\equiv P_{i,\bar{n}_i}^I(t)$

  - Determine extreme flows from wind uncertainties by considering signs of GSFs and extreme Markovian nodal injections

$$\sum_{i:a_l^i>0} [a_l^i \cdot \min_{n_i} P_{i,n_i}^M(t)] + \sum_{i:a_l^i<0} [a_l^i \cdot \max_{n_i} P_{i,n_i}^M(t)] \le \sum_i [a_l^i \cdot P_{i,n_i}^M(t)]$$

- Ramp rate constraints
  - For possible states, state transitions, and $p_{i,k,\bar{n}_{i,\min}}^I(t)$ and $p_{i,k,\bar{n}_{i,\max}}^I(t)$

- The objective function
  - With state probabilities and two extreme realizations
  - Want to approximate the expected cost w/o much complexity
    - Include the expected realization with a set of deterministic constraints

$$\min \sum_{t=1}^{T} \sum_{i=1}^{I} \sum_{k=1}^{Ki} \left\{ \sum_{n_i=1}^{Ni} \left[w_{n_i,m_i}(t) C_{i,k}\left(p_{i,k,n_i}^{M}(t) + p_{i,k,m_i}^{I}(t)\right) + w_{n_i,M_i}(t) C_{i,k}\left(p_{i,k,n_i}^{M}(t) + p_{i,k,M_i}^{I}(t)\right) \right] + w_E(t) C_{i,k}\left(p_{i,k,E}(t)\right) + u_{i,k}(t) S_{i,k} + x_{i,k}(t) S_{i,k}^{NL} \right\}$$

Weight for the expected realization, adding up to 1

- Solution methodology – Branch-and-cut

Outline

- Wind integration w/o transmission
- Wind integration with transmission capacity constraints
  - Can be conservative if a big unit does not have a local wind farm $\Rightarrow$ Interval Approach
- An extended hybrid Markovian and interval approach
  - Generation of an isolated unit can depend on a remote wind farm
  - Solved by a synergistic integration of Surrogate Lagrangian Relaxation [7] and branch-and-cut [8]

7. M. A. Bragin, P. B. Luh, J. H. Yan, N. Yu, and G. A. Stern, "Convergence of the Surrogate Lagrangian Relaxation Method," *Journal of Optimization Theory and Applications*, Vol. 164, No. 1, January 2015, pp. 173-201.
8. M. A. Bragin, P. B. Luh, J. H. Yan, and G. A. Stern, "Novel Exploitation of Convex Hull Invariance for Solving Unit Commitment by Using Surrogate Lagrangian Relaxation and Branch-and-cut," to appear in *Proceedings of the IEEE Power and Energy Society 2015 General Meeting*, Denver, CO, USA

Key Ideas

- Allow an isolated unit to depend on a remote wind farm
  - Generation: A Markovian component + an interval component
- Modifications in the formulation?
  - System Demand
  - Ramp rates
  - Transmission capacity ~ Requiring the coordination of a isolated unit with a remote wind farm at a different bus

  $\Rightarrow$ More complicated

$\Rightarrow$ The Extended Formulation

– Simplified extreme Markovian flows – Can be conservative

$$\min f_l^M(t) = \sum_{\substack{i:a_l^i>0 \\ i\neq k}} [a_l^i \cdot \min_{n_i} P_{i,n_i}^M(t)] + \sum_{\substack{i:a_l^i<0 \\ i\neq k}} [a_l^i \cdot \max_{n_i} P_{i,n_i}^M(t)]$$

$$+ \sum_{k:a_l^k>0} [a_l^k \cdot \min_{n_k} P_{k,n_k}^M(t)] + \sum_{k:a_l^k<0} [a_l^k \cdot \max_{n_k} P_{k,n_k}^M(t)]$$

$$+ \sum_{j:a_l^j>0} [a_l^j \cdot \max_{n_k} P_{j,n_k}^M(t)] + \sum_{j:a_l^j<0} [a_l^j \cdot \min_{n_k} P_{j,n_k}^M(t)]$$

k: remote wind farms

j: linked units

$n_k{}^*$ for nodes k and j can be different, but can be derived

– Interval flows

$$f_{l,c}^I(t) = \sum_i [a_l^i \cdot P_{i,c}^I(t)] + \sum_j [a_l^j \cdot P_{j,c}^I(t)]$$

$f_l{}^I$ Interval flow has 2 possible combinations denoted as c

- How to solve the problem? Lagrangian relaxation
- Why? Reversing the property of NP hardness!

- Lagrangian ~ Relaxing all system-wide coupling constraints

$$L = \sum_{t=1}^{T}\{\sum_{i=1}^{I}[p_i(t)\cdot C_i + x_i(t)\cdot S_i^{NL} + u_i(t)\cdot S_i]$$

$$+\lambda(t)(\sum_i P_i) + \sum_l[\mu_{l,-}(t)(-f_l^{\max} - f_l(t))] + \sum_l[\mu_{l,+}(t)(f_l(t) - f_l^{\max})]\}$$

- Individual unit subproblems

$$\min_{\substack{x_i(t)\\ p_i(t)}} L,\ with\ L \equiv \sum_{t=1}^{T}\{[p_i(t)\cdot C_i + x_i(t)\cdot S_i^{NL} + u_i(t)\cdot S_i]$$

$$+\lambda(t)P_i + \sum_{l=1}^{L}\mu_{l,+}(t)(a_l^i\cdot P_i(t)) - \sum_{l=1}^{L}\mu_{l,-}(a_l^i\cdot P_i(t))\}$$

- Dual problem

$$\max_{\lambda,\mu}\Phi(\lambda,\mu),\ with\ \Phi(\lambda,\mu) \equiv \sum_{i=1}^{I}L_i^*(\lambda,\mu)$$

$$-\sum_{t=1}^{T}\sum_{l=1}^{L}(\mu_{l,+}(t) + \mu_{l,-}(t))f_l^{\max}$$

$$s.t.\ \mu_{l,+}(t)\geq 0, \mu_{l,-}(t)\geq 0$$

- Major difficulties of traditional LR
  - L is difficult to fully optimize
  - λ can suffer from zigzagging
  - Convergence proof and step size require q^*

Surrogate Lagrangian Relaxation [7]

- A new method, proved to converge, and guaranteed for practical implementation without fully optimizing the relaxed problem and without requiring q^*

1) $c^k \sim \prod_{i=1}^{k} \alpha_i \to 0$

2) $\lim_{k \to \infty} \frac{1-\alpha_k}{c^k} = 0$

} Without requiring q^*!

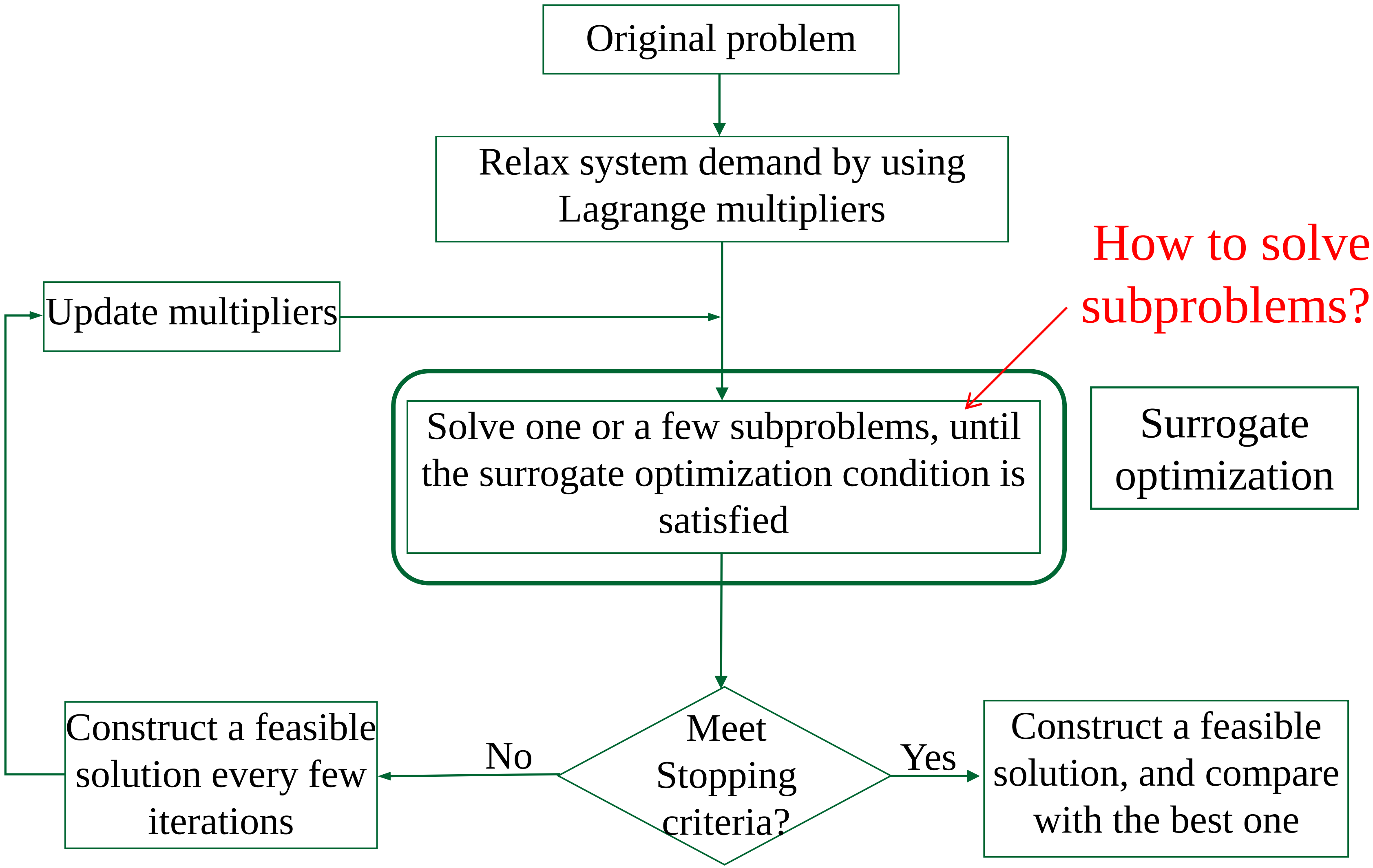
$$\lambda^{k+1} = \lambda^k + c^k \tilde{g}(x^k)$$

- One possible example of α_k that satisfies conditions 1) and 2): $\alpha_k = 1 - \frac{1}{M \cdot k^p}, \ 0 < p < 1, \ M > 1, \ k = 1, 2, \ldots$
- At convergence, the surrogate dual value approaches q^*
  ~ valid lower bound on the feasible cost

~ Overcomes all major difficulties of traditional LR

7. M. A. Bragin, P. B. Luh, J. H. Yan, N. Yu, and G. A. Stern, "Convergence of the Surrogate Lagrangian Relaxation Method," *Journal of Optimization Theory and Applications*, Vol. 164, No. 1, January 2015, pp. 173-201.

Schematic of Surrogate Lagrangian Relaxation



Difficulties of Standard Branch-and-Cut

- Branch-and-cut (B&C) can suffer from slow convergence
  - Facet-defining cuts and even valid inequalities are problem-dependent and can be difficult to obtain
    - When facet-defining cuts are not available, a large number of branching operations will be performed
  - No "local" concept $\Rightarrow$ Constraints associated with one subsystem (e.g., a combined cycle unit with complicated state transition constraints) are treated as global constraints and affect the entire solution process

Synergistic Combination with Branch and Cut [8]

- SLR relaxation and B&C are synergistically combined to simultaneously exploit separability and linearity:
  - Relax coupling constraints (system demand/transmission)
  - Solve a subproblem by using branch-and-cut w/ warm start
    - Subproblem complexity is drastically reduced
    - Subproblem convex hull are much easier to obtain and not affected by local constraints of other subproblems
    - Subproblem cuts are effective and remain valid for the future
    - Convex hulls for a subproblem never changes (if obtained, then solving this subproblem in the future is **a piece of cake!!**)
  - Update multiplies by SLR – fast convergence w/o q^*

- subproblem

$$L = \sum_{i=1}^{I} \left\{ \sum_{t=1}^{T} \left(C_i(p_i(t), t) + S_i(t) - \lambda(t) p_i(t) \right) \right\} + \sum_{t=1}^{T} \lambda(t) P_d(t)$$

8. M. A. Bragin, P. B. Luh, J. H. Yan, and G. A. Stern, "Novel Exploitation of Convex Hull Invariance for Solving Unit Commitment by Using Surrogate Lagrangian Relaxation and Branch-and-cut," to appear in *Proceedings of the IEEE Power and Energy Society 2015 General Meeting*, Denver, CO, USA

Implementation of SLR + Branch-and-Cut

- Testing system – IEEE 30-bus 41-branch 24-period
  - Relax system demand & transmission capacity constraints
  - Form individual unit subproblems s.t. unit-wise constraints
  - Configurations: 10 wind farms, 10 co-located units, 2 non-colocated cheap units
  - A penalty of $5000/MWh on wind curtailment beyond a threshold
- Implementation – In CPLEX 12.6.0.0 on Dell Precision M4500
  - SLR implemented using ILOG Script for OPL
    - Flow control, load data, generate models, update multipliers, warm start …
  - Subproblems solved by the CPLEX using branch-and-cut
  - Multipliers are initialized according to priority list

Units' characteristics

Unit #	pmin	pmax	Offer price	No-load cost	Start-up cost	Associated wind farm
Co-located units						
1	5	157	62.6	786.8	50	1
2	8	100	56.7	945.6	100	2
3	14	157	62.6	700	50	3
4	22	100	56.7	800	40	4
5	10	60	42.1	1000	40	5
6	3	157	62.6	650	40	6
7	15	100	56.7	950	39	7
8	10	80	41.1	1243.5	110	8
9	5	157	62.6	600	40	9
10	25	100	56.7	750	50	10
Non-co-located units						
11	10	80	37.2	900	440	2
12	10	90	39	1000	500	8

Testing results

- With 5% wind penetration (p_{max} = 4 mw for a wind farm)

	Non-extended case	Extended case	
Method	B&C	SLR+B&C	B&C
Lower bound (k$)	324,190	318,920	319953
Feasible cost (k$)	327,259	325,915**	N/A
Gap (%)	0.94	2.1	N/A
Clock time* (s) — Iterations	44	847	1200
Clock time* (s) — Heuristics		53	
Wind Curtailment (k$)	0	0	N/A
Load Shedding (k$)	0	766.264	N/A

1000 Simulation runs	Non-extended case	Extended case
E(Cost) (k$)	307,451	309,839
STD(Cost) (k$)	2.33	2.12
Wind Curtailment (k$)	0	0
Load Shedding (k$)	0.65	0.43

Clock time* : solving time + other time (19 iterations)
**: Feasible solution obtained after 297 seconds

Testing results

- With 15% wind penetration (p_{max} = 12 mw for a wind farm)

		Non-extended case	Extended case	
Method		B&C	SLR+B&C	B&C
Lower bound (k$)		293,748	289,035	285305
Feasible cost (k$)		296,808	294,859**	N/A
Gap (%)		1	1.97	N/A
Clock time* (s)	Iterations	1054	1046	1200
	Heuristics		154	
Wind Curtailment (k$)		0	0	N/A
Load Shedding (k$)		0	1795.47	N/A

1000 Simulation runs	Non-extended case	Extended case
E(Cost) (k$)	297,035	286,417
STD(Cost) (k$)	42.04	17.47
Wind Curtailment (k$)	0	0
Load Shedding (k$)	19.15	8.17

Clock time* : solving time + other time (25 iterations)
**: Feasible solution was obtained after 369 seconds

Conclusion

- An important but difficult problem w/ no practical solutions
- A major breakthrough for effective grid integration of intermittent wind (and solar), with key innovations:
  - Markov processes as opposed to scenarios to model wind generation for reduced complexity
  - Markov + interval-based optimization to overcome the complexity caused by transmission capacity constraints
  - The extended approach further reduces the conservativeness
- Opens a new and effective way to address stochastic problems w/o scenario analysis or over conservativeness
- The innovative SLR + B&C opens a new direction on solving large mixed-integer linear programming problems
- What is the role of FERC on intermittent renewables?

Thank You!