First Order Line Loss Approximation for LMP Calculation

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Increasing Market & Planning Efficiency through Improved Software

FERC, June 22-24, 2015

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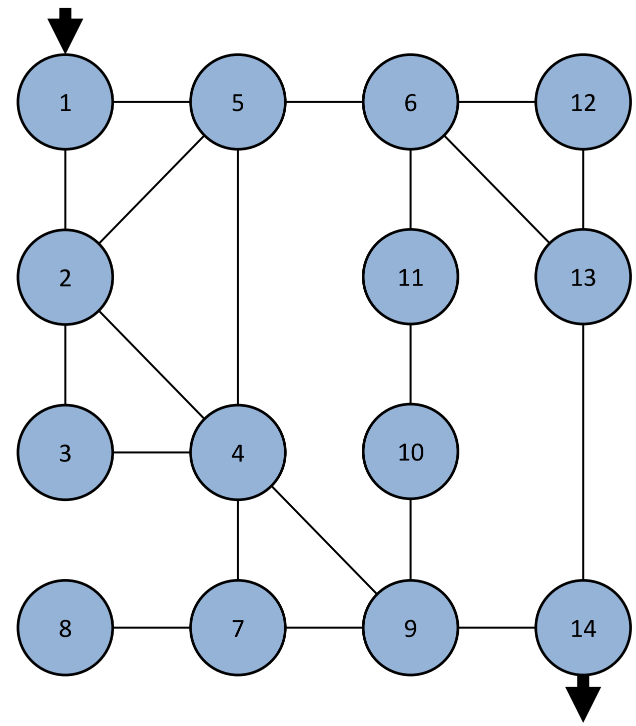
Topics

1. How a "lossless" model estimates line losses
2. Loss factor comparisons
   – Reference bus and an alternative approach
3. Sequential linear programming
   – Two implementations
   – Results and unsettled problems
4. New method
   – Updates to implicit parameters

Shift Factor / Distribution Factor

- Resistance << reactance
- Small voltage angles
- Voltage is close to one

DC power flow approximation



Model applies superposition principle

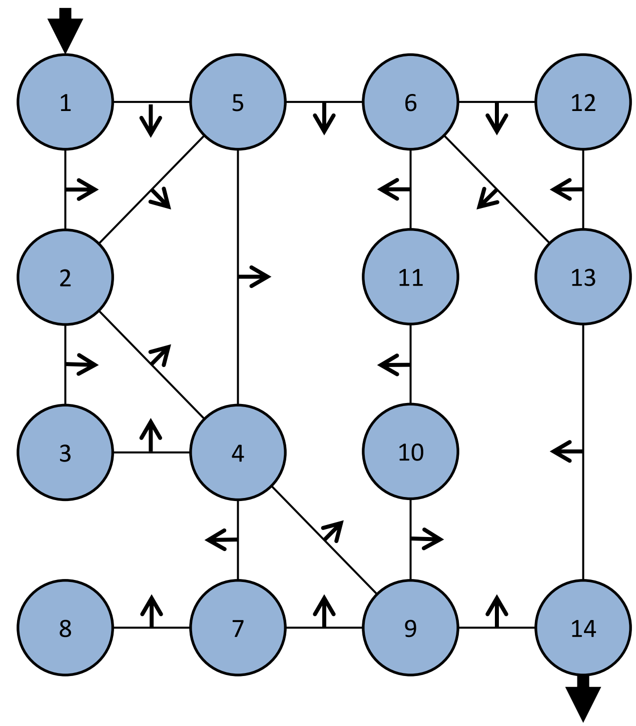
Loss Factor

- "Base Point" solution
- Actual (AC) physics
- Solve new power flow with a small injection

Change in system losses

as a numerical derivative

1 MW injection



< 1 MW withdrawal at the reference bus

$$\ell f_i = \delta\ell / \delta p_i$$
$$\approx \text{(injection – withdrawal) / injection}$$

ISO Current Practices

ISO	Loss Function	Frequency	DAM base point	RTM base point
CAISO	Linear	After each AC power flow	AC power flow	AC power flow
MISO	Linear	Hourly DAM / 5-minute RTM	State estimator	State estimator
NYISO	Linear	Hourly DAM / 5-minute RTM with intermediate passes	Dispatch optimization	Dispatch optimization
ISO-NE	Linear	Hourly DAM / 5-minute RTM	State estimator	State estimator

Model Formulation

maximize $-\Sigma_i c_i(p_i)$

subject to $\Sigma_i(p_i - d_i) - \ell = 0$

$\ell = \Sigma_i(\ell f_i(p_i - d_i)) - \ell^0$

$f_k = \Sigma_i(s_{ki}(p_i - d_i - \ell e_i)) \quad \forall\, k \in K$

$l_i \leq p_i \leq u_i,\ l_k \leq f_k \leq u_k \quad \forall\, i \in N, \forall\, k \in K$

Variables:

p_i = power generation at node i

ℓ = system losses

f_k = power flow on branch k

Parameters:

d_i = power demand at node i

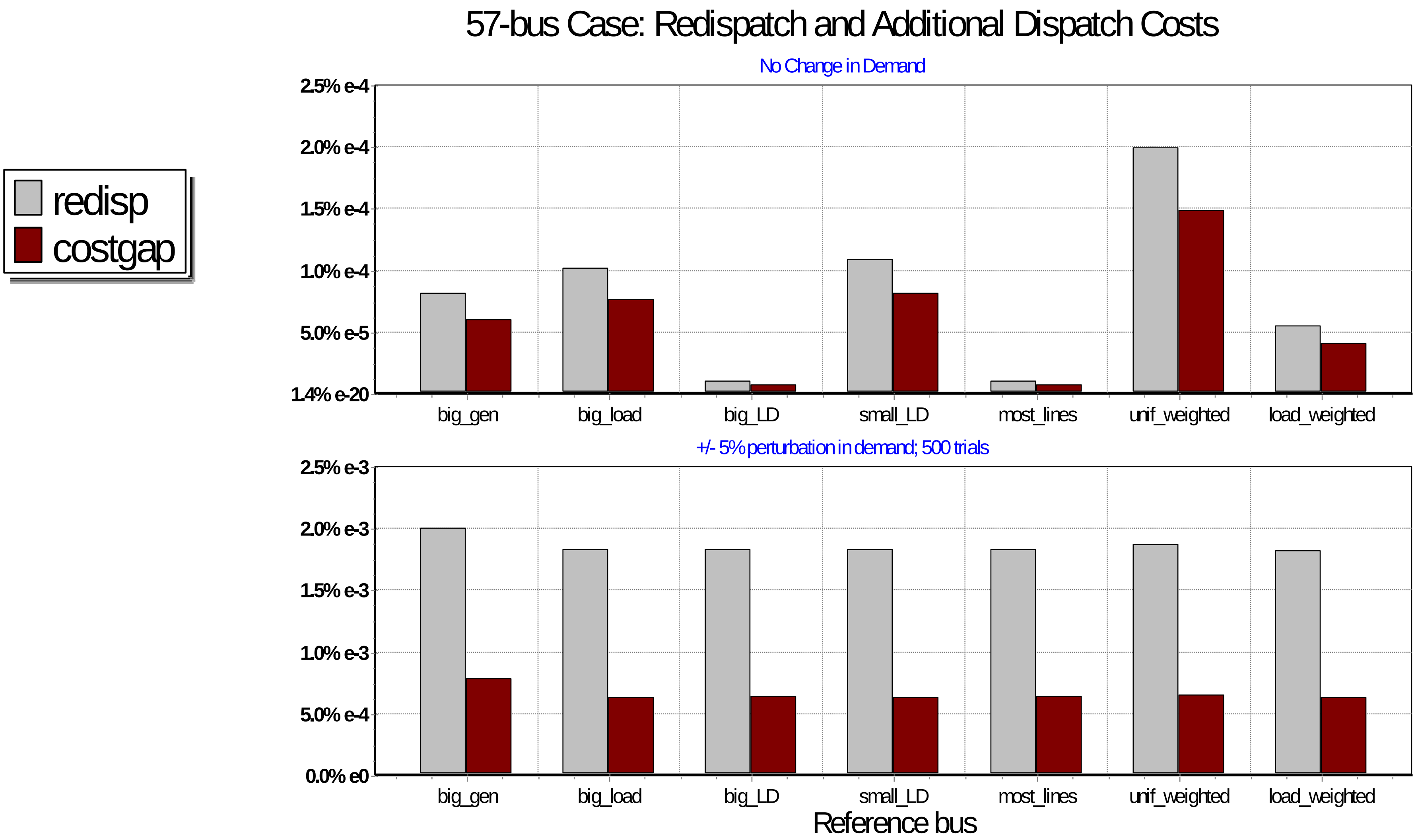
ℓf_i = loss factor ($\delta\ell / \delta p_i$)

ℓ^0 = loss constant

s_{ki} = shift factor ($\delta f_k / \delta p_i$)

e_i = loss distribution factor

Reference Bus Sensitivity



Proposed Loss Factor

From Ohm's law: $f_k = v_k i_k$ $\quad$ $\ell_k = (i_k)^2 r_k$

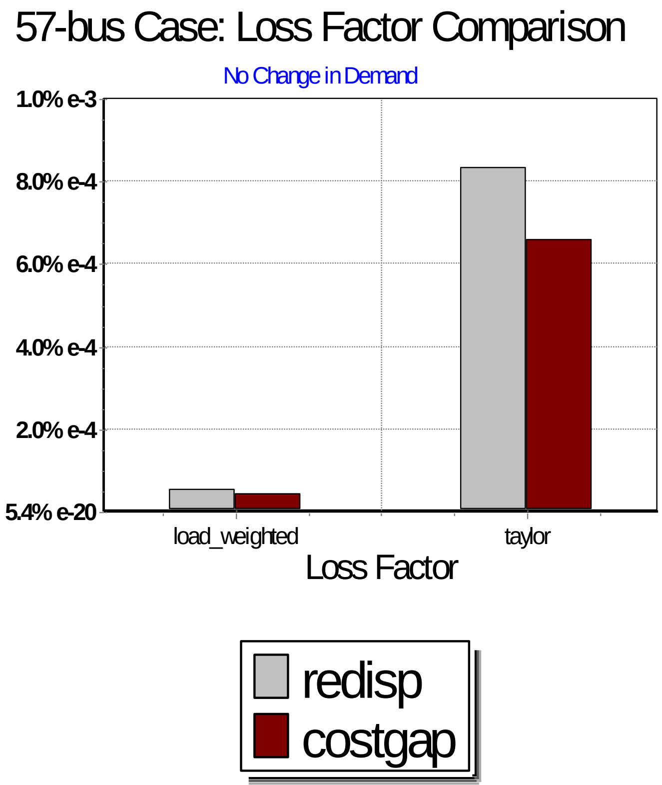
assume $v_k = 1$ $\quad \Rightarrow \quad$ $\ell_k = (f_k)^2 r_k$

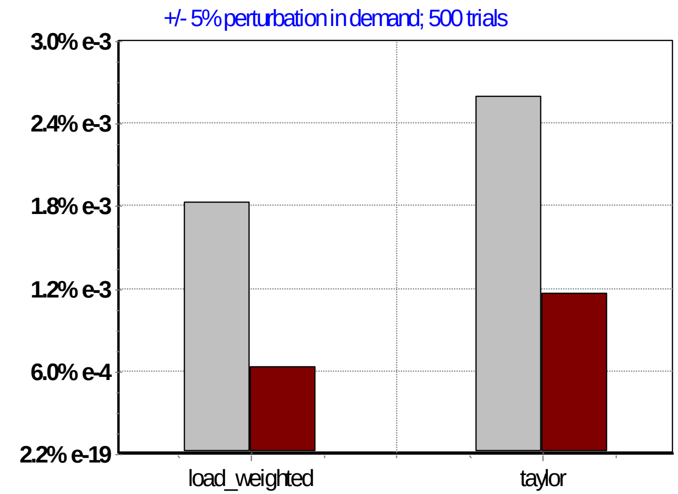
$$\ell f_i = \delta\ell / \delta p_i$$
$$= \delta\ell_1 / \delta p_i + \ldots + \delta\ell_k / \delta p_i$$

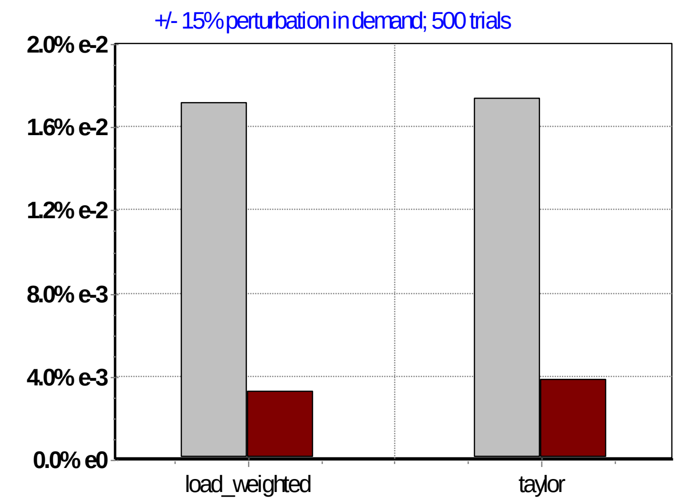
$$\delta\ell_k / \delta p_i = 2 r_k f^0{}_k (\delta f_k / \delta p_i)$$
$$= 2 r_k f^0{}_k s_{ki}$$

$$\ell f_i = \Sigma_k (2 r_k f^0{}_k s_{ki})$$

Numerical Derivative vs. Proposed Loss Factor

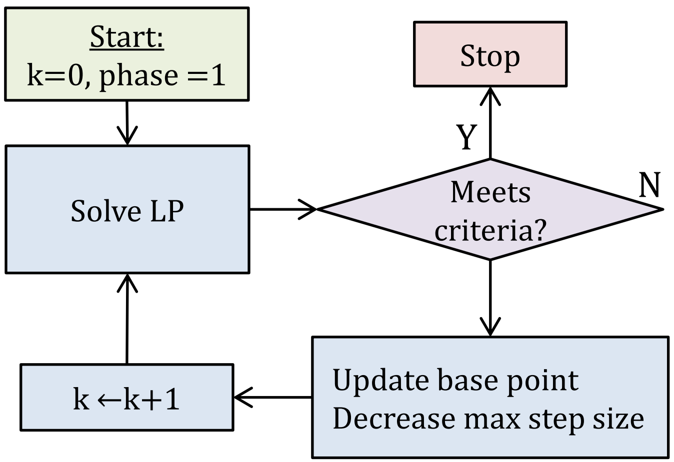






Sequential Linear Programming

Simple SLP Approach



Quadratic NLP

max $\quad -\Sigma_i c_i(p_i)$

s.t. $\quad \Sigma_i(p_i - d_i) - \ell = 0$

$\ell = \Sigma_k(r_k f_k^2)$

$f_k = \Sigma_i(s_{ki}(p_i - d_i - \ell e_i))$

$l_i \leq p_i \leq u_i, l_k \leq f_k \leq u_k$

$\forall i \in N, \forall k \in K$

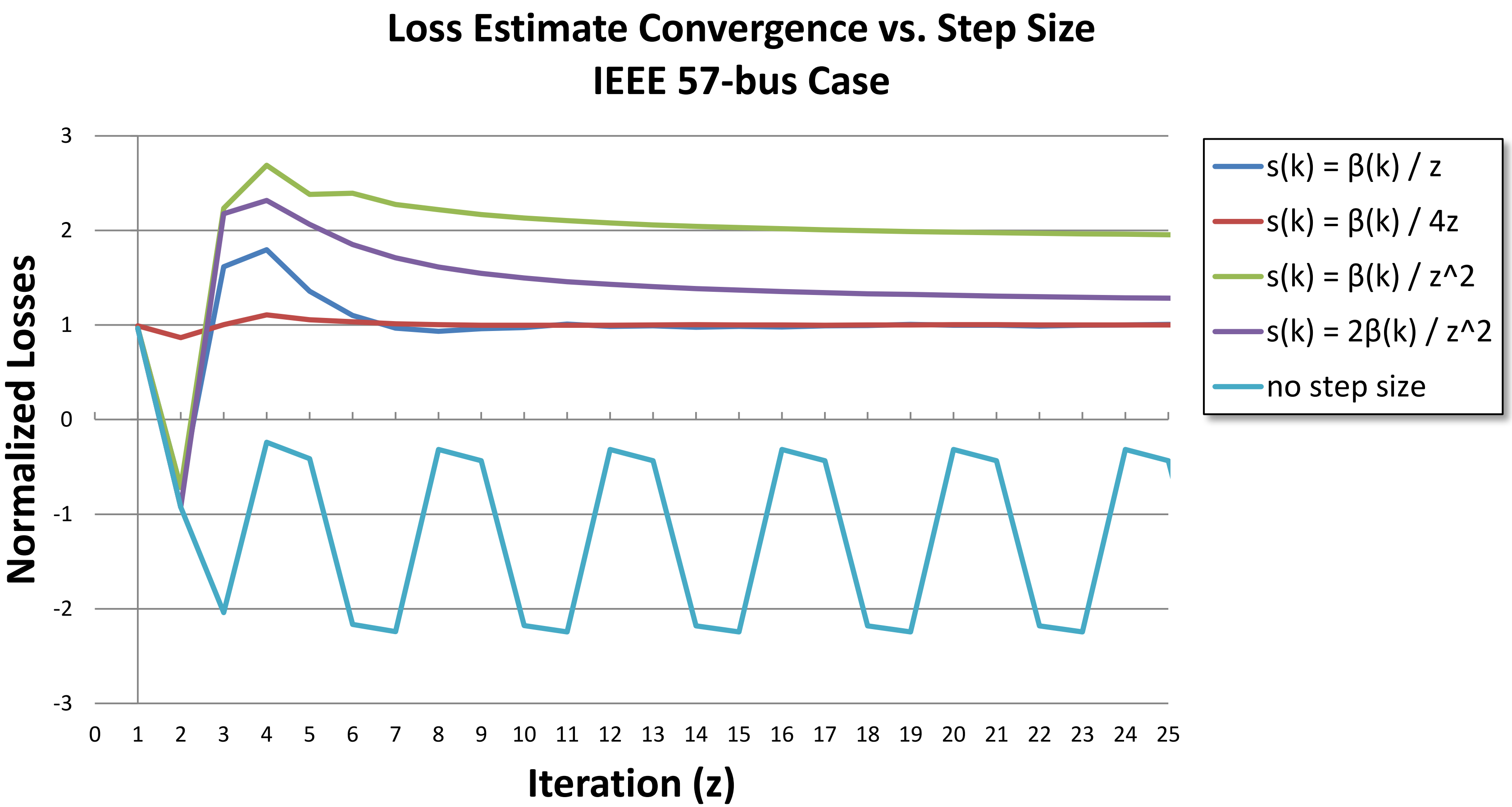
First order Taylor series of the quadratic constraint at f^0_k:

New constraint: $\ell = \Sigma_i(\ell f_i(p_i - d_i)) - \ell^0$

Where: $\ell f_i = \Sigma_k(2 r_k f^0_k s_{ki})$

$\ell^0 = \Sigma_k(r_k f^0_k f^0_k)$

Simple SLP Convergence



Desired Attributes

No cycling – Common of tendancy similar methods

Consistent pricing – Binding step size constraints affect LMP

Sensible loss values – Negative line losses, etc.

Feasibility – Avoid overly constraining the problem

Improvement – Does the update make sense?

Convergence speed – Would we rather solve something else?

Model losses implicitly on each branch

$\ell f_i = \delta\ell_1 / \delta p_i + \ldots + \delta\ell_k / \delta p_i$

$\ell f_{ki} = \delta\ell_k / \delta p_i$

$= 2 r_k f^0{}_k s_{ki}$

Solve LP for solution $\{\bar{p}_i, \bar{f}_k, \bar{\ell}\}$

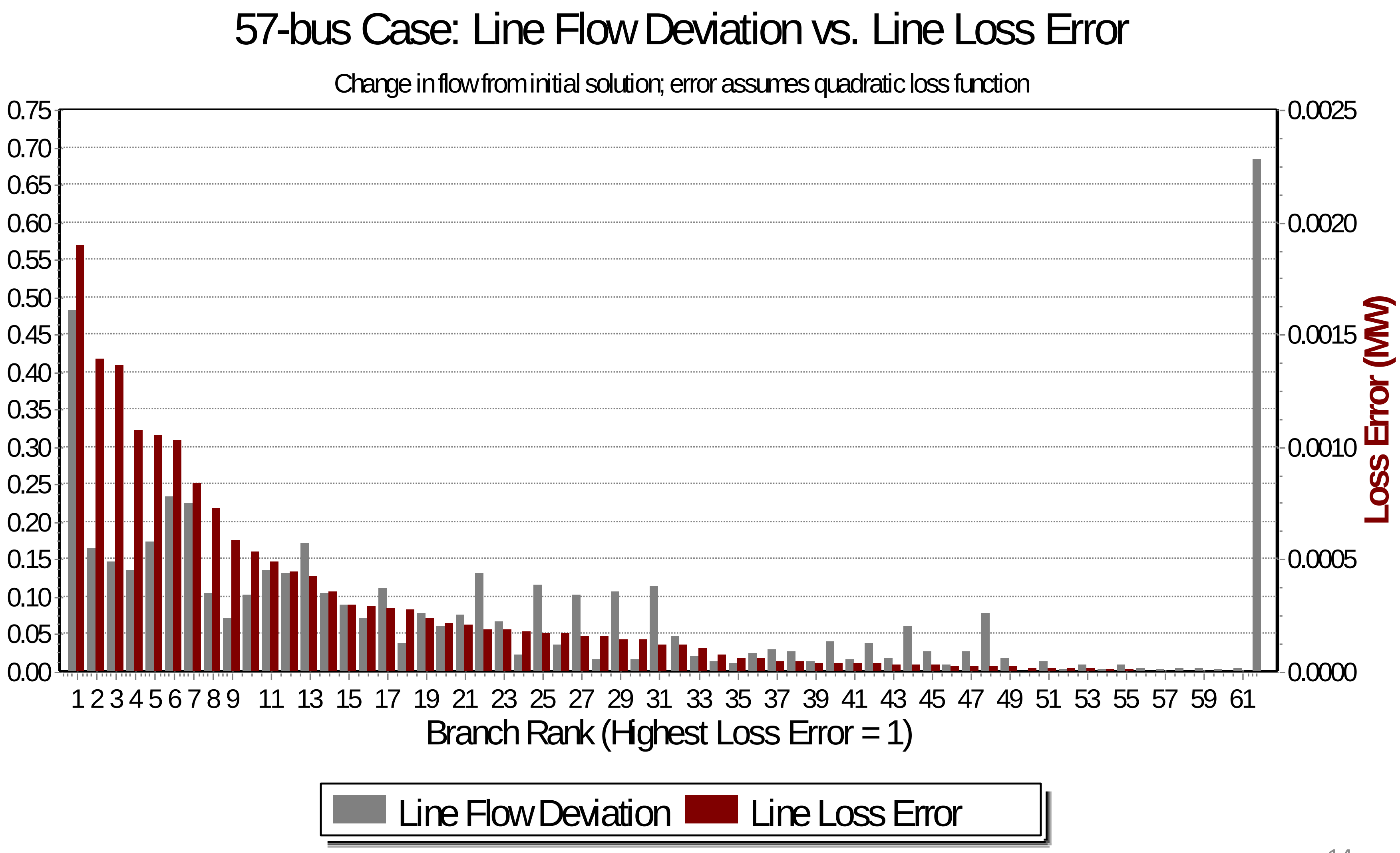
$\bar{\ell}_k = \Sigma_i(\ell f_{ki}(\bar{p}_i - d_i)) - \ell_k{}^0$ Linear losses on 'k'

$\tilde{\ell}_k = r_k(\bar{f}_k)^2$ Quad. losses on 'k'

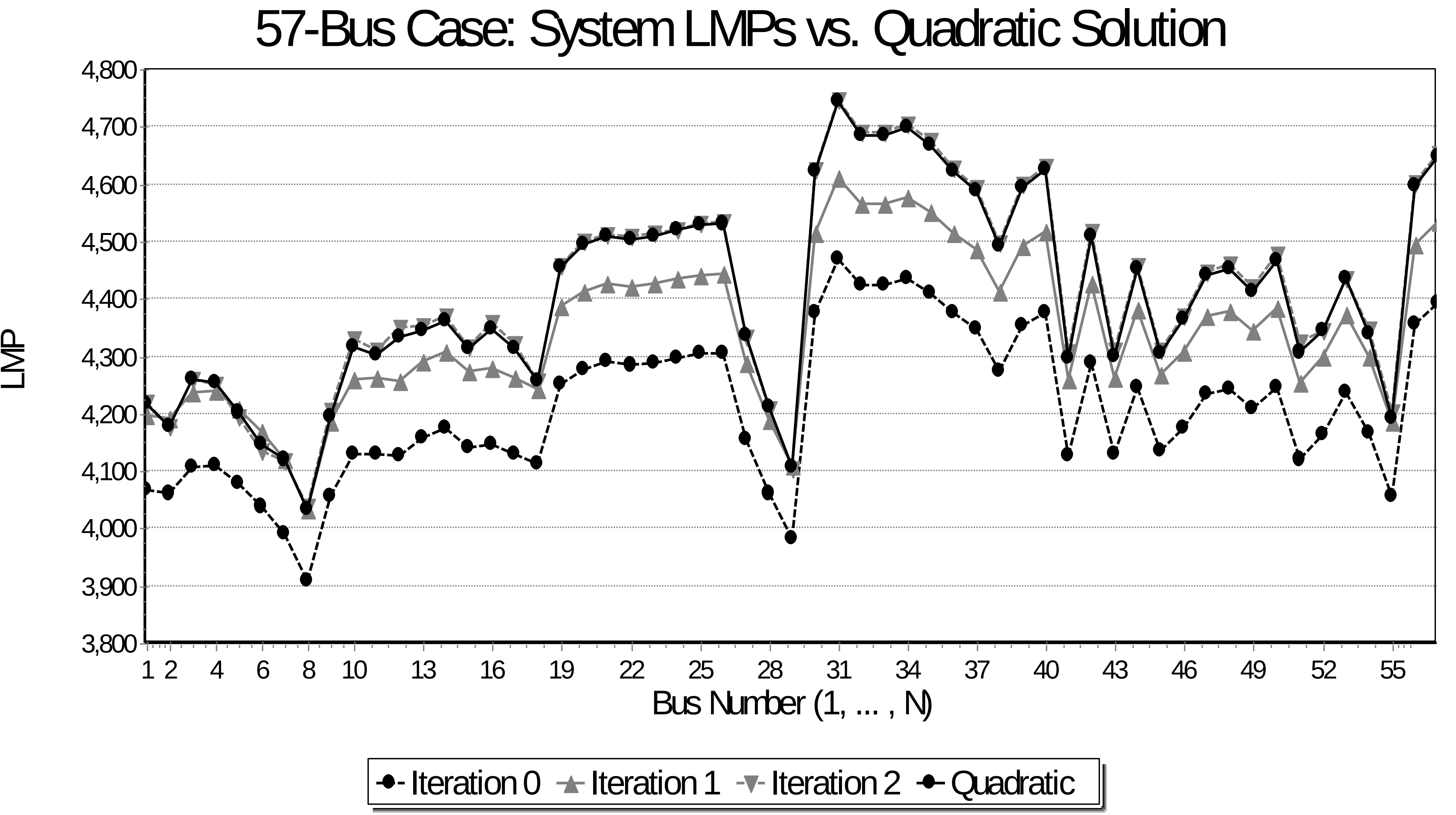
$\delta_k = |\bar{\ell}_k - \tilde{\ell}_k|$ Error on 'k'

$\Delta = \Sigma_k \delta_k$ Total error

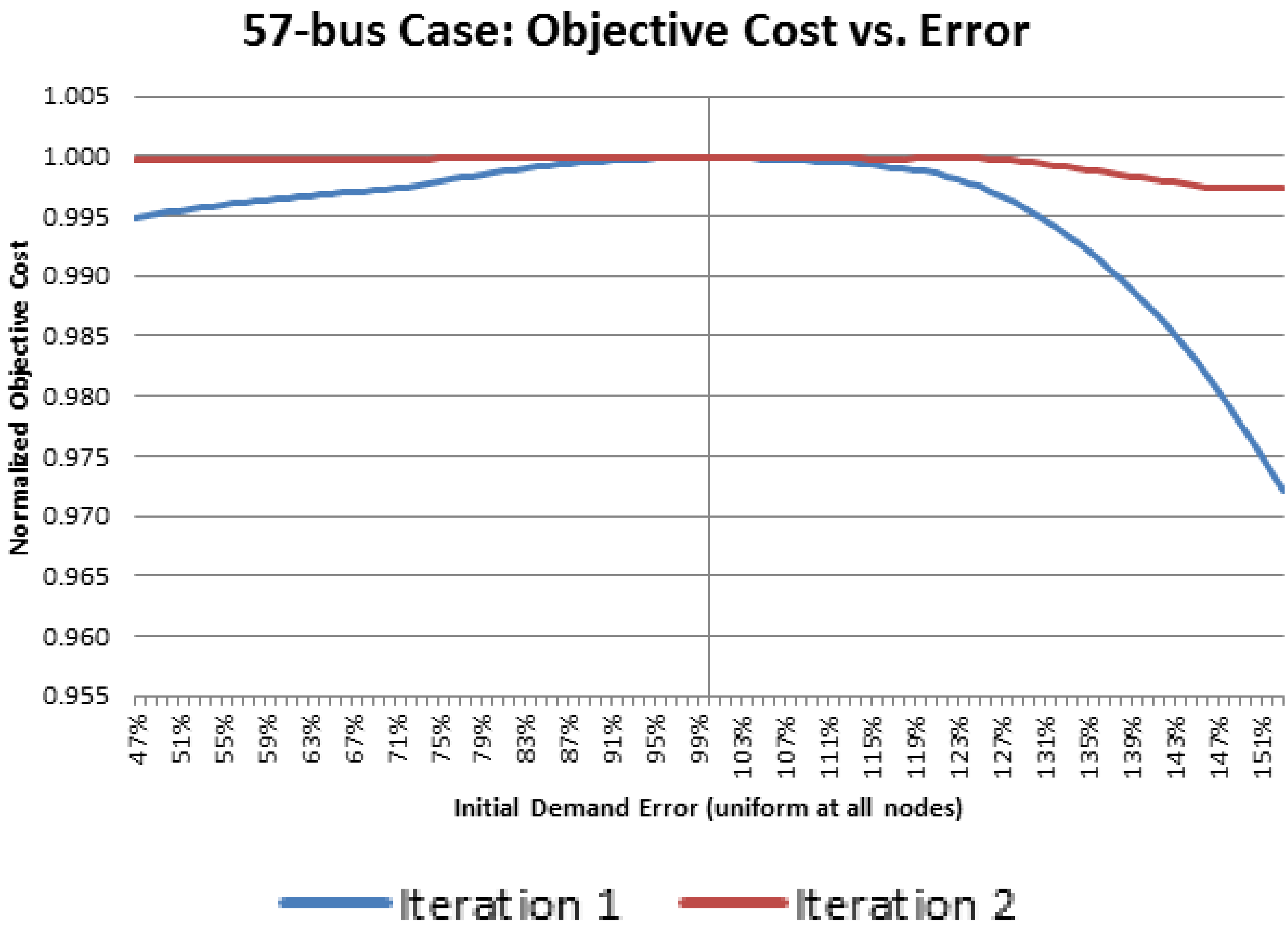
Power flow poorly predicts power loss



Improvement after one iteration



Initial Error Sensitivity

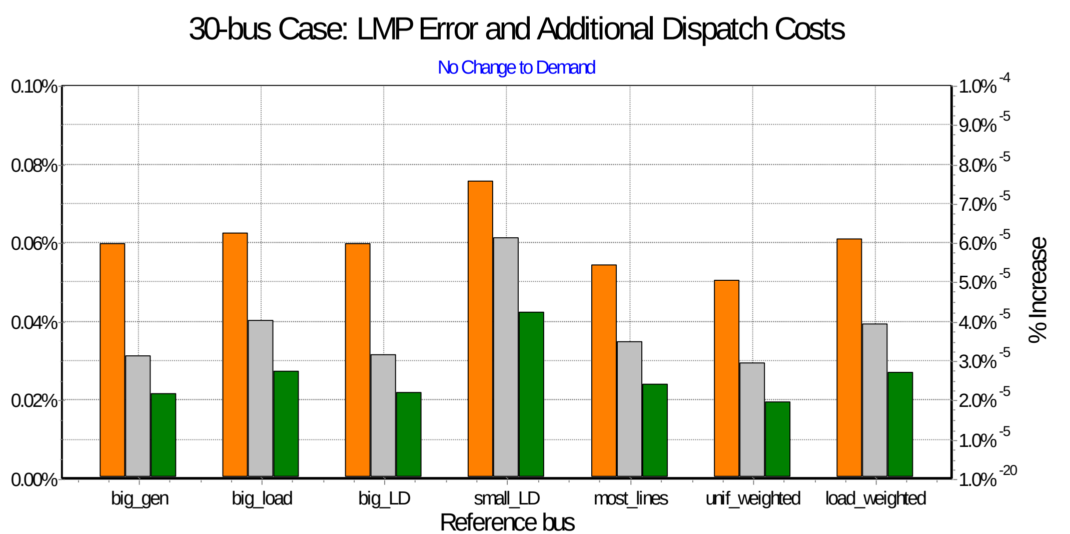


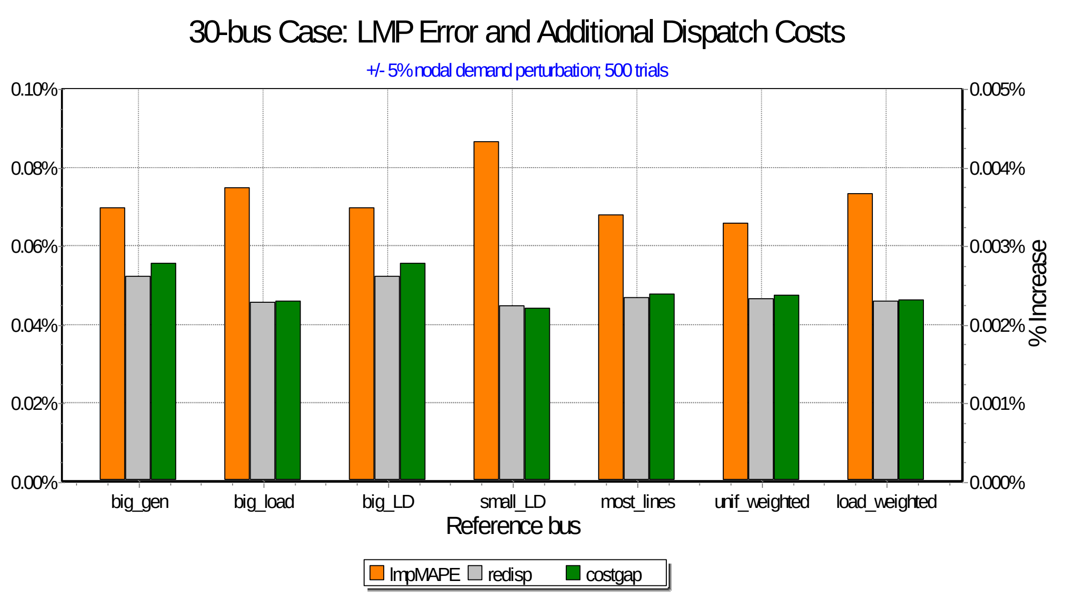
Summary

- Reformulated loss factor
- Improvement w/out solving AC equations
- Benefits to accurate loss modeling
  - prevent ad hoc adjustments, gaming
- Future work:
  - AC-feasible results
  - RTO scale modeling

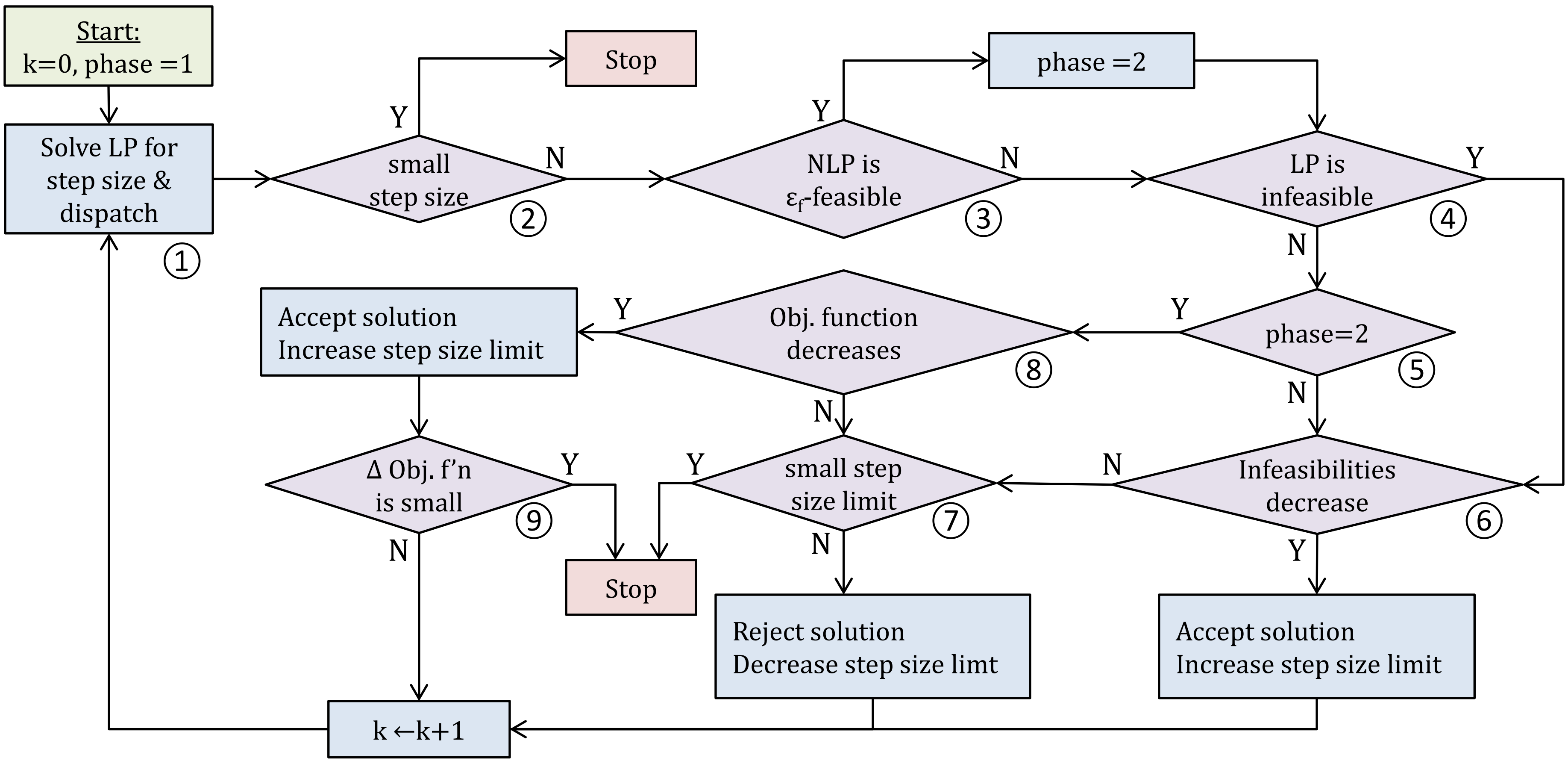
End

Reference Bus Sensitivity





Palacios-Gomez SLPR Algorithm

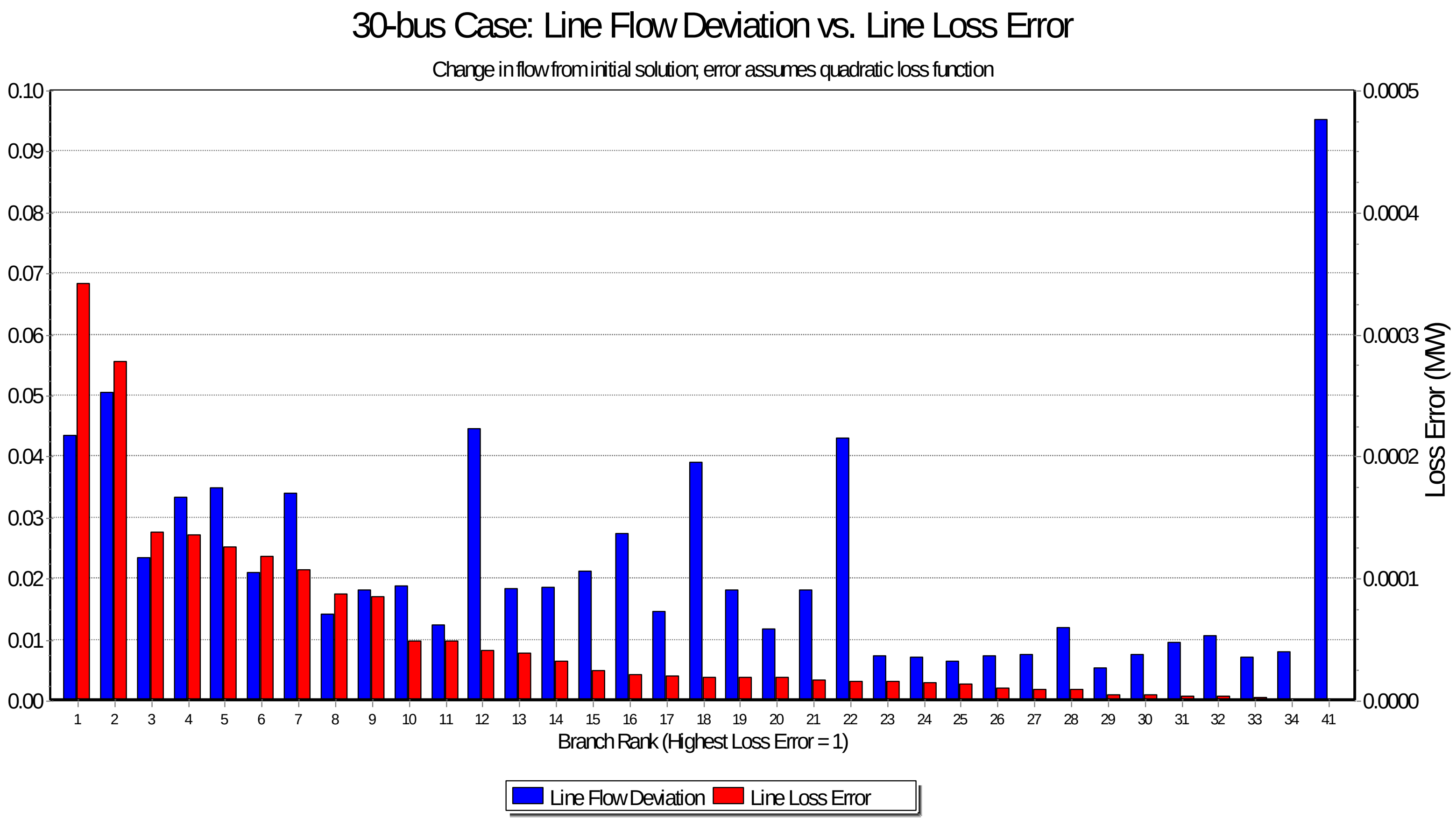


Palacios-Gomez SLPR Convergence

Case	Iterations	Accepted Solutions	Convergence Criteria	ε-Feasibility
14-bus	15	3	Small step size	No
30-bus	1	0	Small step size	No
57-bus	27	11	Small step size	No
118-bus	32	14	Small step size limit	No
300-bus	22	0	Small step size	No

*Algorithm begins with the ACOPF solution

Power flow poorly predicts power loss



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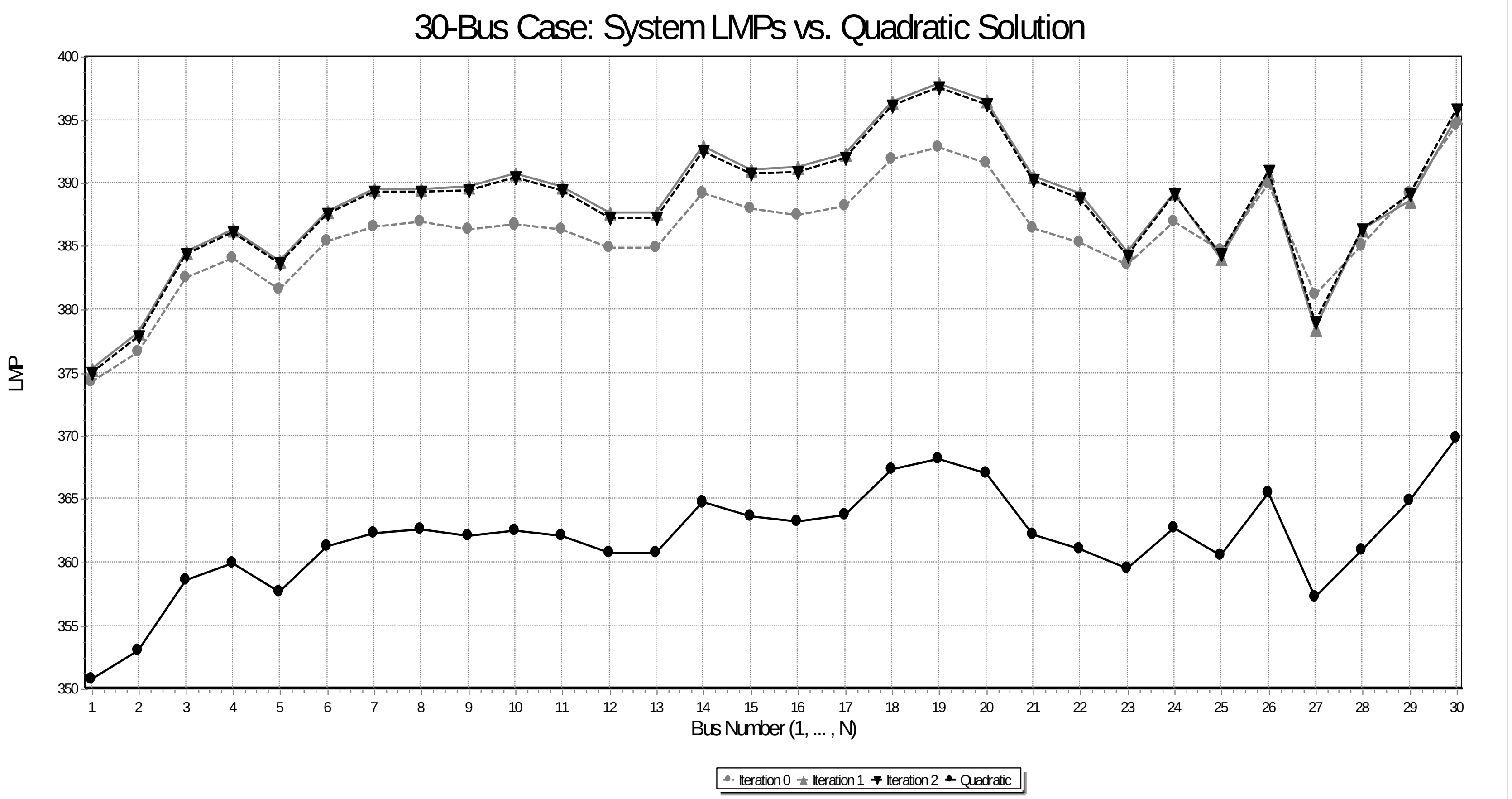

Initial Error Sensitivity