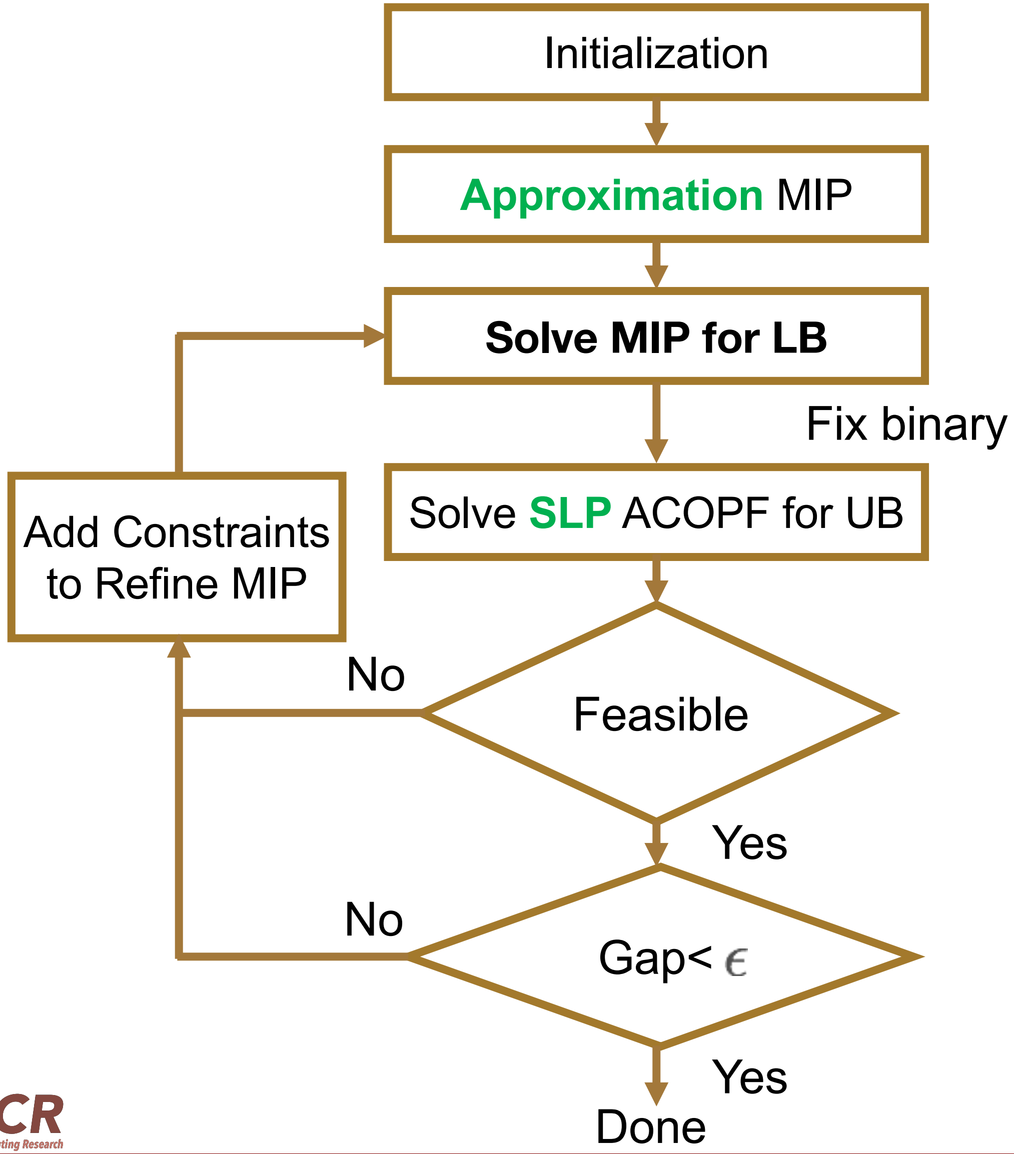
MINLP solved by Outer Approximation (OA)



Local Solution [R2]

CONTRIBUTIONS
GLOBAL SOLUTION METHOD

ACOPF Second-Order Cone Relaxation § (SOCR)

$$c_{b,b} \equiv (v_b^r)^2 + (v_b^j)^2 = v_b^2$$

$$c_{b,k} \equiv v_b^r v_k^r + v_b^j v_k^j = |v_b||v_k| \cos\theta_{b,k}$$

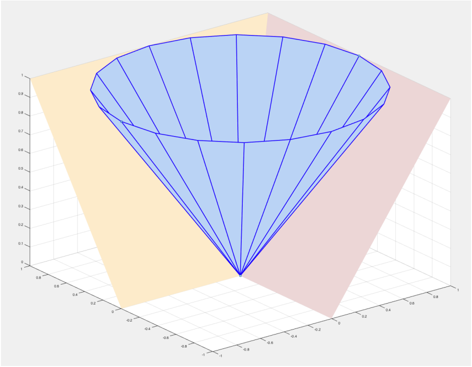
$$s_{b,k} \equiv v_b^r v_k^j - v_k^r v_b^j = -|v_b||v_k| \sin\theta_{b,k}$$

$$c_{b,k}^2 + s_{b,k}^2 = c_{b,b} c_{k,k} \quad \forall\, l = (b,k)$$

KVL-based constraints (next slide)

$$c_{b,k}^2 + s_{b,k}^2 \le c_{b,b} c_{k,k} \quad \forall\, l = (b,k)$$

$$c_{b,k} = c_{k,b},\; s_{b,k} = -s_{k,b} \quad \forall\, l = (b,k)$$



$$\min \sum_{g \in \mathcal{G}} [A_g^2 (p_g^G)^2 + A_g^1 p_g^G + A_g^0]$$

s.t.

$$\sum_{l \in \mathcal{L}_b^{in}} p_l^t + \sum_{l \in \mathcal{L}_b^{out}} p_l^f + G_b^{sh} c_{b,b} + P_b^D - \sum_{g \in \mathcal{G}_b} p_g^G = 0 \qquad \forall\, b$$

$$\sum_{l \in \mathcal{L}_b^{in}} q_l^t + \sum_{l \in \mathcal{L}_b^{out}} q_l^f - B_b^{sh} c_{b,b} + Q_b^D - \sum_{g \in \mathcal{G}_b} q_g^G = 0 \qquad \forall\, b$$

$$p_l^f = G_l^{ff} c_{b,b} + G_l^{ft} c_{b,k} - B_l^{ft} s_{b,k} \qquad \forall\, l$$

$$q_l^f = -B_l^{ff} c_{b,b} - B_l^{ft} c_{b,k} - G_l^{ft} s_{b,k} \qquad \forall\, l$$

$$p_l^t = G_l^{tt} c_{k,k} + G_l^{tf} c_{k,b} - B_l^{tf} s_{k,b} \qquad \forall\, l$$

$$q_l^t = -B_l^{tt} c_{k,k} - B_l^{tf} c_{k,b} - G_l^{tf} s_{k,b} \qquad \forall\, l$$

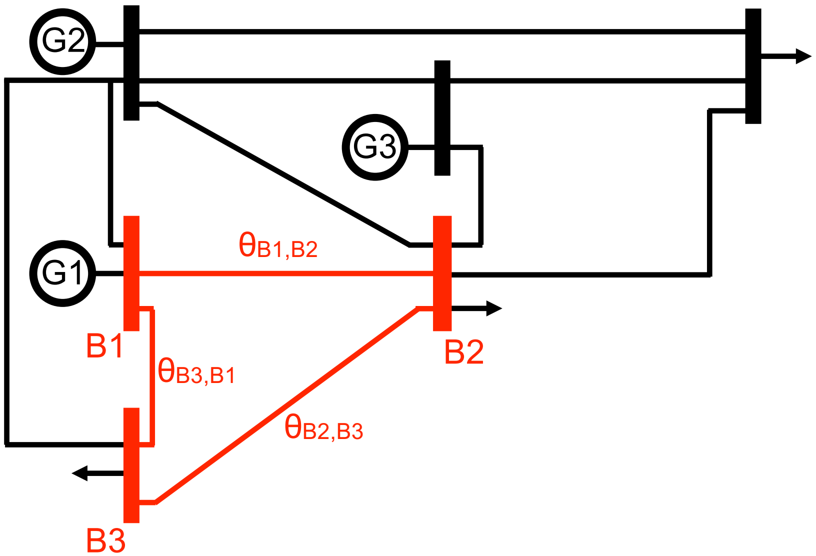
$$(p_l^f)^2 + (q_l^f)^2 \le (S_l^{max})^2,\; (p_l^t)^2 + (q_l^t)^2 \le (S_l^{max})^2 \qquad \forall\, l$$

$$(V_b^{min})^2 \le c_{b,b} \le (V_b^{max})^2 \qquad \forall\, b$$

$$P_g^{G,min} \le p_g^G \le P_g^{G,max},\; Q_g^{G,min} \le q_g^G \le Q_g^{G,max} \qquad \forall\, g$$

§ Second-Order Cone Relaxation (Jabr, 2006; Kocuk, 2015)

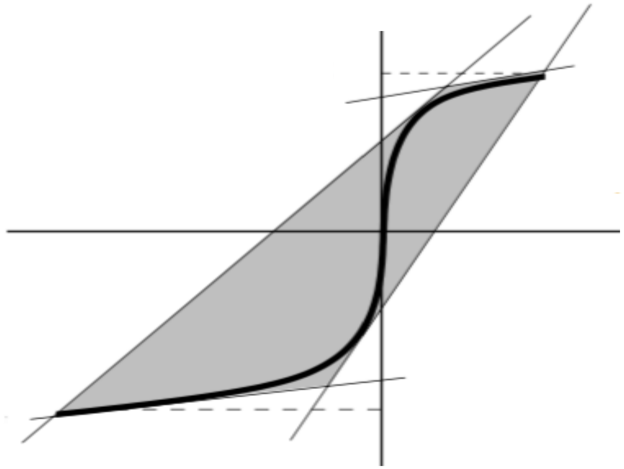
Improving the Lower Bound of SOCR [R3]



Cycle Constraints:
the sum of angle differences on each cycle equals to zero

$$\sum_{l \in \mathcal{L}_c} \theta_l = 0 \qquad \forall\, \mathcal{L}_c$$

$$\theta_l \equiv \theta_{b,k} = -\arctan\left(\frac{s_{b,k}}{c_{b,k}}\right) \qquad \forall\, l = (b,k)$$



Convex Relaxation of *arctan*:
(PW)Linear Over- and Under-Estimators
Optimality-Based Bound Tightening (OBBT)
Gradually Adding Cycle Constraints

Global ACOPF Performance

Case Name	Optimal Solution	Optimality Gap (%)	CPU Time (s)	Iteration Number
Case6ww	3126.36	0.008	0.26	4
Case14	8081.52	0.003	0.43	3
Case30	574.52	0.000	0.95	5
Case39	41864.18	0.005	1.21	3
Case57	41737.79	0.006	7.29	12
Case89	5817.60	0.009	46.2	44
Case118	129660.69	0.006	18.5	14
Case300	719725.10	0.009	82.7	49
NESTA Case6ww	3143.97	0.000	0.74	7
NESTA Case14	244.05	0.003	0.22	3
NESTA Case30	204.97	0.000	0.57	4
NESTA Case39	96505.52	0.009	3.00	8
NESTA Case57	1143.27	0.006	9.62	20
NESTA Case89	5819.81	0.009	55.8	57
NESTA Case118	3718.64	0.000	93.7	55
NESTA Case300	16891.28	0.000	138.2	26

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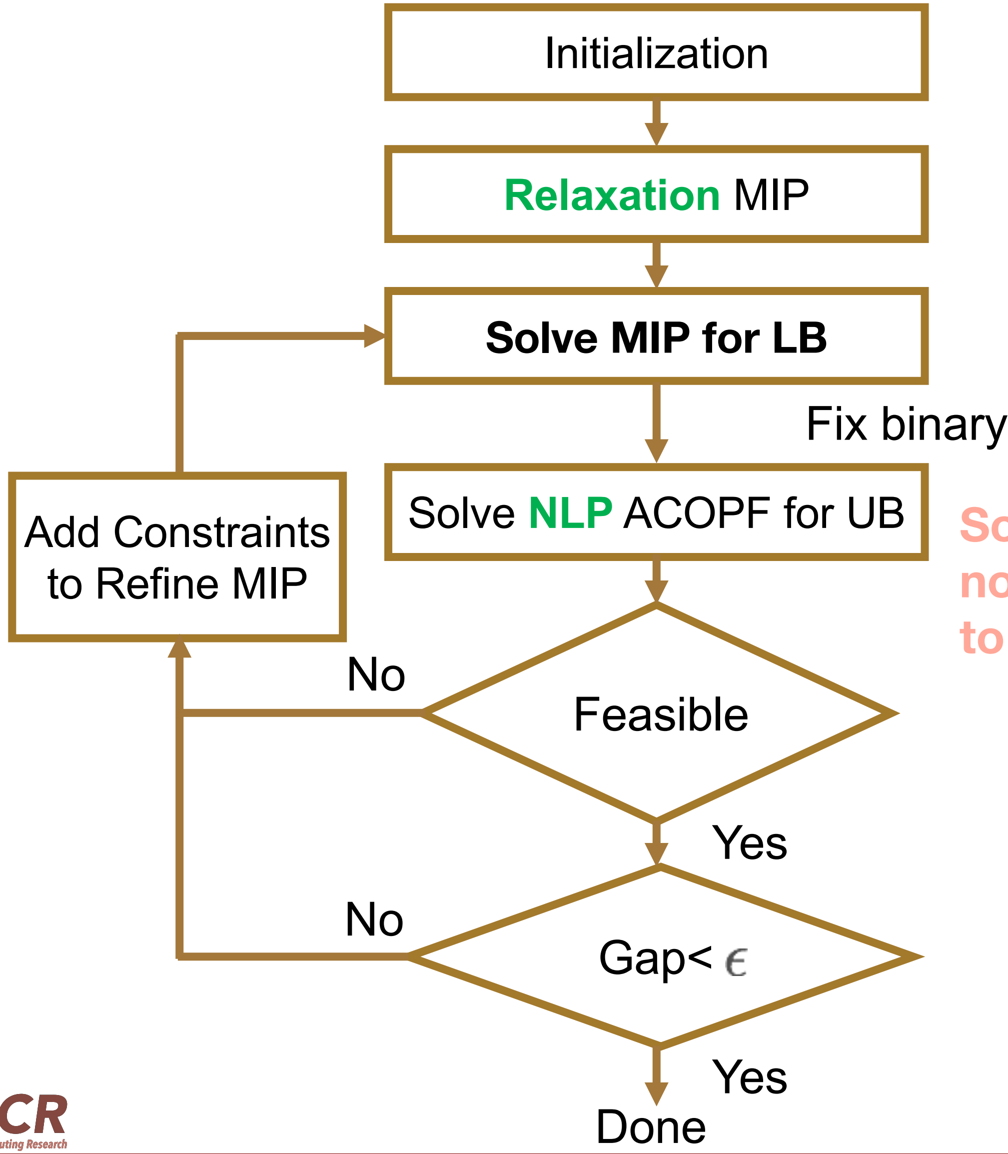
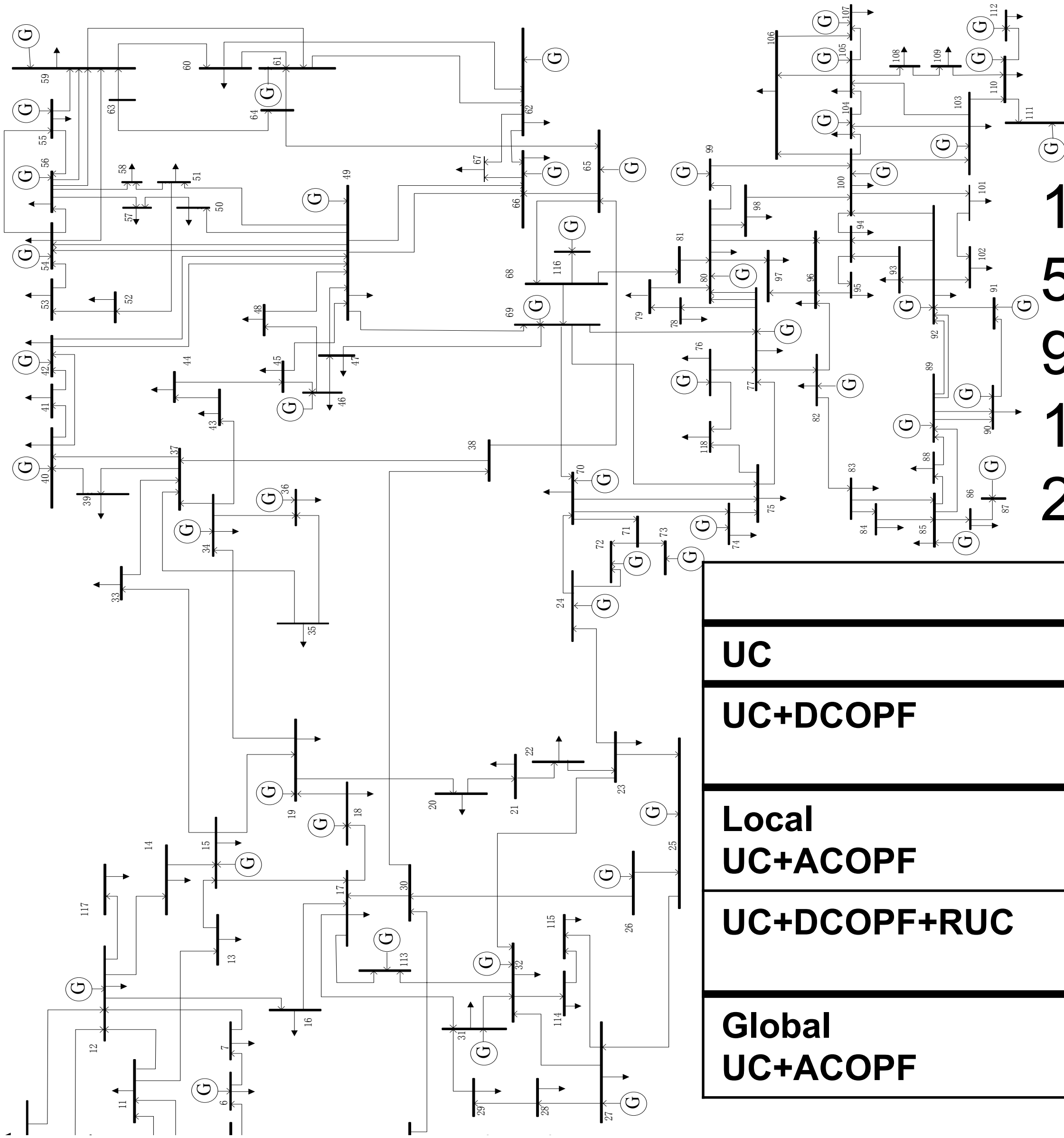


Solving nonlinear, non-convex AC OPF to global optimality? ✔

Global Solution [R4]

CONTRIBUTIONS
UC+ACOPF RESULTS



IEEE-118 §

- 118 nodes
- 54 generators
- 91 loads
- 186 network elements/lines
- 24-hour hourly commitment

	Cost ($)	AC Feasible?
UC	811,658 (base)	NO
UC+DCOPF	814,715 (+0.4%)	NO
Local UC+ACOPF	843,591 (+3.9%)	YES
UC+DCOPF+RUC	844,922 (+4.1%)	YES
Global UC+ACOPF	835,926 (+3.0%)	YES

- Key Takeaway: Results indicate considerable divergence between the market settlements and stability/reliability requirements

§ Data from Fu et al. (2006)

Computational Results (Local Method)

	UC	UC+DCOPF	UC+ACOPF		UC+DCOPF+RUC	
	MILP	MILP	MILP	SLP	MILP	SLP
Solution Time (s)						
6-Bus	0.13	0.21	0.88(3)	0.07(50)	1.02(1, 1)	0.06(33)
RTS-79	1.86	6.76	88.71(3)	0.75(36)	10.37(1, 2)	0.45(26)
IEEE-118	5.04	21.42	110.17(2)	5.06(46)	57.2(1, 1)	3.71(33)
Cost ($)						
6-Bus	101,270	106,987	101,763		102,523	
RTS-79	823,145	823,894	895,281		896,169	
IEEE-118	811,658	814,715	843,591		844,922	

- Most of the OA algorithm time spent in the MILP (MIP gap tolerance 0.1%)
- UC+ACOPF: 5x-15x slower than the UC+DCOPF
- UC+DCOPF+RUC: 1.5x-5x slower than the UC+DCOPF

Local v. Global UC+ACOPF Method

Case	Problem Formulation	Upper Bound	Lower Bound	Relative Gap (%)	CPU Time (s)
6-Bus	Global Local	101,763 101,763	101,655 -	0.11% *0.11%*	3.6 0.95
RTS-79	Global Local	895,096 895,281	893,967 -	0.13% *0.15%*	266.4 89.46
IEEE-118	Global Local	835,926 843,591	833,057 -	0.34% *1.25%*	8480 115.23

- **Note**: Thermal limits different in global solution method (apparent power thermal limit) and local solution method (current thermal limit) so a direct comparison (above) is *inexact*
- On the largest test case, the approximation method is over 70x faster, at the cost of 0.91% in relative optimality gap change

ONGOING WORK

Ongoing Work

- Study of global solution techniques applied to the PSV, RSV and RIV ACOPF formulations
- Implications on market settlements for including AC network constraints in the day-ahead
- Improving the performance of the MIP solution time in the OA algorithm (e.g., hybrid OA + branch-and-bound)
- Comparing the fidelity and computational performance to current market practices on larger scale, more realistic networks (GRIDDATA)

References

- [R1] A. Castillo, P. Lipka, J.-P. Watson, S.S. Oren, and R.P. O'Neill. "A Successive Linear Programming Approach to Solving the IV-ACOPF." Transactions on Power Systems (2015).
- [R2] A. Castillo, C. Laird, C. A. Silva-Monroy, J.-P. Watson, and R.P. O'Neill. "The Unit Commitment Problem with AC Optimal Power Flow Constraints." Transactions on Power Systems (2016).
- [R3] J. Liu, M. Bynum, A. Castillo, J.-P. Watson and C. Laird. "Global Solution of ACOPF Problems Using a Piecewise Outer-Approximation Approach Based on SOCP Relaxations." (2017) submitted.
- [R4] J. Liu, A. Castillo, J.-P. Watson, and C. Laird. "Global Solution Strategies for the Network-Constrained Unit Commitment (NCUC) Problem with Nonlinear AC Transmission Models." (2017) submitted.