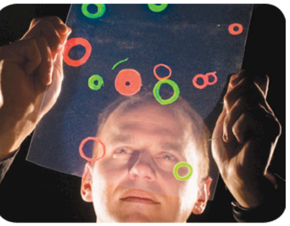
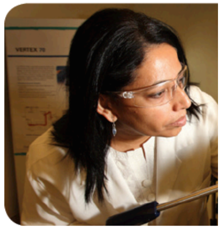


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Solving the Mixed Integer Non-linear Programming Problem of Unit Commitment on AC Power Systems

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U.S. DEPARTMENT OF ENERGY

NNSA National Nuclear Security Administration

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Overview

- Today's Practice
- Contributions
  - Overview
  - Local Solution Method
  - Global Solution Method
  - UC+ACOPF Results
- Ongoing Work

TODAY'S PRACTICE

Issues in Day-Ahead Markets

- Operational Challenges
  - Committing Least Cost + Maintaining Reliability
  - Out-of-Merit Reliability Commitments
  - Improving convergence between day-ahead and real-time prices

- Algorithmic Challenges
  - Accounting for reliability needs in dispatch and pricing optimization
  - Better physical representation of the generating units and underlying network

Unit Commitment in the Day-Ahead Market

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Current Practices	Proposed Approach
UC/Security-Constrained UC • Copper-plate (no network/single node) • Ignores congestion; requires cutsets to proxy capacity limits on network • Most tractable **SCUC DCOPF** • Real power flows only (proportional to current) • BΘ (full) or PTDF (compact) approach **Extensions:** • Accounts for losses • Nomograms/cutsets to proxy reliability requirements	**SCUC ACOPF** • Co-optimizes real and reactive power dispatch • Accounts for commitments needed for blackstart service, reactive support, voltage support, and interface control • Nonlinear, nonconvex on meshed networks

The link between physics and prices

- Locational marginal pricing (LMP) is the spot price of electricity
- Dual variable/Lagrange multiplier (λ_n) to real power balancing at all buses

$$p_n - p_n^d + p_n^g = 0 \qquad (\lambda_n)$$

ACOPF
$$p_n = |v_n| \sum_{m \in \mathcal{N}} |v_m| \left(G_{nm} \cos \theta_{nm} + B_{nm} \sin \theta_{nm} \right)$$

DCOPF
$$p_n = \sum_{m \in \mathcal{N}} \left(B_{nm} \theta_{nm} \right) \approx |\tilde{v}_n| \sum_{m \in \mathcal{N}} |\tilde{v}_m| \left(B_{nm} \theta_{nm} \right)$$

DCOPF with losses
$$p_n = \sum_{m \in \mathcal{N}} \left(G_{nm} \left(\theta_{nm} \right)^2 \Big/ 2 + B_{nm} \theta_{nm} \right)$$

The LMP incorporates the marginal cost of supplying the next MW of load for a given location in time; includes

1. marginal unit cost,
2. cost of network congestion (due to thermal line limits), and
3. cost of real power losses on the network

CONTRIBUTIONS

CONTRIBUTIONS
OVERVIEW

UC+ACOPF: MINLP

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Min Production Costs + Startup Costs + No-Load Costs

subject to

AC Network Limits

Real power balancing

Reactive power balancing

Voltage magnitude bounds

Thermal line limits

Spinning reserves

Apparent Power Production Limits §

Max/min real/reactive power generation

Ramp up/down rates on real power

Minimum up/down time

System Data

Nodal voltage limits

Reserve requirements

Real/reactive power load

Transformer tap ratio and phase-shifters

Thermal line limits and line R/X/B

Shunts

Generator Data

Synchronous condensers

T0 state and startup lags

Minimum up/down time

Ramp up/down limits

Startup/shutdown ramp limits

Min/max real/reactive power limits

§ Extends Morales-España, Latorre, and Ramos, "Tight and compact MILP formulation for the thermal unit commitment problem," *IEEE Trans. on Power Syst.*, vol. 28, no. 4, pp. 4897–4908, 2013.

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Nodal Power Balancing is Nonconvex

- Polar Power-Voltage Power Flow Formulation (**PSV**)

$$|v_{n,t}| \sum_{m \in \mathcal{N}} |v_{m,t}| \left(G_{nm} \cos\theta_{nm,t} + B_{nm} \sin\theta_{nm,t}\right) - p^+_{n,t} + p^-_{n,t} = 0, \qquad \forall n \in \mathcal{N}$$

$$|v_{n,t}| \sum_{m \in \mathcal{N}} |v_{m,t}| \left(G_{nm} \sin\theta_{nm,t} - B_{nm} \cos\theta_{nm,t}\right) - q^+_{n,t} + q^-_{n,t} = 0, \qquad \forall n \in \mathcal{N}$$

- Rectangular Power-Voltage Power Flow Formulation (**RSV**)

$$v^r_{n,t} \sum_{m \in \mathcal{N}} \left(G_{nm} v^r_{m,t} - B_{nm} v^j_{m,t}\right) + v^j_{n,t} \sum_{m \in \mathcal{N}} \left(G_{nm} v^j_{m,t} + B_{nm} v^r_{m,t}\right) - p^+_{n,t} + p^-_{n,t} = 0, \quad \forall n \in \mathcal{N}$$

$$v^j_{n,t} \sum_{m \in \mathcal{N}} \left(G_{nm} v^r_{m,t} - B_{nm} v^j_{m,t}\right) - v^r_{n,t} \sum_{m \in \mathcal{N}} \left(G_{nm} v^j_{m,t} + B_{nm} v^r_{m,t}\right) - q^+_{n,t} + q^-_{n,t} = 0, \quad \forall n \in \mathcal{N}$$

- Rectangular Current Injection Formulation (**RIV**)

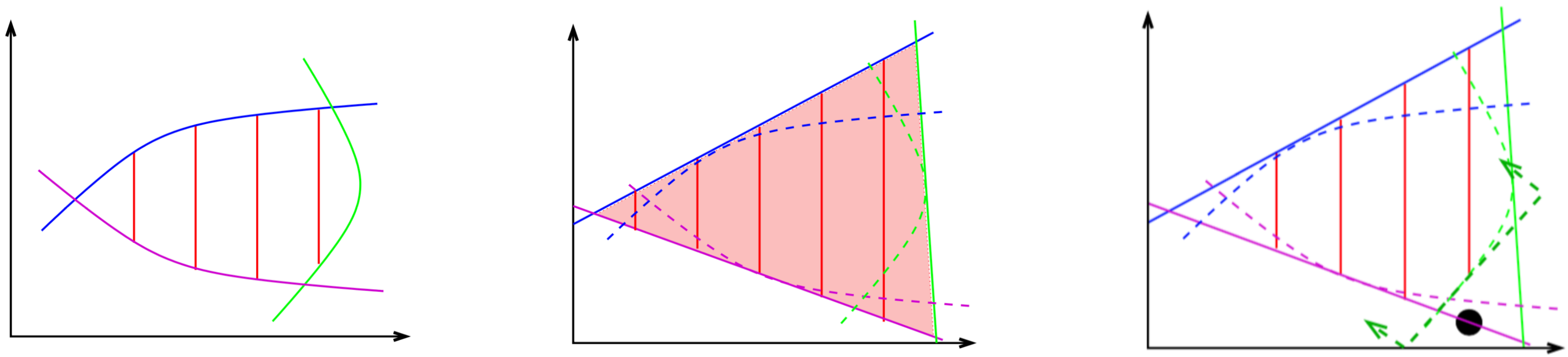
$$i^r_{n,t} - \left(\sum_{k(n,\cdot) \in \mathcal{F}} i^r_{k(n,m),t} + G^s_n v^r_{n,t} - B^s_n v^j_{n,t} \right) = 0, \left(v^r_{n,t} i^r_{n,t} + v^j_{n,t} i^j_{n,t} \right) - p^+_{n,t} + p^-_{n,t} = 0, \quad \forall n \in \mathcal{N}$$

$$i^j_{n,t} - \left(\sum_{k(n,\cdot) \in \mathcal{F}} i^j_{k(n,m),t} + G^s_n v^j_{n,t} + B^s_n v^r_{n,t} \right) = 0, \left(v^j_{n,t} i^r_{n,t} - v^r_{n,t} i^j_{n,t} \right) - q^+_{n,t} + q^-_{n,t} = 0, \quad \forall n \in \mathcal{N}$$

MINLP solved by Outer Approximation § (OA)

$$\begin{cases} minimize_x \ f(x), \\ subject\ to \ g(x) \leq 0, \\ \qquad x \in X, \\ \qquad x_i \in \mathbb{Z}, \forall i \in I \end{cases}$$

$f: \mathbb{R}^n \rightarrow \mathbb{R}, g: \mathbb{R}^n \rightarrow \mathbb{R}^m$ are twice continuously differentiable functions,
$X \subset \mathbb{R}^n$ is a bounded polyhedral set, and
$I \subseteq \{1, \dots, n\}$ is the index set of integer variables



§ Outer Approximation Algorithm (Duran and Grossman, 1986); Graphics (Belotti et al., 2013)

CONTRIBUTIONS

LOCAL SOLUTION METHOD

Successive Linear Programming (SLP) [R1]

MIN Piecewise linear cost function with penalty factors

s.t.

Line Current Flows

$$i^r_{k(n,m)} = \mathrm{Re}\left(Y^k_{1,1} v_n + Y^k_{1,2} v_m\right),\ i^r_{k(m,n)} = \mathrm{Re}\left(Y^k_{2,1} v_n + Y^k_{2,2} v_m\right) \quad \forall k \in K$$

$$i^j_{k(n,m)} = \mathrm{Im}\left(Y^k_{1,1} v_n + Y^k_{1,2} v_m\right),\ i^j_{k(m,n)} = \mathrm{Im}\left(Y^k_{2,1} v_n + Y^k_{2,2} v_m\right) \quad \forall k \in K$$

Network Current Balancing

$$i^r_n - \left(\sum_{k(n,\cdot)} i^r_{k(n,m)} + G^{sh}_n v^r_n - B^{sh}_n v^j_n\right) = 0 \quad \forall n \in N$$

$$i^j_n - \left(\sum_{k(n,\cdot)} i^j_{k(n,m)} + G^{sh}_n v^j_n + B^{sh}_n v^r_n\right) = 0 \quad \forall n \in N$$

Nodal Power Injections

First-order Taylor series

Generator Limits

Inequality constraints with slack variables

Nodal Voltage Magnitude Limits

Outer approximation,
First-order Taylor series,
Step-size bounds,
Tangential cutting planes, &
Inequality constraints with slack variables

Thermal Line (Flowgate) Limits

Set reduction, Outer approximation,
First-order Taylor series,
Tangential cutting planes, &
Inequality constraints with slack variables

SLP Convergence Properties §

(1) A KKT point to the ACOPF is found

(2) The SLP optimal solution is ACOPF feasible but not optimal

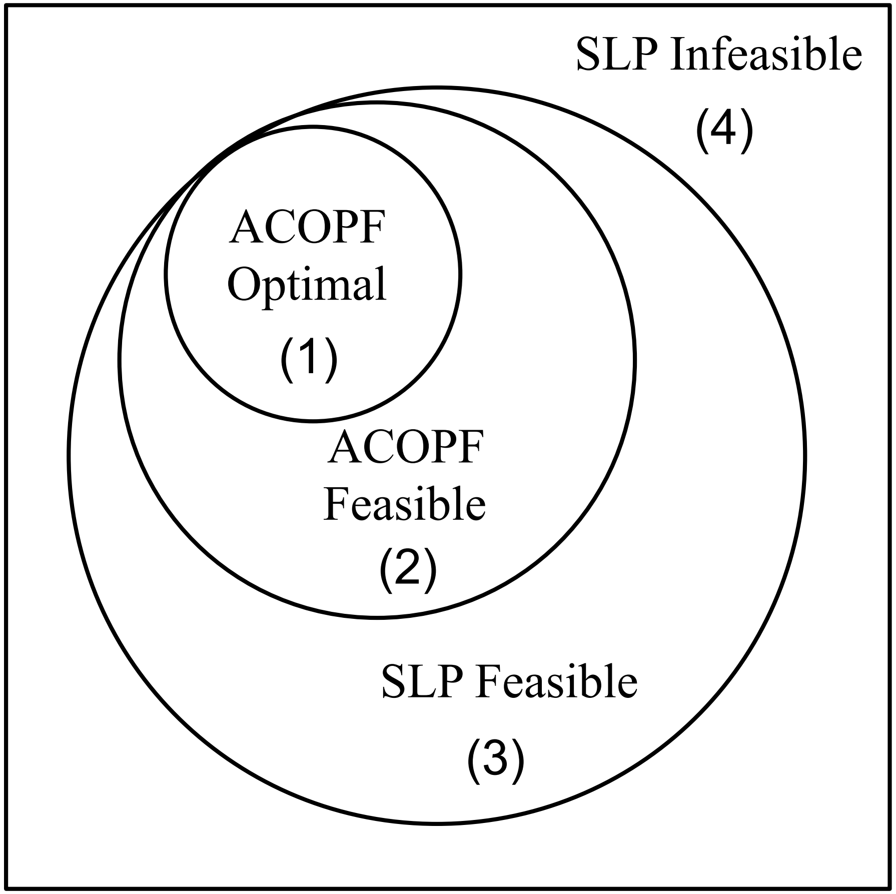
- Still a useful solution; may be better than a DCOPF with AC feasibility or decoupled OPF solution

(3) The SLP optimal solution is ACOPF infeasible

- Active penalties present
- Solution may be useful depending upon whether the violated limits are “soft” or “hard”

(4) The SLP is infeasible

- The ACOPF may have no solution
- The SLP requires a better initialization



§ Extends Theorem 10.3.1 of Bazaraa et al. (2006)

Time Complexity Performance $\Theta\left(|\mathcal{N}|^p\right)$

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Baseline	Best-Case Simulations p	R^2	RMSE (s)	All Converged Simulations p	R^2	RMSE (s)
NLP/KNITRO	1.42	0.83	1.46	1.47	0.82	1.40
NLP/IPOPT	1.13	0.95	0.60	1.34	0.97	0.50
SLP/CPLEX	0.97	0.99	0.20	1.01	0.98	0.33
SLP/Gurobi	1.01	0.99	0.21	1.03	0.98	0.33
Thermally Constrained						
NLP/KNITRO	1.39	0.88	1.13	1.39	0.89	1.08
NLP/IPOPT	1.11	0.98	0.36	1.22	0.97	0.50
SLP/CPLEX	0.99	0.99	0.17	1.00	0.98	0.31
SLP/Gurobi	1.06	0.99	0.23	1.05	0.97	0.36

- Running time increases linearly with the network size (p=1 corresponds to a linear algorithmic scaling) for the SLP algorithm
- *Potentially applicable in the strict time frames of the real-time markets*

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