Error Bounds on Power Flow Linearizations: A Convex Relaxation Approach

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The Power Flow Equations

- Model the relationship between the voltages and the power injections

AC power flow equations	Voltages: $V_i = \|V_i\| \angle \theta_i$

$$P_i = |V_i| \sum_{k=1}^{n} |V_k| \left(\mathbf{G}_{ik} \cos\left(\theta_i - \theta_k\right) + \mathbf{B}_{ik} \sin\left(\theta_i - \theta_k\right)\right)$$

$$Q_i = |V_i| \sum_{k=1}^{n} |V_k| \left(\mathbf{G}_{ik} \sin\left(\theta_i - \theta_k\right) - \mathbf{B}_{ik} \cos\left(\theta_i - \theta_k\right)\right)$$

- Central to many power system optimization problems
  - Optimal power flow, unit commitment, voltage stability, contingency analysis, transmission switching, etc.

The Power Flow Equations

> "Today, 50 years after the problem was formulated, we still do not have a fast, robust solution technique for the full ACOPF."
>
> R.P. O'Neill, Chief Economic Advisor, US Federal Energy Regulatory Commission, 2013.

DC Power Flow Approximation

- Linearization of the power flow equations

$$P_i = |V_i| \sum_{k=1}^{n} |V_k| \left(\mathbf{G}_{ik} \cos(\theta_i - \theta_k) + \mathbf{B}_{ik} \sin(\theta_i - \theta_k)\right)$$

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- Advantages:
  - Fast and reliable solution using linear programming
- Disadvantages:
  - No consideration of voltage magnitudes or reactive power
  - Approximation error

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DC Power Flow Approximation

- Linearization of the power flow equations

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DC Power Flow Approximation

- Linearization of the power flow equations

$$\color{blue}{P_i} = \overset{1}{|V_i|} \sum_{k=1}^{n} \overset{1}{|V_k|} \left(\overset{0}{\mathbf{G}_{ik}} \cos\left(\theta_i - \theta_k\right) + \mathbf{B}_{ik} \overset{(\theta_i - \theta_k)}{\sin} \left(\theta_i - \theta_k\right)\right)$$

$$\cancel{Q_i = |V_i| \sum_{k=1}^{n} |V_k| \left(\mathbf{G}_{ik} \sin\left(\theta_i - \theta_k\right) - \mathbf{B}_{ik} \cos\left(\theta_i - \theta_k\right)\right)}$$

$$\Rightarrow \quad \color{blue}{P_i} = \sum_{k=1}^{n} \mathbf{B}_{ik} \left(\theta_i - \theta_k\right)$$

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  - Fast and reliable solution using linear programming
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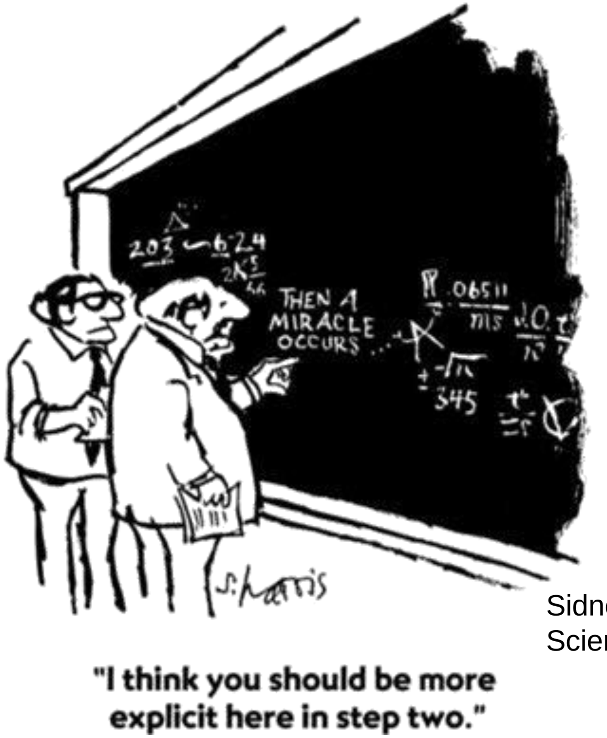
Current Industry Practice

Clear markets using the DC power flow approximation	→ Engineering intuition and heuristics →	Feasible operating point for the AC power flow equations



Sidney Harris,
Science Cartoons Plus

Summary

$$\min_{|V|,\theta} \sum_{i \in \mathcal{G}} \left(c_{2i} {P_{Gi}}^2 + c_{1i} P_{Gi} + c_{0i} \right)$$

Generation Cost

$$\text{subject to} \quad P_{Gi}^{\min} \leq P_{Gi} \leq P_{Gi}^{\max}$$

$$Q_{Gi}^{\min} \leq Q_{Gi} \leq Q_{Gi}^{\max}$$

$$V_i^{\min} \leq |V_i| \leq V_i^{\max}$$

$$\theta_{ik}^{\min} \leq \theta_i - \theta_k \leq \theta_{ik}^{\max}$$

$$|P_{flow,ik}| \leq P_{flow,ik}^{\max}$$

Engineering Constraints

Physical Laws

$$P_i = |V_i| \sum_{k=1}^{n} |V_k| \left(\mathbf{G}_{ik} \cos\left(\theta_i - \theta_k\right) + \mathbf{B}_{ik} \sin\left(\theta_i - \theta_k\right) \right)$$

$$Q_i = |V_i| \sum_{k=1}^{n} |V_k| \left(\mathbf{G}_{ik} \sin\left(\theta_i - \theta_k\right) - \mathbf{B}_{ik} \cos\left(\theta_i - \theta_k\right) \right)$$

Summary

$$\min_{|V|,\theta} \quad \sum_{i\in\mathcal{G}} \left(c_{2i}{P_{Gi}}^2 + c_{1i}P_{Gi} + c_{0i}\right)$$

Generation Cost

$$\text{subject to} \qquad P_{Gi}^{\min} \leq P_{Gi} \leq P_{Gi}^{\max}$$

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$$P_i = \sum_{k=1}^{n} \mathbf{B}_{ik}\left(\theta_k - \theta_i\right)$$

Accuracy of the DC Power Flow Approximation

OR

How good is what we do now?

DC Power Flow Accuracy

- Many studies of DC power flow accuracy:
  - [Yan & Sekar ’02], [Liu & Gross ’02], [Baldick ’03], [Overbye, Cheng, & Sun ’04], [Baldick, Dixit & Overbye ’05], [Purchala, Meeus, Van Dommelen & Belmans ’05], [Van Hertem, Verboomen, Purchala, Belmans & Kling ’06], [Li & Bo ’07], [Duthaler, Emery, Andersson & Kurzidem ’08], [Stott, Jardim & Alsac ‘09], [Lesieutre, Schlindwein & Beglin ‘10], [Qi, Shi & Tylavsky ’12]
- Accuracy depends on the application and test case

> “At no stage in the tests were we able to discern any statistical pattern in the dc-flow error scatters. This defeated all our attempts to find concise, meaningful indices with which to characterize and display dc-model accuracies.” [Stott, Jardim & Alsac ‘09]

Assessing DC Power Flow Accuracy

- Goal: bound the worst-case error in the active power flows between the DC and AC power flow models

Power Injections

$P_i,\ Q_i$

DC Power Flow

$$P_i = \sum_{k=1}^{n} \mathbf{B}_{ik} \left(\theta_i - \theta_k\right)$$

$$P_{flow,ik}^{DC} = \mathbf{B}_{ik} \left(\theta_i - \theta_k\right)$$

AC Power Flow

$$P_i = |V_i| \sum_{k=1}^{n} |V_k| \left(\mathbf{G}_{ik} \cos\left(\theta_i - \theta_k\right) + \mathbf{B}_{ik} \sin\left(\theta_i - \theta_k\right)\right)$$

$$Q_i = |V_i| \sum_{k=1}^{n} |V_k| \left(\mathbf{G}_{ik} \sin\left(\theta_i - \theta_k\right) - \mathbf{B}_{ik} \cos\left(\theta_i - \theta_k\right)\right)$$

$$P_{flow,ik}^{AC} = |V_i| |V_k| \left(\mathbf{G}_{ik} \cos\left(\theta_i - \theta_k\right) + \mathbf{B}_{ik} \sin\left(\theta_i - \theta_k\right)\right) - |V_k|^2 \mathbf{G}_{ik}$$

Error

$$\left|\left| P_{flow}^{DC} - P_{flow}^{AC} \right|\right|$$

Worst-Case Error Formulation

$$\max_{|V|,\theta^{AC},\theta^{DC}} \quad \left|\left| P_{flow}^{DC} - P_{flow}^{AC} \right|\right|_\infty$$

Maximize Error

$$\text{subject to} \quad \left|V_i^{min}\right| \leq |V_i| \leq \left|V_i^{max}\right|$$

$$\theta_{ik}^{min} \leq \theta_i^{AC} - \theta_k^{AC} \leq \theta_{ik}^{max}$$

$$P_i^{min} \leq P_i \leq P_i^{max}$$

$$Q_i^{min} \leq Q_i \leq Q_i^{max}$$

Operational Constraints

$$P_i = \sum_{k=1}^{n} \mathbf{B}_{ik} \left(\theta_i^{DC} - \theta_k^{DC}\right)$$

$$P_{flow,ik}^{DC} = \mathbf{B}_{ik} \left(\theta_i^{DC} - \theta_k^{DC}\right)$$

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AC Power Flow

$$P_{flow,ik}^{AC} = |V_i|\,|V_k| \left(\mathbf{G}_{ik} \cos\left(\theta_i^{AC} - \theta_k^{AC}\right) + \mathbf{B}_{ik} \sin\left(\theta_i^{AC} - \theta_k^{AC}\right)\right) - |V_k|^2 \mathbf{G}_{ik}$$

Worst-Case Error Formulation

$$
\begin{aligned}
\max_{|V|,\theta^{AC},\theta^{DC}} \quad & \left|\left|P_{flow}^{DC} - P_{flow}^{AC}\right|\right|_{\infty} \\
\text{subject to} \quad & \left|V_i^{min}\right| \leq |V_i| \leq \left|V_i^{max}\right| \\
& \theta_{ik}^{min} \leq \theta_i^{AC} - \theta_k^{AC} \leq \theta_{ik}^{max} \\
& P_i^{min} \leq P_i \leq P_i^{max} \\
& Q_i^{min} \leq Q_i \leq Q_i^{max} \\
& P_i = \sum_{k=1}^{n} \mathbf{B}_{ik} \left(\theta_i^{DC} - \theta_k^{DC}\right) \\
& P_{flow,ik}^{DC} = \mathbf{B}_{ik} \left(\theta_i^{DC} - \theta_k^{DC}\right) \\
& P_i = |V_i| \sum_{k=1}^{n} |V_k| \left(\mathbf{G}_{ik} \cos\left(\theta_i^{AC} - \theta_k^{AC}\right) + \mathbf{B}_{ik} \sin\left(\theta_i^{AC} - \theta_k^{AC}\right)\right) \\
& Q_i = |V_i| \sum_{k=1}^{n} |V_k| \left(\mathbf{G}_{ik} \sin\left(\theta_i^{AC} - \theta_k^{AC}\right) - \mathbf{B}_{ik} \cos\left(\theta_i^{AC} - \theta_k^{AC}\right)\right) \\
& P_{flow,ik}^{AC} = |V_i||V_k| \left(\mathbf{G}_{ik} \cos\left(\theta_i^{AC} - \theta_k^{AC}\right) + \mathbf{B}_{ik} \sin\left(\theta_i^{AC} - \theta_k^{AC}\right)\right) - |V_k|^2 \mathbf{G}_{ik}
\end{aligned}
$$

Non-Convex!

Handling the Objective Function

- Maximize the infinity norm by solving
  4 × (number of lines) optimization problems:

$$\boxed{\max_{|V|,\theta^{AC},\theta^{DC}} \quad ||P_{flow}^{DC} - P_{flow}^{AC}||_\infty} \rightarrow \boxed{\max_{ik\in\mathcal{L}} \left\{ \max_{|V|,\theta^{AC},\theta^{DC}} \quad \left|P_{flow,ik}^{DC} - P_{flow,ik}^{AC}\right| \right\}}$$

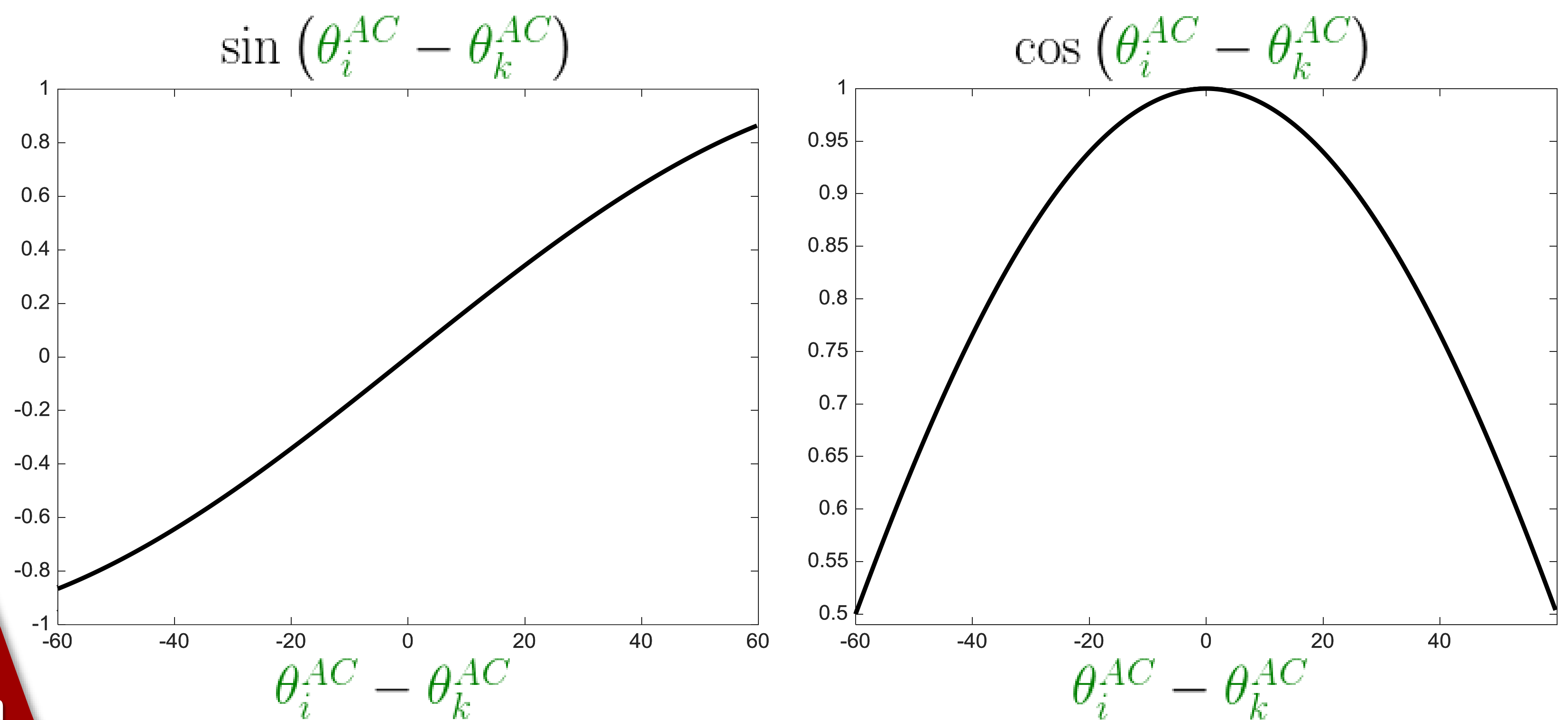
For each line $ik \in \mathcal{L}$ and $\sigma = \{-1, 1\}$,
solve (in parallel):

$$\max_{|V|,\theta^{AC},\theta^{DC}} \quad \sigma \cdot \left(P_{flow,ik}^{DC} - P_{flow,ik}^{AC}\right)$$

Select the largest absolute value among all the solutions

Handling the Power Flow Equations via Convex Relaxations

- A variety of recently developed relaxations
- One example: The QC Relaxation [Coffrin, Hijazi & Van Hentenryck '15]



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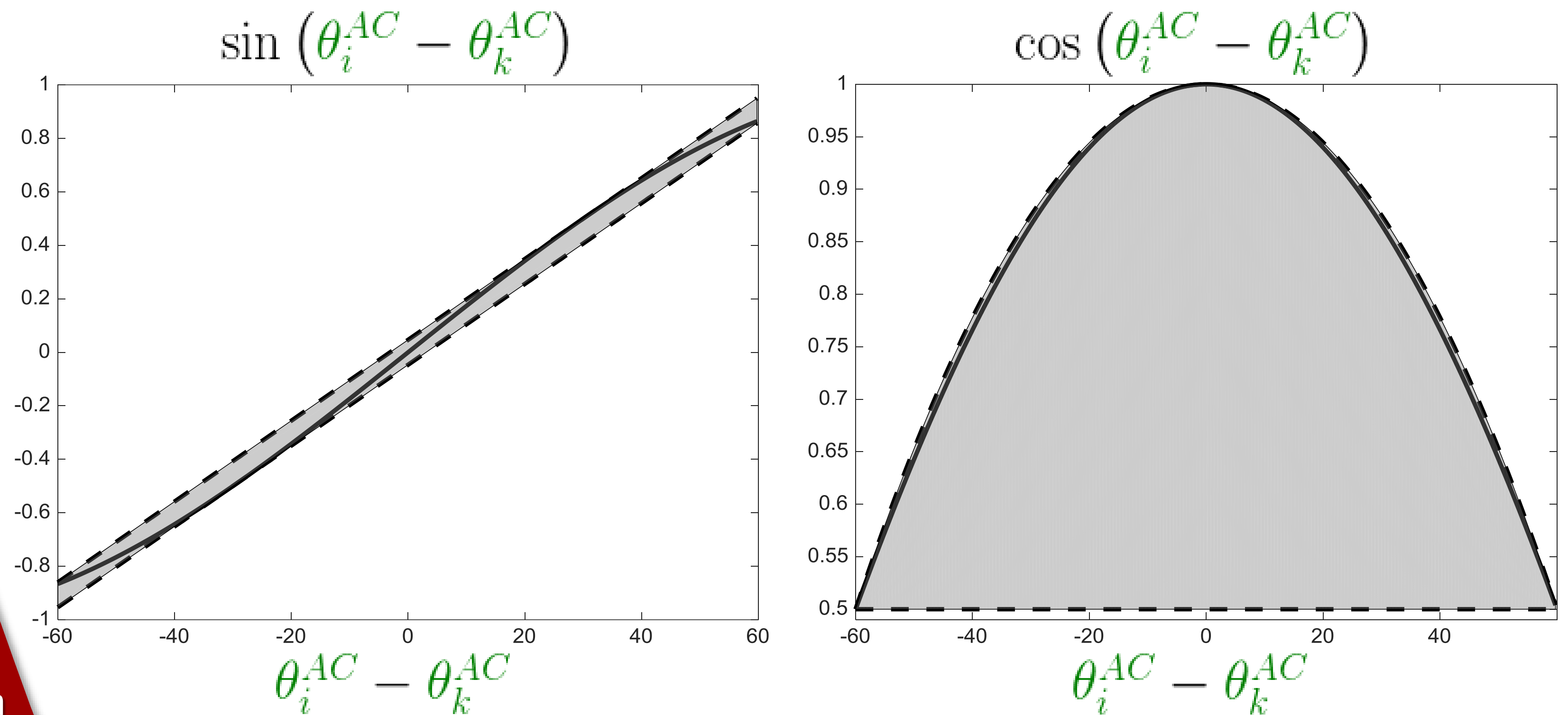

Further Tightening the Relaxation

- Augment the QC relaxation with
  - A Semidefinite Programming Relaxation of the power flow equations in rectangular coordinates [Lavaei & Low ‘12]
  - Sparsity-Exploiting Moment Relaxations from the Lasserre hierarchy [Lasserre ‘01], [Molzahn & Hiskens ’15], [Josz & Molzahn ‘16]
  - Lifted Nonlinear Cuts implied by the angle difference and voltage magnitude limits [Coffrin, Hijazi & Van Hentenryck ‘15], [Chen, Atamturk & Oren ‘15]
  - Arctangent Envelopes [Kocuk, Dey & Sun ‘16]
- Apply a bound tightening algorithm to improve upon the specified operational limits [Kocuk, Dey & Sun ‘15], [Chen, Atamturk & Oren ‘15], [Coffrin, Hijazi & Van Hentenryck ‘16]

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Semidefinite Relaxation of the Power Flow Equations

- Write power flow equations as $z^H \mathbf{A}_i z = c_i$
  where $z = \begin{bmatrix} V_1 & \ldots & V_n \end{bmatrix}^\intercal$ with voltage phasors $V \in \mathbb{C}^n$

- Define matrix $\mathbf{W} = zz^H$

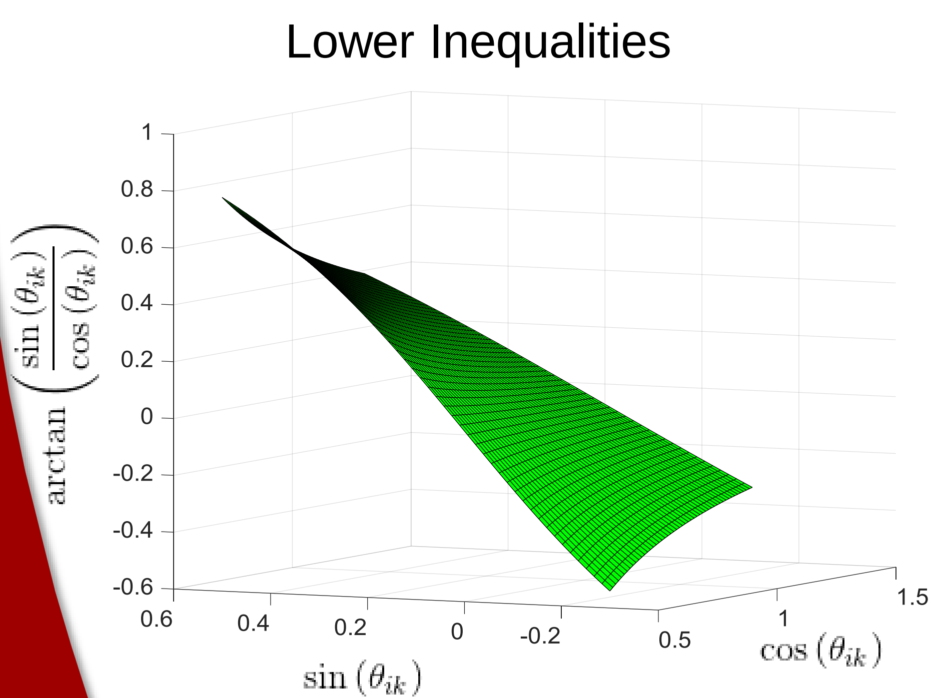
- Rewrite as $\text{rank}\left(\mathbf{W}\right) = 1$ and $\begin{cases} \text{trace}\left(\mathbf{A}_i \mathbf{W}\right) = c_i \\ \mathbf{W} \succeq 0 \end{cases}$

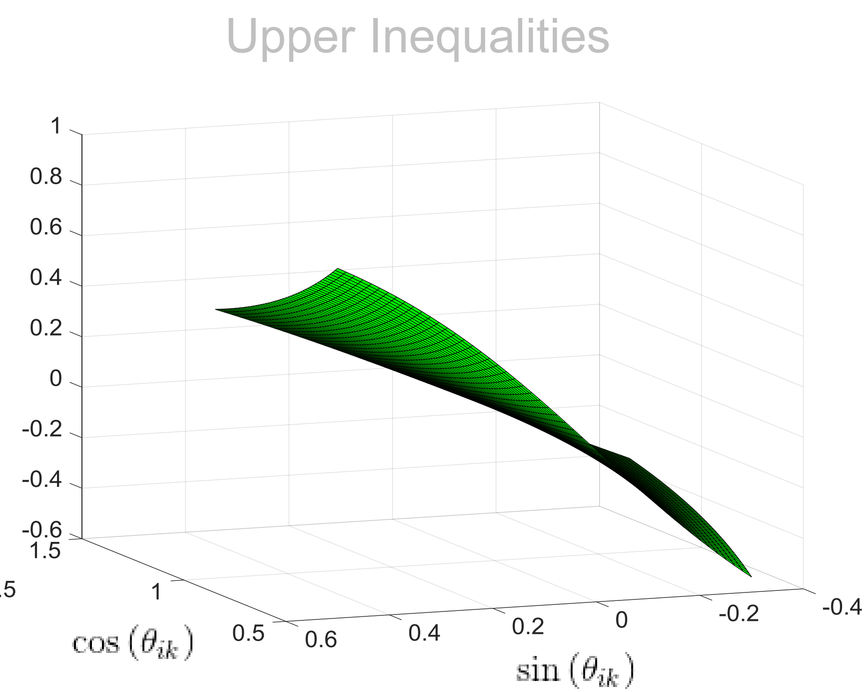
- Relaxation:
  Do not enforce $\text{rank}\left(\mathbf{W}\right) = 1$ [Lavaei & Low '12]

  – A solution with $\text{rank}\left(\mathbf{W}\right) = 1$ implies zero relaxation gap and recovery of the globally optimal voltage profile. This is not necessary for our problem: we only require a lower bound.

Arctangent Envelopes

- Enclose the arctangent function using linear inequalities [Kocuk, Dey, & Sun '16]





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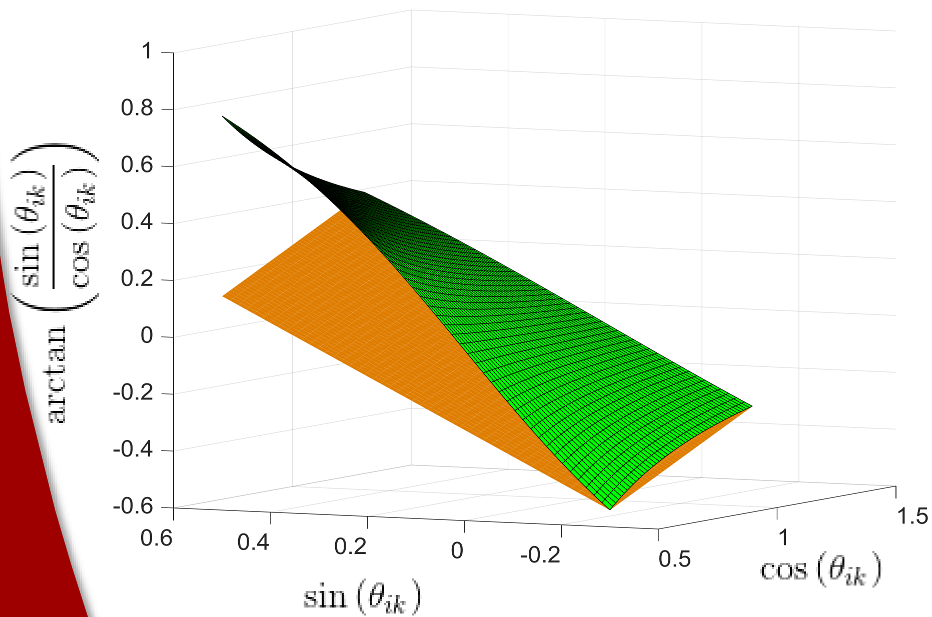
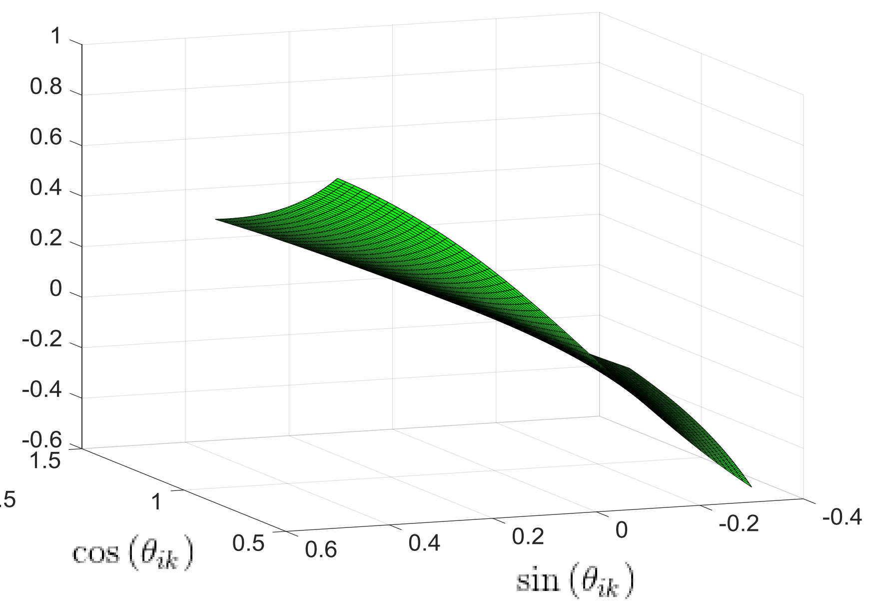
Lower Inequalities



Upper Inequalities



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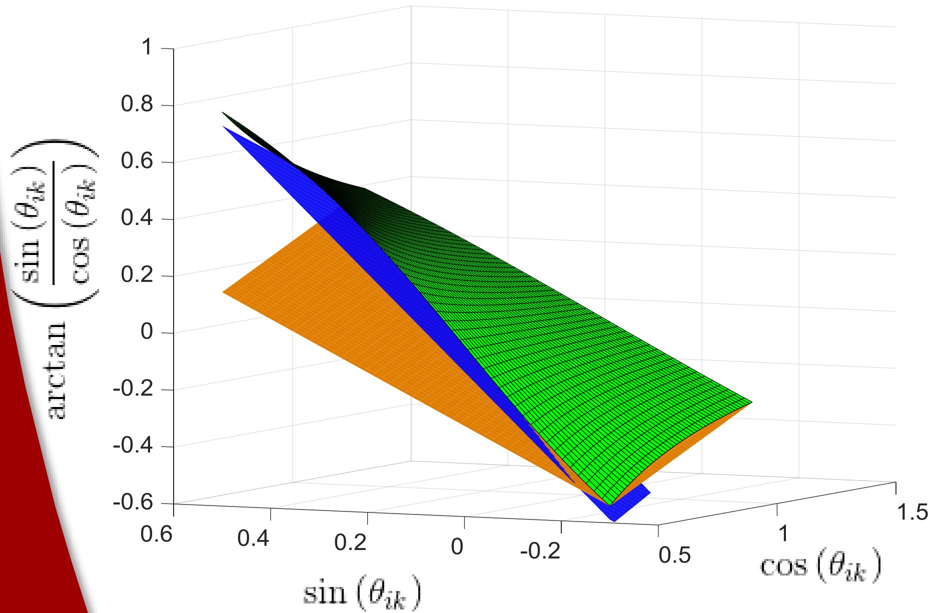

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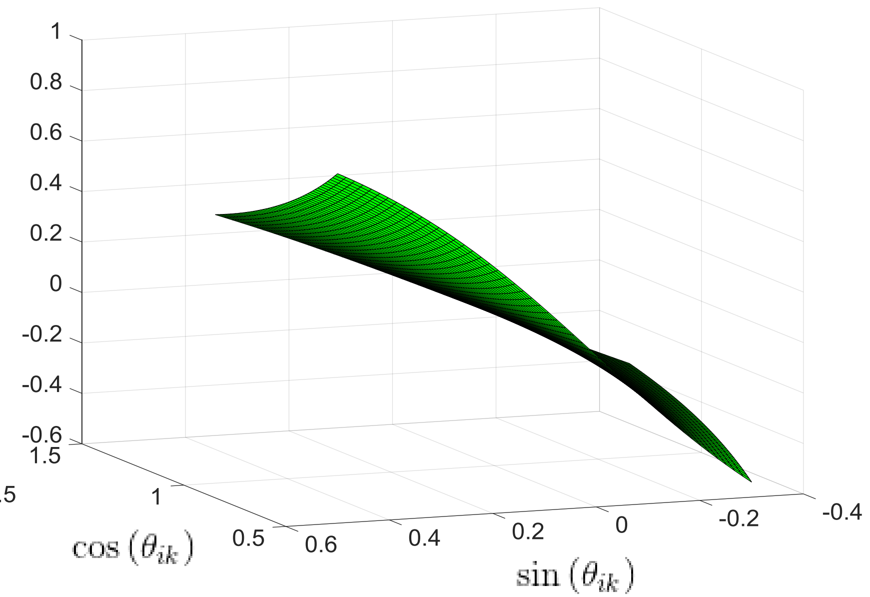

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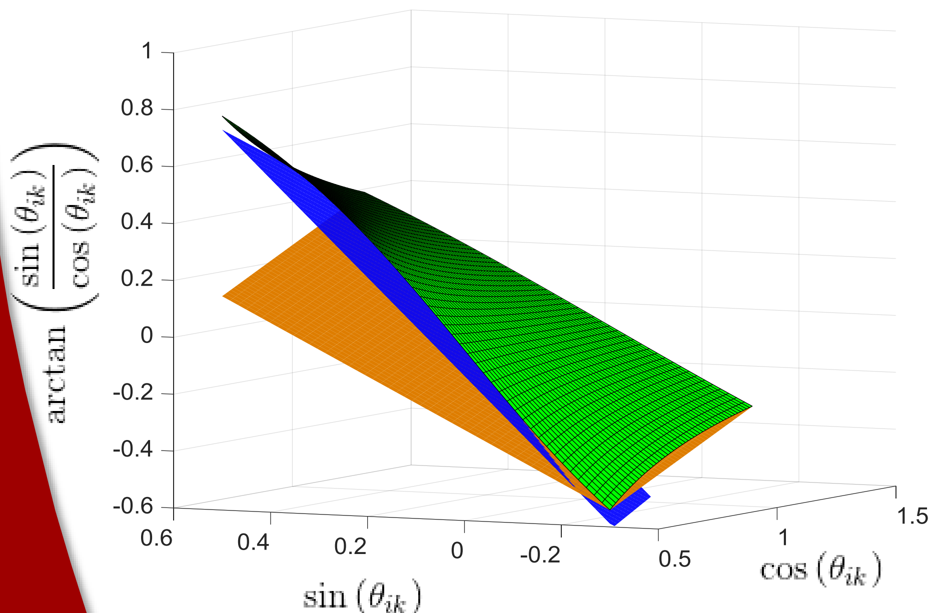
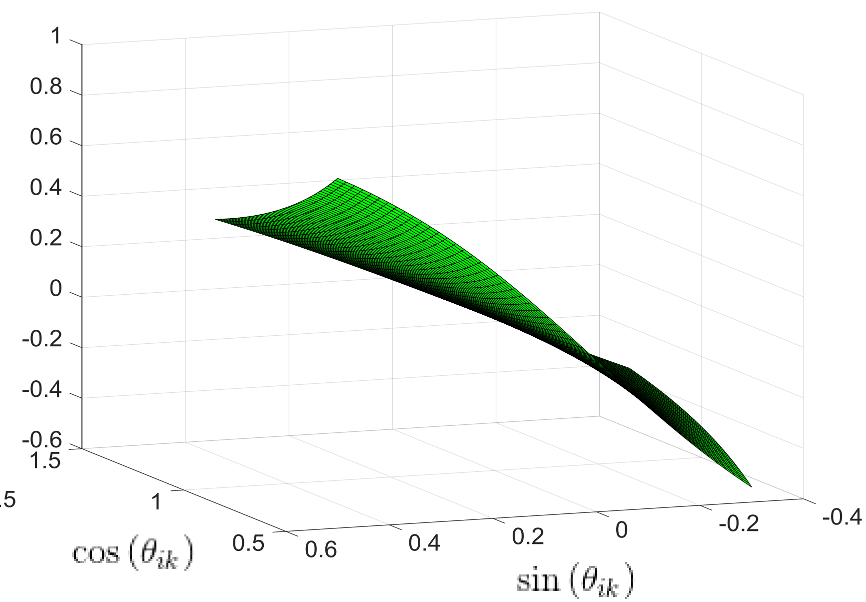


Upper Inequalities



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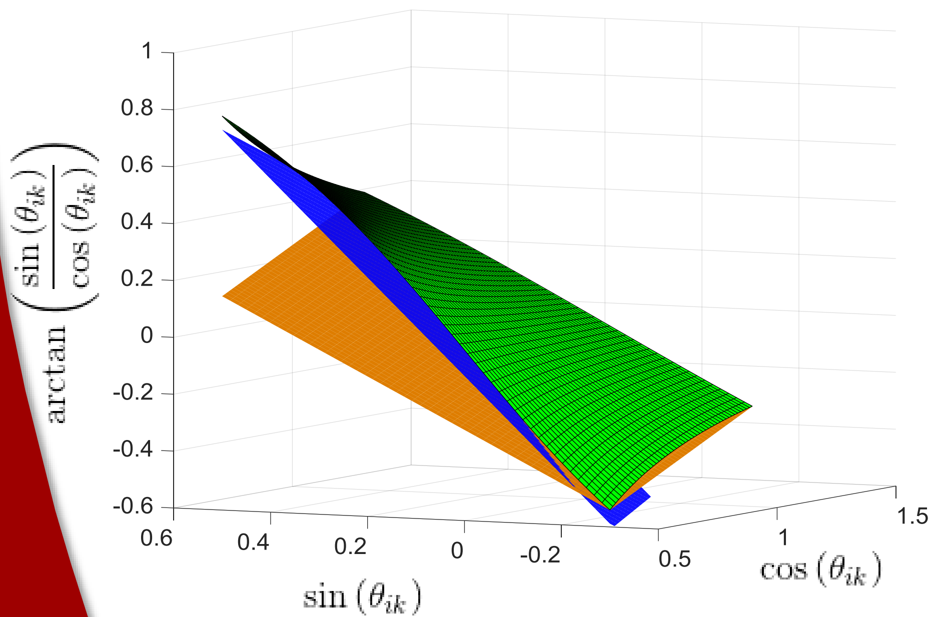

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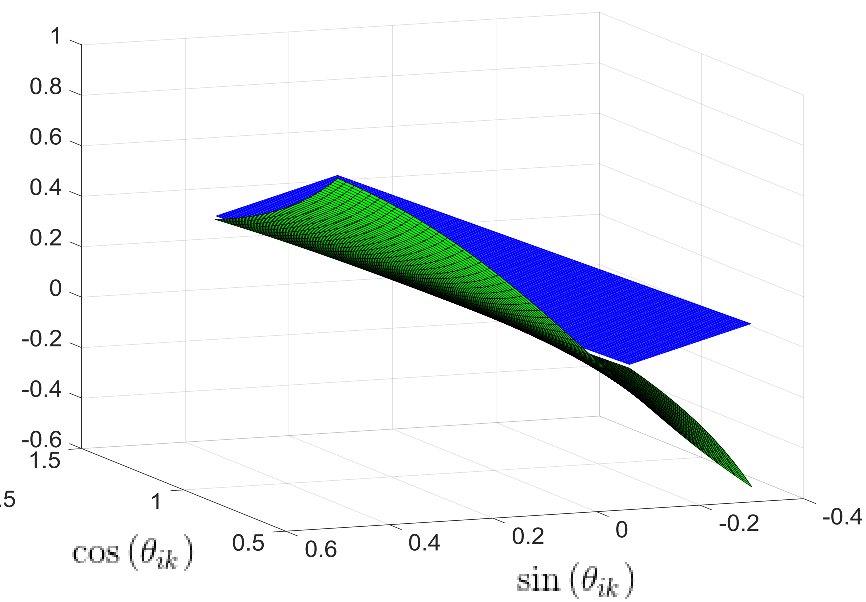

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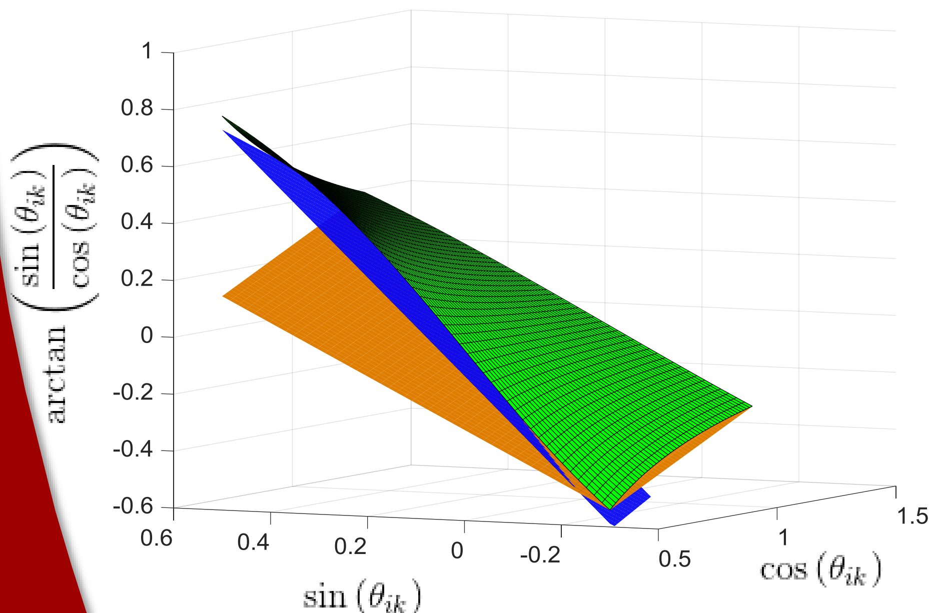

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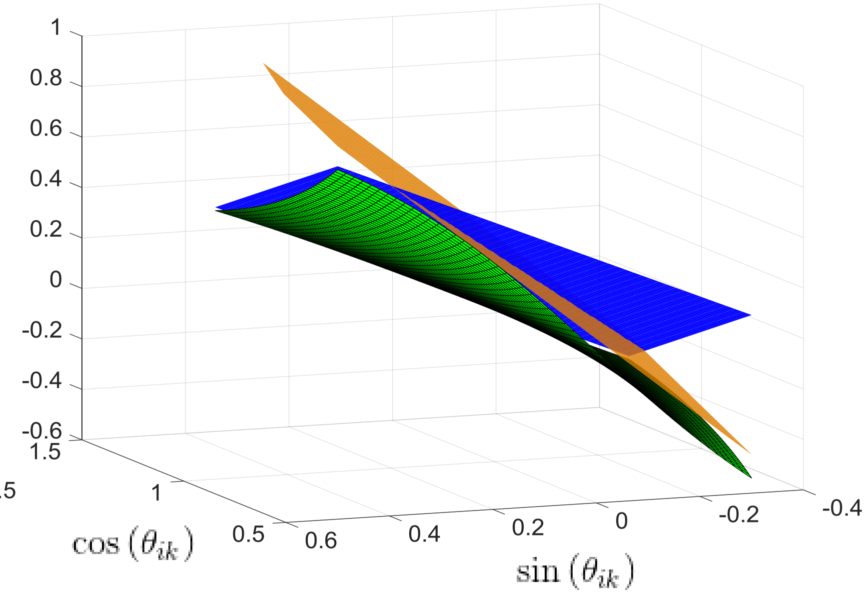


Formulation

Formulation Summary

For each $ik \in \mathcal{L}$ and $\sigma = \{-1, 1\}$, solve (in parallel):

$$\max_{|V|,\theta^{AC},\theta^{DC}} \quad \sigma \cdot \left(P^{DC}_{flow,ik} - P^{AC}_{flow,ik}\right)$$

Maximize the absolute value of the error at each line terminal

$$\text{subject to} \quad \left|V_i^{min}\right| \leq |V_i| \leq \left|V_i^{max}\right|$$

$$\theta_{ik}^{min} \leq \theta_i^{AC} - \theta_k^{AC} \leq \theta_{ik}^{max}$$

$$P_i^{min} \leq P_i \leq P_i^{max}$$

$$Q_i^{min} \leq Q_i \leq Q_i^{max}$$

Operational constraints

$$P_i = \sum_{k=1}^{n} \mathbf{B}_{ik} \left(\theta_i^{DC} - \theta_k^{DC}\right)$$

$$P^{DC}_{flow,ik} = \mathbf{B}_{ik} \left(\theta_i^{DC} - \theta_k^{DC}\right)$$

DC Power Flow

$$P_i = |V_i| \sum_{k=1}^{n} |V_k| \left(\mathbf{G}_{ik} \cos\left(\theta_i^{AC} - \theta_k^{AC}\right) + \mathbf{B}_{ik} \sin\left(\theta_i^{AC} - \theta_k^{AC}\right)\right)$$

$$Q_i = |V_i| \sum_{k=1}^{n} |V_k| \left(\mathbf{G}_{ik} \sin\left(\theta_i^{AC} - \theta_k^{AC}\right) - \mathbf{B}_{ik} \cos\left(\theta_i^{AC} - \theta_k^{AC}\right)\right)$$

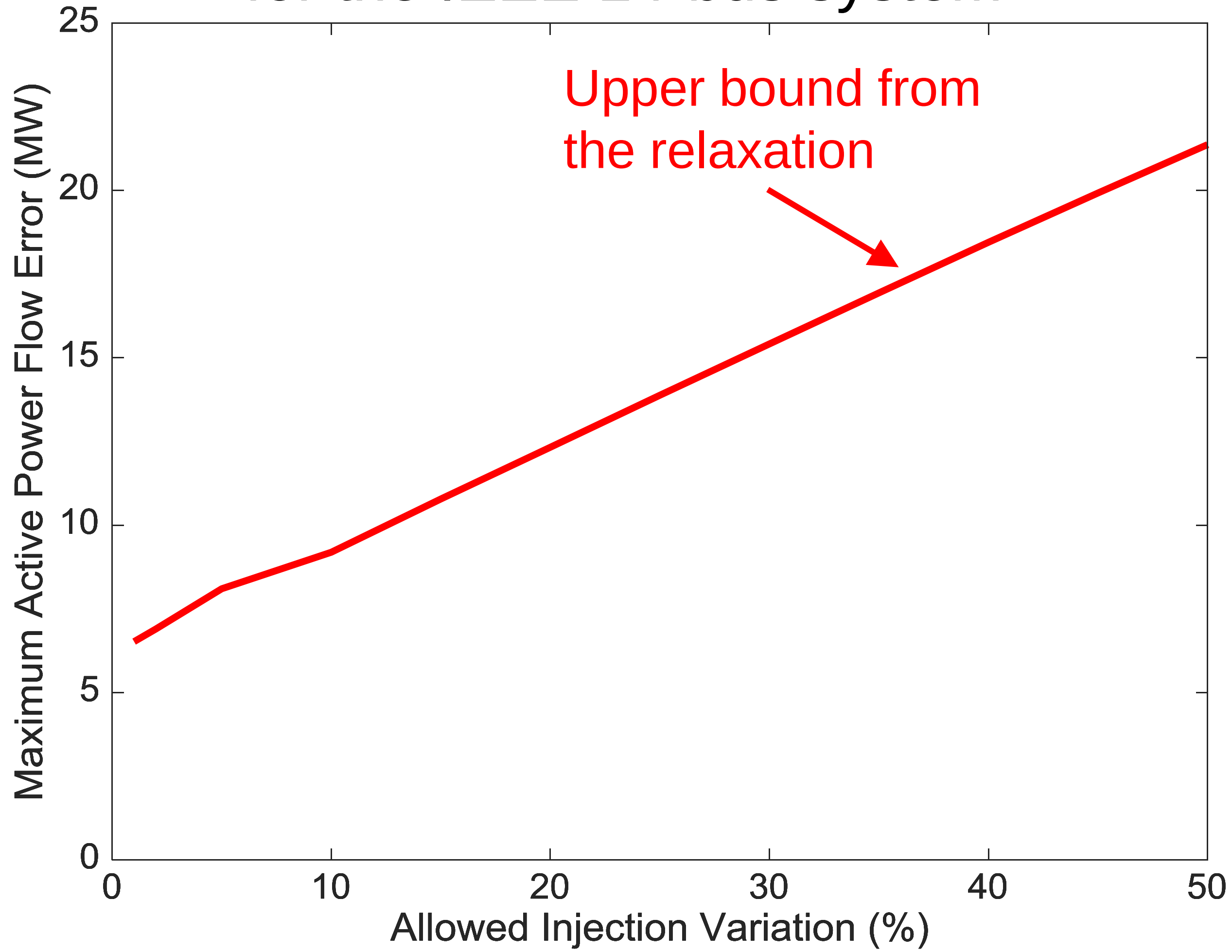
$$P^{AC}_{flow,ik} = |V_i|\,|V_k| \left(\mathbf{G}_{ik} \cos\left(\theta_i^{AC} - \theta_k^{AC}\right) + \mathbf{B}_{ik} \sin\left(\theta_i^{AC} - \theta_k^{AC}\right)\right) - |V_k|^2 \mathbf{G}_{ik}$$

Convex relaxations

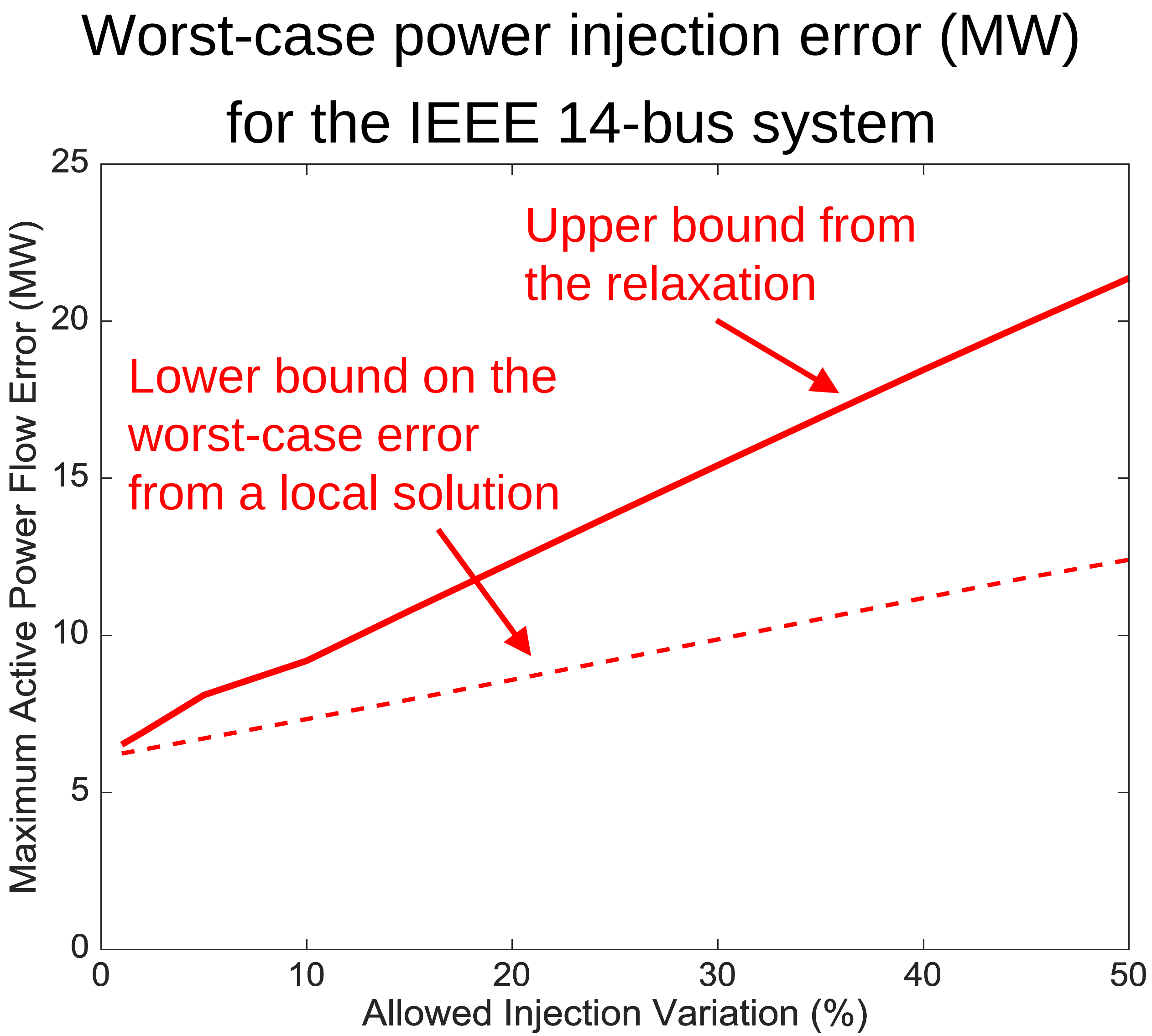
Results for the IEEE Test Cases

Results

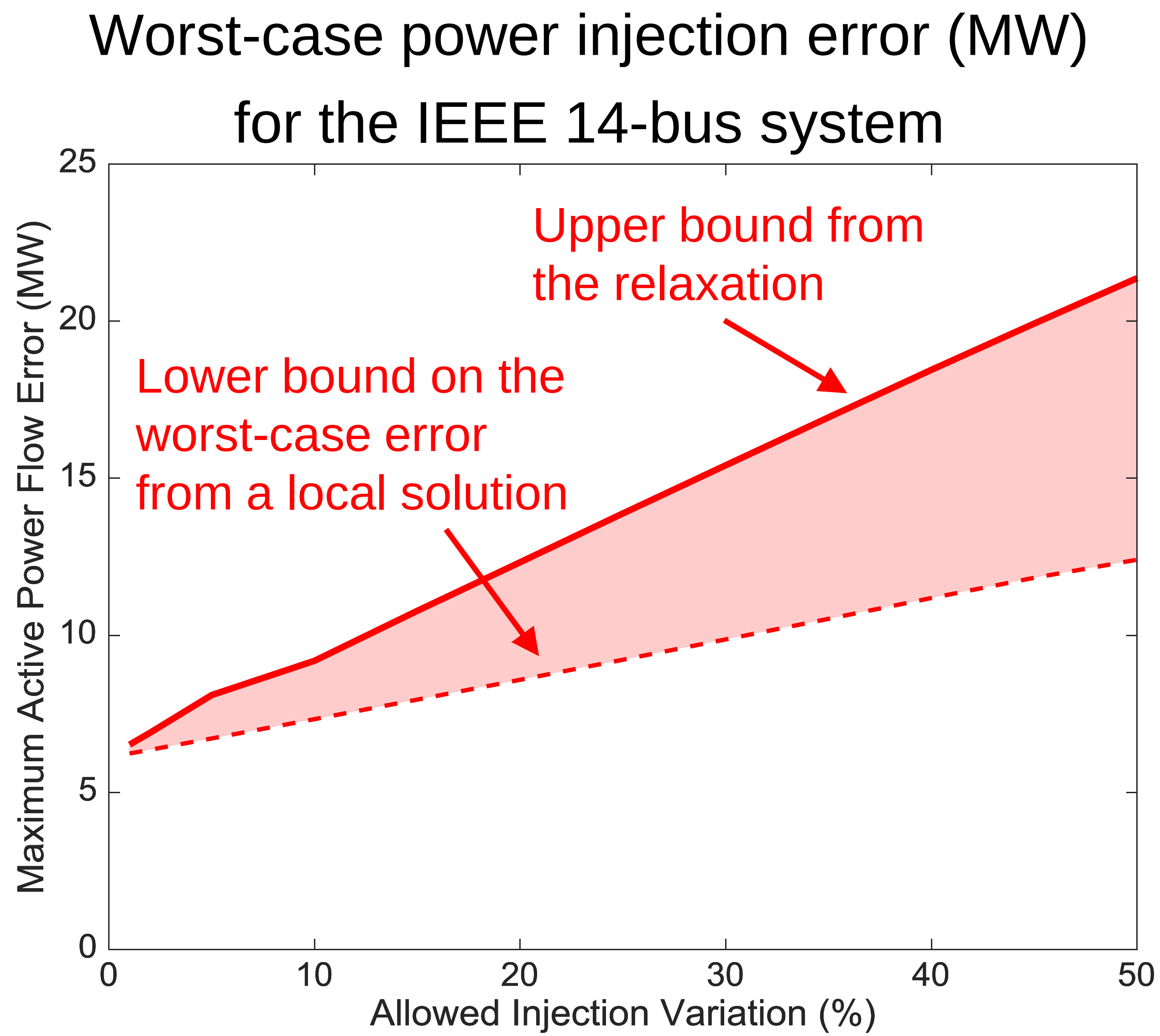
Worst-case power injection error (MW)
for the IEEE 14-bus system



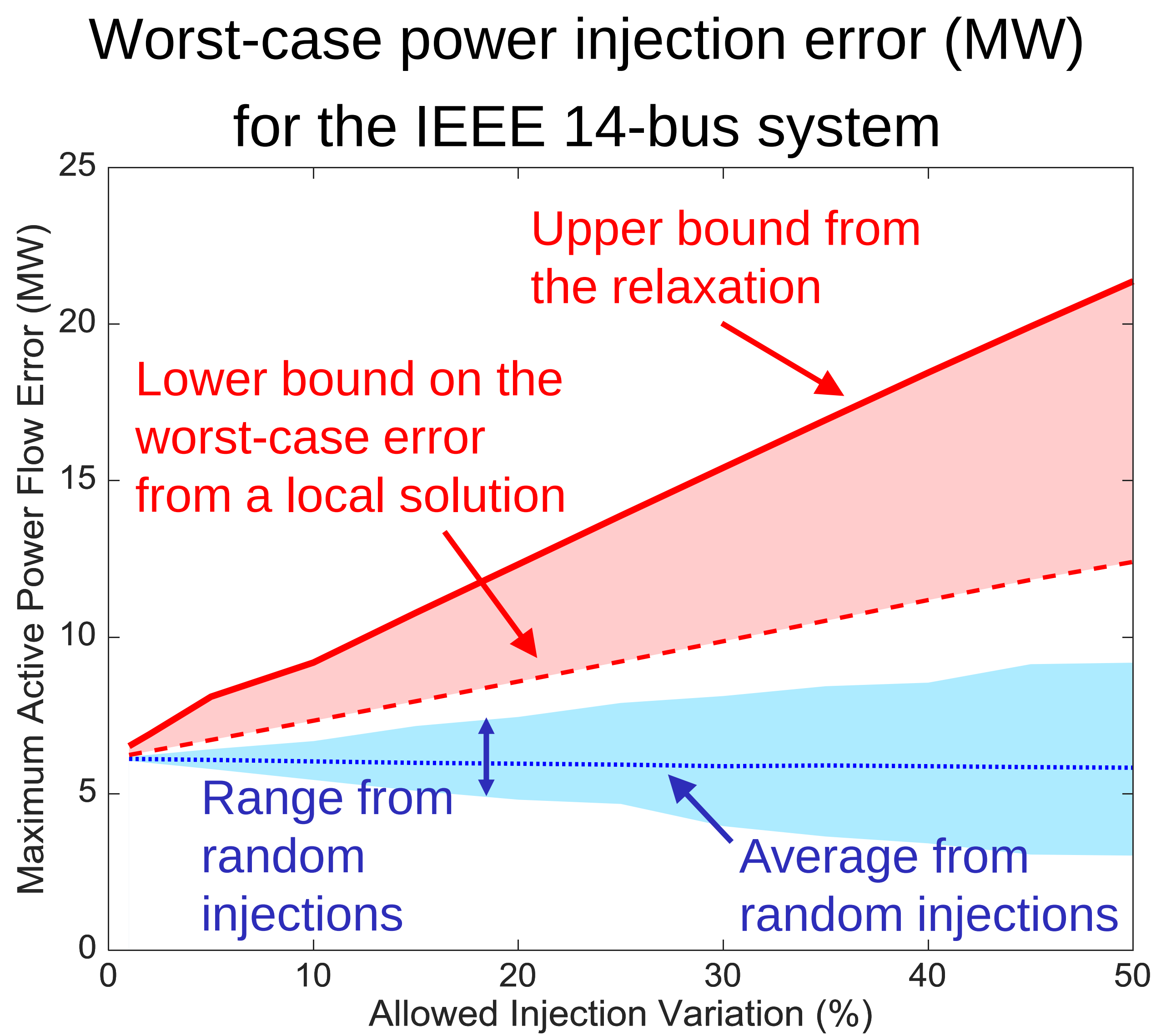
Results


Worst-case power injection error (MW)
for the IEEE 14-bus system
Upper bound from
the relaxation
Lower bound on the
worst-case error
from a local solution
Maximum Active Power Flow Error (MW)
Allowed Injection Variation (%)
0
5
10
15
20
25
0
10
20
30
40
50

Results

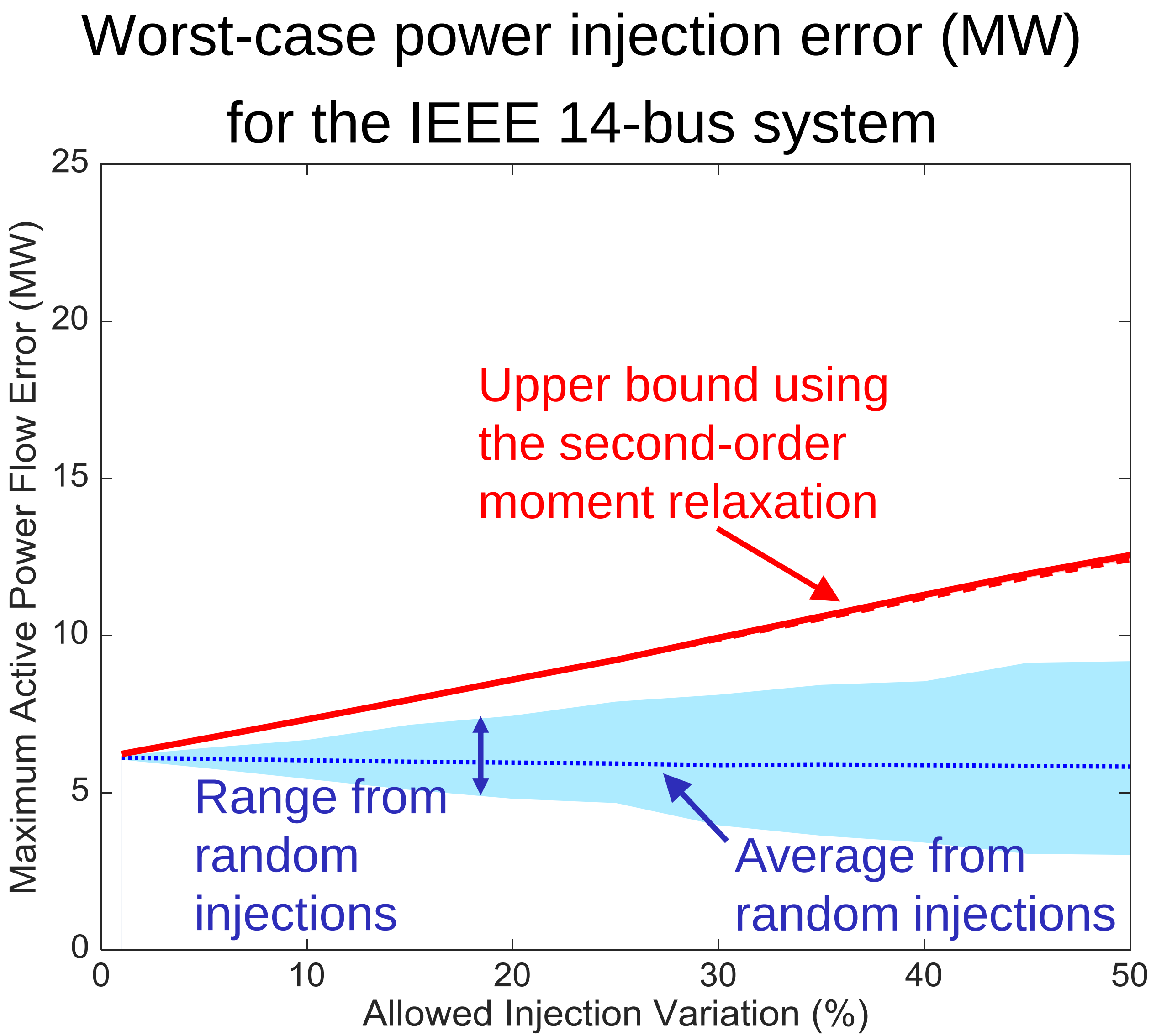

Worst-case power injection error (MW)
for the IEEE 14-bus system
Maximum Active Power Flow Error (MW)
Allowed Injection Variation (%)
0
5
10
15
20
25
0
10
20
30
40
50
Upper bound from
the relaxation
Lower bound on the
worst-case error
from a local solution

Results



Worst-case power injection error (MW)
for the IEEE 14-bus system
Maximum Active Power Flow Error (MW)
Allowed Injection Variation (%)
0
5
10
15
20
25
0
10
20
30
40
50
Upper bound from
the relaxation
Lower bound on the
worst-case error
from a local solution
Range from
random
injections
Average from
random injections

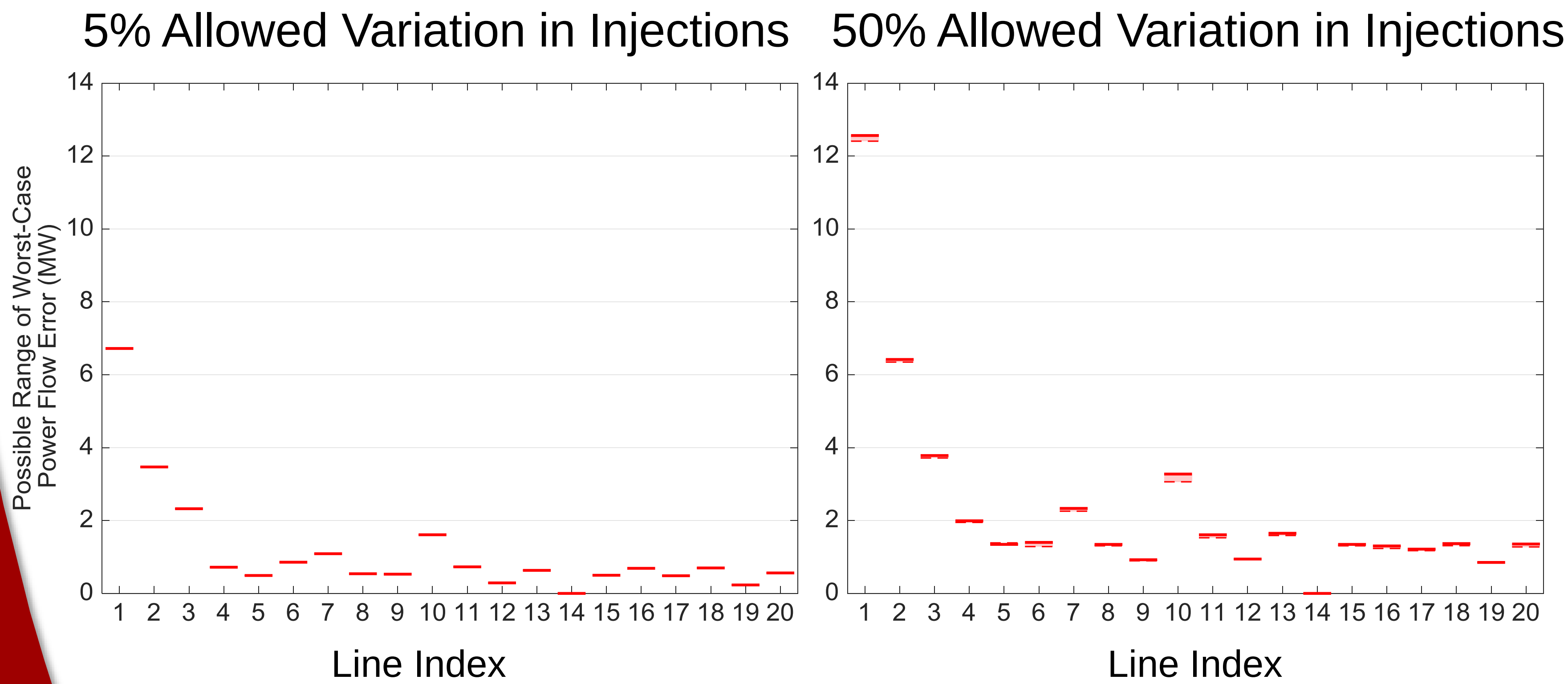
Results


Worst-case power injection error (MW)
for the IEEE 14-bus system
Maximum Active Power Flow Error (MW)
Allowed Injection Variation (%)
0
5
10
15
20
25
0
10
20
30
40
50
Upper bound using
the second-order
moment relaxation
Range from
random
injections
Average from
random injections

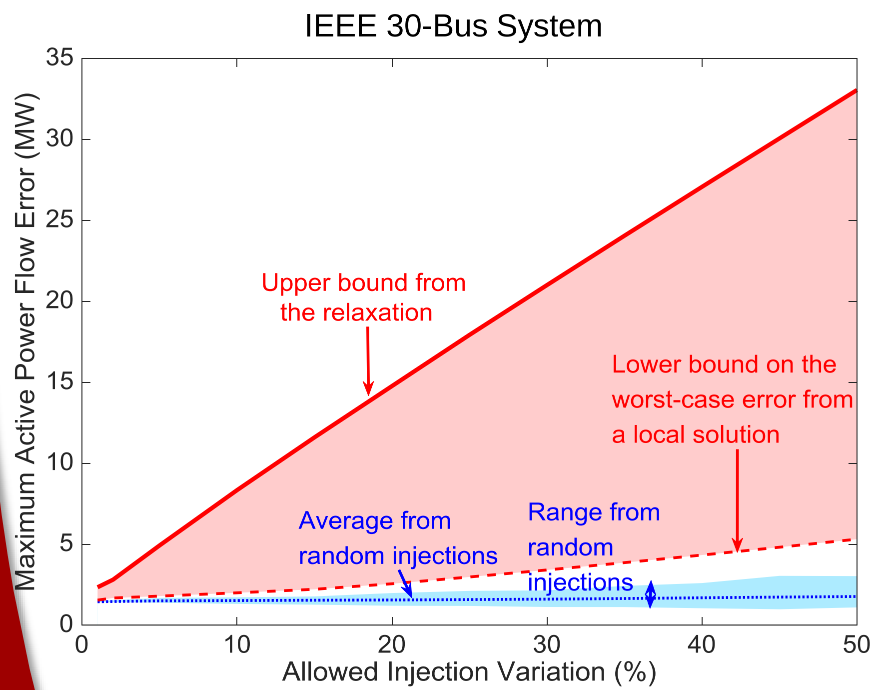
Results

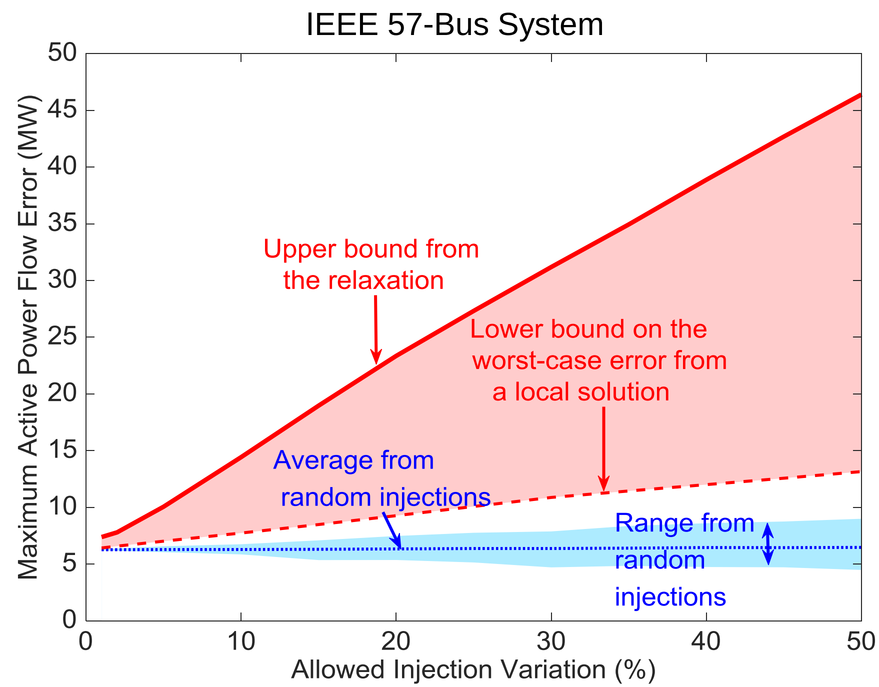
Worst-case power flow error (MW)

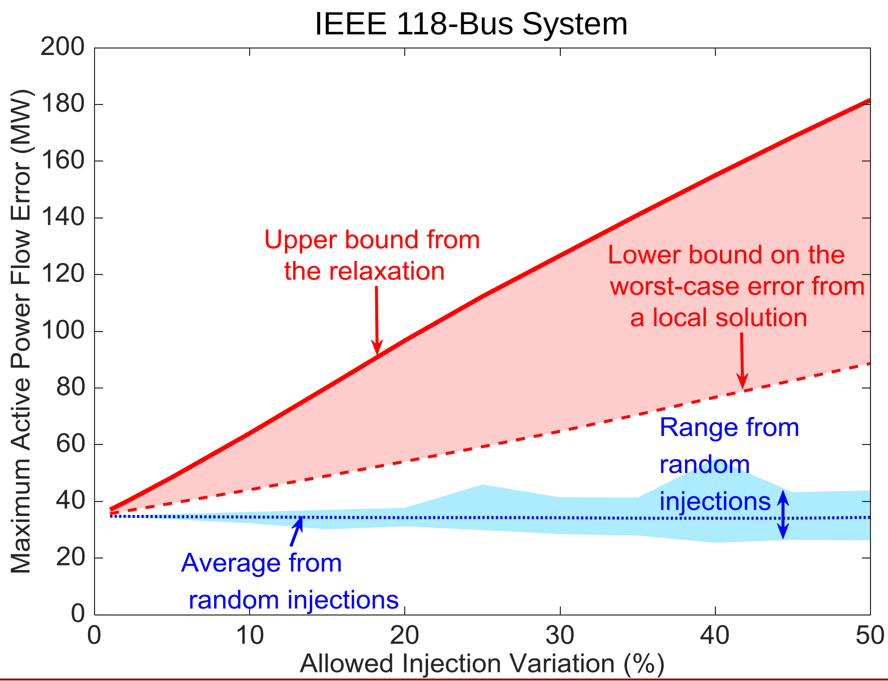
for the IEEE 14-bus system, by line



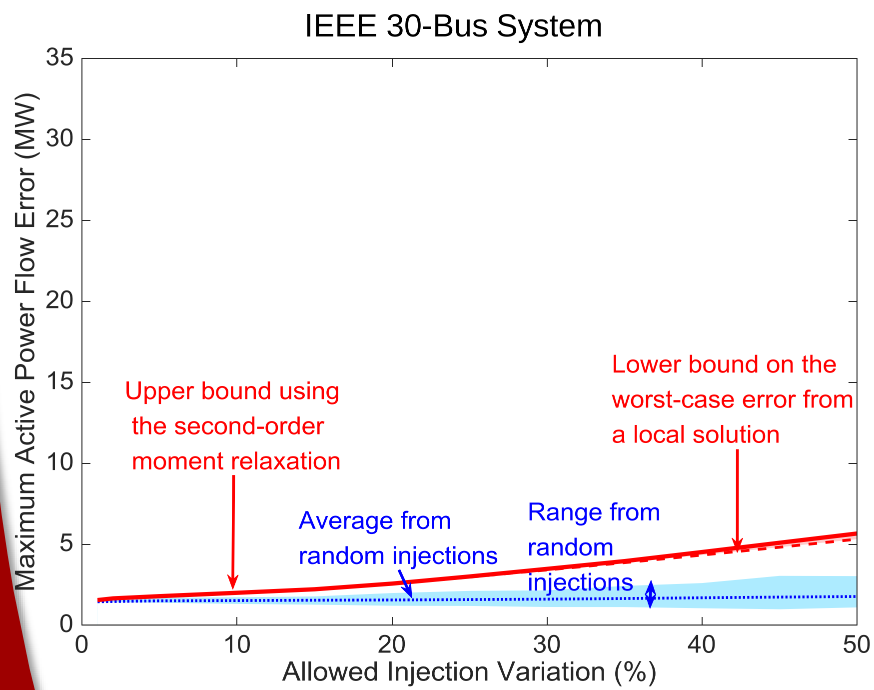
Results


IEEE 30-Bus System
Maximum Active Power Flow Error (MW)
Allowed Injection Variation (%)
Upper bound from the relaxation
Lower bound on the worst-case error from a local solution
Average from random injections
Range from random injections
0
5
10
15
20
25
30
35
0
10
20
30
40
50

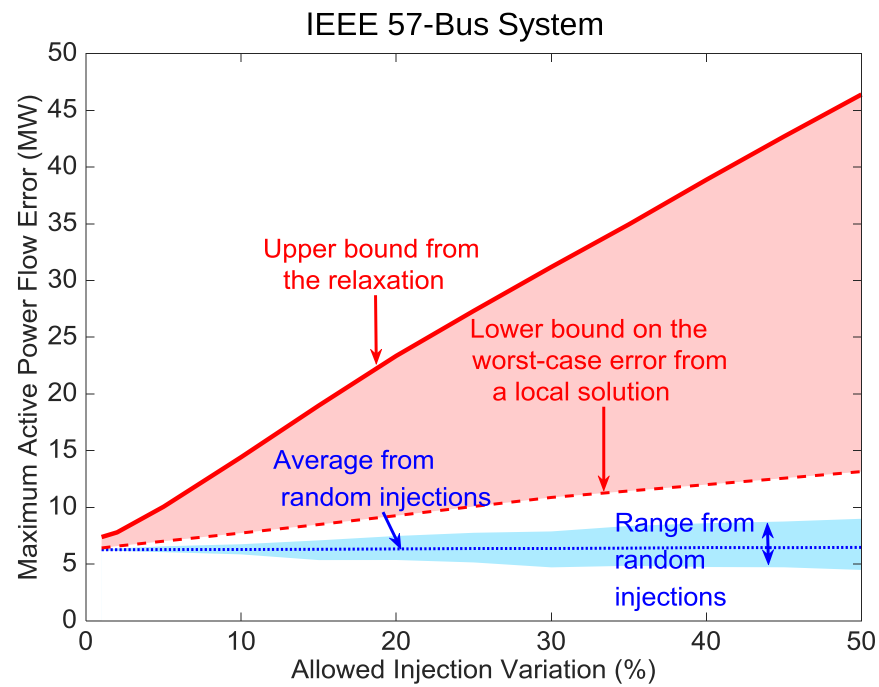

IEEE 57-Bus System
Maximum Active Power Flow Error (MW)
Allowed Injection Variation (%)
Upper bound from the relaxation
Lower bound on the worst-case error from a local solution
Average from random injections
Range from random injections
0
5
10
15
20
25
30
35
40
45
50
0
10
20
30
40
50


IEEE 118-Bus System
Maximum Active Power Flow Error (MW)
Allowed Injection Variation (%)
Upper bound from the relaxation
Lower bound on the worst-case error from a local solution
Range from random injections
Average from random injections
0
20
40
60
80
100
120
140
160
180
200
0
10
20
30
40
50

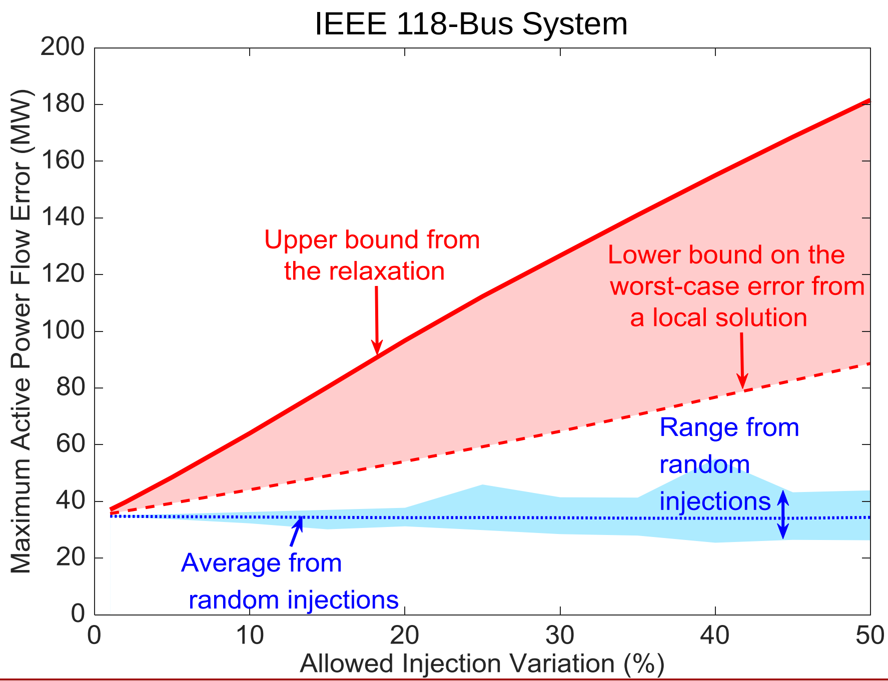
Results



IEEE 30-Bus System
Maximum Active Power Flow Error (MW)
Allowed Injection Variation (%)
Upper bound using the second-order moment relaxation
Average from random injections
Range from random injections
Lower bound on the worst-case error from a local solution



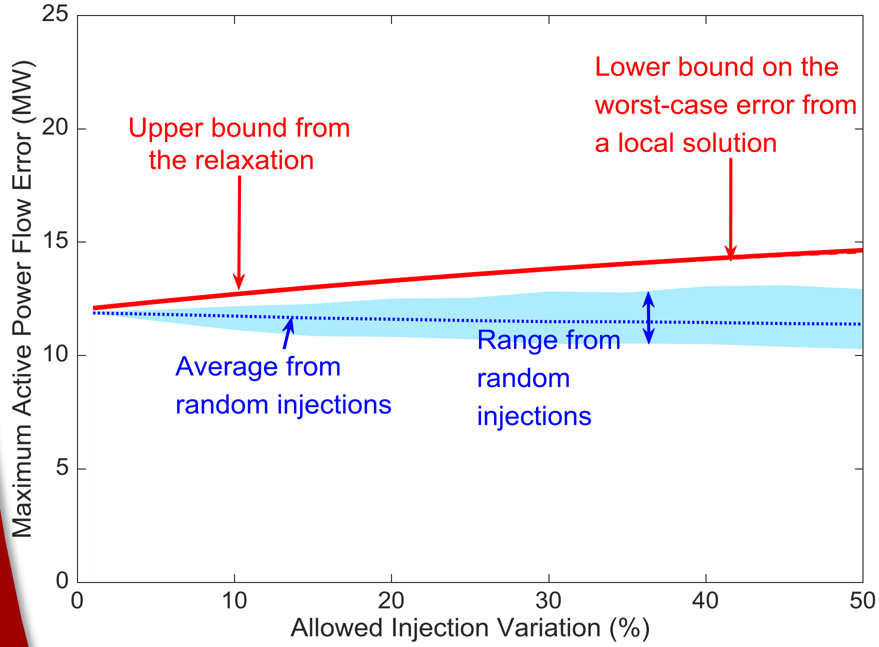
IEEE 57-Bus System
Maximum Active Power Flow Error (MW)
Allowed Injection Variation (%)
Upper bound from the relaxation
Lower bound on the worst-case error from a local solution
Average from random injections
Range from random injections



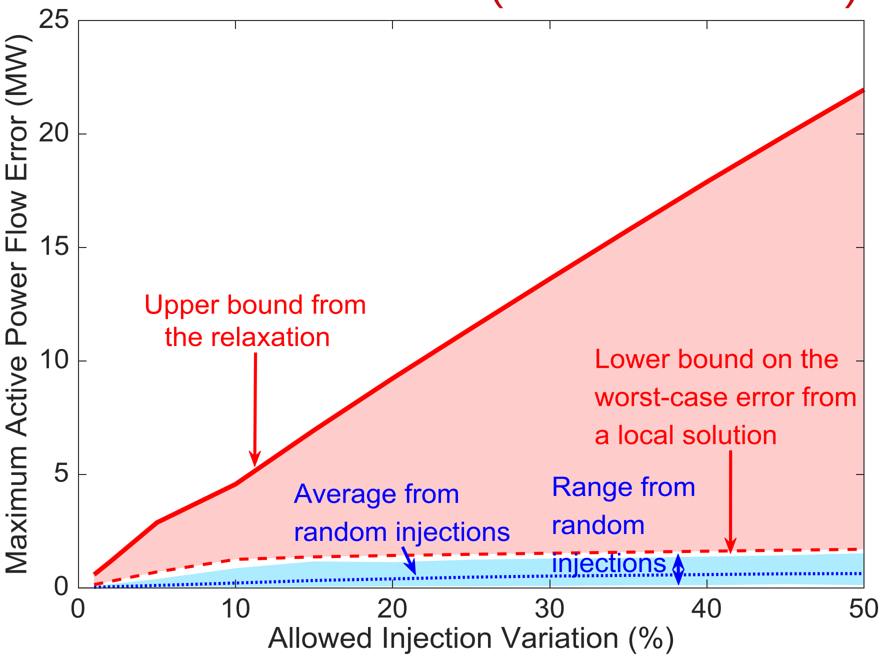
IEEE 118-Bus System
Maximum Active Power Flow Error (MW)
Allowed Injection Variation (%)
Upper bound from the relaxation
Lower bound on the worst-case error from a local solution
Range from random injections
Average from random injections

Other Linearizations

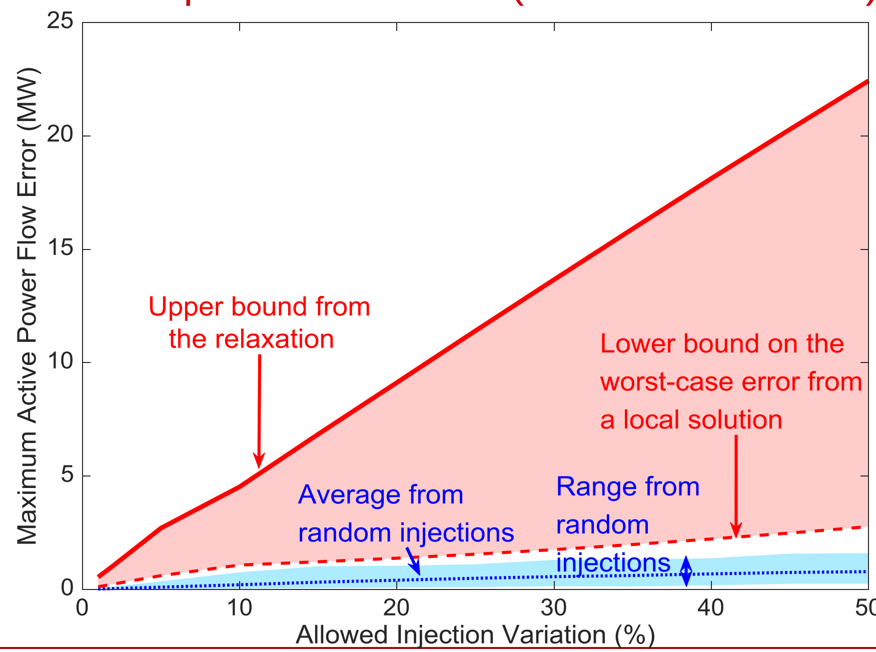
Hot-Start DC Power Flow



Power Flow Jacobian (Polar Coordinates)

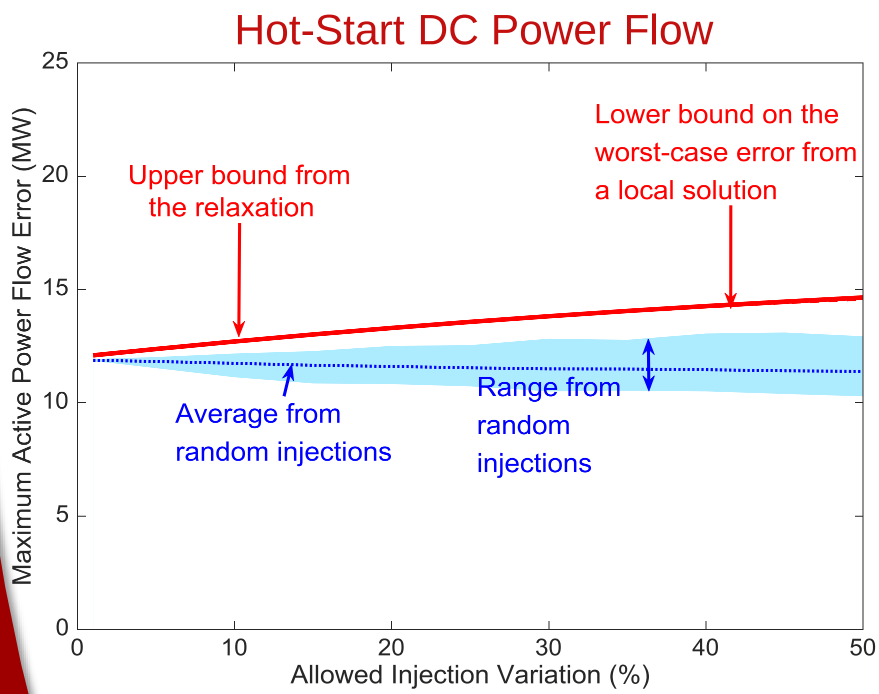


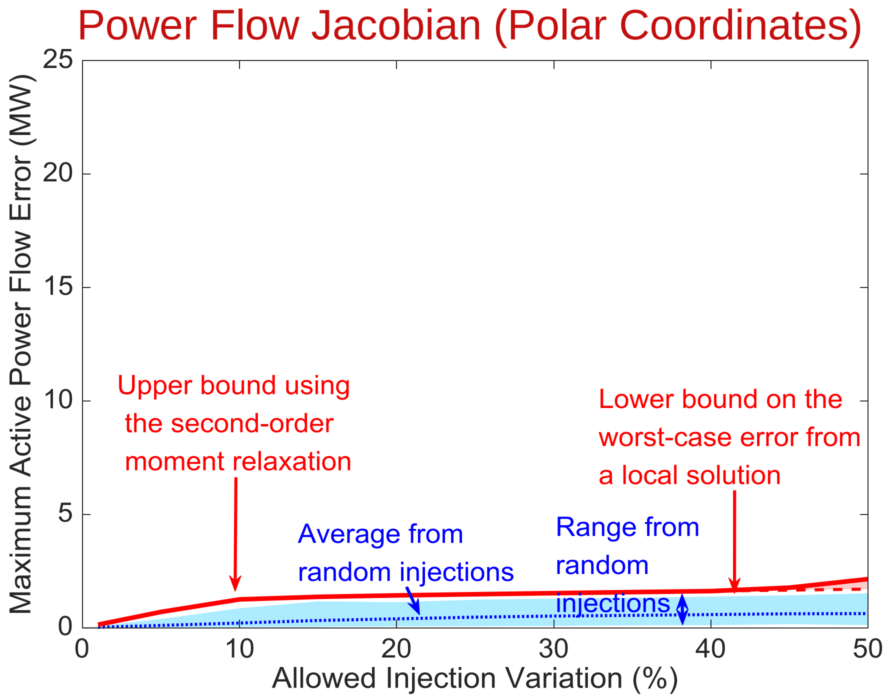
Decoupled Power Flow (Polar Coordinates)

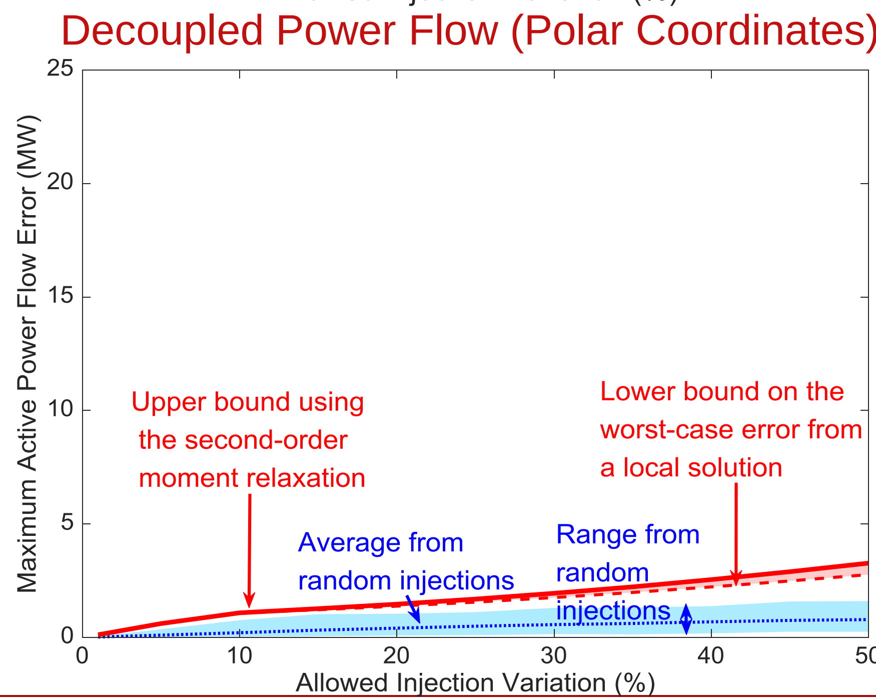


IEEE 14-Bus System

Other Linearizations







IEEE 14-Bus System

Conclusions

- We proposed an algorithm that uses convex relaxations to bound the worst-case error of power flow linearizations
- Results for several IEEE test cases show:
  - The bound is reasonably tight
  - The DC power flow can have large errors for some operating conditions
- Next steps:
  - Application to other linear approximations and test cases
  - Comparison with other error bounds
  - Determination of physical explanations for large errors
  - Design of new linearizations informed by the worst-case error

Questions?

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