

Scenario-Based Decomposition for Parallel Solution of the Contingency-Constrained Alternating Current Optimal Power Flow

Jean-Paul Watson

With Carl Laird, Anya Castillo, Cesar Silva-Monroy, and Gabe Hackebeil

Discrete Math and Algorithms Department

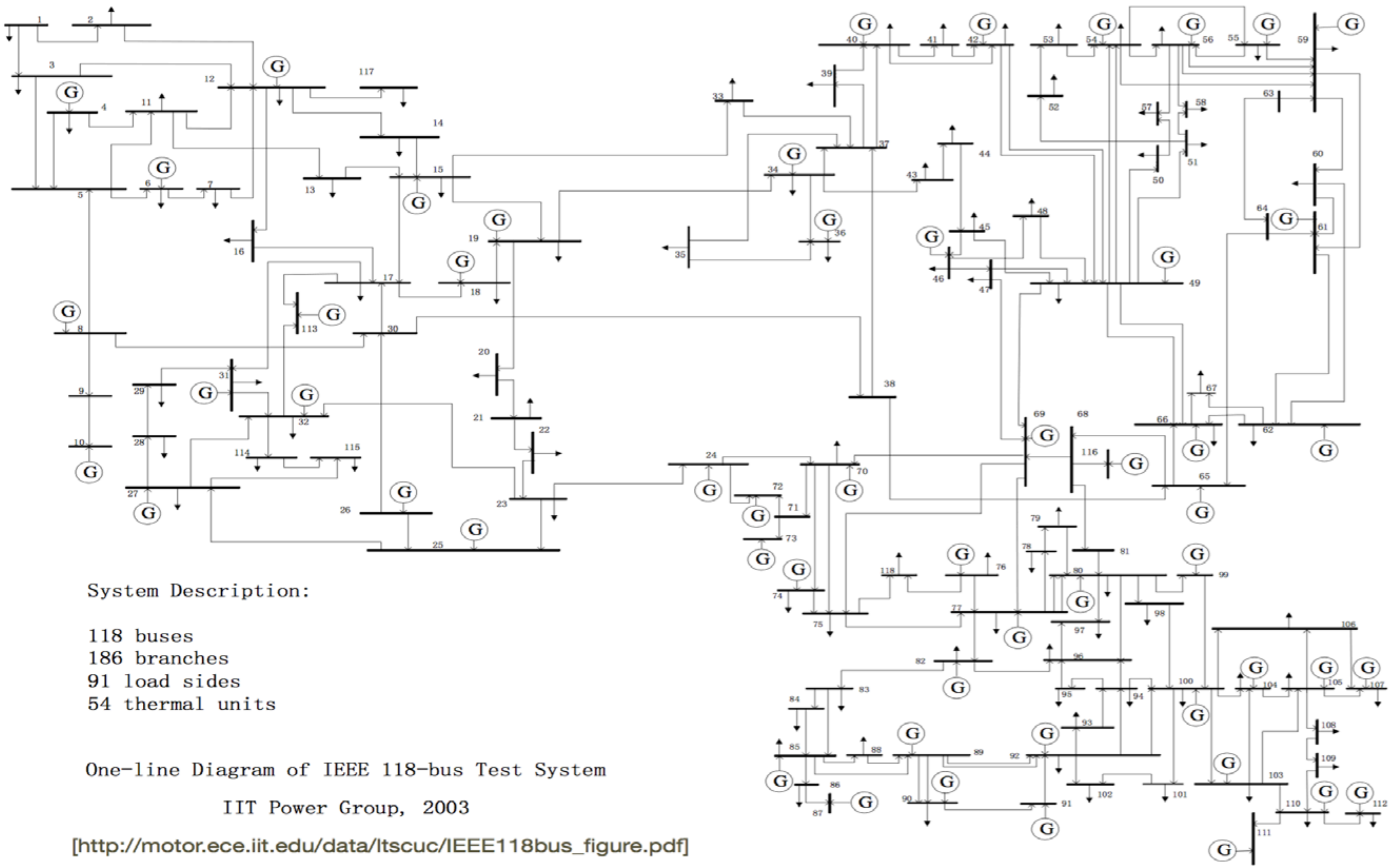
Sandia National Laboratories, Albuquerque, NM

CCR Center for Computing Research

U.S. DEPARTMENT OF ENERGY NNSA National Nuclear Security Administration

Sandia National Laboratories is a multi-program laboratory managed and operated by Sandia Corporation, a wholly owned subsidiary of Lockheed Martin Corporation, for the U.S. Department of Energy's National Nuclear Security Administration under contract DE-AC04-94AL85000.

Power Grid Operations



[http://motor.ece.iit.edu/data/ltscuc/IEEE118bus_figure.pdf]

Power Grid Operations At Sandia

Wind and Solar Forecasting and Scenario Generation…

Optimal Power Flow (DC and AC)

Security Constrained Unit Commitment (Linear/Nonlinear)

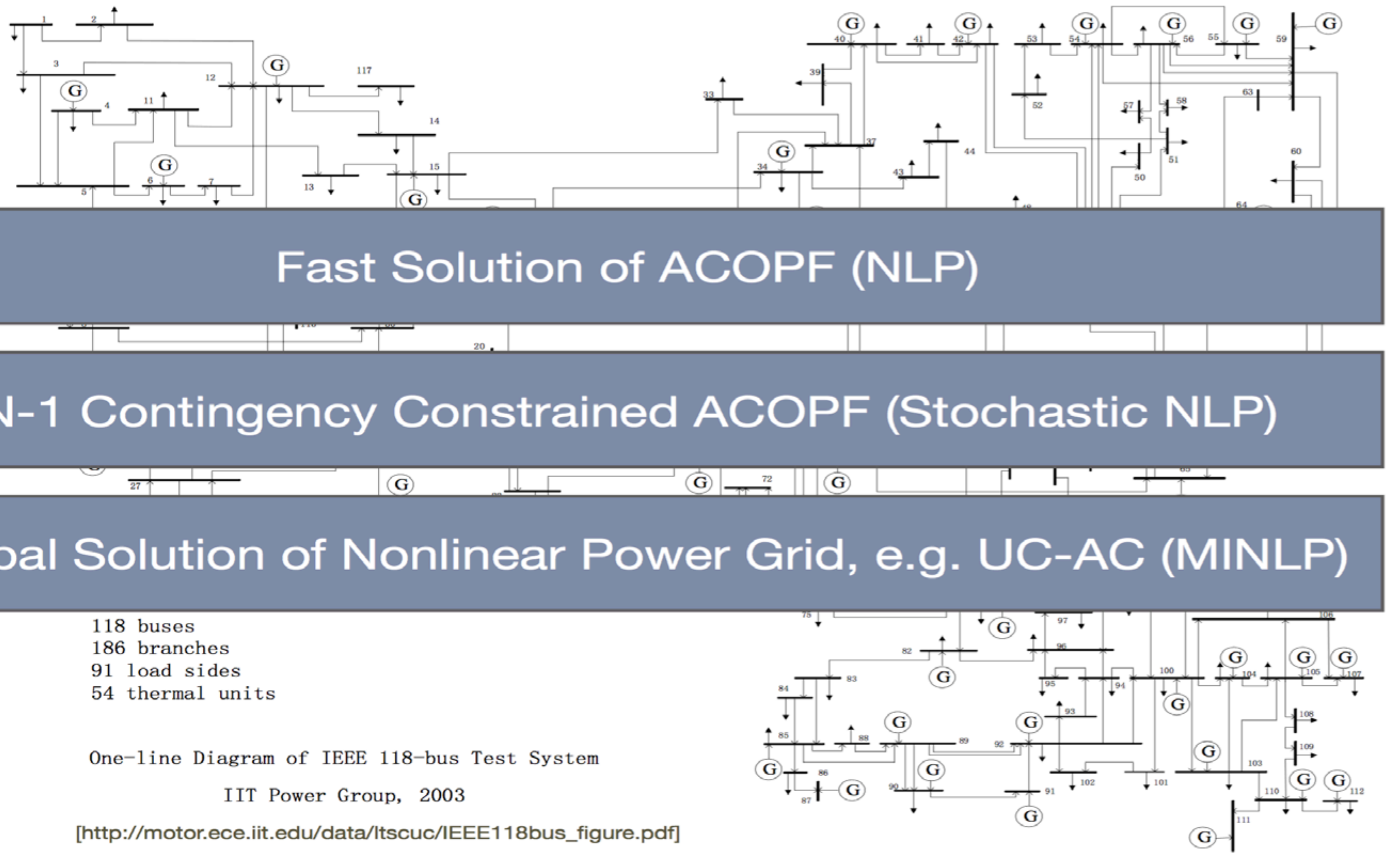
Reliable and Resilient Operation
(adverse weather, element failure, GMD)

Global/MINLP Solution of Nonlinear Power Systems Problems

Impact of Renewables …

Nonlinear Optimization for Power Grid Systems

Sandia National Laboratories

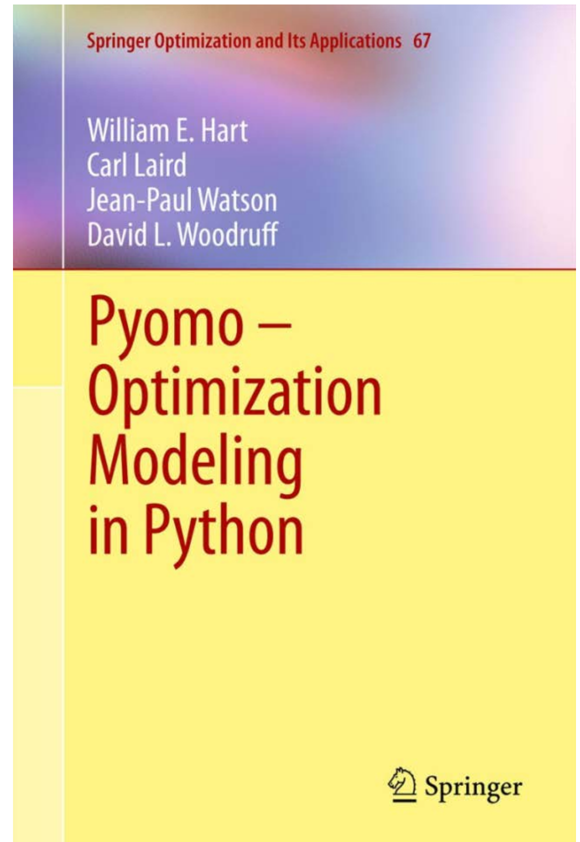


[http://motor.ece.iit.edu/data/ltscuc/IEEE118bus_figure.pdf]

- Fast Solution of ACOPF (NLP)
- N-1 Contingency Constrained ACOPF (Stochastic NLP)
- Global Solution of Nonlinear Power Grid, e.g. UC-AC (MINLP)

Our Software Environment: Pyomo

- Project homepage
  - http://software.sandia.gov/pyomo
- “The Book”
  - 2nd Edition released in June 2017



- Mathematical Programming Computation papers
  - Pyomo: Modeling and Solving Mathematical Programs in Python (Vol. 3, No. 3, 2011)
  - PySP: Modeling and Solving Stochastic Programs in Python (Vol. 4, No. 2, 2012)

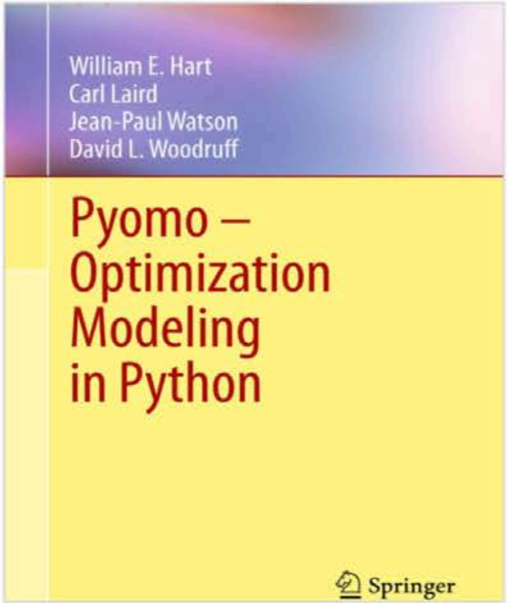
Our Hardware Environments

- Our objective is to run on commodity clusters
  - Utilities don't have, and don't want, supercomputers
  - But they do or might have multi-hundred node clusters
- Various HPC platforms we use and have used

SYSTEM	YEAR	VENDOR	CORES	RMAX (GFLOP/S)	RPEAK (GFLOP/S)
Sky Bridge - Cray CS300-LC, Xeon E5-2670 8C 2.6GHz, Intel Truscale	2015	Cray Inc.	29,584	532,900	615,347
Pecos - Xtreme-X , Xeon E5-2670 8C 2.600GHz, Infiniband QDR	2012	Cray Inc.	19,712	336,800	410,010
Chama - Xtreme-X GreenBlade GB512X, Xeon E5-2670 8C 2.600GHz, Infiniband QDR	2011	Cray Inc.	19,680	332,000	409,344
Dark Sand - Appro Xtreme-X Supercomputer, Xeon E5-2670 8C	2012	Cray Inc.	14,720	268,100	306,176

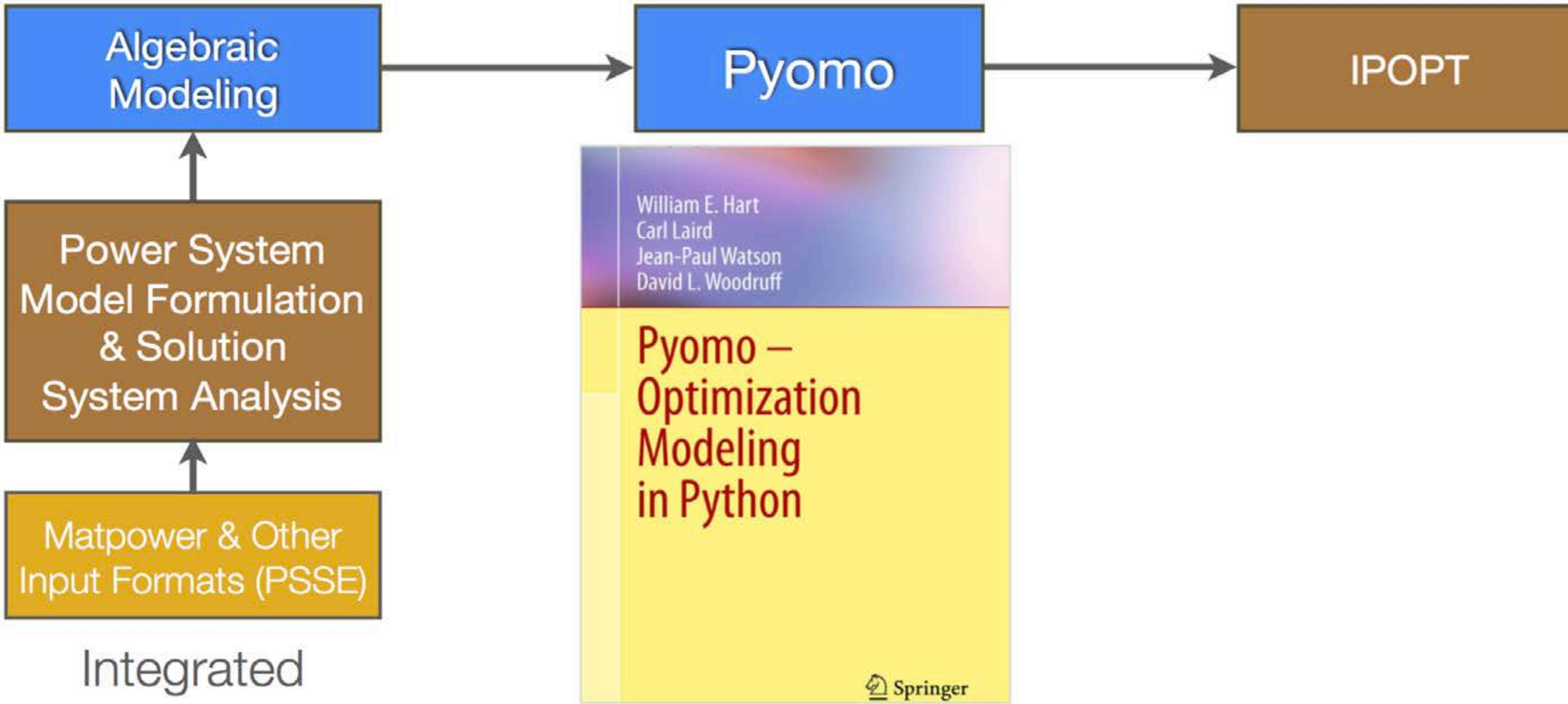
- Multi-Core SMP Workstation
  - 64-core AMD, 512GB of RAM
  - For only $17K from Dell....

Modeling Power Systems With Pyomo



Hackebeil, Hart, Laird, Siirola, Watson, Woodruff, and many others…

Modeling Power Systems With Pyomo



Integrated Python-based toolchain

Hackebeil, Hart, Laird, Siirola, Watson, Woodruff, and many others…

ACOPF Problem Formulation

$$\min \sum_{g\in\mathcal{G}} \text{GeneratorCost}_g(P_g, Q_g)$$

$$\begin{bmatrix} i^l_{fr} \\ i^l_{fj} \\ i^l_{tr} \\ i^l_{tj} \end{bmatrix} = Y^l_{br} \begin{bmatrix} v^l_{fr} \\ v^l_{fj} \\ v^l_{tr} \\ v^l_{tj} \end{bmatrix} \quad \forall\, l \in \mathcal{L}$$

PI Transmission Model

$$P^l_t = (v^{bt(l)}_r \cdot i^l_{tr} + v^{bt(l)}_j \cdot i^l_{tj}) \quad \forall\, l \in \mathcal{L}$$

$$Q^l_t = (v^{bt(l)}_j \cdot i^l_{tr} - v^{bt(l)}_r \cdot i^l_{tj}) \quad \forall\, l \in \mathcal{L}$$

$$P^l_f = (v^{bf(l)}_r \cdot i^l_{fr} + v^{bf(l)}_j \cdot i^l_{fj}) \quad \forall\, l \in \mathcal{L}$$

$$Q^l_f = (v^{bf(l)}_j \cdot i^l_{fr} - v^{bf(l)}_r \cdot i^l_{fj}) \quad \forall\, l \in \mathcal{L}$$

$$S^l_t \geq (P^l_t)^2 + (Q^l_t)^2 \quad \forall\, l \in \mathcal{L}$$

$$S^l_f \geq (P^l_f)^2 + (Q^l_f)^2 \quad \forall\, l \in \mathcal{L}$$

Real Power, Reactive Power, Apparent Power Constraints From the Branches

$$0 = \sum_{l\in\mathcal{B}^b_{in}} P^l_t + \sum_{l\in\mathcal{B}^b_{out}} P^l_f + \sum_{d\in\mathcal{D}^b} P^d_L - \sum_{g\in\mathcal{G}^b} P^g_G + Y^b_{sh} \cdot [(v^b_r)^2 + (v^b_j)^2] \quad \forall\, b \in \mathcal{B}$$

$$0 = \sum_{l\in\mathcal{B}^b_{in}} Q^l_t + \sum_{l\in\mathcal{B}^b_{out}} Q^l_f + \sum_{d\in\mathcal{D}^b} Q^d_L - \sum_{g\in\mathcal{G}^b} Q^g_G + Y^b_{sh} \cdot [(v^b_r)^2 + (v^b_j)^2] \quad \forall\, b \in \mathcal{B}$$

Power Balance Constraints

$$v^b_m = (v^b_r)^2 + (v^b_j)^2 \quad \forall\, b \in \mathcal{B}$$

$$v^{ref}_j = 0$$

Voltage Maximum/Reference

bounds on $v^b_m, P^g_G, Q^g_G, S^l_f, S^l_t$

ACOPF Solution With Pyomo/Ipopt

Sandia National Laboratories

Case Name	Number of Variables	Solution Time (CPU Seconds)
case4gs	67	0.015
case5	67	0.003
case9	95	0.004
case9Q	95	0.004
case6ww	105	0.004
nesta_case_14_ieee	194	0.007
case14	197	0.005
case30	399	0.028
case24_ieee_rts	416	0.016
case39	465	0.015
case57	767	0.015
case118	1832	0.037
case89pegase	1881	0.067
case300	4025	0.13
case30Q	4025	0.14
case2383wp	28456	2.6
case2737sop	31846	2.3
case2736sp	31927	2
case2746wp	32183	2
case2746wop	32357	2.1
case3012wp	35242	2.6

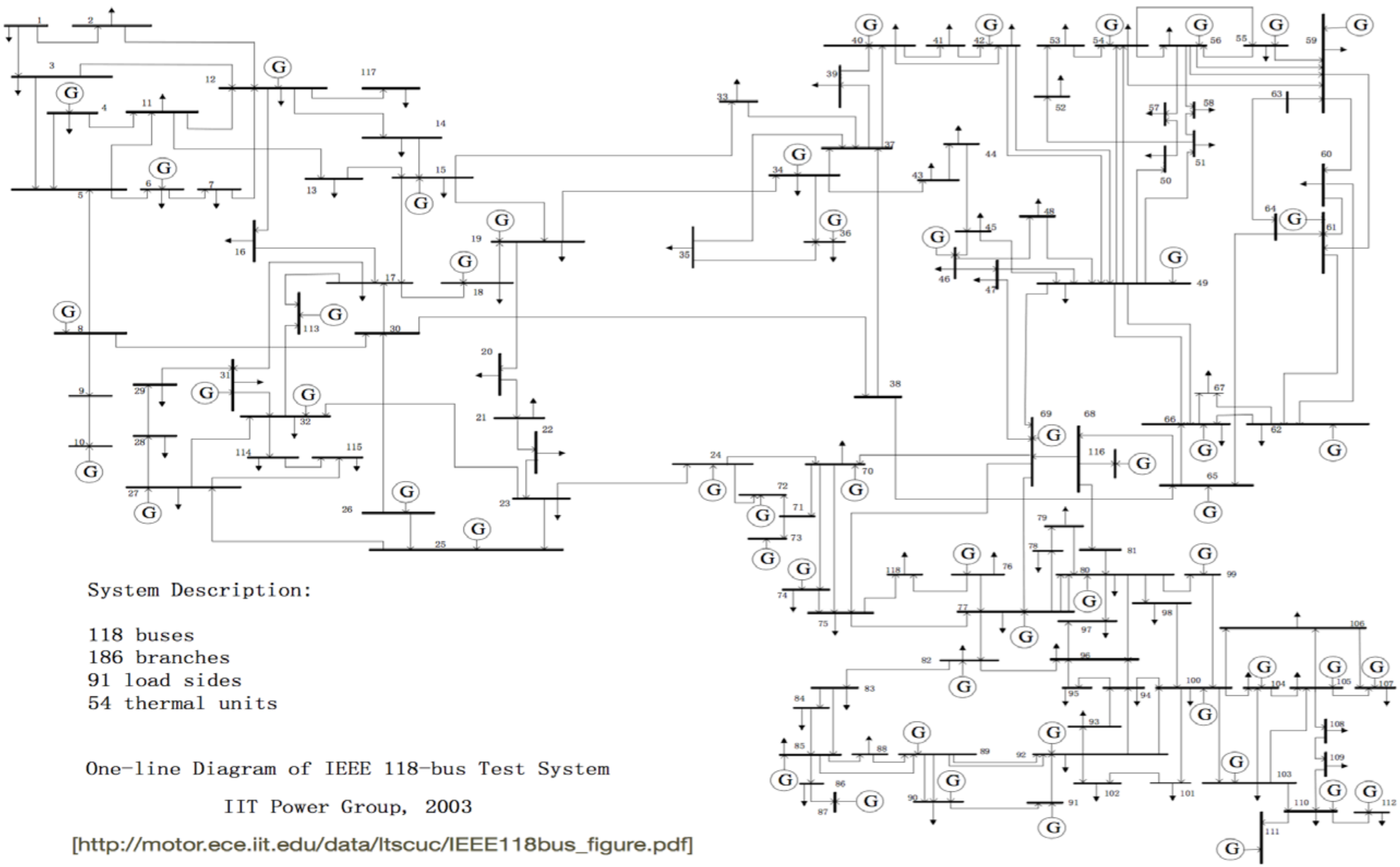
Global Solution of ACOPF With Pyomo/Ipopt

Table 1: Problem Size and Performance Results

Case Name	Optimal Solution	Optimality Gap (%)	CPU Time (s)	Iterations
Case6ww	3126.36	8×10^{-3}	0.26	4
Case14	8081.52	3×10^{-3}	0.43	3
Case30	574.52	0.0	0.95	5
Case39	41864.18	5×10^{-3}	1.21	3
Case57	41737.79	6×10^{-3}	7.29	12
Case89	5817.60	9×10^{-3}	46.2	44
Case118	129660.69	6×10^{-3}	18.5	14
Case300	719725.10	9×10^{-3}	82.7	49
NESTA Case6ww	3143.97	0.0	0.74	7
NESTA Case14	244.05	3×10^{-3}	0.22	3
NESTA Case30	204.97	0.0	0.57	4
NESTA Case39	96505.52	9×10^{-3}	3.00	8
NESTA Case57	1143.27	6×10^{-3}	9.62	20
NESTA Case89	5819.81	9×10^{-3}	55.8	57
NESTA Case118	3718.64	0.0	93.7	55
NESTA Case300	16891.28	0.0	138.2	26

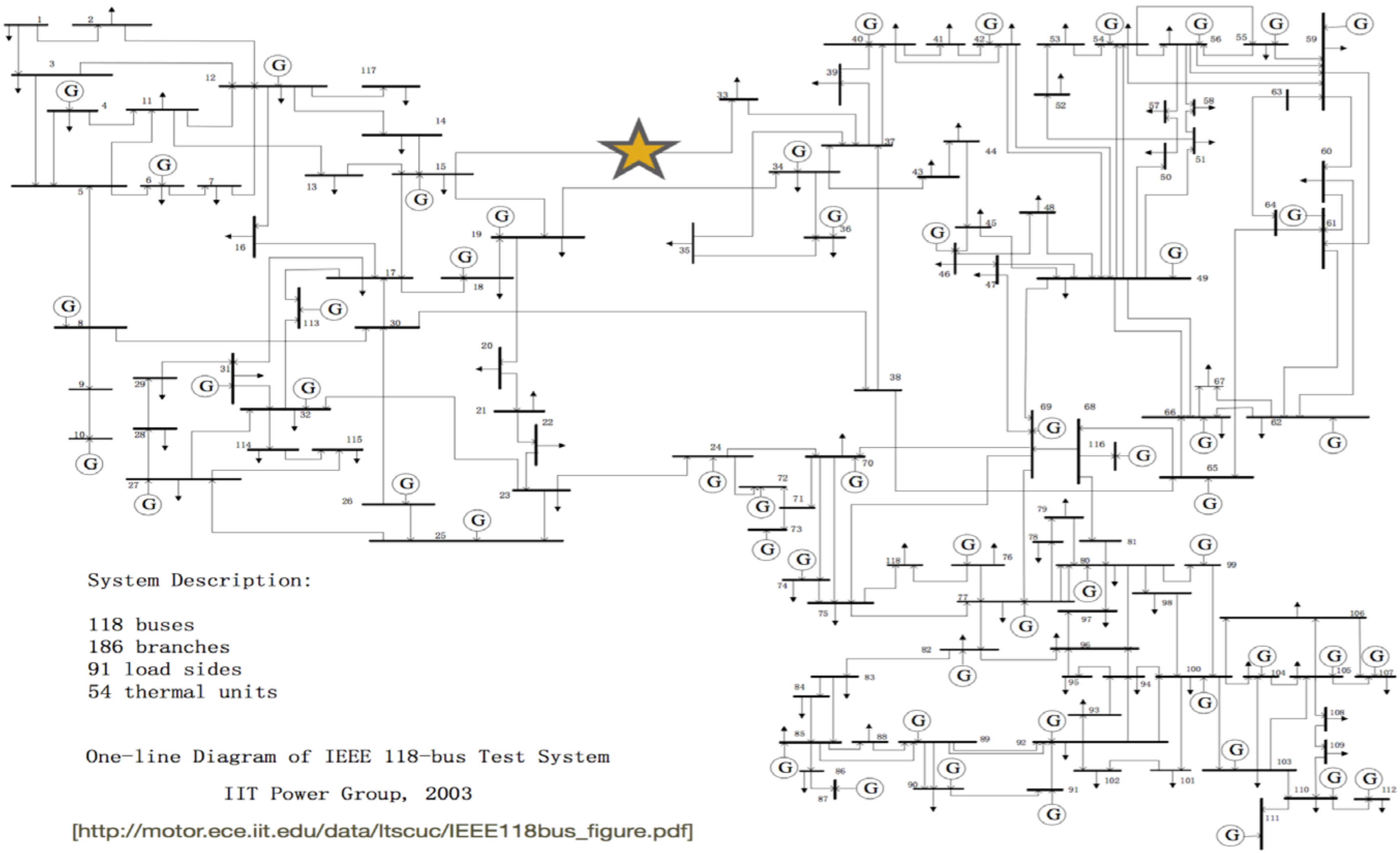
Taken from: Global Solution Strategies for the Network-Constrained Unit Commitment (NCUC) Problem with Nonlinear AC Transmission Models. Liu, Castillo, Watson, and Laird – Optimization Online, under review.

N-1 Contingency-Constrained ACOPF Problem



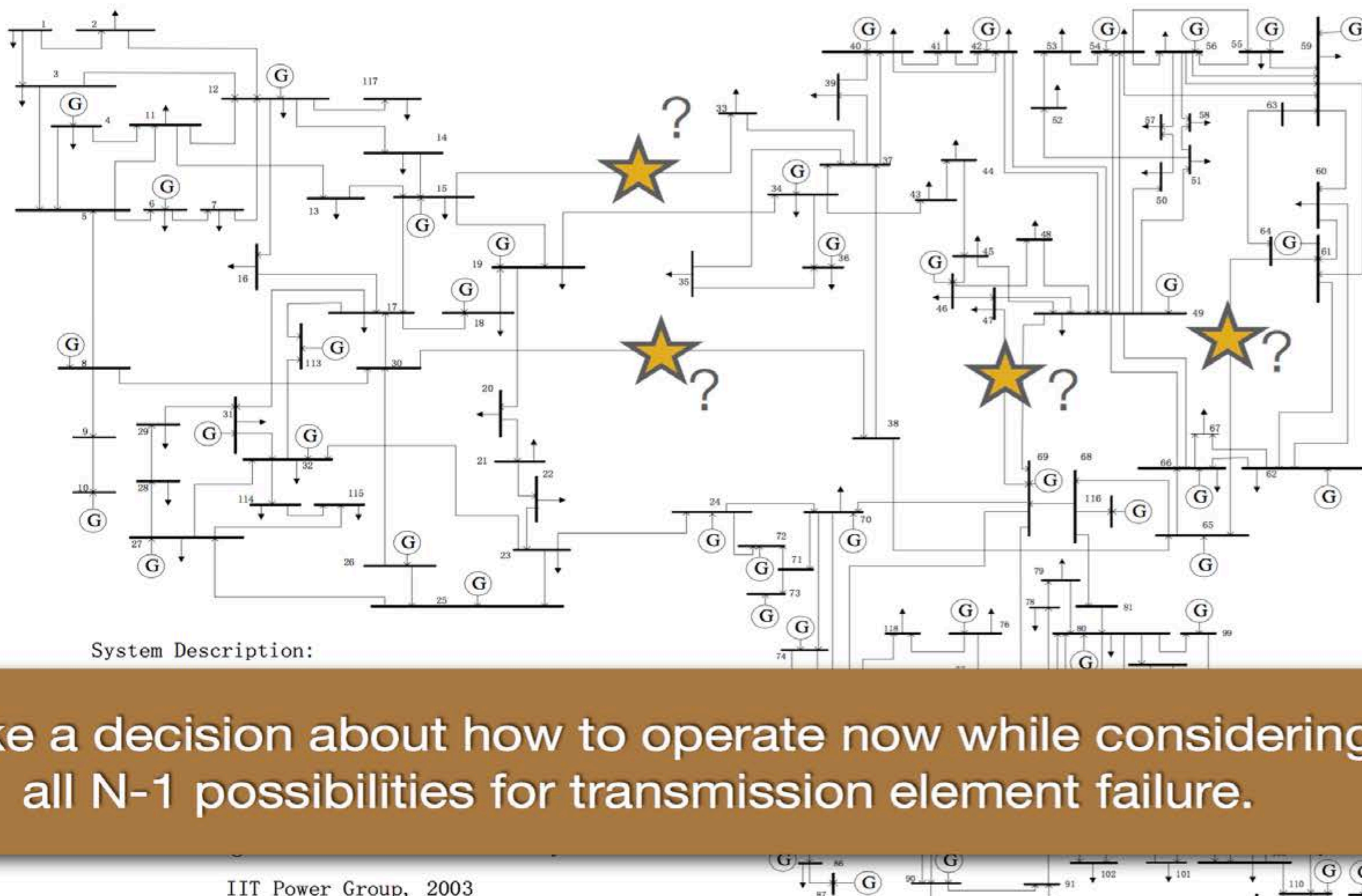
[http://motor.ece.iit.edu/data/ltscuc/IEEE118bus_figure.pdf]

N-1 Contingency-Constrained ACOPF Problem



[http://motor.ece.iit.edu/data/ltscuc/IEEE118bus_figure.pdf]

N-1 Contingency-Constrained ACOPF Problem

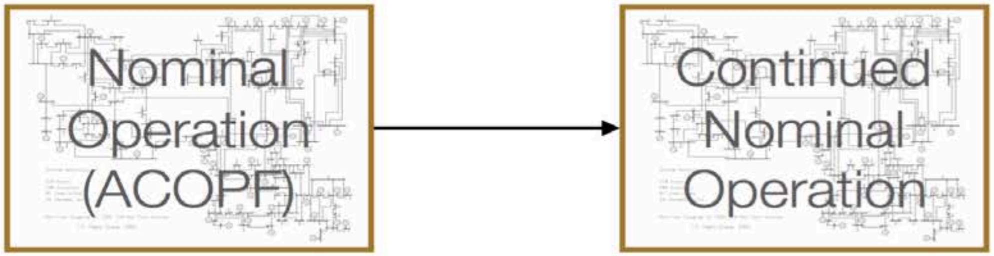


Make a decision about how to operate now while considering all N-1 possibilities for transmission element failure.

[http://motor.ece.iit.edu/data/ltscuc/IEEE118bus_figure.pdf]

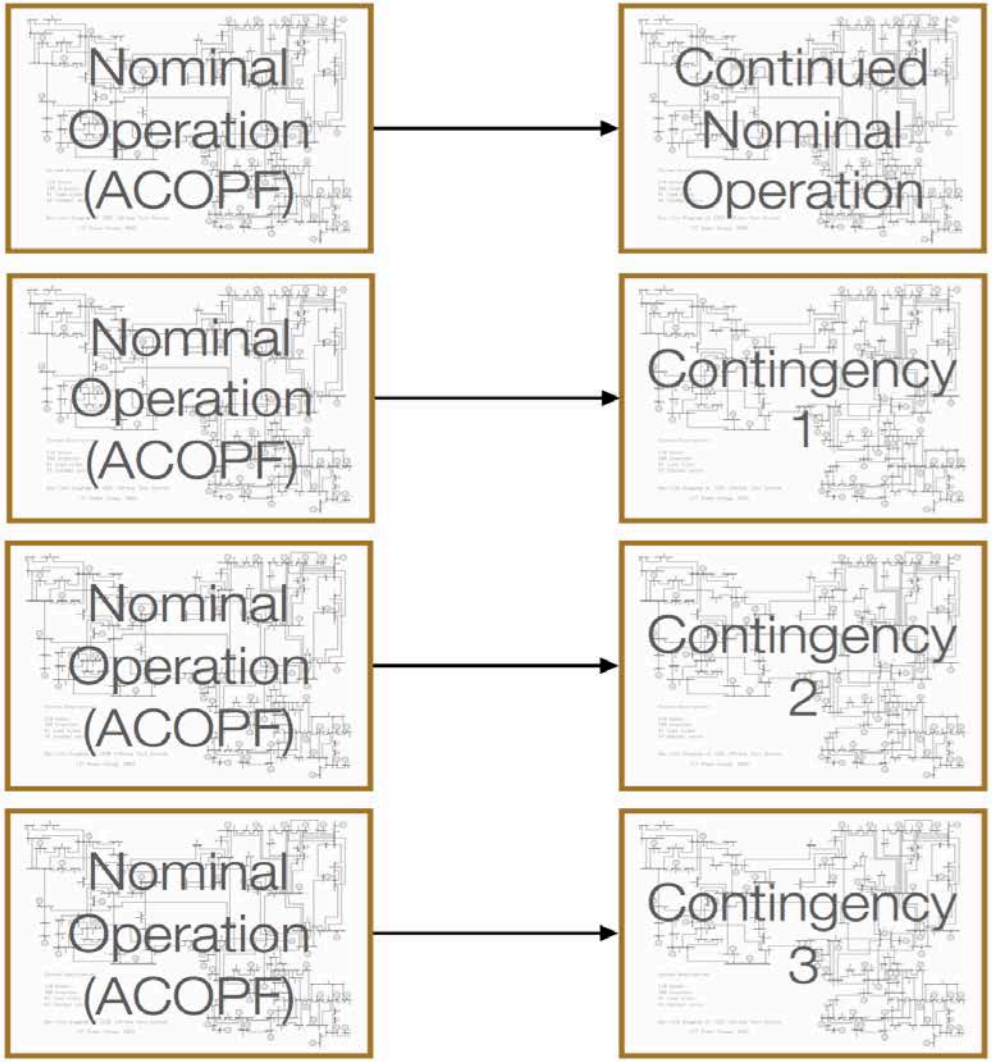
Contingency-Constrained ACOPF Problem

Contingency-Constrained ACOPF Problem



Contingency-Constrained ACOPF Problem

Sandia National Laboratories



Contingency-Constrained Stochastic Programming ACOPF

$\mathcal{L}$	Lines	$\mathcal{G}$	Generators
$\mathcal{B}$	All Buses	$\mathcal{D}$	Load Buses
$\mathcal{S}$	Scenarios	$\mathcal{T}$	Time periods

$$\min \sum_{s\in S} p_s \sum_{t\in T} \Bigg[\sum_{g\in\mathcal{G}} C_g^G(P_{g,t,s}^G, Q_{g,t,s}^G) + \rho_1 \sum_{g\in\mathcal{G}} \left[\left(P_{g,t,s}^G - P_{g,t,s}^{\star G}\right)^2 + \left(Q_{g,t,s}^G - Q_{g,t,s}^{\star G}\right)^2 \right]$$

$$+\rho_2 \sum_{b\in\mathcal{D}} \left[\left(P_{b,t,s}^L - P_b^{\star L}\right)^2 + \left(Q_{b,t,s}^L - Q_b^{\star L}\right)^2 \right] \Bigg]$$

$$\text{s.t.} \quad \begin{bmatrix} i_{l,t,s}^{fr} \\ i_{l,t,s}^{fj} \\ i_{l,t,s}^{tr} \\ i_{l,t,s}^{tj} \end{bmatrix} = Y_{l,t,s} \begin{bmatrix} v_{bf(l),t,s}^r \\ v_{bf(l),t,s}^j \\ v_{bt(l),t,s}^r \\ v_{bt(l),t,s}^j \end{bmatrix} \qquad \forall\, l\in\mathcal{L},\ t\in\mathcal{T},\ s\in\mathcal{S}$$

$$P_{b,t,s}^S = Y_b^S \left[(v_{b,t,s}^r)^2 + (v_{b,t,s}^j)^2 \right] \qquad \forall\, b\in\mathcal{H},\ t\in\mathcal{T},\ s\in\mathcal{S}$$

$$Q_{b,t,s}^S = -Y_b^S \left[(v_{b,t,s}^r)^2 + (v_{b,t,s}^j)^2 \right] \qquad \forall\, b\in\mathcal{H},\ t\in\mathcal{T},\ s\in\mathcal{S}$$

$$0 = \sum_{l\in\mathcal{I}_b} P_{l,t,s}^t + \sum_{l\in\mathcal{O}_b} P_{l,t,s}^f + P_{b,t,s}^S + P_{b,t,s}^L - P_{b,t,s}^G \qquad \forall\, b\in\mathcal{B},\ t\in\mathcal{T},\ s\in\mathcal{S}$$

$$0 = \sum_{l\in\mathcal{I}_b} Q_{l,t,s}^t + \sum_{l\in\mathcal{O}_b} Q_{l,t,s}^f + Q_{b,t,s}^S + Q_{b,t,s}^L - Q_{b,t,s}^G \qquad \forall\, b\in\mathcal{B},\ t\in\mathcal{T},\ s\in\mathcal{S}$$

Contingency-Constrained Stochastic Programming ACOPF

$\mathcal{L}$	Lines	$\mathcal{G}$	Generators
$\mathcal{B}$	All Buses	$\mathcal{D}$	Load Buses
$\mathcal{S}$	Scenarios	$\mathcal{T}$	Time periods

$$\min \sum_{s\in S} p_s \sum_{t\in T} \Bigg[\sum_{g\in\mathcal{G}} C_g^G(P_{g,t,s}^G, Q_{g,t,s}^G) + \rho_1 \sum_{g\in\mathcal{G}} \left[\left(P_{g,t,s}^G - P_{g,t,s}^{\star G}\right)^2 + \left(Q_{g,t,s}^G - Q_{g,t,s}^{\star G}\right)^2 \right] + \rho_2 \sum_{b\in\mathcal{D}} \left[\left(P_{b,t,s}^L - P_b^{\star L}\right)^2 + \left(Q_{b,t,s}^L - Q_b^{\star L}\right)^2 \right] \Bigg]$$

Objective: Expected Value of Generator Op. Costs + Penalty

$$\text{s.t.} \quad \begin{bmatrix} i_{l,t,s}^{fr} \\ i_{l,t,s}^{fj} \\ i_{l,t,s}^{tr} \\ i_{l,t,s}^{tj} \end{bmatrix} = Y_{l,t,s} \begin{bmatrix} v_{bf(l),t,s}^{r} \\ v_{bf(l),t,s}^{j} \\ v_{bt(l),t,s}^{r} \\ v_{bt(l),t,s}^{j} \end{bmatrix} \qquad \forall\, l\in\mathcal{L},\ t\in\mathcal{T},\ s\in\mathcal{S}$$

$$P_{b,t,s}^S = Y_b^S \left[(v_{b,t,s}^r)^2 + (v_{b,t,s}^j)^2 \right] \qquad \forall\, b\in\mathcal{H},\ t\in\mathcal{T},\ s\in\mathcal{S}$$

$$Q_{b,t,s}^S = -Y_b^S \left[(v_{b,t,s}^r)^2 + (v_{b,t,s}^j)^2 \right] \qquad \forall\, b\in\mathcal{H},\ t\in\mathcal{T},\ s\in\mathcal{S}$$

$$0 = \sum_{l\in\mathcal{I}_b} P_{l,t,s}^t + \sum_{l\in\mathcal{O}_b} P_{l,t,s}^f + P_{b,t,s}^S + P_{b,t,s}^L - P_{b,t,s}^G \qquad \forall\, b\in\mathcal{B},\ t\in\mathcal{T},\ s\in\mathcal{S}$$

$$0 = \sum_{l\in\mathcal{I}_b} Q_{l,t,s}^t + \sum_{l\in\mathcal{O}_b} Q_{l,t,s}^f + Q_{b,t,s}^S + Q_{b,t,s}^L - Q_{b,t,s}^G \qquad \forall\, b\in\mathcal{B},\ t\in\mathcal{T},\ s\in\mathcal{S}$$

Contingency-Constrained Stochastic Programming ACOPF

$\mathcal{L}$	Lines	$\mathcal{G}$	Generators
$\mathcal{B}$	All Buses	$\mathcal{D}$	Load Buses
$\mathcal{S}$	Scenarios	$\mathcal{T}$	Time periods

$$\min \sum_{s \in S} p_s \sum_{t \in T} \left[\sum_{g \in \mathcal{G}} C_g^G(P_{g,t,s}^G, Q_{g,t,s}^G) + \rho_1 \sum_{g \in \mathcal{G}} \left[(P_{g,t,s}^G - P_{g,t,s}^{\star G})^2 + (Q_{g,t,s}^G - Q_{g,t,s}^{\star G})^2 \right] + \rho_2 \sum_{b \in \mathcal{D}} \left[(P_{b,t,s}^L - P_b^{\star L})^2 + (Q_{b,t,s}^L - Q_b^{\star L})^2 \right] \right]$$

$$\text{s.t.} \quad \begin{bmatrix} i_{l,t,s}^{fr} \\ i_{l,t,s}^{fj} \\ i_{l,t,s}^{tr} \\ i_{l,t,s}^{tj} \end{bmatrix} = Y_{l,t,s} \begin{bmatrix} v_{bf(l),t,s}^{r} \\ v_{bf(l),t,s}^{j} \\ v_{bt(l),t,s}^{r} \\ v_{bt(l),t,s}^{j} \end{bmatrix} \quad \forall\, l \in \mathcal{L},\ t \in \mathcal{T},\ s \in \mathcal{S}$$

IV Relationships (to/from) Every Line (all S and T)

$$P_{b,t,s}^S = Y_b^S \left[(v_{b,t,s}^r)^2 + (v_{b,t,s}^j)^2 \right] \quad \forall\, b \in \mathcal{H},\ t \in \mathcal{T},\ s \in \mathcal{S}$$

$$Q_{b,t,s}^S = -Y_b^S \left[(v_{b,t,s}^r)^2 + (v_{b,t,s}^j)^2 \right] \quad \forall\, b \in \mathcal{H},\ t \in \mathcal{T},\ s \in \mathcal{S}$$

PI Transmission Model

$$0 = \sum_{l \in \mathcal{I}_b} P_{l,t,s}^t + \sum_{l \in \mathcal{O}_b} P_{l,t,s}^f + P_{b,t,s}^S + P_{b,t,s}^L - P_{b,t,s}^G \quad \forall\, b \in \mathcal{B},\ t \in \mathcal{T},\ s \in \mathcal{S}$$

$$0 = \sum_{l \in \mathcal{I}_b} Q_{l,t,s}^t + \sum_{l \in \mathcal{O}_b} Q_{l,t,s}^f + Q_{b,t,s}^S + Q_{b,t,s}^L - Q_{b,t,s}^G \quad \forall\, b \in \mathcal{B},\ t \in \mathcal{T},\ s \in \mathcal{S}$$

Contingency-Constrained Stochastic Programming ACOPF

$\mathcal{L}$	Lines	$\mathcal{G}$	Generators
$\mathcal{B}$	All Buses	$\mathcal{D}$	Load Buses
$\mathcal{S}$	Scenarios	$\mathcal{T}$	Time periods

$$\min \sum_{s \in S} p_s \sum_{t \in T} \left[\sum_{g \in \mathcal{G}} C_g^G(P_{g,t,s}^G, Q_{g,t,s}^G) + \rho_1 \sum_{g \in \mathcal{G}} \left[\left(P_{g,t,s}^G - P_{g,t,s}^{\star G}\right)^2 + \left(Q_{g,t,s}^G - Q_{g,t,s}^{\star G}\right)^2 \right] + \rho_2 \sum_{b \in \mathcal{D}} \left[\left(P_{b,t,s}^L - P_b^{\star L}\right)^2 + \left(Q_{b,t,s}^L - Q_b^{\star L}\right)^2 \right] \right]$$

$$\text{s.t.} \quad \begin{bmatrix} i_{l,t,s}^{fr} \\ i_{l,t,s}^{fj} \\ i_{l,t,s}^{tr} \\ i_{l,t,s}^{tj} \end{bmatrix} = Y_{l,t,s} \begin{bmatrix} v_{bf(l),t,s}^r \\ v_{bf(l),t,s}^j \\ v_{bt(l),t,s}^r \\ v_{bt(l),t,s}^j \end{bmatrix} \qquad \forall\, l \in \mathcal{L},\ t \in \mathcal{T},\ s \in \mathcal{S}$$

$$P_{b,t,s}^S = Y_b^S \left[(v_{b,t,s}^r)^2 + (v_{b,t,s}^j)^2 \right] \qquad \forall\, b \in \mathcal{H},\ t \in \mathcal{T},\ s \in \mathcal{S}$$

$$Q_{b,t,s}^S = -Y_b^S \left[(v_{b,t,s}^r)^2 + (v_{b,t,s}^j)^2 \right] \qquad \forall\, b \in \mathcal{H},\ t \in \mathcal{T},\ s \in \mathcal{S}$$

$$0 = \sum_{l \in \mathcal{I}_b} P_{l,t,s}^t + \sum_{l \in \mathcal{O}_b} P_{l,t,s}^f + P_{b,t,s}^S + P_{b,t,s}^L - P_{b,t,s}^G \qquad \forall\, b \in \mathcal{B},\ t \in \mathcal{T},\ s \in \mathcal{S}$$

$$0 = \sum_{l \in \mathcal{I}_b} Q_{l,t,s}^t + \sum_{l \in \mathcal{O}_b} Q_{l,t,s}^f + Q_{b,t,s}^S + Q_{b,t,s}^L - Q_{b,t,s}^G \qquad \forall\, b \in \mathcal{B},\ t \in \mathcal{T},\ s \in \mathcal{S}$$

Power Balance (to/from) Every Bus (all S and T)

Contingency-Constrained Stochastic Programming ACOPF

$$P^f_{l,t,s} = v^r_{bf(l),t,s} \cdot i^{fr}_{l,t,s} + v^j_{bf(l),t,s} \cdot i^{fj}_{l,t,s} \quad \forall\, l \in \mathcal{L},\ t \in \mathcal{T},\ s \in \mathcal{S}$$
$$Q^f_{l,t,s} = v^j_{bf(l),t,s} \cdot i^{fr}_{l,t,s} - v^r_{bf(l),t,s} \cdot i^{fj}_{l,t,s} \quad \forall\, l \in \mathcal{L},\ t \in \mathcal{T},\ s \in \mathcal{S}$$
$$P^t_{l,t,s} = v^r_{bt(l),t,s} \cdot i^{tr}_{l,t,s} + v^j_{bt(l),t,s} \cdot i^{tj}_{l,t,s} \quad \forall\, l \in \mathcal{L},\ t \in \mathcal{T},\ s \in \mathcal{S}$$
$$Q^t_{l,t,s} = v^j_{bt(l),t,s} \cdot i^{tr}_{l,t,s} - v^r_{bt(l),t,s} \cdot i^{tj}_{l,t,s} \quad \forall\, l \in \mathcal{L},\ t \in \mathcal{T},\ s \in \mathcal{S}$$

Power (to/from)
Every Line
(all S and T)

$$(P^f_{l,t,s})^2 + (Q^f_{l,t,s})^2 \le (S^U)^2 \quad \forall\, l \in \mathcal{L},\ t \in \mathcal{T},\ s \in \mathcal{S}$$
$$(P^t_{l,t,s})^2 + (Q^t_{l,t,s})^2 \le (S^U)^2 \quad \forall\, l \in \mathcal{L},\ t \in \mathcal{T},\ s \in \mathcal{S}$$
$$(V^L_b)^2 \le (v^r_{b,t,s})^2 + (v^j_{b,t,s})^2 \le (V^U_b)^2 \quad \forall\, b \in \mathcal{B},\ t \in \mathcal{T},\ s \in \mathcal{S}$$
$$P^{GL}_g \le P^G_{g,t,s} \le P^{GU}_g \quad \forall\, g \in \mathcal{G},\ t \in \mathcal{T},\ s \in \mathcal{S}$$
$$Q^{GL}_g \le Q^G_{g,t,s} \le Q^{GU}_g \quad \forall\, g \in \mathcal{G},\ t \in \mathcal{T},\ s \in \mathcal{S}$$
$$v^j_{rb,t,s} = 0 \quad \forall\, t \in \mathcal{T},\ s \in \mathcal{S}$$
$$P^L_{b,1,s} = P^{\star L}_b \quad \forall\, b \in \mathcal{D},\ s \in \mathcal{S}$$
$$Q^L_{b,1,s} = Q^{\star L}_b \quad \forall\, b \in \mathcal{D},\ s \in \mathcal{S}$$
$$-P^{GR}_g \le P^G_{g,1,s} - P^G_{g,2,s} \le P^{GR}_g \quad \forall\, g \in \mathcal{G},\ s \in \mathcal{S}$$
$$P^G_{g,1,0} = P^G_{g,2,0} \quad \forall\, g \in \mathcal{G}$$
$$Q^G_{g,1,0} = Q^G_{g,2,0} \quad \forall\, g \in \mathcal{G}$$
$$P^G_{g,1,0} = P^G_{g,1,s} \quad \forall\, g \in \mathcal{G},\ s \in \mathcal{S}/\{0\}$$
$$Q^G_{g,1,0} = Q^G_{g,1,s} \quad \forall\, g \in \mathcal{G},\ s \in \mathcal{S}/\{0\}$$

Contingency-Constrained Stochastic Programming ACOPF

$$P^f_{l,t,s} = v^r_{bf(l),t,s} \cdot i^{fr}_{l,t,s} + v^j_{bf(l),t,s} \cdot i^{fj}_{l,t,s} \qquad \forall\, l \in \mathcal{L},\ t \in \mathcal{T},\ s \in \mathcal{S}$$

$$Q^f_{l,t,s} = v^j_{bf(l),t,s} \cdot i^{fr}_{l,t,s} - v^r_{bf(l),t,s} \cdot i^{fj}_{l,t,s} \qquad \forall\, l \in \mathcal{L},\ t \in \mathcal{T},\ s \in \mathcal{S}$$

$$P^t_{l,t,s} = v^r_{bt(l),t,s} \cdot i^{tr}_{l,t,s} + v^j_{bt(l),t,s} \cdot i^{tj}_{l,t,s} \qquad \forall\, l \in \mathcal{L},\ t \in \mathcal{T},\ s \in \mathcal{S}$$

$$Q^t_{l,t,s} = v^j_{bt(l),t,s} \cdot i^{tr}_{l,t,s} - v^r_{bt(l),t,s} \cdot i^{tj}_{l,t,s} \qquad \forall\, l \in \mathcal{L},\ t \in \mathcal{T},\ s \in \mathcal{S}$$

$$(P^f_{l,t,s})^2 + (Q^f_{l,t,s})^2 \leq (S^U)^2 \qquad \forall\, l \in \mathcal{L},\ t \in \mathcal{T},\ s \in \mathcal{S}$$

$$(P^t_{l,t,s})^2 + (Q^t_{l,t,s})^2 \leq (S^U)^2 \qquad \forall\, l \in \mathcal{L},\ t \in \mathcal{T},\ s \in \mathcal{S}$$

$$(V^L_b)^2 \leq (v^r_{b,t,s})^2 + (v^j_{b,t,s})^2 \leq (V^U_b)^2 \qquad \forall\, b \in \mathcal{B},\ t \in \mathcal{T},\ s \in \mathcal{S}$$

$$P^{GL}_g \leq P^G_{g,t,s} \leq P^{GU}_g \qquad \forall\, g \in \mathcal{G},\ t \in \mathcal{T},\ s \in \mathcal{S}$$

$$Q^{GL}_g \leq Q^G_{g,t,s} \leq Q^{GU}_g \qquad \forall\, g \in \mathcal{G},\ t \in \mathcal{T},\ s \in \mathcal{S}$$

$$v^j_{rb,t,s} = 0 \qquad \forall\, t \in \mathcal{T},\ s \in \mathcal{S}$$

$$P^L_{b,1,s} = P^{\star L}_b \qquad \forall\, b \in \mathcal{D},\ s \in \mathcal{S}$$

$$Q^L_{b,1,s} = Q^{\star L}_b \qquad \forall\, b \in \mathcal{D},\ s \in \mathcal{S}$$

$$-P^{GR}_g \leq P^G_{g,1,s} - P^G_{g,2,s} \leq P^{GR}_g \qquad \forall\, g \in \mathcal{G},\ s \in \mathcal{S}$$

$$P^G_{g,1,0} = P^G_{g,2,0} \qquad \forall\, g \in \mathcal{G}$$

$$Q^G_{g,1,0} = Q^G_{g,2,0} \qquad \forall\, g \in \mathcal{G}$$

$$P^G_{g,1,0} = P^G_{g,1,s} \qquad \forall\, g \in \mathcal{G},\ s \in \mathcal{S}/\{0\}$$

$$Q^G_{g,1,0} = Q^G_{g,1,s} \qquad \forall\, g \in \mathcal{G},\ s \in \mathcal{S}/\{0\}$$

Line limits on (to/from) of every line (all S and T)

Contingency-Constrained Stochastic Programming ACOPF

$$P^f_{l,t,s} = v^r_{bf(l),t,s} \cdot i^{fr}_{l,t,s} + v^j_{bf(l),t,s} \cdot i^{fj}_{l,t,s} \quad \forall\, l \in \mathcal{L},\ t \in \mathcal{T},\ s \in \mathcal{S}$$

$$Q^f_{l,t,s} = v^j_{bf(l),t,s} \cdot i^{fr}_{l,t,s} - v^r_{bf(l),t,s} \cdot i^{fj}_{l,t,s} \quad \forall\, l \in \mathcal{L},\ t \in \mathcal{T},\ s \in \mathcal{S}$$

$$P^t_{l,t,s} = v^r_{bt(l),t,s} \cdot i^{tr}_{l,t,s} + v^j_{bt(l),t,s} \cdot i^{tj}_{l,t,s} \quad \forall\, l \in \mathcal{L},\ t \in \mathcal{T},\ s \in \mathcal{S}$$

$$Q^t_{l,t,s} = v^j_{bt(l),t,s} \cdot i^{tr}_{l,t,s} - v^r_{bt(l),t,s} \cdot i^{tj}_{l,t,s} \quad \forall\, l \in \mathcal{L},\ t \in \mathcal{T},\ s \in \mathcal{S}$$

$$(P^f_{l,t,s})^2 + (Q^f_{l,t,s})^2 \le (S^U)^2 \quad \forall\, l \in \mathcal{L},\ t \in \mathcal{T},\ s \in \mathcal{S}$$

$$(P^t_{l,t,s})^2 + (Q^t_{l,t,s})^2 \le (S^U)^2 \quad \forall\, l \in \mathcal{L},\ t \in \mathcal{T},\ s \in \mathcal{S}$$

$$(V^L_b)^2 \le (v^r_{b,t,s})^2 + (v^j_{b,t,s})^2 \le (V^U_b)^2 \quad \forall\, b \in \mathcal{B},\ t \in \mathcal{T},\ s \in \mathcal{S}$$

$$P^{GL}_g \le P^G_{g,t,s} \le P^{GU}_g \quad \forall\, g \in \mathcal{G},\ t \in \mathcal{T},\ s \in \mathcal{S}$$

$$Q^{GL}_g \le Q^G_{g,t,s} \le Q^{GU}_g \quad \forall\, g \in \mathcal{G},\ t \in \mathcal{T},\ s \in \mathcal{S}$$

$$v^j_{rb,t,s} = 0 \quad \forall\, t \in \mathcal{T},\ s \in \mathcal{S}$$

$$P^L_{b,1,s} = P^{\star L}_b \quad \forall\, b \in \mathcal{D},\ s \in \mathcal{S}$$

$$Q^L_{b,1,s} = Q^{\star L}_b \quad \forall\, b \in \mathcal{D},\ s \in \mathcal{S}$$

$$-P^{GR}_g \le P^G_{g,1,s} - P^G_{g,2,s} \le P^{GR}_g \quad \forall\, g \in \mathcal{G},\ s \in \mathcal{S}$$

$$P^G_{g,1,0} = P^G_{g,2,0} \quad \forall\, g \in \mathcal{G}$$

$$Q^G_{g,1,0} = Q^G_{g,2,0} \quad \forall\, g \in \mathcal{G}$$

$$P^G_{g,1,0} = P^G_{g,1,s} \quad \forall\, g \in \mathcal{G},\ s \in \mathcal{S}/\{0\}$$

$$Q^G_{g,1,0} = Q^G_{g,1,s} \quad \forall\, g \in \mathcal{G},\ s \in \mathcal{S}/\{0\}$$

Voltage limits on every bus (all S and T)

Contingency-Constrained Stochastic Programming ACOPF

$$P^f_{l,t,s} = v^r_{bf(l),t,s} \cdot i^{fr}_{l,t,s} + v^j_{bf(l),t,s} \cdot i^{fj}_{l,t,s} \qquad \forall\, l \in \mathcal{L},\ t \in \mathcal{T},\ s \in \mathcal{S}$$

$$Q^f_{l,t,s} = v^j_{bf(l),t,s} \cdot i^{fr}_{l,t,s} - v^r_{bf(l),t,s} \cdot i^{fj}_{l,t,s} \qquad \forall\, l \in \mathcal{L},\ t \in \mathcal{T},\ s \in \mathcal{S}$$

$$P^t_{l,t,s} = v^r_{bt(l),t,s} \cdot i^{tr}_{l,t,s} + v^j_{bt(l),t,s} \cdot i^{tj}_{l,t,s} \qquad \forall\, l \in \mathcal{L},\ t \in \mathcal{T},\ s \in \mathcal{S}$$

$$Q^t_{l,t,s} = v^j_{bt(l),t,s} \cdot i^{tr}_{l,t,s} - v^r_{bt(l),t,s} \cdot i^{tj}_{l,t,s} \qquad \forall\, l \in \mathcal{L},\ t \in \mathcal{T},\ s \in \mathcal{S}$$

$$(P^f_{l,t,s})^2 + (Q^f_{l,t,s})^2 \le (S^U)^2 \qquad \forall\, l \in \mathcal{L},\ t \in \mathcal{T},\ s \in \mathcal{S}$$

$$(P^t_{l,t,s})^2 + (Q^t_{l,t,s})^2 \le (S^U)^2 \qquad \forall\, l \in \mathcal{L},\ t \in \mathcal{T},\ s \in \mathcal{S}$$

$$(V^L_b)^2 \le (v^r_{b,t,s})^2 + (v^j_{b,t,s})^2 \le (V^U_b)^2 \qquad \forall\, b \in \mathcal{B},\ t \in \mathcal{T},\ s \in \mathcal{S}$$

$$P^{GL}_g \le P^G_{g,t,s} \le P^{GU}_g \qquad \forall\, g \in \mathcal{G},\ t \in \mathcal{T},\ s \in \mathcal{S}$$

$$Q^{GL}_g \le Q^G_{g,t,s} \le Q^{GU}_g \qquad \forall\, g \in \mathcal{G},\ t \in \mathcal{T},\ s \in \mathcal{S}$$

$$v^j_{rb,t,s} = 0 \qquad \forall\, t \in \mathcal{T},\ s \in \mathcal{S}$$

$$P^L_{b,1,s} = P^{\star L}_b \qquad \forall\, b \in \mathcal{D},\ s \in \mathcal{S}$$

$$Q^L_{b,1,s} = Q^{\star L}_b \qquad \forall\, b \in \mathcal{D},\ s \in \mathcal{S}$$

$$-P^{GR}_g \le P^G_{g,1,s} - P^G_{g,2,s} \le P^{GR}_g \qquad \forall\, g \in \mathcal{G},\ s \in \mathcal{S}$$

$$P^G_{g,1,0} = P^G_{g,2,0} \qquad \forall\, g \in \mathcal{G}$$

$$Q^G_{g,1,0} = Q^G_{g,2,0} \qquad \forall\, g \in \mathcal{G}$$

$$P^G_{g,1,0} = P^G_{g,1,s} \qquad \forall\, g \in \mathcal{G},\ s \in \mathcal{S}/\{0\}$$

$$Q^G_{g,1,0} = Q^G_{g,1,s} \qquad \forall\, g \in \mathcal{G},\ s \in \mathcal{S}/\{0\}$$

Generator bounds on every generator (all S and T)

Contingency-Constrained Stochastic Programming ACOPF

$$
\begin{aligned}
P^f_{l,t,s} &= v^r_{bf(l),t,s} \cdot i^{fr}_{l,t,s} + v^j_{bf(l),t,s} \cdot i^{fj}_{l,t,s} && \forall\, l \in \mathcal{L},\ t \in \mathcal{T},\ s \in \mathcal{S} \\
Q^f_{l,t,s} &= v^j_{bf(l),t,s} \cdot i^{fr}_{l,t,s} - v^r_{bf(l),t,s} \cdot i^{fj}_{l,t,s} && \forall\, l \in \mathcal{L},\ t \in \mathcal{T},\ s \in \mathcal{S} \\
P^t_{l,t,s} &= v^r_{bt(l),t,s} \cdot i^{tr}_{l,t,s} + v^j_{bt(l),t,s} \cdot i^{tj}_{l,t,s} && \forall\, l \in \mathcal{L},\ t \in \mathcal{T},\ s \in \mathcal{S} \\
Q^t_{l,t,s} &= v^j_{bt(l),t,s} \cdot i^{tr}_{l,t,s} - v^r_{bt(l),t,s} \cdot i^{tj}_{l,t,s} && \forall\, l \in \mathcal{L},\ t \in \mathcal{T},\ s \in \mathcal{S} \\
(P^f_{l,t,s})^2 + (Q^f_{l,t,s})^2 &\le (S^U)^2 && \forall\, l \in \mathcal{L},\ t \in \mathcal{T},\ s \in \mathcal{S} \\
(P^t_{l,t,s})^2 + (Q^t_{l,t,s})^2 &\le (S^U)^2 && \forall\, l \in \mathcal{L},\ t \in \mathcal{T},\ s \in \mathcal{S} \\
(V^L_b)^2 \le (v^r_{b,t,s})^2 + (v^j_{b,t,s})^2 &\le (V^U_b)^2 && \forall\, b \in \mathcal{B},\ t \in \mathcal{T},\ s \in \mathcal{S} \\
P^{GL}_g \le P^G_{g,t,s} &\le P^{GU}_g && \forall\, g \in \mathcal{G},\ t \in \mathcal{T},\ s \in \mathcal{S} \\
Q^{GL}_g \le Q^G_{g,t,s} &\le Q^{GU}_g && \forall\, g \in \mathcal{G},\ t \in \mathcal{T},\ s \in \mathcal{S} \\
v^j_{rb,t,s} &= 0 && \forall\, t \in \mathcal{T},\ s \in \mathcal{S} \\
P^L_{b,1,s} &= P^{\star L}_b && \forall\, b \in \mathcal{D},\ s \in \mathcal{S} \\
Q^L_{b,1,s} &= Q^{\star L}_b && \forall\, b \in \mathcal{D},\ s \in \mathcal{S} \\
-P^{GR}_g \le P^G_{g,1,s} - P^G_{g,2,s} &\le P^{GR}_g && \forall\, g \in \mathcal{G},\ s \in \mathcal{S} \\
P^G_{g,1,0} &= P^G_{g,2,0} && \forall\, g \in \mathcal{G} \\
Q^G_{g,1,0} &= Q^G_{g,2,0} && \forall\, g \in \mathcal{G} \\
P^G_{g,1,0} &= P^G_{g,1,s} && \forall\, g \in \mathcal{G},\ s \in \mathcal{S}/\{0\} \\
Q^G_{g,1,0} &= Q^G_{g,1,s} && \forall\, g \in \mathcal{G},\ s \in \mathcal{S}/\{0\}
\end{aligned}
$$

Fix load values for Nominal stage (all S)

Contingency-Constrained Stochastic Programming ACOPF

$$P^f_{l,t,s} = v^r_{bf(l),t,s} \cdot i^{fr}_{l,t,s} + v^j_{bf(l),t,s} \cdot i^{fj}_{l,t,s} \quad \forall\, l \in \mathcal{L},\ t \in \mathcal{T},\ s \in \mathcal{S}$$
$$Q^f_{l,t,s} = v^j_{bf(l),t,s} \cdot i^{fr}_{l,t,s} - v^r_{bf(l),t,s} \cdot i^{fj}_{l,t,s} \quad \forall\, l \in \mathcal{L},\ t \in \mathcal{T},\ s \in \mathcal{S}$$
$$P^t_{l,t,s} = v^r_{bt(l),t,s} \cdot i^{tr}_{l,t,s} + v^j_{bt(l),t,s} \cdot i^{tj}_{l,t,s} \quad \forall\, l \in \mathcal{L},\ t \in \mathcal{T},\ s \in \mathcal{S}$$
$$Q^t_{l,t,s} = v^j_{bt(l),t,s} \cdot i^{tr}_{l,t,s} - v^r_{bt(l),t,s} \cdot i^{tj}_{l,t,s} \quad \forall\, l \in \mathcal{L},\ t \in \mathcal{T},\ s \in \mathcal{S}$$
$$(P^f_{l,t,s})^2 + (Q^f_{l,t,s})^2 \leq (S^U)^2 \quad \forall\, l \in \mathcal{L},\ t \in \mathcal{T},\ s \in \mathcal{S}$$
$$(P^t_{l,t,s})^2 + (Q^t_{l,t,s})^2 \leq (S^U)^2 \quad \forall\, l \in \mathcal{L},\ t \in \mathcal{T},\ s \in \mathcal{S}$$
$$(V^L_b)^2 \leq (v^r_{b,t,s})^2 + (v^j_{b,t,s})^2 \leq (V^U_b)^2 \quad \forall\, b \in \mathcal{B},\ t \in \mathcal{T},\ s \in \mathcal{S}$$
$$P^{GL}_g \leq P^G_{g,t,s} \leq P^{GU}_g \quad \forall\, g \in \mathcal{G},\ t \in \mathcal{T},\ s \in \mathcal{S}$$
$$Q^{GL}_g \leq Q^G_{g,t,s} \leq Q^{GU}_g \quad \forall\, g \in \mathcal{G},\ t \in \mathcal{T},\ s \in \mathcal{S}$$
$$v^j_{rb,t,s} = 0 \quad \forall\, t \in \mathcal{T},\ s \in \mathcal{S}$$
$$P^L_{b,1,s} = P^{\star L}_b \quad \forall\, b \in \mathcal{D},\ s \in \mathcal{S}$$
$$Q^L_{b,1,s} = Q^{\star L}_b \quad \forall\, b \in \mathcal{D},\ s \in \mathcal{S}$$
$$-P^{GR}_g \leq P^G_{g,1,s} - P^G_{g,2,s} \leq P^{GR}_g \quad \forall\, g \in \mathcal{G},\ s \in \mathcal{S}$$
$$P^G_{g,1,0} = P^G_{g,2,0} \quad \forall\, g \in \mathcal{G}$$
$$Q^G_{g,1,0} = Q^G_{g,2,0} \quad \forall\, g \in \mathcal{G}$$
$$P^G_{g,1,0} = P^G_{g,1,s} \quad \forall\, g \in \mathcal{G},\ s \in \mathcal{S}/\{0\}$$
$$Q^G_{g,1,0} = Q^G_{g,1,s} \quad \forall\, g \in \mathcal{G},\ s \in \mathcal{S}/\{0\}$$

Ramping constraint on all generators from stage 1 to stage 2 (all S)

Contingency-Constrained Stochastic Programming ACOPF

$$P^f_{l,t,s} = v^r_{bf(l),t,s} \cdot i^{fr}_{l,t,s} + v^j_{bf(l),t,s} \cdot i^{fj}_{l,t,s} \qquad \forall\, l \in \mathcal{L},\ t \in \mathcal{T},\ s \in \mathcal{S}$$

$$Q^f_{l,t,s} = v^j_{bf(l),t,s} \cdot i^{fr}_{l,t,s} - v^r_{bf(l),t,s} \cdot i^{fj}_{l,t,s} \qquad \forall\, l \in \mathcal{L},\ t \in \mathcal{T},\ s \in \mathcal{S}$$

$$P^t_{l,t,s} = v^r_{bt(l),t,s} \cdot i^{tr}_{l,t,s} + v^j_{bt(l),t,s} \cdot i^{tj}_{l,t,s} \qquad \forall\, l \in \mathcal{L},\ t \in \mathcal{T},\ s \in \mathcal{S}$$

$$Q^t_{l,t,s} = v^j_{bt(l),t,s} \cdot i^{tr}_{l,t,s} - v^r_{bt(l),t,s} \cdot i^{tj}_{l,t,s} \qquad \forall\, l \in \mathcal{L},\ t \in \mathcal{T},\ s \in \mathcal{S}$$

$$(P^f_{l,t,s})^2 + (Q^f_{l,t,s})^2 \le (S^U)^2 \qquad \forall\, l \in \mathcal{L},\ t \in \mathcal{T},\ s \in \mathcal{S}$$

$$(P^t_{l,t,s})^2 + (Q^t_{l,t,s})^2 \le (S^U)^2 \qquad \forall\, l \in \mathcal{L},\ t \in \mathcal{T},\ s \in \mathcal{S}$$

$$(V^L_b)^2 \le (v^r_{b,t,s})^2 + (v^j_{b,t,s})^2 \le (V^U_b)^2 \qquad \forall\, b \in \mathcal{B},\ t \in \mathcal{T},\ s \in \mathcal{S}$$

$$P^{GL}_g \le P^G_{g,t,s} \le P^{GU}_g \qquad \forall\, g \in \mathcal{G},\ t \in \mathcal{T},\ s \in \mathcal{S}$$

$$Q^{GL}_g \le Q^G_{g,t,s} \le Q^{GU}_g \qquad \forall\, g \in \mathcal{G},\ t \in \mathcal{T},\ s \in \mathcal{S}$$

$$v^j_{rb,t,s} = 0 \qquad \forall\, t \in \mathcal{T},\ s \in \mathcal{S}$$

$$P^L_{b,1,s} = P^{\star L}_b \qquad \forall\, b \in \mathcal{D},\ s \in \mathcal{S}$$

$$Q^L_{b,1,s} = Q^{\star L}_b \qquad \forall\, b \in \mathcal{D},\ s \in \mathcal{S}$$

$$-P^{GR}_g \le P^G_{g,1,s} - P^G_{g,2,s} \le P^{GR}_g \qquad \forall\, g \in \mathcal{G},\ s \in \mathcal{S}$$

$$P^G_{g,1,0} = P^G_{g,2,0} \qquad \forall\, g \in \mathcal{G}$$

$$Q^G_{g,1,0} = Q^G_{g,2,0} \qquad \forall\, g \in \mathcal{G}$$

$$P^G_{g,1,0} = P^G_{g,1,s} \qquad \forall\, g \in \mathcal{G},\ s \in \mathcal{S}/\{0\}$$

$$Q^G_{g,1,0} = Q^G_{g,1,s} \qquad \forall\, g \in \mathcal{G},\ s \in \mathcal{S}/\{0\}$$

Enforce generator power solution equal in stage 1 and 2 for Nominal to Nominal

Contingency-Constrained Stochastic Programming ACOPF

$$P^f_{l,t,s} = v^r_{bf(l),t,s} \cdot i^{fr}_{l,t,s} + v^j_{bf(l),t,s} \cdot i^{fj}_{l,t,s} \qquad \forall\, l \in \mathcal{L},\ t \in \mathcal{T},\ s \in \mathcal{S}$$

$$Q^f_{l,t,s} = v^j_{bf(l),t,s} \cdot i^{fr}_{l,t,s} - v^r_{bf(l),t,s} \cdot i^{fj}_{l,t,s} \qquad \forall\, l \in \mathcal{L},\ t \in \mathcal{T},\ s \in \mathcal{S}$$

$$P^t_{l,t,s} = v^r_{bt(l),t,s} \cdot i^{tr}_{l,t,s} + v^j_{bt(l),t,s} \cdot i^{tj}_{l,t,s} \qquad \forall\, l \in \mathcal{L},\ t \in \mathcal{T},\ s \in \mathcal{S}$$

$$Q^t_{l,t,s} = v^j_{bt(l),t,s} \cdot i^{tr}_{l,t,s} - v^r_{bt(l),t,s} \cdot i^{tj}_{l,t,s} \qquad \forall\, l \in \mathcal{L},\ t \in \mathcal{T},\ s \in \mathcal{S}$$

$$(P^f_{l,t,s})^2 + (Q^f_{l,t,s})^2 \le (S^U)^2 \qquad \forall\, l \in \mathcal{L},\ t \in \mathcal{T},\ s \in \mathcal{S}$$

$$(P^t_{l,t,s})^2 + (Q^t_{l,t,s})^2 \le (S^U)^2 \qquad \forall\, l \in \mathcal{L},\ t \in \mathcal{T},\ s \in \mathcal{S}$$

$$(V^L_b)^2 \le (v^r_{b,t,s})^2 + (v^j_{b,t,s})^2 \le (V^U_b)^2 \qquad \forall\, b \in \mathcal{B},\ t \in \mathcal{T},\ s \in \mathcal{S}$$

$$P^{GL}_g \le P^G_{g,t,s} \le P^{GU}_g \qquad \forall\, g \in \mathcal{G},\ t \in \mathcal{T},\ s \in \mathcal{S}$$

$$Q^{GL}_g \le Q^G_{g,t,s} \le Q^{GU}_g \qquad \forall\, g \in \mathcal{G},\ t \in \mathcal{T},\ s \in \mathcal{S}$$

$$v^j_{rb,t,s} = 0 \qquad \forall\, t \in \mathcal{T},\ s \in \mathcal{S}$$

$$P^L_{b,1,s} = P^{\star L}_b \qquad \forall\, b \in \mathcal{D},\ s \in \mathcal{S}$$

$$Q^L_{b,1,s} = Q^{\star L}_b \qquad \forall\, b \in \mathcal{D},\ s \in \mathcal{S}$$

$$-P^{GR}_g \le P^G_{g,1,s} - P^G_{g,2,s} \le P^{GR}_g \qquad \forall\, g \in \mathcal{G},\ s \in \mathcal{S}$$

$$P^G_{g,1,0} = P^G_{g,2,0} \qquad \forall\, g \in \mathcal{G}$$

$$Q^G_{g,1,0} = Q^G_{g,2,0} \qquad \forall\, g \in \mathcal{G}$$

Non-anticipativity constraints

$$P^G_{g,1,0} = P^G_{g,1,s} \qquad \forall\, g \in \mathcal{G},\ s \in \mathcal{S}/\{0\}$$

$$Q^G_{g,1,0} = Q^G_{g,1,s} \qquad \forall\, g \in \mathcal{G},\ s \in \mathcal{S}/\{0\}$$

Contingency-Constrained ACOPF Results – Extensive Form

Problem data: case118 distributed with Matpower 4.1
- 118 buses, 54 active generators, and 186 branches

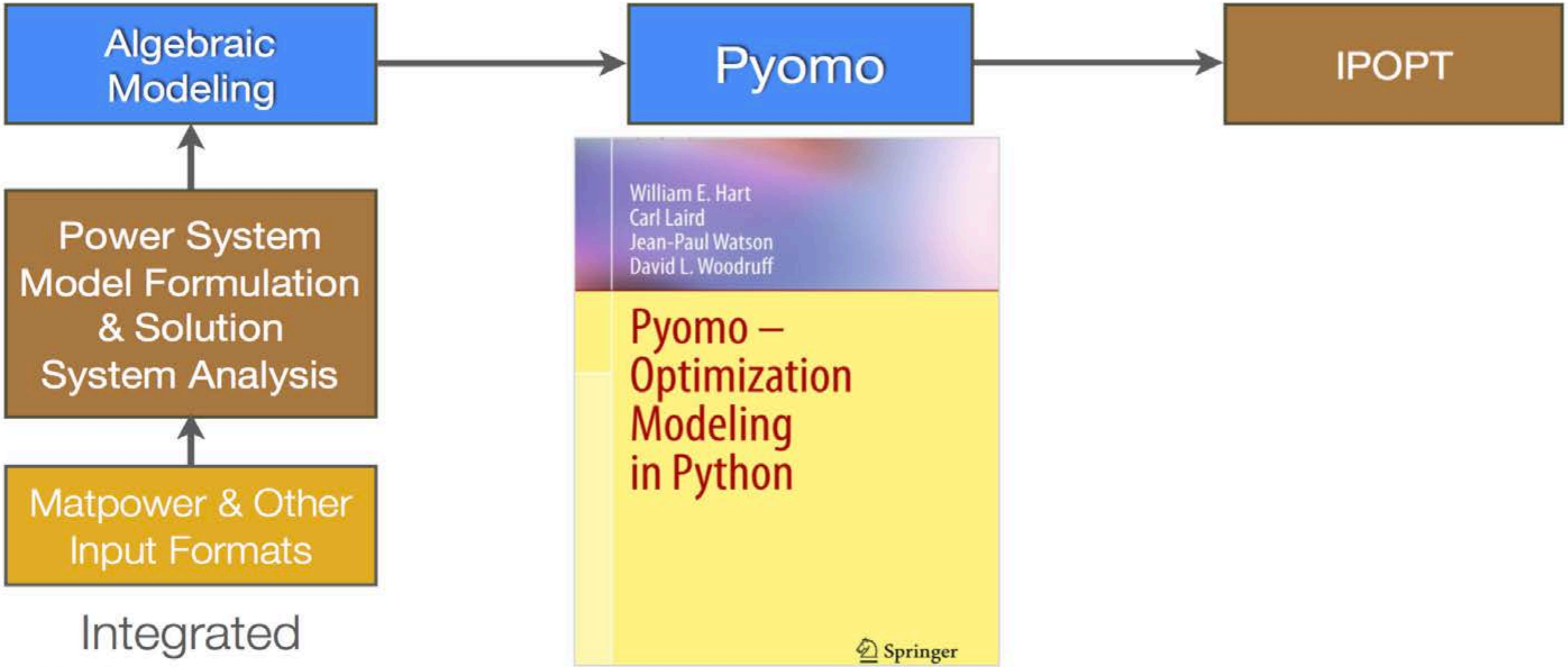
Multi-scenario problem with 128 scenarios in total
- Normal operating scenario and 127 contingencies
- Problem size: ~400,000 variables and ~385,000 constraints

Solution obtained in less than 5 seconds

*Wall-clock time from the Red Mesa supercomputing cluster at Sandia National Lab.
Each node: 12 GB RAM, two 2.93 GHz quad-core, Nehalem X5570 processors

But : Low run-times *do not persist* to 300-bus and larger cases…

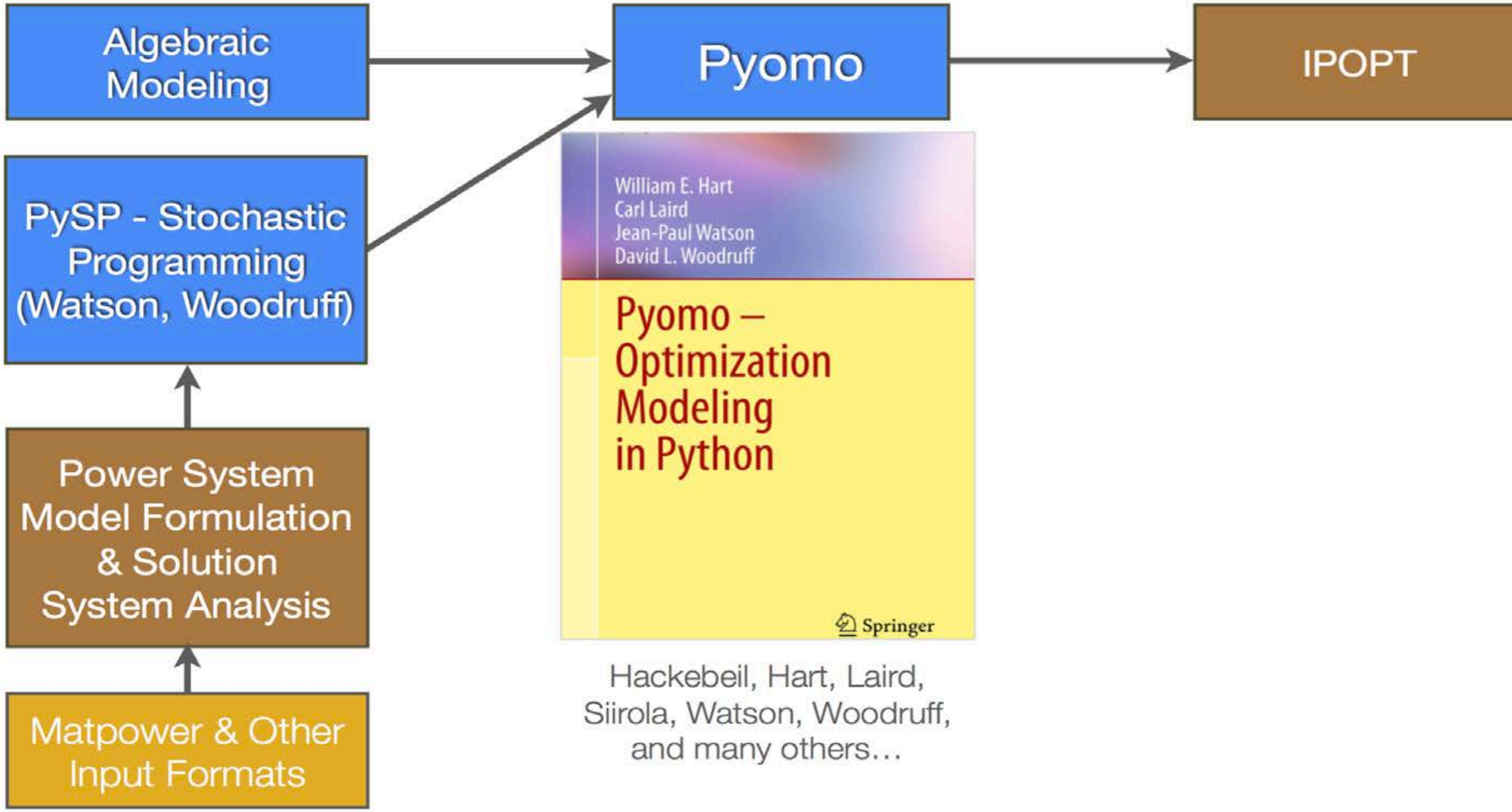
Building the Model with Pyomo and PySP



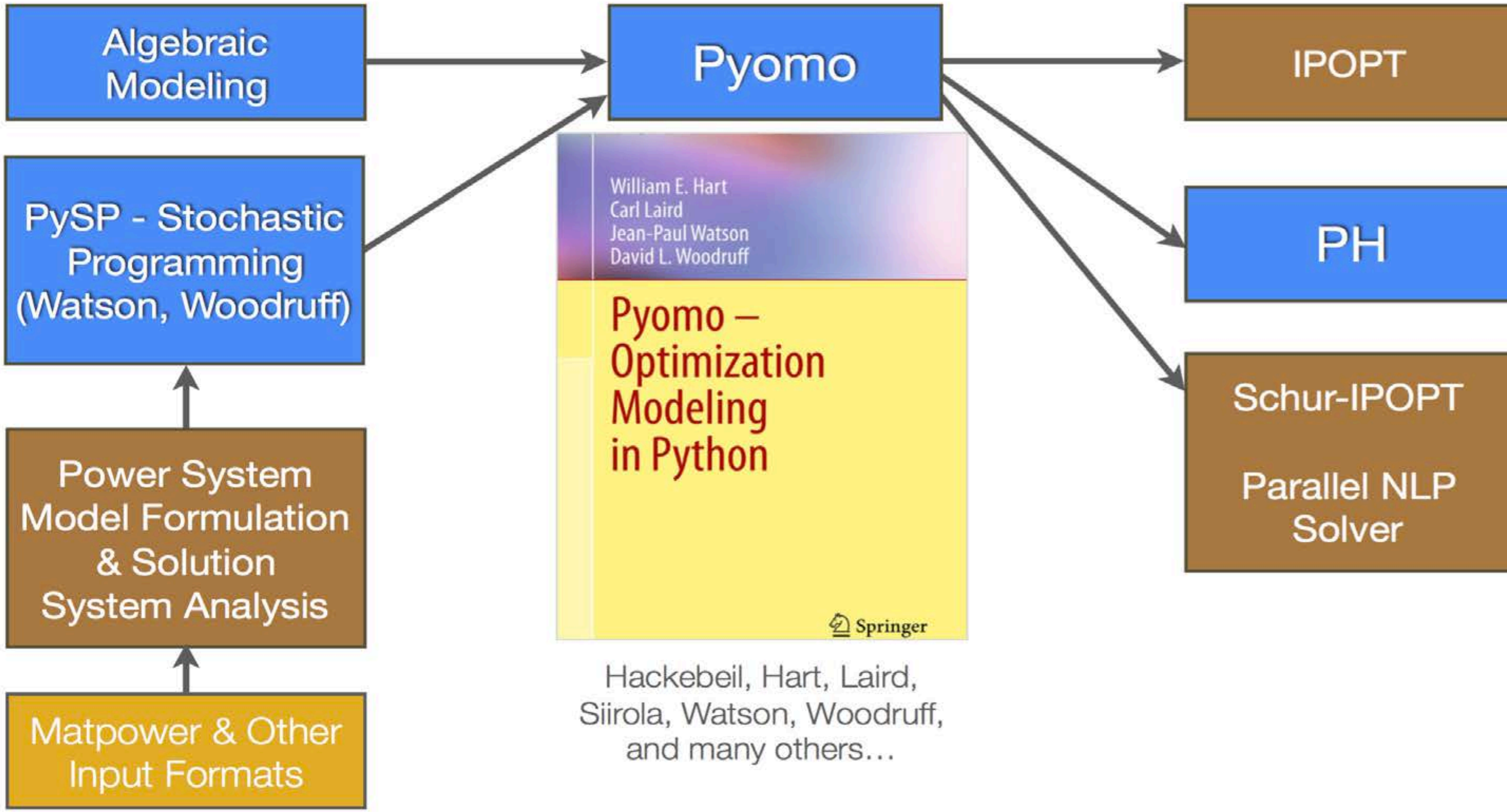
Integrated Python-based toolchain

Hackebeil, Hart, Laird, Siirola, Watson, Woodruff, and many others…

Building the Model with Pyomo and PySP



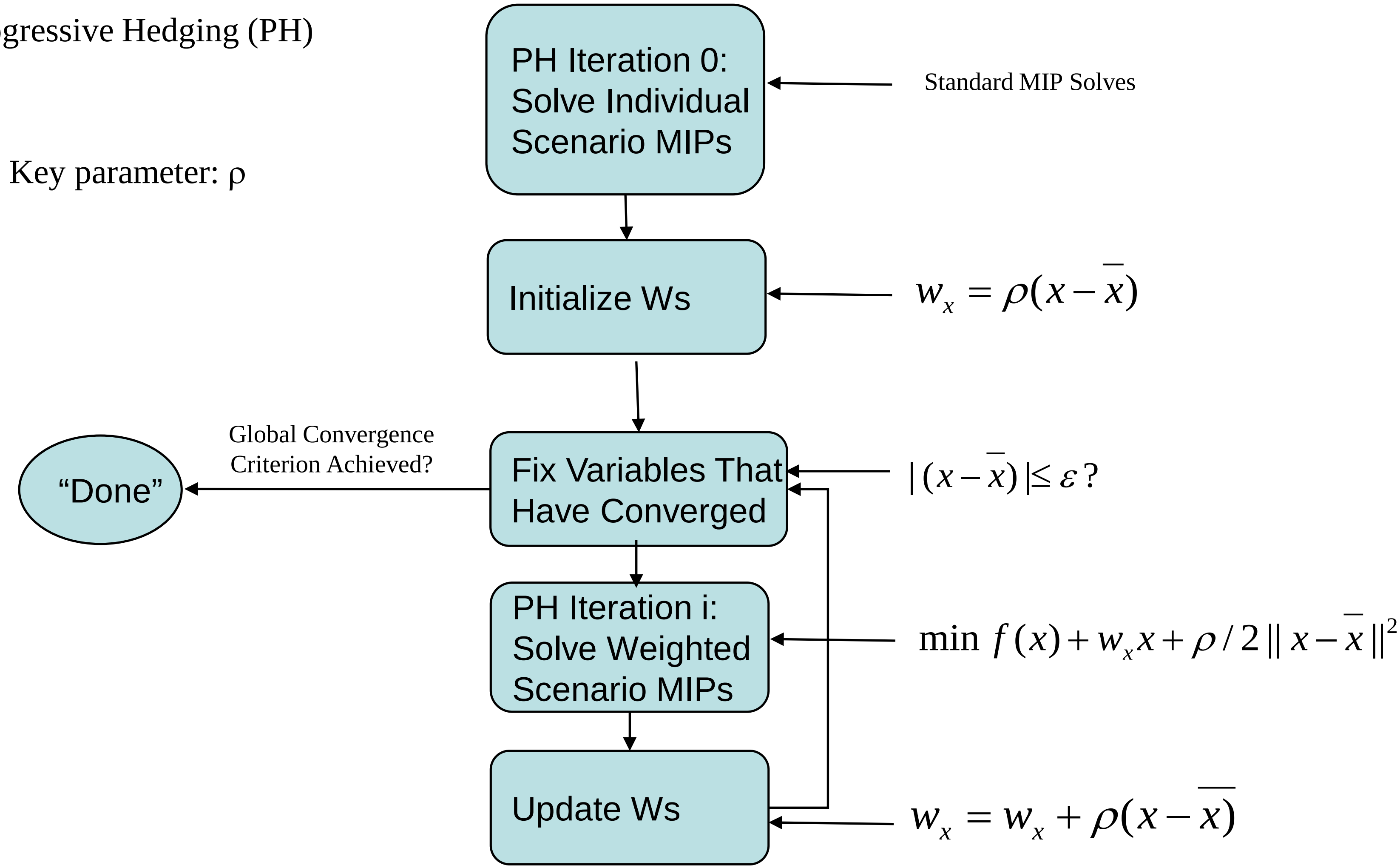
Building the Model with Pyomo and PySP



Scenario-Based Decomposition via Progressive Hedging

Progressive Hedging (PH)

Key parameter: ρ



PH / CCOPF Issues of Note

- Progressive Hedging issues / resolution
  - No ”direct” Python-based interface to Ipopt => significant of file overhead during each iteration (approximately 30% overhead in runs)
  - We are assuming (because it is empirically true) that Ipopt is locating globally optimal solutions
- CCOPF issues / resolution
  - Conduct pre-filtering of contingencies for which no feasible dispatch exists – via graph analysis
  - The above allows for additional contingencies for which no feasible dispatch exists
  - There is no guarantee that there exists a collectively feasible dispatch for the remaining contingency scenarios
    - Consequently, critical to add slack variables for generation/load mis-match in all power balance constraints
  - Thus, we are identifying infeasible contingencies via post-processing, through analysis of slack variables

CCOPF PH Run Times (1)

- Experimental setup
  - One scenario (contingency) sub-problem per core
  - 16 cores per node – Sandia's Skybridge cluster
  - Ipopt configured with MA27
  - Some custom Ipopt option configuration
  - Almost no PH tuning performed
- Performance focus
  - Wall clock time – all that matters for operation
  - Convergence to reasonable tolerances (1e-5)
    - Yields very small discrepancies (in the sixth or greater significant digit) in primal solution characteristics

CCOPF PH Run Times (2)

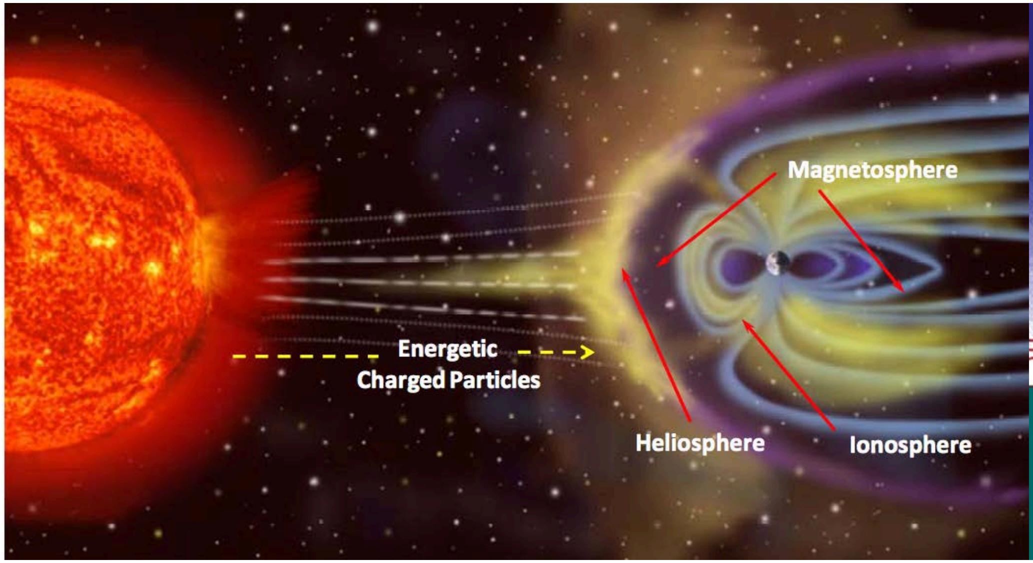
- Sample of results

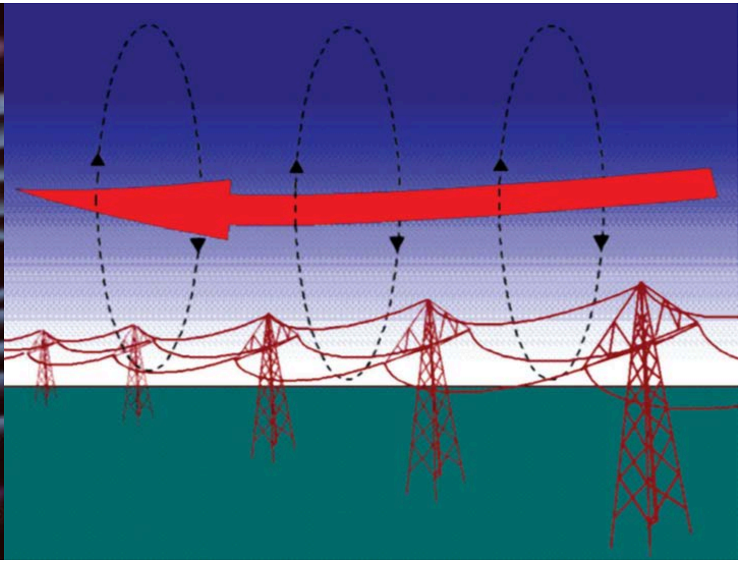
Case Name	# Contingencies	# of PH Iterations	Wall Clock Time
case6ww	11	12	2 s
case57	79	21	12 s
case118	117	14	2m 3s
case300	322	8	2m 54s
case2383wp	2252	6	4m 50s

- Key message
  - Using "modestly" sized clusters and open source software, we can solve contingency-constrained ACOPF problems of very large scale in (almost) operational time scales

Geomagnetic Disturbances (GMDs)

- Coronal Mass Ejections (CMEs) from the sun send charged particles (i.e., electrons, coronal and solar wind ions) into space
- A GMD occurs when particles from a large Coronal Mass Ejection (CME) escape the sun's halo (corona) and are directed towards earth
- CMEs interact with earth's magnetosphere-ionosphere producing ionospheric currents also called electrojets
- Electrojets perturb earth's geomagnetic field, inducing voltage potential at earth's surface and resulting in geomagnetic induced currents in the grid
- Grid risks:
  - *Loss of reactive power support which could lead to voltage instability and power system collapse*

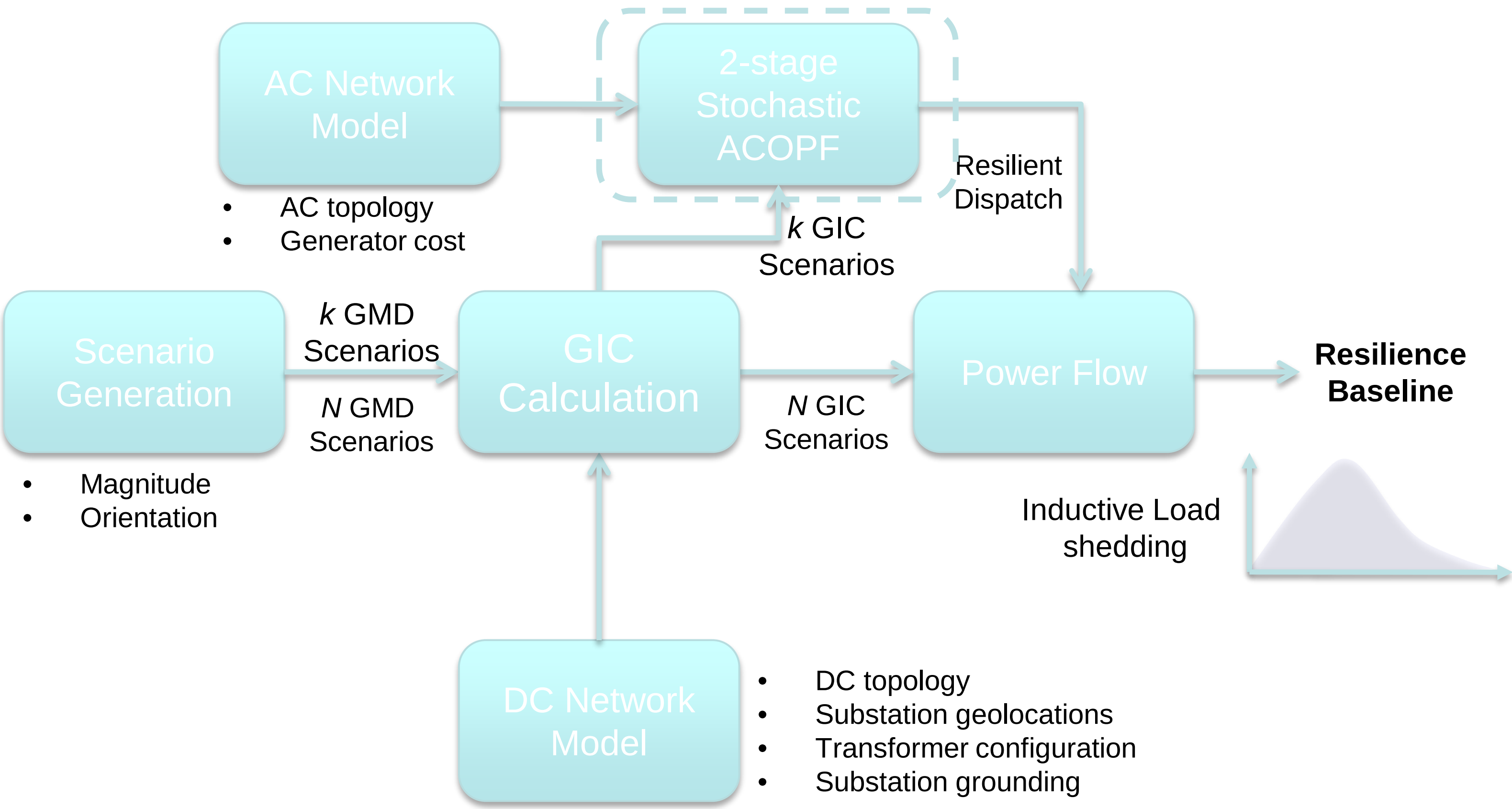




GMD Grid Resilient Performance

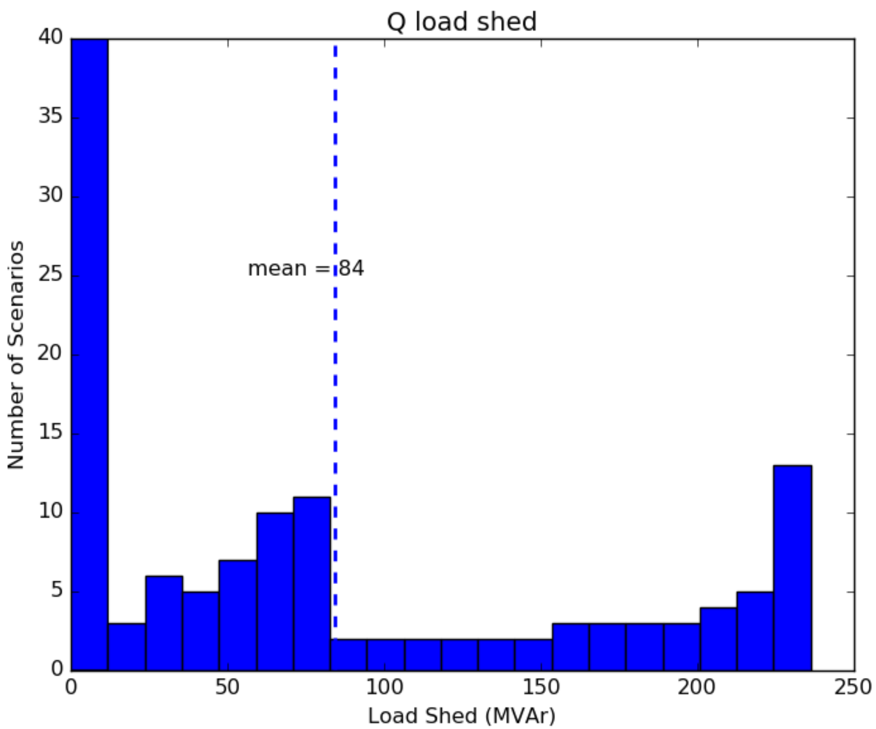
Sandia National Laboratories

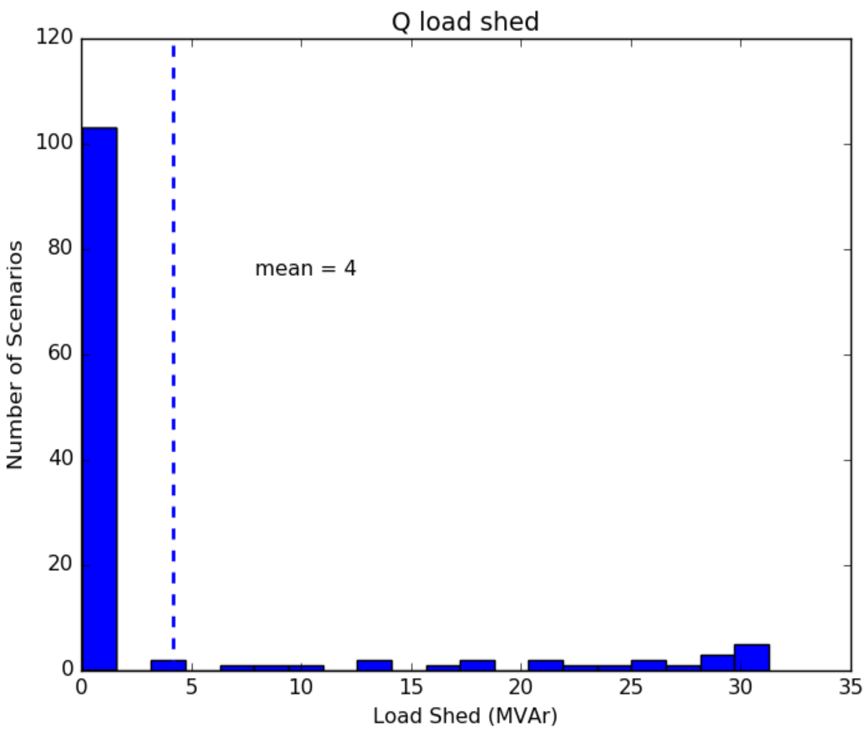
By switching from an economical operation to a resilience-based operation we are able to reduce the probability of system voltage collapse due to a GMD



Mitigating GMDs Through Proactive Dispatch

- Baseline (right) and proactive (left) dispatch
- IEEE 24-bus test case
- Histograms report number of scenarios with given reactive power load





- Proactive dispatch can dramatically reduce the impact of GMD events

Conclusions and Future Directions

- The contingency-constrained ACOPF optimization problem is an extremely relevant problem in power systems operations
  - Only currently solved approximately (at best)
- The contingency-constrained ACOPF can be naturally formulated as a stochastic program, and modeled and solved using existing frameworks and algorithms (Pyomo, PySP, and Ipopt)
- Using modest (<3000) core cluster platforms, large-scale contingency-constrained ACOPF problems can be solved in operational time scales (< 5 minutes)
- Future directions and issues
  - Tuning of PH (likely to have a major impact)
  - Direct / controlled comparison with SchurIpopt as the solver
  - Explore more difficult NESTA cases
  - Probably need to target 1 minute for operational deployment

QUESTIONS