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Stochastic Unit Commitment with Conditional Value-at-Risk Constraints

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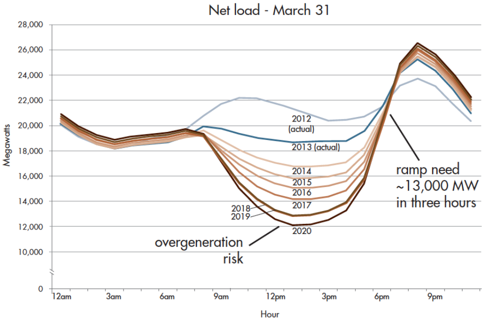
Outline

- Motivation
- Two-stage stochastic unit commitment with CVaR constrains
- Numerical results
- Discussion of using CVaR to price flexible ramping products

Motivation of This Work

- The "duck" curve of net load: The needs of ramp-up and ramp-down resources.



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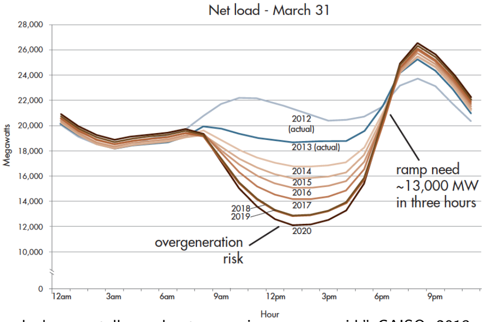
- The "duck" curve of net load: The needs of ramp-up and ramp-down resources.



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- The classic portfolio selection theory (aka the efficient frontier) in dealing with the trade-offs between risks (reliability) and returns (cost)

Targeted Operation in the Day-Ahead/Real-time Market

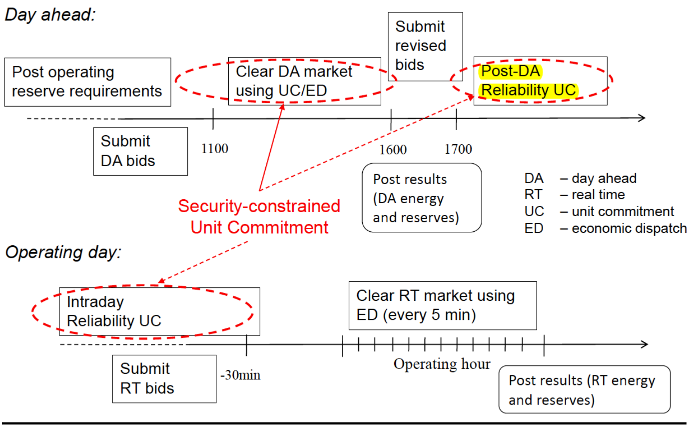

Figure: Market Operations in a Two-Settlement System [ZWL14]

Two-stage Stochastic Unit Commitment – First Stage

Model I: Benchmark: Energy and reserve co-optimization (with a fixed reserve requirement)

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- **First-stage Variables (unit commitment)**

u_{gt} Commitment decision of unit g at period t; *binary*
v_{gt} Start-up action of unit g at period t; *binary*
w_{gt} Shut-down action of unit g at period t; *binary*
rc^{u}_{gt} Regulation-up reserve to commit of unit g at period t
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- **Objective Function**

$$\begin{aligned}\text{Minimize}\quad & \text{Start-up cost } [\sum_{g,t} SU_g v_{gt}] + \text{Shut-down cost } [\sum_{g,t} SD_g w_{gt}] \\ & + \text{Reserve commitment cost } [\sum_{g,t} (C^U_g rc^u_{gt} + C^D_g rc^d_{gt})] \\ & + \mathbb{E}_\xi[\text{2nd-stage cost}]\end{aligned}$$

First Stage (cont.)

First-stage constraints:

- Minimum up and down time

$$u_{gt} - u_{g(t-1)} \leq u_{g\tau}, \qquad \forall g \in G,\ t \in T, \tau = t, \dots, min\{t + L_g - 1\}$$
$$u_{g(t-1)} - u_{gt} \leq 1 - u_{g\tau}, \qquad \forall g \in G,\ t \in T, \tau = t, \dots, min\{t + l_g - 1\}$$

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- Starting up/shutting down

$$v_{gt} \geq u_{gt} - u_{g(t-1)}, \qquad \forall g \in G,\ t \in T$$
$$w_{gt} \geq -u_{gt} + u_{g(t-1)}, \qquad \forall g \in G,\ t \in T$$

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- Fixed reserve requirement (by zones: $z \in Z$)

$$\sum_{g \in G_z} rc_{gt}^{(u,d)} \geq R_{zt}^{(u,d)}, \quad \forall z \in Z, t \in T$$

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- Binary and non-negativity requirements:

$$u_{gt},\ v_{gt},\ w_{gt} \in \{0,1\},\ rc_{gt}^{u}, rc_{gt}^{d} \ge 0,\ \forall g \in G,\ t \in T$$

Second Stage: 24-Hr Real-Time Economic Dispatch

- Second-stage objective function – (Δ_{it}^{ξ}: lost load)

$$\mathbb{E}_{\xi}[\text{2nd-stage cost}] = \mathbb{E}_{\xi} \sum_{t \in T} \sum_{g \in G} \left[Cost_g(p_{gt}^{\xi}) + Cost_r(r_{gt}^{u^{\xi}}) + Cost_r(r_{gt}^{d^{\xi}}) + \sum_{i \in N} VOLL \cdot \Delta_{it}^{\xi} \right]$$

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$$(P_g^{min} + rc_{gt}^{d}) u_{gt} \le p_{gt}^{\xi} \le (P_g^{max} - rc_{gt}^{u}) u_{gt}, \qquad \forall\, g \in G,\ t \in T,\ \xi \in \Xi$$

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$$D_{it}^{\xi} - \Delta_{it}^{\xi} = \sum_{(j,i) \in A_i^{-}} f_{jit}^{\xi} - \sum_{(i,j) \in A_i^{+}} f_{ijt}^{\xi} + \sum_{g \in G_i} (p_{gt}^{\xi} + r_{gt}^{u\xi} - r_{gt}^{d\xi}) + W_{it}^{\xi}, \quad \forall\, i \in N,\ t \in T,\ \xi \in \Xi$$

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- Kirchhoff's voltage law and flow capacity

Value-at-Risk (VaR) and Conditional Value-at-Risk (CVaR)

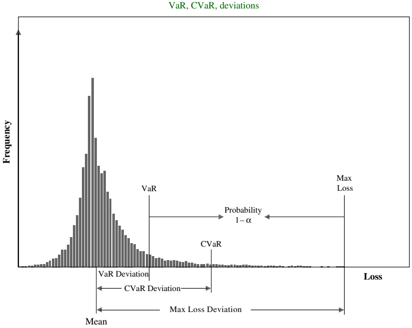

Illustration of VaR and CVaR [SSS08, Figure 1]

- Let $x := (u, v, w, rc^{u,d})$; that is, the first-stage decisions.
- $\Delta(x;\xi)$ denote the total unserved energy in the real-time corresponding to an x.

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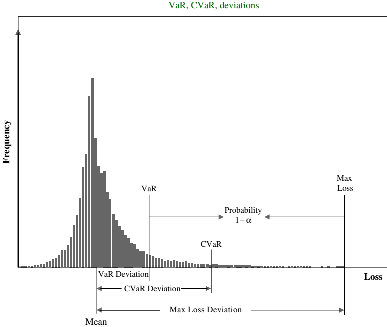

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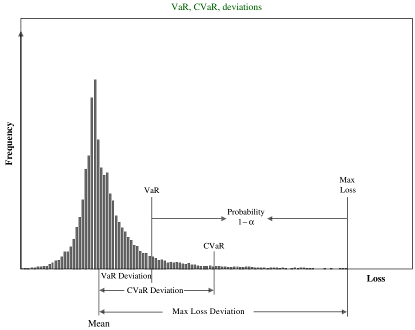

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- $CVaR_\alpha(x) = \mathbb{E}_\xi \left[\Delta(x;\xi) \middle| \Delta(x;\xi) \geq VaR_\alpha \right]$.

Relationship to Reliability Metrics

VaR vs. LOLP

Mathematically, using VaR to ensure reliability leads to chance-constrained UC; that is,

$$P[\Delta(x;\xi) \leq VaR_{\alpha}(x)] \geq \alpha.$$

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CVaR vs. Expected Unserved Energy (EUE)

CVaR is the conditional EUE given that $\Delta(x;\xi) > VaR_{\alpha}(x)$.

Linear Approximation of CVaR

Let $p(\xi)$ be the probability density function of ξ. Consider

$$F_\alpha(x, VaR) = VaR + (1-\alpha)^{-1} \int_{\xi \in \Re^n} \max\{\Delta(x,\xi) - VaR, 0\}\, p(\xi) d\xi.$$

Theorem [KPU02]: Let $\bar{\phi}$ denote a loss allowance (i.e., do not want $CVaR_\alpha > \bar{\phi}$). The following two minimization problems are equivalent:

$$\begin{aligned} \min_{x} \quad & Cost(x) \\ \text{s.t.} \quad & CVaR_\alpha(x) \le \bar{\phi}. \end{aligned} \tag{1}$$

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Linear approximation (Note: the approximation is exact if the support of ξ is finite.)

- Sample ξ from its distribution, and let $\xi_1, \ldots, \xi_j, \ldots \xi_n$ be the n samples (and p_j be the corresponding probability).

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- Use auxiliary variable ζ_j to replace $\max\{\Delta(x,\xi_j) - VaR, 0\}$ in $F_\alpha(x, VaR)$.
- Then $F_\alpha(x, VaR) \le \bar{\phi}$ in Problem (2) can be approximated by the following THREE sets of constraints

$$1.\ \zeta_j \ge \Delta(x,\xi) - VaR, \quad 2.\ \zeta_j \ge 0, \quad 3.\ VaR + (1-\alpha)^{-1} \sum_{i=j}^{n} p_j \zeta_j \le \bar{\phi}, \quad j = 1, \ldots, n.$$

Computational Approach

Model II = Mode I − Fixed Reserve Requirement in Stage-1 + CVaR Constraints in Stage-2

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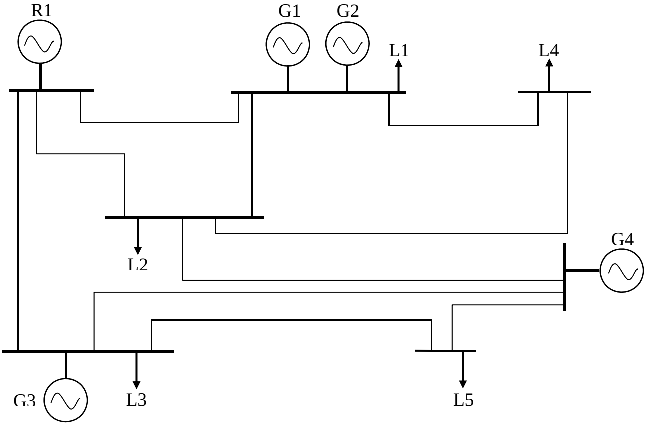
Computational consideration of CVaR constraints

- Advantage (over VaR): linear inequalities are much easier to deal with than chance-constraints
- Disadvantage: the constraint $VaR + (1-\alpha)^{-1} \sum_{i=j}^{n} p_j \zeta_j$ is pooling over all scenarios (the summation); cannot directly apply Benders' decomposition (aka the L-shaped method)

Modified Bender's Decomposition

- Put the CVaR constraints in the Master problem (the first-stage problem plus θ: approximation of the 2nd stage optimal value function)
- Add optimality cuts and feasibility cuts the same way as in Benders'
- Branch-and-cut on the first-stage integer variables
- Provable finite convergence with finite support of ξ and relatively complete recourse ([LL93], [BL11] Prop. 7.4).

Numerical Results: 7-Bus Example



Results: 7-Bus Example – Input Data

Table: Generator Parameters and Costs

	G1	G2	G3	G4
Min-ON (h)	2	1	2	4
Min-OFF (h)	2	2	2	1
Ramp-Up (MW/h)	60	30	60	60
Ramp-Down (MW/h)	60	30	60	60
Pmin (MW)	10	5	9	7
Pmax (MW)	110	50	90	70
Startup ($)	50	500	800	30
Shutdown ($)	50	500	800	20
Fuel Cost a ($)	6.78	6.78	31.67	10.15
Fuel Cost b ($\$/MWh$)	12.888	12.888	26.244	17.820
Fuel Cost c ($\$/MWh^2$)	0.0109	0.0109	0.0697	0.0128

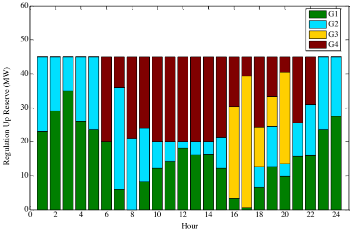
- Reg-up requirement = Reg-down requirement = 50% of largest capacity
- Uncertainty on net-load (did not consider wind-demand correlation or autocorrelation; 100 samples)
- $VOLL$ =1,000 $/MWh, $\alpha = 99\%$, $\bar{\phi} = 4MW$

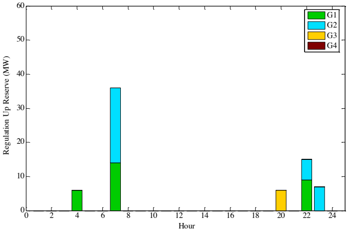
Results: 7-Bus Example

Model	Obj. Val.	Expected Loss (MW)	Max. Loss (MW)	Unit ID	Hour (1-24)
I	$74429	0.686	4	G1	1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1
				G2	1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1
				G3	0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 1 1 1 1 1 0 0 0 0
				G4	1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 0 0
II	$66372	0.695	4	G1	1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1
				G2	1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1
				G3	0 0 0 1 0 0 1 0 0 0 0 0 0 0 0 1 1 1 1 1 0 0 0 0
				G4	1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 0 0

Results: 7-Bus Example (cont.)

Regulation-UP reserves: Model I vs. Model II





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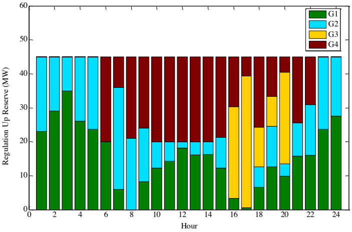
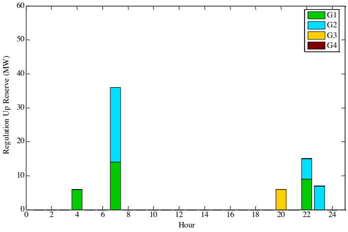
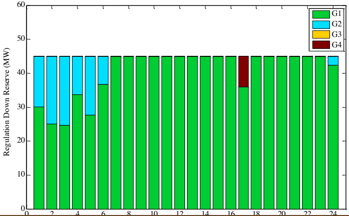
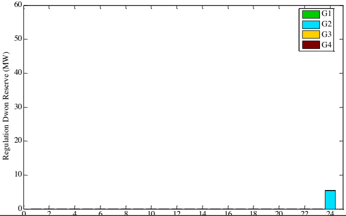




Regulation-DOWN reserves: Model I vs. Model II

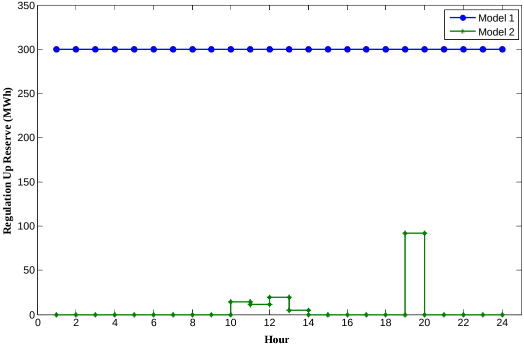




IEEE 118-Bus Example

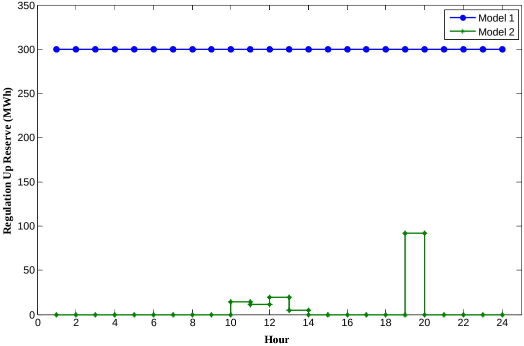
- 54 generators, including 4 wind plants
- 186 transmission lines with 120 MW capacity on each line
- $VOLL = \$100/\text{MWh}$, $\alpha = 99\%$, $\bar{\phi} = 70MW(\approx 1\% peak)$
- 100 scenarios sampled
- Requirement of Reg-Up = Reg-Down = 300 MW (= largest capacity of the generators)
- Use CPLEX: Model I (21,686 sec.); Model II (19,670 sec.)

IEEE 118-Bus Example (cont.)



Comparison of regulation-up reserve capacity between Model I and Model II

IEEE 118-Bus Example (cont.)



Comparison of regulation-up reserve capacity between Model I and Model II

Model II cost reduction $\approx 1.8\%$; Expected Loss for Model I is 0; for Model II is 3.55 MW (really needs to draw the efficient frontier curve to compare! (In progress))

Further Discussions of CVaR

Robust CVaR: What if the distribution $p(\cdot)$ of ξ is not fully known?

Suppose $p(\cdot) \in \mathscr{P}$: a set of distributions. Then the worst-case CVaR ($WCVaR_\alpha(x)$) is: $WCVaR_\alpha(x) := \sup_{p(\cdot)\in\mathscr{P}} CVaR_\alpha(x)$.

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Results [ZF09]:

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Use CVaR to price flexible ramping products (a proposal)

- Shared ramping (up) constraints (using different weights for energy (a_g), reserve (a_r) and flexible ramping (a_f) at <u>different time scales</u> (aka CAISO):

$$a_g(p^\xi_{gt}-p^\xi_{gt-1})+a_r(r^{u^\xi}_{gt}-r^{u^\xi}_{gt-1})+a_f(flexRU^\xi_{gt}-flexRU^\xi_{gt-1}) \leq RU_g,\ \forall\, g \in G, t \in T, \xi \in \Xi$$

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- Define different CVaRs at different time scales
- Use the shadow prices of CVaR constraints to pay for different ramping products

Summary and Future Research

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- Promising results of the CVaR model (in terms of the cost-reliability trade-off) compared to the fixed-reserve case (but based on very limited numerical results)

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Future Research

- Computational experiments of robust CVaR and comparison with distribution-specific CVaR
- Ramp product pricing using CVaR constraints, with intra-hour modeling
- Can we use CVaR as a basis to compensate demand-side resources (DR, DG, etc) and storage for their contribution of reliability?

Thank you!

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