The Hidden Properties of Fast Start Pricing

Tongxin Zheng, Feng Zhao, Dane Schiro, Eugene Litvinov

* The views expressed in this presentation do not represent those of ISO New England
The Concern

• Start-up (SU), no-load (NL), and incremental offers are used to make commitment and dispatch decisions

• Traditionally, prices are determined by the optimal dual variables of the convex dispatch problem
  – SU and NL costs (i.e., commitment costs) are not reflected in prices

• Concern: Traditional prices are unable to “reflect the actual marginal cost of serving load”*
  – This cost presumably includes Fast Start (FS) commitment costs

* FERC Docket No. RM17-3-000 (December 15, 2016)
The Potential Solutions

• To address the concern, ISOs have proposed and/or implemented a variety of “Fast Start Pricing” methods
  – Each method is meant to, at the very least, incorporate FS commitment costs into prices

• Each FS pricing method has unique properties, some of which are not obvious

• Because the fundamental problem here is nonconvexity, there is no perfect solution
Outline

- Evaluation criteria
- Properties of each FS pricing method
- Fundamental questions on FS pricing
- Conclusion
Pricing Criteria

• Before delving into different FS pricing methods, a set of criteria is needed to evaluate them

• Three principles
  1) Efficiency
  2) Transparency
  3) Simplicity
Pricing Criterion: Efficiency

1) **Efficiency**
   a) Assuming truthful offers, cleared quantities maximize social surplus/minimize total production cost
   b) Given prices and uplift (make-whole + LOC), each unit should want to produce its cleared quantity
Pricing Criterion: Transparency

2) **Transparency**
   
a) “Much is known by many” about transaction prices

b) Everyone knows the prices that others receive/pay

• In the context of FS pricing, LMPs are transparent and uplift is not transparent
Pricing Criterion: Simplicity

3) **Simplicity**
   a) As few prices as possible
      • Uniform price at the same location and time
   b) Price formation process should use simple logic
      • Prices are easy to interpret
Categories of Fast Start Pricing

• All FS pricing methods in this presentation derive prices from convex (linear) problems

• Baseline method
  – Fixed commitment pricing

• FS pricing methods
  – Rule-based pricing
  – Convex hull pricing
  – Integer relaxation pricing
Method: Fixed Commitment Pricing

- Unit commitment variables are fixed at optimal values (0 or 1)
  - The resulting linear dispatch problem produces the price
- Prices are derived from incremental costs and do not reflect Commitment costs (SU and NL)
Analysis: Fixed Commitment Pricing

• Efficient
  – Efficient resource allocation
  – Prices and make whole payment ensure online units have adequate dispatch-following incentives

• Not transparent
  – Make-whole payments can be required by online units
  – Lost opportunity costs can be incurred by offline units

• Simple
  – Price obeys the marginal cost pricing concept (i.e., marginal cost of serving the next MW of load)
Method: Rule-based Pricing

- Price is derived from the dispatch problem with modified FS offers
- Typically, variations of the following pricing rules are used
  - Relax $P_{\text{min}}$ to 0 MW
  - Amortize SU cost over minimum run time and $P_{\text{max}}$
  - Amortize NL cost over $P_{\text{max}}$
- These rules do not have a rigorous economic justification
Method: Rule-based Pricing

• **Hidden Property:** *Inconsistent Dispatch & Pricing*
  – The price derived using the modified FS offers may be inconsistent with the cleared quantity
  – Lost opportunity costs/special deviation settlement rules may be needed to ensure dispatch following

\[ P_{\text{min}} \leq P \leq P_{\text{max}} \]

*Unit incremental offer*

\[ LMP \]

\[ LOC \]

*Dispatch point*

\[ 0 \rightarrow P_{\text{min}} \rightarrow P_{\text{max}} \]

*Unit output*
Analysis: Rule-based Pricing

• **Efficient**
  – Combined, prices and uplift ensure that units have adequate dispatch-following incentives

• **Not transparent**
  – Uplift is needed

• **Simple**
  – Price obeys the marginal cost pricing concept (i.e., marginal cost of serving the next MW of load) but is derived from the modified offers
Method: Convex Hull Pricing

- The Lagrangian dual problem for unit commitment is solved
  - Price is the slope of the convex envelope of total cost w.r.t. load

- **Hidden Property**: Minimization of total uplift
  - Price minimizes (make-whole + LOC + transmission/reserve revenue shortfall) over commitment problem’s time horizon

![Graph showing system total cost with convex hull and aggregated original offer.]
Analysis: Convex Hull Pricing

- **Efficient**
  - Combined, prices and uplift ensure that units have adequate dispatch-following incentives

- **Not transparent**
  - Convex Hull Pricing minimizes total uplift (make-whole + LOCs + transmission/reserve collection shortages) but may not eliminate it

- **Not simple**
  - Price does not obey the marginal cost pricing concept
    - Price can be the average cost of one or more units (possibly offline)
  - Computationally difficult to solve for the true convex hull price
Method: Integer Relaxation Pricing

• Relax each binary unit commitment variable
  \[ \{0,1\} \rightarrow [0,1] \]

• While this idea is simple, it has a hidden property

  Price is dependent on the problem formulation!
Example: Integer Relaxation Pricing

- Load = 105MW
- U2 ramp limit = 20MW
- Single interval commitment problem, assume U1 is always “On”
- The optimal commitment/dispatch solution is
  - U1: Output = 95 MW
  - U2: “On”, Output = 10 MW

<table>
<thead>
<tr>
<th></th>
<th>$P_{min}$</th>
<th>$P_{max}$</th>
<th>Inc. Cost</th>
<th>Commitment Cost</th>
<th>Initial State</th>
</tr>
</thead>
<tbody>
<tr>
<td>U1</td>
<td>0</td>
<td>100</td>
<td>$10</td>
<td>0</td>
<td>On</td>
</tr>
<tr>
<td>U2</td>
<td>10</td>
<td>25</td>
<td>$20</td>
<td>$1000</td>
<td>Off</td>
</tr>
</tbody>
</table>
Example: Two Equivalent UC Formulations

• Formulation 1

\[ \begin{align*}
\text{min} & \quad 10p_1 + 20p_2 + 1000x_2 \\
\text{s.t.} & \quad p_1 + p_2 = 105 \\
& \quad p_1 \leq 100 \\
& \quad p_2 \leq 25x_2 \\
& \quad p_2 \geq 10x_2 \\
& \quad p_2 \leq 20 \\
& \quad p_1, p_2 \geq 0 \\
& \quad x_2 \in \{0, 1\}
\end{align*} \]

Formulation difference in ramp constraint

• Formulation 2

\[ \begin{align*}
\text{min} & \quad 10p_1 + 20p_2 + 1000x_2 \\
\text{s.t.} & \quad p_1 + p_2 = 105 \\
& \quad p_1 \leq 100 \\
& \quad p_2 \leq 25x_2 \\
& \quad p_2 \geq 10x_2 \\
& \quad p_2 \leq 20x_2 \\
& \quad p_1, p_2 \geq 0 \\
& \quad x_2 \in \{0, 1\}
\end{align*} \]

• Both formulations have the same feasible region and optimal solution: \((p_1, x_2, p_2) = (95\text{MW}, 1, 10\text{MW})\)

• What happens after integer relaxation?
Example: Integer Relaxation of Two Formulations

• Relaxed Formulation 1

\[
\begin{align*}
\text{min} & \quad 10p_1 + 20p_2 + 1000x_2 \\
\text{s.t.} & \quad p_1 + p_2 = 105 \\
& \quad p_1 \leq 100 \\
& \quad p_2 \leq 25x_2 \\
& \quad p_2 \geq 10x_2 \\
& \quad p_2 \leq 20 \\
& \quad p_1, p_2 \geq 0 \\
& \quad 0 \leq x_2 \leq 1
\end{align*}
\]

• Relaxed Formulation 2

\[
\begin{align*}
\text{min} & \quad 10p_1 + 20p_2 + 1000x_2 \\
\text{s.t.} & \quad p_1 + p_2 = 105 \\
& \quad p_1 \leq 100 \\
& \quad p_2 \leq 25x_2 \\
& \quad p_2 \geq 10x_2 \\
& \quad p_2 \leq 20x_2 \\
& \quad p_1, p_2 \geq 0 \\
& \quad 0 \leq x_2 \leq 1
\end{align*}
\]

Equivalently,

\[
\begin{align*}
\text{min} & \quad 1050 + 10p_2 + 1000x_2 \\
\text{s.t.} & \quad p_2 \geq 5 \\
& \quad p_2 \leq 25x_2 \\
& \quad p_2 \geq 10x_2 \\
& \quad p_2 \leq 20 \\
& \quad p_2 \leq 105 \\
& \quad p_2 \geq 0 \\
& \quad 0 \leq x_2 \leq 1
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& \quad p_2 \leq 20x_2 \\
& \quad p_2 \leq 105 \\
& \quad p_2 \geq 0 \\
& \quad 0 \leq x_2 \leq 1
\end{align*}
\]
Example: Feasible Regions of Relaxed Formulations

• Relaxed Formulation 1
  - Optimal solution
    - U2: Commitment = 0.2, Output = 5 MW
    - U1: Output = 100 MW

• Relaxed Formulation 2
  - Optimal solution
    - U2: Commitment = 0.25, Output = 5 MW
    - U1: Output = 100 MW
Example: Integer Relaxation Prices

• What is the LMP for Formulation 1?
  – The next MW of load would be satisfied by U2
  – The binding constraint $p_2 \leq 25x_2$
    implies a fractional U2 commitment increase ($1/25$) associated with a 1 MW output increase

  \[
  \text{LMP} = 20 + \frac{1000}{25} = 60
  \]

• What is the LMP for Formulation 2?
  – The next MW of load would be satisfied by U2
  – The binding constraint $p_2 \leq 20x_2$
    implies a fractional U2 commitment increase ($1/20$) associated with a 1 MW output increase

  \[
  \text{LMP} = 20 + \frac{1000}{20} = 70
  \]
Example Conclusion: Integer Relaxation Pricing

• Integer relaxation pricing **depends on the UC formulation**
  – Reformulating the UC problem is not unusual; ISOs use reformulations to improve computational performance
  – With integer relaxation pricing, the ISO has to consider the potential effects of UC reformulations on prices

• Without the complete mathematical formulation, integer relaxation is **not a well-defined pricing scheme**
  – The problem formulation should not impact the market outcome

• Uplift is still necessary
Analysis: Integer Relaxation Pricing

• **Efficient**
  – Combined, prices and uplift ensure that units have adequate dispatch-following incentives

• **Not transparent**
  – Uplift is needed

• **Not simple**
  – Price depends on the UC formulation and is hard to explain
  – For real-time single-interval pricing, the ISO cannot directly relax the multi-interval commitment problem
    • Instead, a single-interval “commitment-type” problem that amortizes commitment costs (similar to Rule-based Pricing) must be formulated and relaxed
## Summary of FS Pricing Methods

<table>
<thead>
<tr>
<th></th>
<th>Efficiency</th>
<th>Transparency</th>
<th>Simplicity</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fixing Commitment</td>
<td>Yes</td>
<td>No</td>
<td>Yes</td>
</tr>
<tr>
<td>Rule-based</td>
<td>Yes</td>
<td>No</td>
<td>Yes</td>
</tr>
<tr>
<td>Convex Hull</td>
<td>Yes</td>
<td>No*</td>
<td>No</td>
</tr>
<tr>
<td>Integer Relaxation</td>
<td>Yes</td>
<td>No</td>
<td>No</td>
</tr>
</tbody>
</table>

*If the size of total uplift is the only measure of transparency, Convex Hull Pricing is the “most transparent” approach.

There is no perfect price for a nonconvex problem!
Fundamental Questions on FS Pricing

• What costs should be reflected in price? Is the answer dependent on length of the market interval (e.g., DAM or RTM)?

• How does FS pricing relate to the missing money issue?

• How should Transparency and Simplicity be balanced?

• Does FS pricing inadvertently mimic one-part bidding?

No clear answers from economic theory!
Conclusion

- FS pricing is an imperfect solution for a **nonconvex pricing** problem
- The **Efficiency-Transparency-Simplicity** criteria can be used to compare different FS pricing methods
- **All** existing FS pricing methods have drawbacks
- **Hidden properties** of FS pricing were discussed
- **Broader questions** on FS pricing remain unanswered
Questions