Dual Pricing Algorithm in Non-Convex Electricity Markets

Robin Broder Hytowitz,[1,2] Richard P. O'Neill,[1]
Anya Castillo,[3] and Brent Eldridge[1,2]

[1]Federal Energy Regulatory Commission, Washington, DC
[2]Johns Hopkins University, Baltimore, MD
[3]Sandia National Laboratories, Albuquerque, NM

Outline

- Non-Convex Pricing & Cost Allocation
- Historical Examples
- Basic Formulation
  – Assumptions
  – Unit commitment
  – Dual Pricing Algorithm Constraints
- Sample Problems and Comparisons
  – Convex Hull
  – Extended LMP

Background

- Day-ahead markets aim to maximize surplus
  - Unit commitment model (MILP)
  - Contain continuous and binary bid functions
  - Startup and no load costs create non-convexities
- Generators are guaranteed non-negative profits
  - Not guaranteed by LMP

Background

- Pay generators uplift or make-whole payments
- Who pays for these additional uplift costs?
  - Generally spread across load
  - No clear criteria
  - Only ‘roughly’ allocated to beneficiaries

Literature

- Many proposals for non-convex pricing
  - LMP with uplift payments (O'Neill, Sotkiewicz, Hobbs, Rothkopf, Stewart)
  - Convex hull (Gribik, Hogan & Pope)
  - Extended LMP (Wang, Luh, Gribik, Zhang & Peng)
  - Modified LMP (Bjørndal & Jörnsten)
  - General uplift with zero-sum transfers (Motto & Galiana)
  - Semi-Lagrangean approach (Araoz & Jörnsten)
  - Primal-dual approach (Ruiz, Conejo, & Gabriel)
  - Review and internal zero-sum uplifts (Liberopoulos & Andrianesis)

Cost Allocation Principles

Maximize surplus

- Assumes demand can bid their value

Non-confiscation

- Incent participants to stay in the market
- Generator profits ≥ 0
- Net demand value ≥ 0

Revenue neutrality

- For each market payments equal receipts
- Money out = money in

Incentivize efficient investments

- New resources such as consumption efficiency, generation or transmission lines

Outline

- Cost Allocation
- Historical Examples
- Basic Formulation
- Sample Problems and Comparisons

Historical Example: Canal Units

- Canal Units on Cape Cod run daily due to long startup times and regional specifications
- Units support customers on Cape Cod
  - Without that demand, they would not be needed
- Uplift broadly allocated including Lower Southeastern Massachusetts (SEMA)
  - SEMA does not benefit
  - Costs should have been allocated primarily to Cape Cod to find a cheaper alternative much sooner

Source: http://www.iso-ne.com/markets-operations/market-performance/load-costs

Historical Example: Upper Peninsula

- Presque Isle Power Plant mainly powers the Upper Peninsula (UP)
  - Generates 90% of power in UP, 12% in Wisconsin Energy system
  - Sells 50% to Empire and Tilden mines
- Used for reliability in UP
  - Costs allocated to all LSEs in Wisconsin and UP on a pro rata basis
  - FERC found this unjust and unreasonable

Source: Phizzy
https://commons.wikimedia.org/wiki/File:MichiganUpperPeninsula.svg

Outline

- Cost Allocation
- Historical Examples
- Basic Formulation
- Sample Problems and Comparisons

Assumptions

- Demand is not infinitely valued
- Penalties will be imposed for deviating from optimal dispatch
  - Others suggest paying lost opportunity costs, which can be revenue inadequate
- Linear constraints will not significantly change this analysis and are omitted
  - Transmission
  - Reserves
  - Reliability

Unit Commitment Market Model

$$\max \ \sum_{i \in D} b_i d_i - \sum_{i \in G} \left(c_i p_i + c_i^{\mathrm{SU}} z_i \right)$$ Market surplus

$$\sum_{i \in D} d_i - \sum_{i \in G} p_i = 0$$ Market clearing

$$p_i^{\min} z_i \leq p_i \leq p_i^{\max} z_i \qquad \forall i \in G$$ Generation bounds

$$0 \leq d_i \leq d_i^{\max} \qquad \forall i \in D$$ Demand bounds

$$z_i \in \{0,1\} \qquad \forall i \in G$$ Commitment

<u>Decision variables</u>

p_i Cleared energy

d_i Cleared demand

z_i Startup commitment

Post-UC Pricing Model

$\max \ \sum_{i \in D} b_i d_i - \sum_{i \in G} \left(c_i p_i + c_i^{\mathrm{SU}} z_i \right)$			Market surplus
$\sum_{i \in D} d_i - \sum_{i \in G} p_i = 0$		λ	Market clearing
$p_i^{\min} z_i \leq p_i \leq p_i^{\max} z_i$	$\forall i \in G$	$\beta_i^{\max}, \beta_i^{\min}$	Generation bounds
$0 \leq d_i \leq d_i^{\max}$	$\forall i \in D$	$\alpha_i^{\max}$	Demand bounds
$z_i = z_i^*$	$\forall i \in G$	δ_i	Fix optimal schedule

Decision variables

p_i Cleared energy

d_i Cleared demand

z_i Startup commitment

Dual Model

$\min \ \sum_{i \in D} d_i^{\max} \alpha_i^{\max} + \sum_{i \in G} z_i^* \delta_i$			Resource valuation
$\lambda + \alpha_i^{\max} \geq b_i$	$\forall i \in D$	d_i	Value condition
$-\lambda + \beta_i^{\max} - \beta_i^{\min} \geq -c_i$	$\forall i \in G$	p_i	Profit condition
$\delta_i - p_i^{\max} \beta_i^{\max} + p_i^{\min} \beta_i^{\min} = -c_i^{\mathrm{SU}}$	$\forall i \in G$	v_i	Startup economics
$\alpha_i^{\max}, \beta_i^{\max}, \beta_i^{\min} \geq 0$	$\forall i \in D \cup G$		Non-negativity

New Variables

- λ^{DPA} : new LMP
- u_i^p, u_i^{pd}: make-whole payment
- u_i^c, u_i^{cd} : make−whole charge

  – Allocated by resource

Objective

- Minimize uplift payments
  - $\min \sum_{i \in D^+} d_i^* u_i^{\mathrm{pd}} + \sum_{i \in G^+} p_i^* u_i^{\mathrm{p}}$
  - Uplift payments from demand and generation

Market Surplus

- Maintain optimal market surplus
  - $\sum_{i \in D} \Psi_i + \sum_{i \in G} \Pi_i = MS^*$
  - Use optimal dispatch, making it a redundant constraint

Maximize market surplus

Profit Definition

- From complementary slackness of the generation bounds and the profit condition, combining with the startup economics, we calculate the linear surplus of generator *i*
  - $\delta_i = p_i^*(\lambda - c_i) - c_i^{SU}$
  - dispatch*(LMP – marginal cost) – startup cost
- To ensure non-confiscation, the linear surplus and uplift payments must be non-negative
  - $\Pi_i = \delta_i + p_i^*(u_i^p - u_i^c) \geq 0$

Non-confiscation

Value Definition

- From complementary slackness of the value condition, and non-negativity of variables, demand *i*
  - $d_i^*(b_i - \lambda) = d_i^* \alpha_i^{max*} \geq 0$
- To ensure non-confiscation, the value and uplift payments must be non-negative
  - $\Psi_i = d_i^* \alpha_i^{max*} + d_i^*\left(u_i^p - u_i^c\right) \geq 0$

Non-confiscation

Additional constraints

- Revenue neutrality
  - $\sum_{i \in D^+} d_i^* \left(u_i^{\mathrm{pd}} - u_i^{\mathrm{cd}}\right) + \sum_{i \in G^+} p_i^* \left(u_i^{\mathrm{p}} - u_i^{\mathrm{c}}\right) = 0$
- Non-recourse of demand not selected
  - $\lambda^{\mathrm{DPA}} \geq b_i$
  - Value of new LMP not entice out-of-market demand to consume

Revenue neutrality

Formulation: Dual Pricing Algorithm

$\min \sum_{i\in D} d_i^* u_i^{\mathrm{pd}} + \sum_{i\in G} p_i^* u_i^{\mathrm{p}}$		Uplift minimization
$\sum_{i\in D} d_i^* \left(u_i^{\mathrm{pd}} - u_i^{\mathrm{cd}}\right) + \sum_{i\in G} p_i^* \left(u_i^{\mathrm{p}} - u_i^{\mathrm{c}}\right) = 0$		Uplift revenue neutrality
$\Psi_i = d_i^* \left(b_i - \lambda^{\mathrm{DPA}} + u_i^{\mathrm{pd}} - u_i^{\mathrm{cd}}\right)$	$\forall i \in D^+$	Value definition
$\Pi_i = p_i^* \left(\lambda^{\mathrm{DPA}} - c_i + u_i^{\mathrm{p}} - u_i^{\mathrm{c}}\right) - c_i^{\mathrm{SU}}$	$\forall i \in G^+$	Profit definition
$\lambda^{\mathrm{DPA}} \geq b_i$	$\forall i \in D^0$	Non-recourse condition
$\Psi_i, \Pi_i \geq 0$	$\forall i \in D^+ \cup G^+$	Value conditions
$u_i^{\mathrm{p}}, u_i^{\mathrm{c}}, u_i^{\mathrm{pd}}, u_i^{\mathrm{cd}} \geq 0$	$\forall i \in D^+ \cup G^+$	Non-negativity

Properties of the DPA

- Non-confiscation
- Revenue neutral (and adequate)
- Feasible solution with optimal feasible UC
- Does not change optimal dispatch solution
- Easy to implement in present ISO software
- Problem is linear – computationally efficient
- Solution is non-unique
  - Can be conditioned depending on operator preference

Non-Unique Prices

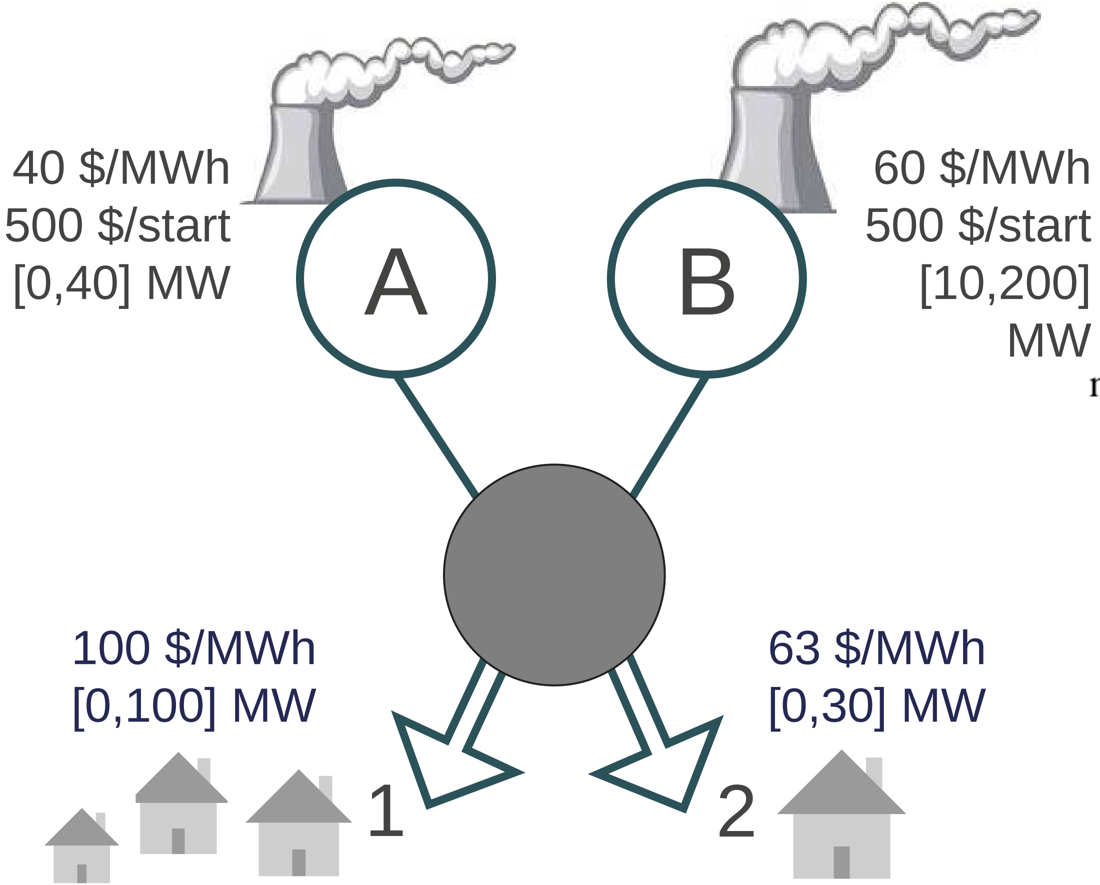
- Conditioning
  - Allows the market operator to adjust LMP based on regional policies
- Example: tie new LMP to LMP from dispatch run
  - New constraint: $\frac{(\lambda^{\mathrm{DPA}} - \lambda^{*})}{\lambda^{*}} - \lambda^{\mathrm{up}} + \lambda^{\mathrm{dn}} = 0$
  - New Objective:

$$\min \sum_{i \in D} d_i^* u_i^{\mathrm{pd}} + \sum_{i \in G} p_i^* u_i^{\mathrm{p}} + c^{\mathrm{up}} \lambda^{\mathrm{up}} + c^{\mathrm{dn}} \lambda^{\mathrm{dn}}$$

Outline

- Cost Allocation
- Historical Examples
- Basic Formulation
- Sample Problems and Comparisons

Example: Single node, single period



$$\max MS = \sum_{i \in D} b_i d_i - \sum_{i \in G} (c_i p_i + c_i^{su} z_i)$$

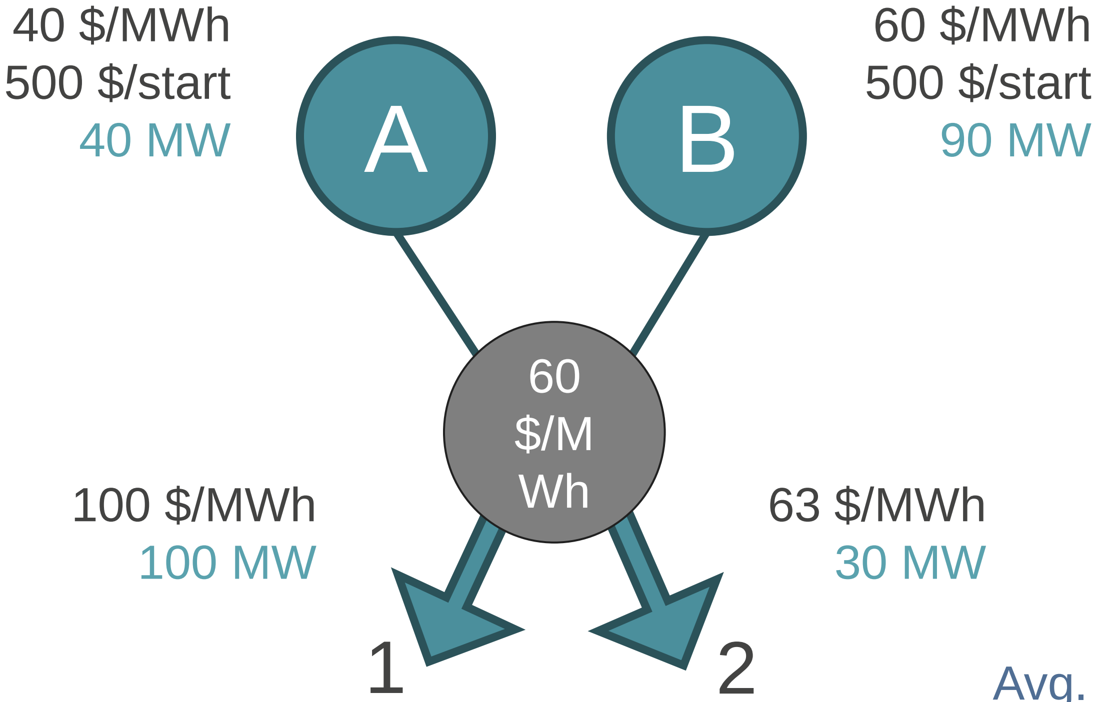
$$\sum_{i \in D} d_i - \sum_{i \in G} p_i = 0$$

$$p_i^{\min} z_i \le p_i \le p_i^{\max} z_i \quad \forall i \in G$$

$$0 \le d_i \le d_i^{\max} \quad \forall i \in D$$

$$z_i = z_i^{*} \quad \forall i \in G$$

Resulting UC Solution



Payment = 63.85 $/MWh

Market surplus = $3830		
Gen	Margin ($/MWh)	Profit ($)
A	20	300
B	0	-500
Buyer	Margin ($/MWh)	Net Value ($)
1	40	4000
2	3	90

Price = $60/MWh

Uplift = $500

Avg. socialized uplift = $3.85/MWh

Results of DPA

λ^{DPA}	Make whole payment	Unallocated make whole payment
65.56	76.67	0

Gen	Marg. Cost	u^p	u^c
A	40	0	0
B	60	0	0
Buyer	**Value**	**u^p**	**u^c**
1	100	0	0.767
2	63	2.556	0

u_i^p Make whole payment

u_i^c Make whole charge

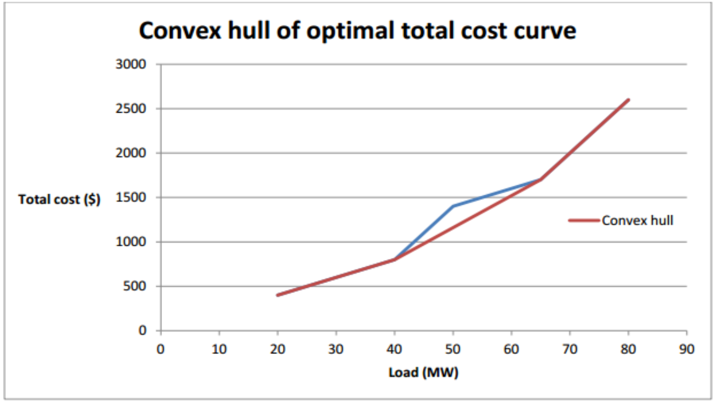
λ^{DPA} New LMP

Results of DPA

		Post-UC Value ($)		Value under DPA ($)	
LMP (λ)		60		65.56	
		Unit ($/MWh)	Total	Unit ($/MWh)	Total
Profit	Gen A	20	300	25.56 (+28%)	522.22 (+74%)
	Gen B	0	-500	5.56	0

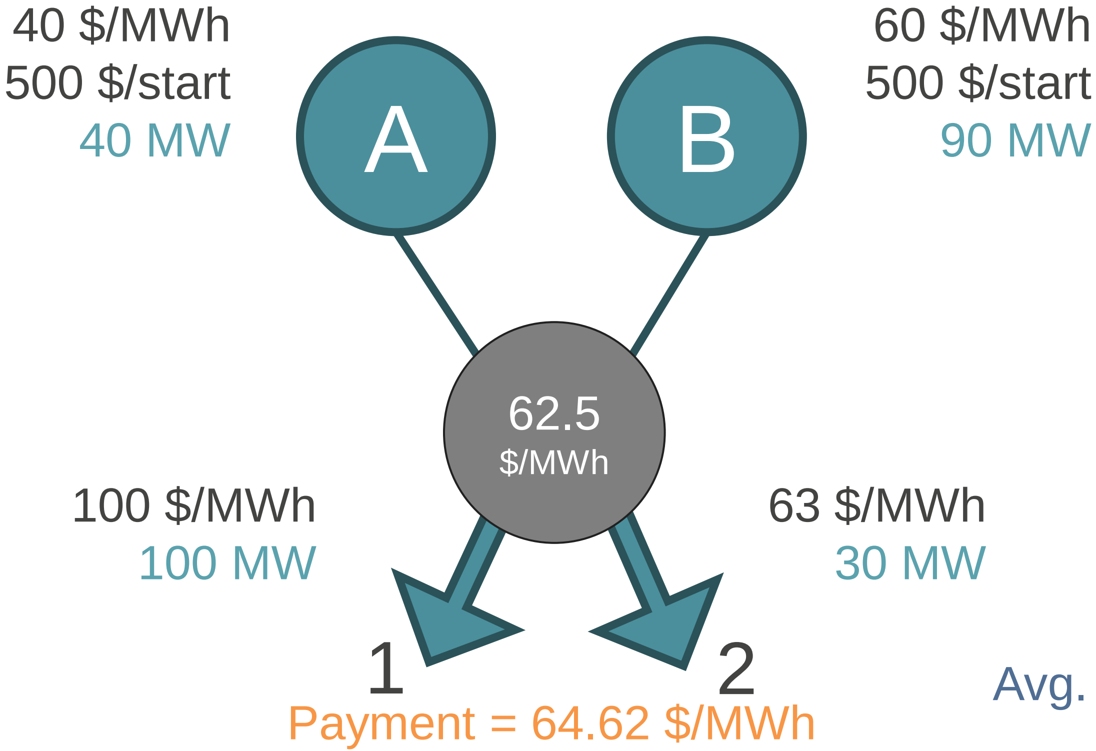
Comparison to Convex Hull

- Convex hull formulation finds a uniform price that minimizes side payments
  - Not all side payments minimized
  - Not well understood
- Formulation based on [1]



[1] D.A. Schiro, T. Zheng, F. Zhao, and E. Litvinov, “Convex Hull Pricing in Electricity Markets: Formulation, Analysis, and Implementation Challenges,” ISO-NE. [Online] Available: http://www.optimization-online.org/DB_FILE/2015/03/4830.pdf

Resulting CH Solution



Market surplus = $4165		
Gen	Margin ($/MWh)	Profit ($)
A	20	400
B	0	-275
Buyer	Margin ($/MWh)	Net Value ($)
1	40	3750
2	3	15

Price = $62.50/MWh

Uplift = $275

Avg. socialized uplift = $2.12/MWh

Results Comparison

		Original Value		Value under DPA		Value under Convex Hull	
LMP λ ($/MWh)		60		65.56		62.50	
		Unit ($/MWh)	Total	Unit ($/MWh)	Total	Unit ($/MWh)	Total
Profit	Gen A ($40/MWh)	20 (-)	300 (-)	25.56 (+28%)	522.22 (+74%)	22.50 (+13%)	400 (+33%)
	Gen B ($60/MWh)	0	-500	5.56	0	2.50	-275
Value	Buyer 1 ($100/MWh)	40 (-)	4000 (-)	33.678 (-19%)	3367 (-19%)	37.50 (-6%)	3750 (-6%)
	Buyer 2 ($63/MWh)	3	90	0	0	0.50	15

ELMP Pricing Run

- ELMP determined after the dispatch run
- Relaxes the commitment variables
  - $0 \leq z_i \leq 1$
- Uses incremental costs instead of marginal costs
  - $IncCost_i = Mc_i + (Su_i^{cost} / dispatch_i)$

Price Comparison

	Price
Post-UC	60
DPA	65.56
Convex Hull	62.50
ELMP	68.056

- ELMP is above the bid for demand 2
- Gens A&B are dispatched to meet demand 1
- With inelastic demand, not problematic

Conclusions

- Cost allocation can be problematic if costs are spread across all load
- DPA is
  - Easy to implement
  - Linear and computationally efficient
  - Revenue neutral
  - Non-confiscatory
  - Does not change optimal solution
  - Performs well against other formulations
- Additional simulations and extensions (multi-period, multi-node) can further explore dual pricing

Thank you!

Questions?

robin.hytowitz@ferc.gov

Revenue Adequacy and LOCs

Market surplus = $200						
Gen	Marginal Cost ($/MWh)	Start Up Cost	Linear Profit ($)	Dispatch (MWh)	Max Capacity (MW)	Total Cost ($)
A	30	900	1100	0	200	0
B	40	100	-100	60	200	2500
LMP = **$40/MWh** Uplift = **-$100** Avg. socialized uplift = **-$1.67/MWh**						
Buyer	Value ($/MWh)	Load (MWh)	Max demand (MW)	Marginal Value ($/MWh)	Total Value ($)	Gross Value ($)
1	45	60	60	5	300	2700

200 MWh($40/MWh-$30/MWh)-$900 = $1100 = LOC > MS = $200