Strengthened MILP Formulation for the Edge-based Combined-Cycle Unit Model

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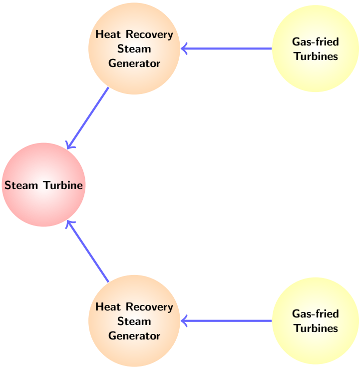
Outline

Outline

Combined-Cycle Units

- Combustion Turbines: use natural gas, produce electricity and heat.
- Heat Recovery Steam Generator: produce steam.
- Steam Turbines: use steam to produce electricity.



Current Practice

- Aggregated modeling approach [1]:
  - Treats the whole combined-cycle unit as a traditional thermal unit.
  - Less decision variables.
  - Cannot reflect the relationship between CT and ST.
- Pseudo unit approach [2]:
  - Associates each combustion turbine (CT) with a portion of the steam turbine (ST).
  - Less decision variables.
  - Cannot capture the transition process.
- Configuration-based model [3],[4],[5],[6]:
  - Represents each combination of CTs and STs as a configuration.
  - Cannot capture the operating constraints such as min-up/-down constraints for each turbine.

Configuration-Based Model

- Work at different typical configurations: 0CT + 0ST, 1CT + 0ST, 2CTs + 0ST, 1CT + 1ST, and 2CTs + 1ST
- Each configuration is treated as a pseudo unit: generation limits, ramping rates, and min-up/-down constraints.

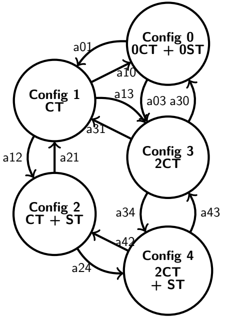


Figure: Transition Graph for 2CTs + 1ST

Challenges

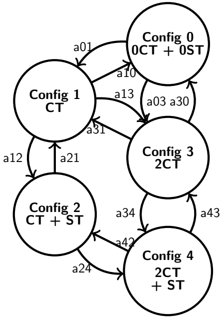


Figure: Transition Graph for 2CTs + 1ST

- Time t, Configuration 1 online
- Time $t+2$ Load increases, Generation amount increases, ST starts up, Configuration 2 online
- Time $t+3$ Works at Configuration 2 for several time periods (e.g., 4 time periods).
- If load increases dramatically, it might be more than the capacity of Configuration 2 at time $t+3, t+4$.
- Second CT can start up, if this CT satisfies it own min-down time requirement.

Challenges

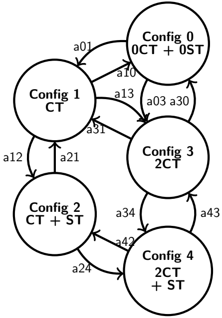


Figure: Transition Graph for 2CTs + 1ST

- Improve flexibility?
- Design the min-up/-down constraints for each turbine instead of each configuration.

Edge-Based Combined-Cycle Unit Model

- **Motivation**
  - Improved the accuracy of the configuration-based model [7].
- **Method**
  - Proposed an edge-based formulation based on the transition graph.
- **Contribution**
  - Exactly described the physical constraints (in particular, min-up/-down restrictions for each turbine) and transition costs between different configurations.
  - Increased the flexibility of the combined-cycle units in terms of unit commitment.
  - Explored the structure of the state transition graph for combined-cycle units (such as the network flow structure) that commercial optimization solvers, e.g., CPLEX, can recognize.

Strengthened Edge-Based Combined-Cycle Unit Model

- **Motivation**
  - Reduce the computational time in the day-ahead unit commitment engine caused by combined-cycle units.
- **Method**
  - Cutting plane method.
- **Contribution**
  - Derived tighter min-up/-down and ramping rate constraints for a combined-cycle unit.
  - Provided several families of stronger valid inequalities of ramping rates for a combined-cycle unit by exploring the structure of the transition graph.

Outline

Transition Graph

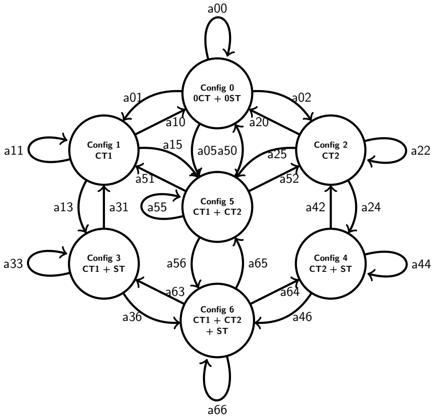


Figure: Complete Transition Graph for 2CTs + 1ST

- Use complete transition graph (distinguish two CTs).
- Edge binary variables (z_t^a): transition action at each time period.
- Unique constraints:

$$\sum_{a \in \mathcal{A}} z_t^a = 1, \forall t. \qquad (1)$$

Network Flow

Configuration Status

$$\sum_{a \in (\mathcal{A}_k^{\text{in}} \bigcup \mathcal{A}_k^{\text{sl}})} z_t^a \tag{2}$$

Logical Constraints

$$\sum_{a \in (\mathcal{A}_k^{\text{in}} \bigcup \mathcal{A}_k^{\text{sl}})} z_t^a = \sum_{a \in (\mathcal{A}_k^{\text{out}} \bigcup \mathcal{A}_k^{\text{sl}})} z_{t+1}^a, \forall k \in \mathcal{C}, \forall t. \tag{3}$$

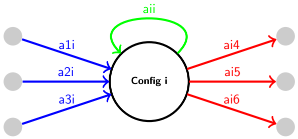


Figure: Edges of One Node

Min-up Time

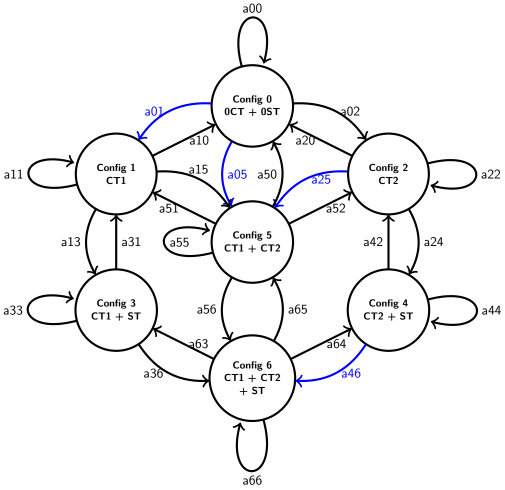


Figure: Start-up CT1

- CT1 Starts up: $a01, a05, a25, a46$

Min-up Time

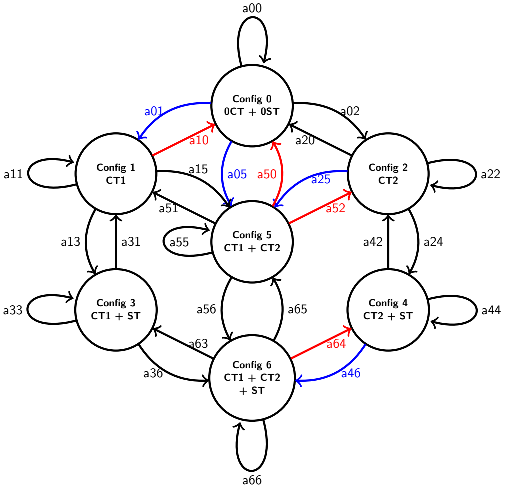


Figure: Start-up CT1

- CT1 starts up: $a01, a05, a25, a46$
- CT1 cannot shut down: $a10, a50, a52, a64$

Min-up Time

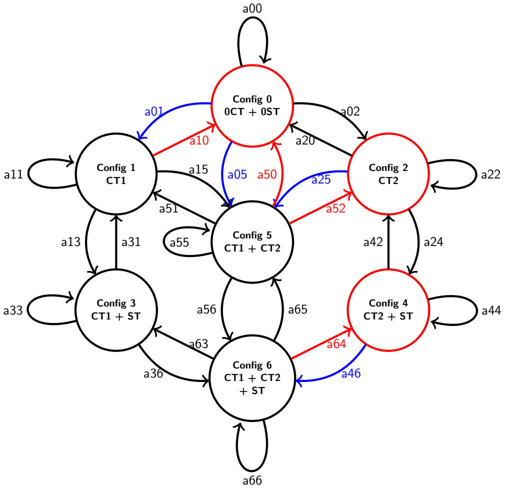


Figure: Start-up CT1

- CT1 starts up: $a01, a05, a25, a46$
- CT1 cannot shut down: $a10, a50, a52, a64$
- Configurations without CT1 cannot be online: Config 0, Config 2, Config 4

Min-up Time

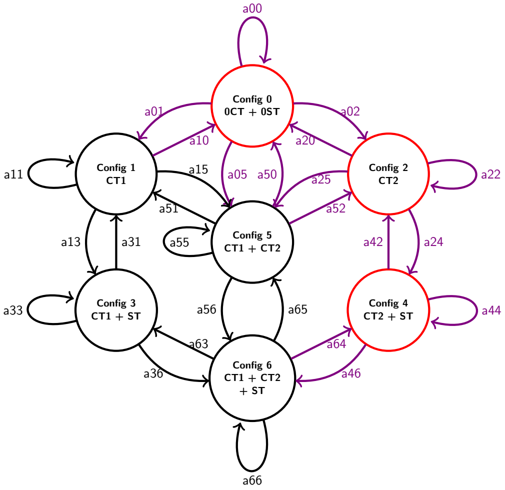
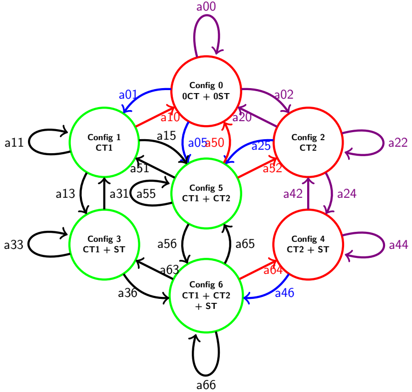


Figure: Start-up CT1

- CT1 starts up: $a01, a05, a25, a46$
- CT1 cannot shut down: $a10, a50, a52, a64$
- Configurations without CT1 cannot be online: Config 0, Config 2, Config 4
- Edges connected with Red configurations cannot be active.

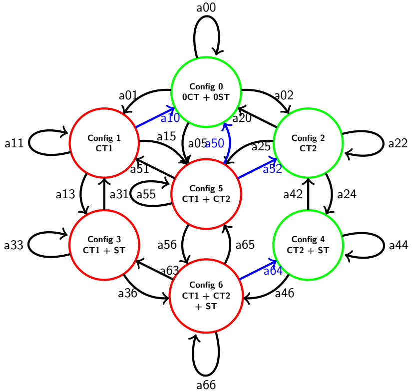
Min-up Time Constraints



$$\sum_{a \in \bigcup_{k \in \mathcal{C}_i^{\text{off}}} \mathcal{A}_k^{\text{all}}} z_\tau^a \leq 1 - \sum_{a \in \mathcal{A}_i^{\text{su}}} z_t^a, \forall i \in \mathcal{U}^{\text{CT}} \cup \mathcal{U}^{\text{ST}},$$

$$\forall \tau \in \{t+1, \cdots, \min\{\mathcal{T}_{\text{end}}, T_{\text{mu}}^i + t - 1\}\}, \forall t. \quad (4)$$

Min-down Time Constraints



$$\sum_{a\in\bigcup_{k\in\mathcal{C}_i^{\text{on}}}\mathcal{A}_k^{\text{all}}} z_\tau^a \le 1 - \sum_{a\in\mathcal{A}_i^{\text{sd}}} z_t^a, \forall i \in \mathcal{U}^{\text{CT}}\cup\mathcal{U}^{\text{ST}},$$

$$\tau \in \{t+1,\cdots,\min\{\mathcal{T}_{\text{end}}, T_{\text{md}}^i + t-1\}\}, \forall t. \qquad (5)$$

Ramping Constraints

- If this particular edge is not active (i.e., $z^a_{t+1} = 0$), this ramping constraint is relaxed, following the definition of P^{cap}
- Otherwise, if this edge is active (i.e., $z^a_{t+1} = 1$), this edge provides the ramping limit for the whole combined-cycle unit, because only one of the edges can be active at each time period.

$$p_{t+1} - p_t \leq \text{RU}_a z^a_{t+1} + P^{\text{cap}}(1 - z^a_{t+1}), \forall a \in \mathcal{A}, \forall t, \tag{6}$$

$$p_t - p_{t+1} \leq \text{RD}_a z^a_{t+1} + P^{\text{cap}}(1 - z^a_{t+1}), \forall a \in \mathcal{A}, \forall t. \tag{7}$$

Reduced State Transition Graph

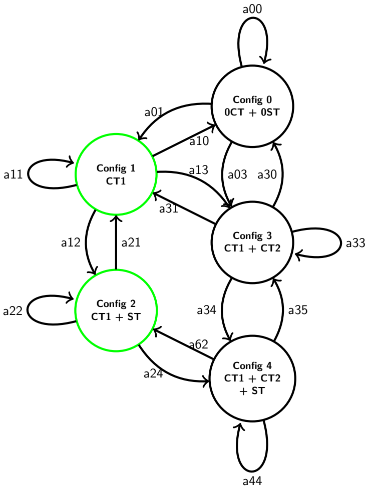
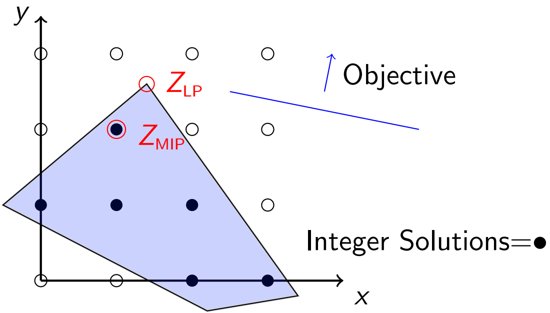


Figure: Reduced State Transition Graph for 2CT+1ST

Outline

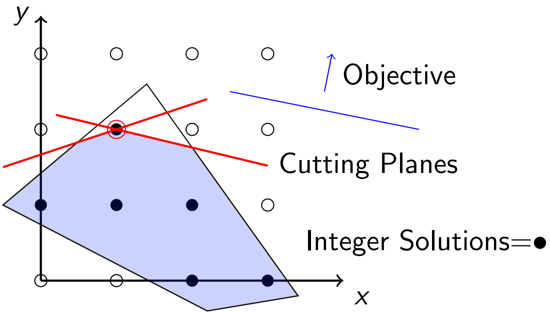
Cutting Planes

Strong valid inequalities to cut off fractional solutions.



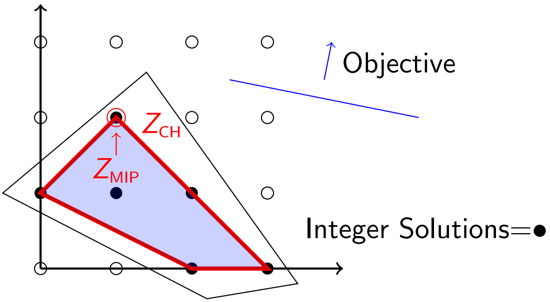
Cutting Planes

Strong valid inequalities to cut off fractional solutions.



Convex Hull

The smallest convex feasible region containing all feasible integer solutions



Min-up Time Constraints

- If turbine i is online at time period t, then this turbine starts up at most once during time interval $[t - T^i_{\text{mu}} + 1, t - 1]$.
- If turbine i starts up at time interval $[t - T^i_{\text{mu}} + 1, t - 1]$, then the configurations without turbine i cannot be online at time period t.

$$\sum_{\kappa=1}^{T^i_{\text{mu}}-1} \sum_{a \in \mathcal{A}^{\text{su}}_i} z^a_{t-\kappa} \leq 1 - \sum_{a \in \bigcup_{k \in \mathcal{C}^{\text{off}}_i} \mathcal{A}^{\text{all}}_k} z^a_t, \tag{8}$$

$$\forall i \in \mathcal{U}^{\text{CT}} \cup \mathcal{U}^{\text{ST}}, \forall t \in \{T^i_{\text{mu}}, \cdots, \mathcal{T}_{\text{end}}\}.$$

Min-down Time Constraints

- If one of the arcs in $\mathcal{A}^i_{\text{sd}}$, representing the shut-down process of turbine i, is active during time interval $[t - T^i_{\text{md}} + 1, t - 1]$, then arcs $\bigcup_{k \in \mathcal{C}^{\text{on}}_i} \mathcal{A}^{\text{all}}_k$ connected to the configurations ($\mathcal{C}^{\text{on}}_i$) with turbine i cannot be active.
- The configurations with turbine i must be offline at time period t when turbine i shuts down at time interval $[t - T^i_{\text{md}} + 1, t - 1]$.

$$\sum_{\kappa=1}^{T^i_{\text{md}}-1} \sum_{a \in \mathcal{A}^{\text{sd}}_i} z^a_{t-\kappa} \leq 1 - \sum_{a \in \bigcup_{k \in \mathcal{C}^{\text{on}}_i} \mathcal{A}^{\text{all}}_k} z^a_t,$$

$$\forall i \in \mathcal{U}^{\text{CT}} \cup \mathcal{U}^{\text{ST}}, \forall t \in \{T^i_{\text{md}}, \cdots, \mathcal{T}_{\text{end}}\}. \qquad (9)$$

Ramping Rate Constraints

- Since only one of the arcs in the transition graph can be active at each time period t, only one item in the right-hand side of (10) can be positive and all others would be zeros.
- The positive item represents the active arc that provides the ramping up rate limit. The same analysis can be applied to ramping down constraints (11)

$$p_{t+1} - p_t \leq \sum_{a \in \mathcal{A}} \mathrm{RU}_a z^a_{t+1}, \forall t \in \mathcal{T}, \tag{10}$$

$$p_t - p_{t+1} \leq \sum_{a \in \mathcal{A}} \mathrm{RD}_a z^a_{t+1}, \forall t \in \mathcal{T}. \tag{11}$$

Single-Arc Ramping Up Rate Inequalities

$$p^m_{t+1} - p^n_t \leq \mathrm{RU}^{a(n,m)} z^{a(n,m)}_{t+1} + \overline{P_m}\Big(\sum_{a \in (\mathcal{A}^{\text{in}}_m \bigcup \mathcal{A}^{\text{sl}}_m)} z^a_{t+1} \Big)$$
$$- \underline{P_n}\Big(\sum_{a \in (\mathcal{A}^{\text{in}}_n \bigcup \mathcal{A}^{\text{sl}}_n)} z^a_t \Big) + (\underline{P_n} - \overline{P_m}) z^{a(n,m)}_{t+1},$$
$$\forall a(n,m) \in \mathcal{A}, \forall t \in \mathcal{T}, \qquad (12)$$

Table: Validity of Ramping Up Inequalities (12)

Case	Value of Binary Variables			Inequality	
	$\sum_{a \in (\mathcal{A}^{\text{in}}_n \bigcup \mathcal{A}^{\text{sl}}_n)} z^a_t$	$\sum_{a \in (\mathcal{A}^{\text{in}}_m \bigcup \mathcal{A}^{\text{sl}}_m)} z^a_{t+1}$	$z^{a(n,m)}_{t+1}$	LHS	RHS
1	1	1	1	$p^m_{t+1} - p^n_t$	$\mathrm{RU}^{a(n,m)}$
2	1	0	0	$-p^n_t$	$-\underline{P_n}$
3	0	1	0	p^m_{t+1}	$\overline{P_m}$
4	0	0	0	0	0

Single-Arc Ramping Down Rate Inequalities

$$p_t^n - p_{t+1}^m \leq \mathrm{RD}^{a(n,m)} z_{t+1}^{a(n,m)} + \overline{P_n}\Big(\sum_{a\in(\mathcal{A}_n^{\text{in}} \bigcup \mathcal{A}_n^{\text{sl}})} z_t^a\Big)$$
$$- \underline{P_m}\Big(\sum_{a\in(\mathcal{A}_m^{\text{in}} \bigcup \mathcal{A}_m^{\text{sl}})} z_{t+1}^a\Big) + (\underline{P_m} - \overline{P_n}) z_{t+1}^{a(n,m)},$$
$$\forall a(n,m) \in \mathcal{A}, \forall t \in \mathcal{T}. \quad (13)$$

Table: Validity of Ramping Down Inequalities (13)

Case	Value of Binary Variables			Inequality	
	$\sum_{a\in(\mathcal{A}_n^{\text{in}} \bigcup \mathcal{A}_n^{\text{sl}})} z_t^a$	$\sum_{a\in(\mathcal{A}_m^{\text{in}} \bigcup \mathcal{A}_m^{\text{sl}})} z_{t+1}^a$	$z_{t+1}^{a(n,m)}$	LHS	RHS
1	1	1	1	$p_t^n - p_{t+1}^m$	$\mathrm{RD}^{a(n,m)}$
2	1	0	0	p_t^n	$\overline{P_n}$
3	0	1	0	$-p_{t+1}^m$	$-\underline{P_m}$
4	0	0	0	0	0

Multi-Configuration Ramping Rate Inequalities

- Suppose that the combined-cycle unit works on Configuration m at time period $t+1$. As shown in the following figure, we know one of the incoming arcs ($a_{n_1,m}$, $a_{n_2,m}$, $a_{n_3,m}$) or the self-loop arc $a_{m,m}$ must be active at time period $t+1$.

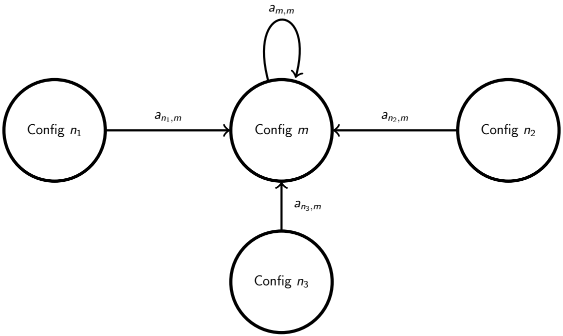


Figure: Configuration Transition Graph for Configuration m

Multi-Configuration Ramping Up Rate Inequalities

$$p_{t+1}^m - \sum_{n \in \mathcal{C}_{\to m}} p_t^n \leq \sum_{n \in \mathcal{C}_{\to m}} \mathrm{RU}^{a(n,m)} z_{t+1}^{a(n,m)} - \sum_{n \in \mathcal{C}_{\to m}} \underline{P}_n \Big(\Big(\sum_{a \in (\mathcal{A}_n^{\mathrm{in}} \bigcup \mathcal{A}_n^{\mathrm{sl}})} z_t^a \Big) - z_{t+1}^{a(n,m)} \Big), \forall m \in \mathcal{C}, \forall t. \quad (14)$$

Table: Validity of Ramping Up Inequalities (14)

Case	Value of Binary Variables		Inequality	
	$\sum_{n \in \mathcal{C}_{\to m}} z_{t+1}^{a(n,m)}$	$\sum_{n \in \mathcal{C}_{\to m}} \sum_{a \in (\mathcal{A}_n^{\mathrm{in}} \bigcup \mathcal{A}_n^{\mathrm{sl}})} z_t^a$	LHS	RHS
1	1	1	$p_{t+1}^m - p_t^{\bar{n}}$	$\mathrm{RU}^{a(\bar{n},m)}$
2	0	1	$-p_t^{\bar{n}}$	$-\underline{P}_{\bar{n}}$
3	0	0	0	0

Multi-Configuration Ramping Down Rate Inequalities

$$\sum_{n \in \mathcal{C}_{\to m}} p_t^n - p_{t+1}^m \leq \sum_{n \in \mathcal{C}_{\to m}} \mathrm{RD}^{a(n,m)} z_{t+1}^{a(n,m)} + \sum_{n \in \mathcal{C}_{\to m}} \overline{P}_n \Bigg(\Big(\sum_{a \in (\mathcal{A}_n^{\mathrm{in}} \bigcup \mathcal{A}_n^{\mathrm{sl}})} z_t^a \Big) - z_{t+1}^{a(n,m)} \Bigg), \forall m \in \mathcal{C}, \forall t. \quad (15)$$

Table: Validity of Ramping Down Inequalities (15)

Case	Value of Binary Variables		Inequality	
	$\sum_{n \in \mathcal{C}_{\to m}} z_{t+1}^{a(n,m)}$	$\sum_{n \in \mathcal{C}_{\to m}} \sum_{a \in (\mathcal{A}_n^{\mathrm{in}} \bigcup \mathcal{A}_n^{\mathrm{sl}})} z_t^a$	LHS	RHS
1	1	1	$p_t^{\bar{n}} - p_{t+1}^m$	$\mathrm{RD}^{a(\bar{n},m)}$
2	0	1	$p_t^{\bar{n}}$	$\overline{P}_{\bar{n}}$
3	0	0	0	0

Multi-Configuration Ramping Rate Inequalities

- Suppose that the combined-cycle unit works on Configuration n at time period t in the following figure. Then, one of the outgoing arcs $(a_{n,m_1}, a_{n,m_2}, a_{n,m_3})$ or the self-loop arc $a_{n,n}$ must be active at time period $t+1$.

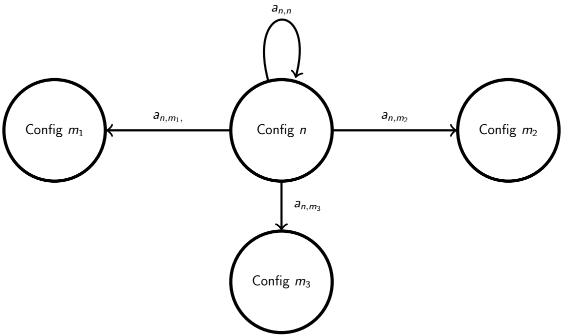


Figure: Configuration Transition Graph for Configuration n

Multi-Configuration Ramping Up Rate Inequalities

$$\sum_{m\in\mathcal{C}_{n\to}} p_{t+1}^{m} - p_t^n \leq \sum_{m\in\mathcal{C}_{n\to}} \mathrm{RU}^{a(n,m)} z_{t+1}^{a(n,m)} + \sum_{m\in\mathcal{C}_{n\to}} \overline{P}_m \Big(\Big(\sum_{a\in(\mathcal{A}_m^{\mathrm{in}} \bigcup \mathcal{A}_m^{\mathrm{sl}})} z_{t+1}^{a} \Big) - z_{t+1}^{a(n,m)} \Big), \forall n \in \mathcal{C}, \forall t. \quad (16)$$

Table: Validity of Ramping Up Inequalities (16)

Case	Value of Binary Variables		Inequality	
	$\sum_{m\in\mathcal{C}_{n\to}} z_{t+1}^{a(n,m)}$	$\sum_{m\in\mathcal{C}_{n\to}} \sum_{a\in(\mathcal{A}_m^{\mathrm{in}} \bigcup \mathcal{A}_m^{\mathrm{sl}})} z_{t+1}^{a}$	LHS	RHS
1	1	1	$p_{t+1}^{\bar{m}} - p_t^n$	$\mathrm{RU}^{a(n,\bar{m})}$
2	0	1	$p_t^{\bar{m}}$	$\overline{P}_{\bar{m}}$
3	0	0	0	0

Multi-Configuration Ramping Down Rate Inequalities

$$p_n^t - \sum_{m \in \mathcal{C}_{n\to}} p_{t+1}^m \le \sum_{m \in \mathcal{C}_{n\to}} \mathrm{RD}^{a(n,m)} z_{t+1}^{a(n,m)} - \sum_{m \in \mathcal{C}_{n\to}} \underline{P}_m \Big(\Big(\sum_{a \in (\mathcal{A}_m^{\text{in}} \bigcup \mathcal{A}_m^{\text{sl}})} z_{t+1}^a \Big) - z_{t+1}^{a(n,m)} \Big), \forall n \in \mathcal{C}, \forall t. \quad (17)$$

Table: Validity of Ramping Down Inequalities (17)

Case	Value of Binary Variables		Inequality	
	$\sum_{m \in \mathcal{C}_{n\to}} z_{t+1}^{a(n,m)}$	$\sum_{m \in \mathcal{C}_{n\to}} \sum_{a \in (\mathcal{A}_m^{\text{in}} \bigcup \mathcal{A}_m^{\text{sl}})} z_{t+1}^a$	LHS	RHS
1	1	1	$p_t^n - p_{t+1}^{\bar{m}}$	$\mathrm{RD}^{a(n,\bar{m})}$
2	0	1	$-p_t^{\bar{m}}$	$-\underline{P}_{\bar{m}}$
3	0	0	0	0

Outline

Experiment Setting

- IEEE 118 Bus System: 54 traditional thermal units and 12 combined-cycle units.
- 10 different load scenarios.
- Intel(R) Core(TM) i7-4500U 1.8GHz with 8G memory and CPLEX 12.5.
- EBF: Edge-based formulation.
- TEBF: The edge-based formulation with min-up/-down constraints (4) and (5) replaced by tighter min-up/-down constraints (8) and (9).
- REBF: The edge-based formulation with ramping constraints (6) and (7) replaced by tighter ramping constraints (10) - (17).
- SEBF: Strengthened edge-based formulation.

Computational Results

Table: Root Node Information

Cases		LP Objective Values ($)				Integrality Gap (10^{-4})			
		EBF	TEBF	REBF	SEBF	EBF	TEBF	REBF	SEBF
G-I	1	1879876	1880212	1880451	1880774	9.92	8.11	6.85	5.14
	2	1879103	1879456	1879698	1880031	10.15	8.30	7.10	5.34
	3	1885160	1885489	1885739	1886056	10.01	8.29	6.93	5.26
	4	1876169	1876512	1876746	1877070	10.01	8.19	6.96	5.15
	5	1887136	1887470	1887715	1888032	9.39	7.84	6.53	4.74
G-II	1	3615129	3615571	3616036	3616440	9.18	8.08	6.90	6.68
	2	3606929	3607372	3607865	3608274	9.98	7.72	6.68	6.05
	3	3602757	3603224	3603670	3604094	11.47	8.79	7.30	6.2
	4	3609224	3609688	3610134	3610562	9.89	9.09	7.34	6.75
	5	3607151	3607576	3608070	3608472	11.45	9.25	7.89	6.17

Computational Results

Table: Computational Times

Cases		Time				Number of Nodes			
		EBF	TEBF	REBF	SEBF	EBF	TEBF	REBF	SEBF
G-I	1	1668.92	1478.32	1208.08	650.23	4526	3315	2537	1064
	2	1383.41	985.74	538.77	604.61	4570	2562	644	828
	3	1474.76	1569.19	483.59	400.15	3683	5895	1218	952
	4	1282.29	903.13	502.99	335.69	2899	2471	640	442
	5	1240.13	811.17	317.38	407.5	4375	2154	299	572
G-II	1	1114.08	945.24	999.92	797.65	1184	1197	377	316
	2	***	884.34	489.7	666.86	1206	1248	0	0
	3	***	***	833.12	828.68	1213	1190	174	126
	4	2512.31	3411.7	1820.3	798.59	1251	1233	1134	204
	5	***	3231.19	702.01	834.63	1157	1486	0	152

Convergence Process

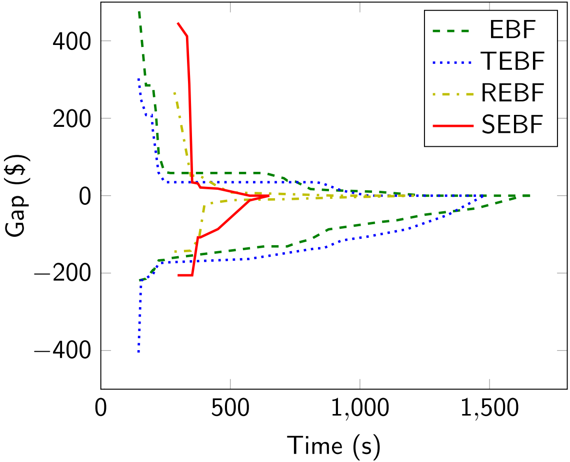


Figure: Convergence evolution of Case 1 in One-Day UC

Convergence Process

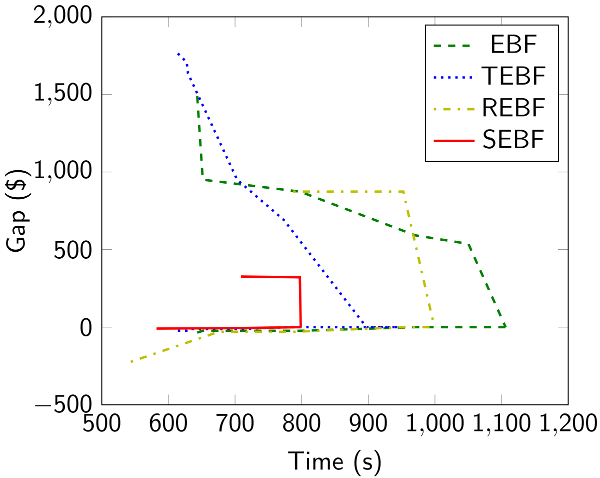


Figure: Convergence evolution of Case 1 in Two-Day UC

Outline

Contributions

- Increase the accuracy. Exactly describe the physical constraints (in particular, min-up/-down restrictions for each turbine) and transition costs between different configurations.
- Increase the flexibility by tracking the status of each turbine.
- A better computational performance.

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Q&A

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Thank you!