Robust Risk-Constrained Unit Commitment With Large-Scale Wind Generation : An Adjustable Uncertainty Set Approach

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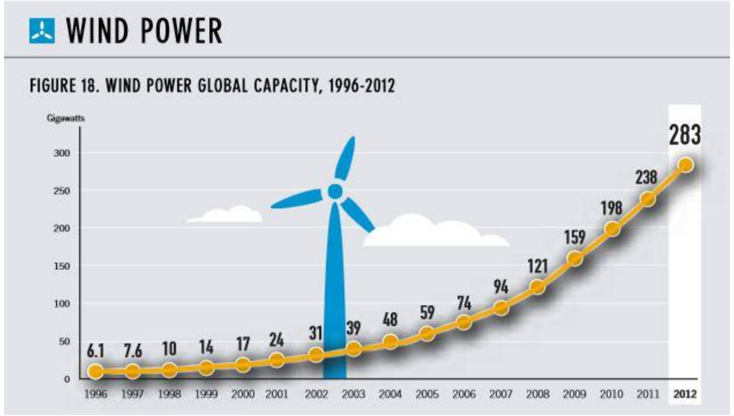
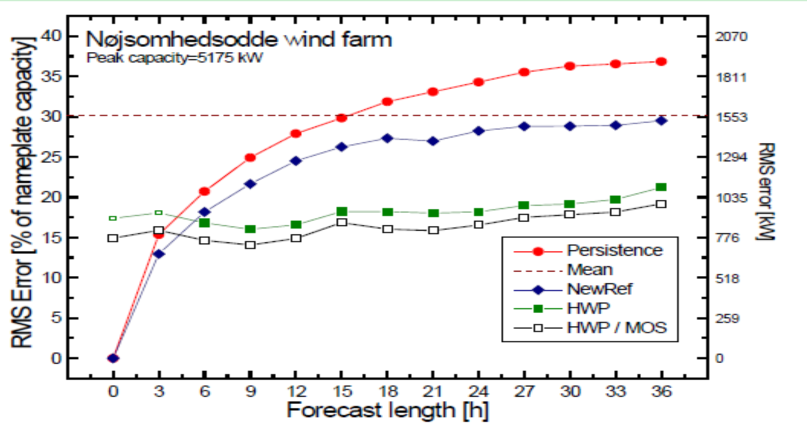
Outline

- Background, Concepts & Related Works
- Mathematical Formulation
- Solution Methodology
- An Illustrative Example
- Summary

Background: Integration of Wind Power

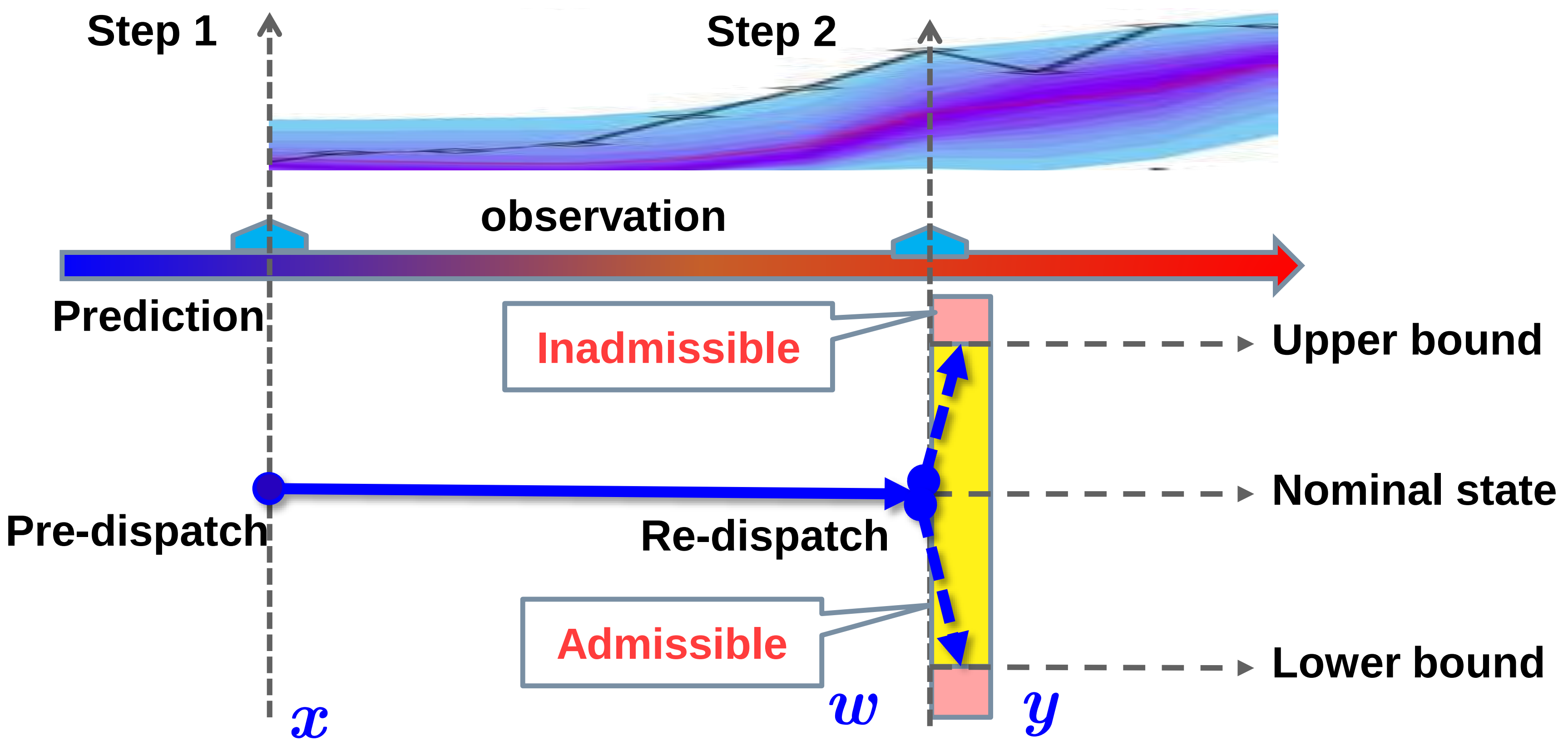
- The share of volatile and uncertain wind generation increase
  - Global capacity: 337 GW, 5.8% (Jun. 2014)
  - Potential: 95 TW, supply for 1.5 Earth (2000 h/year)





- Wind Generation
  - **Uncertainty and variability**: day-ahead forecast is poor: 15~20% error
  - **Non-dispatchability** : not being treated as dispatchable resources in most markets

Background: Two-Step Dispatch Framework



x : pre-dispatch decision (UC, ED) w : uncertainties (wind generation)

y : re-dispatch decision (reserve)

Related Works: Robust Approach

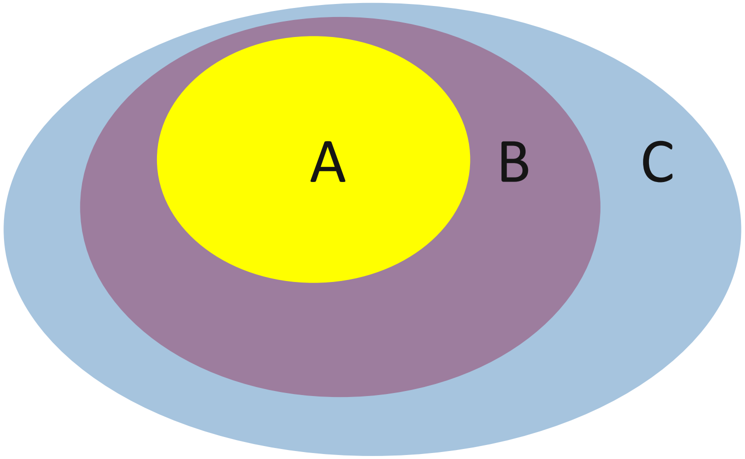
- Robust approach to handle uncertainty
  - A solution is feasible for all possible values in an uncertainty set
- Two-stage fully adaptive
  - Classic model: Jiang, Wang, and Guan (2012), Zhao and Zeng (2012) Bertsimas, Litvinov, and Sun, et al (2013)
  - Unified stochastic and robust: Zhao and Guan (2013)
  - Hybrid stochastic/interval: Dvorkin, Pandzic, and Ortega-Vazquez, et al (2015)
  - Dynamic uncertainty set: Lorca and Sun (2015)
  - Multi-band uncertainty set: Hu and Wu (2016), Dai, Wu, and Wu (2016)
- Multi-stage linear decision rule: Lorca, Sun, and Litvinov et al (2016)
- The uncertainty sets in all those works are prescribed
  - confidence intervals, budgets, polyhedral sets, ellipsoidal sets, and so on

Related Works: Uncertainty Sets

- Abstract formulation for two-stage robust models

$$\min_{x \in X} c^T x + \max_{w \in W} \min_{y \in Y(x,w)} d^T w + f^T y$$

W is the prescribed uncertainty set.

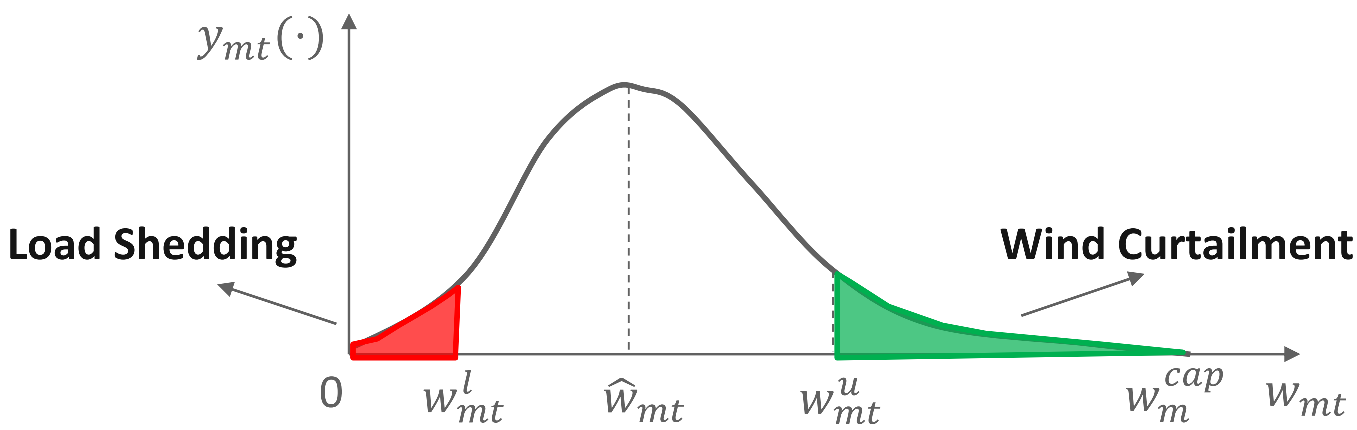


- **A** : prescribed uncertainty W, ***uncertainty set***
- **B** : uncertainty x can cover, ***admissibility region***
- **C** : possible region of uncertainty

- The uncertainty set determines the amount of committed flexibility resources
  - Operational cost
  - Operational risk

Dynamic Risk-Based Uncertainty Set

- Drawback of prescribed uncertainty sets
  - Disconnection between benefits and costs
    - Unnecessarily large uncertainty sets incur excessive flexibility resources
  - Ignorance of consequence
    - Out-of-range events might be highly costly

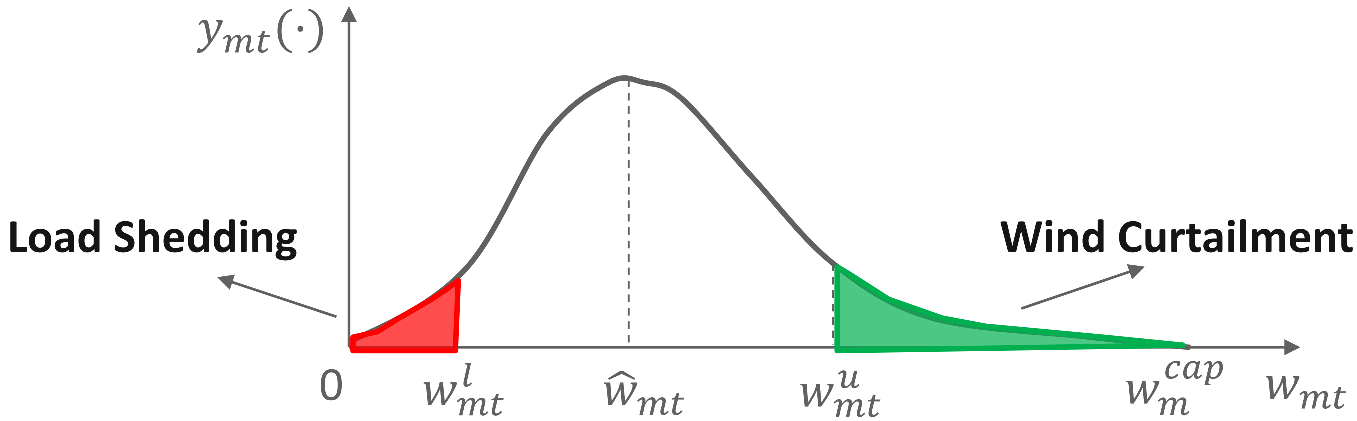


- Risk-based uncertainty set design
  - Risk = Consequence * Probability
  - Uncertainty sets are dynamically determined based on risk requirements
  - Consider both operational costs and operational risk

An Adjustable Uncertainty Set Approach

- Application settings
  - Day-ahead UCED
- Uncertainty sets
  - Bounds of random parameters: renewable generation
  - Uncertain parameters are independent
- Define risks
  - Consequences: production costs, load shedding, interruption of service, social welfare
  - Probability: probabilistic models, historical data
- Optimization model
  - Risk-constrained UCED

Definition of Operational Risks



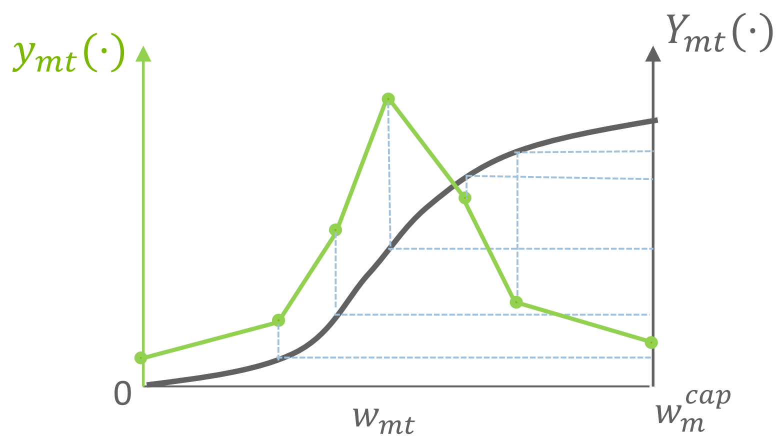
- Operational Risk Index

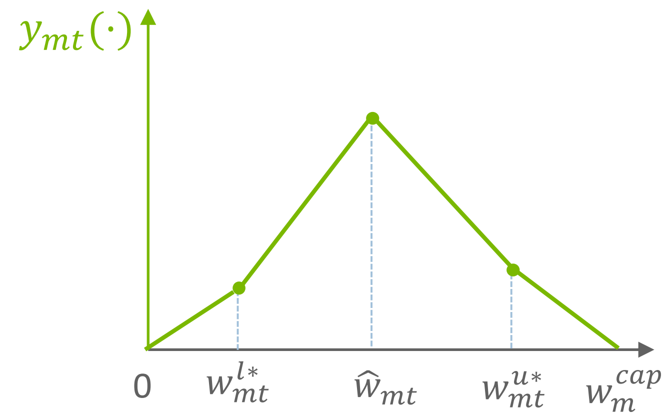
$$Risk = \sum_{t=1}^{T}\sum_{m=1}^{M}\left(g_t^n \int_{w_{mt}^u}^{w_m^{cap}} \left(w_{mt} - w_{mt}^u \right) + g_t^p \int_0^{w_{mt}^l} \left(w_{mt}^l - w_{mt} \right) \right) y_{mt}(w_{mt})\, dw_{mt}$$

  - y_{mt} is probability density function (PDF) of w_{mt}
  - g_t^n, g_t^p are cost coefficient of WC and LS, obtained from contract or estimation

Linearization of PDF

- Probability models are known
  - Cumulative density function is known
  - Piecewise linearization
- Probability models are unknown
  - First & second order moments are known
  - Construct approximations
  - Then linearize





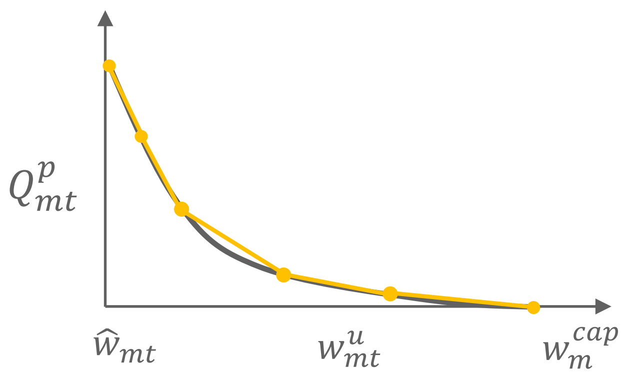
Piecewise Linear PDF of w_{mt}

$$Risk = \sum_{t=1}^{T}\sum_{m=1}^{M}\left(g_t^n \left(\begin{array}{l} \sum_{s^p=0}^{S^p} \int_{Y_{mt}^{-1}(\alpha_{s^p+1}^p)}^{Y_{mt}^{-1}(\alpha_{s^p}^p)} \left(c_{mts^p}^p w_{mt} + d_{mts^p}^p \right) + \\ \int_{w_{mt}^u - \hat{w}_{mt}}^{Y_{mt}^{-1}(\alpha_{S^p+1}^p)} \left(c_{mt(S^p+1)}^p w_{mt} + d_{mt(S^p+1)}^p \right) \end{array} \right) \left(w_{mt} - w_{mt}^u \right) + g_t^p \left(\begin{array}{l} \sum_{s^n=0}^{S^n} \int_{Y_{mt}^{-1}(\alpha_{s^n}^n)}^{Y_{mt}^{-1}(\alpha_{s^n+1}^n)} \left(c_{mts^n}^n w_{mt} + d_{mts^n}^n \right) + \\ \int_{Y_{mt}^{-1}(\alpha_{S^n+1}^n)}^{w_{mt}^l - \hat{w}_{mt}} \left(c_{mt(S^n+1)}^n w_{mt} + d_{mt(S^n+1)}^n \right) \end{array} \right) \left(w_{mt}^l - w_{mt} \right) \right) \mathrm{d}\delta_{mt}$$

Calculation of Operational Risk

$$Risk = \sum_{t=1}^{T}\sum_{m=1}^{M}\left(g_t^n \begin{pmatrix} \displaystyle\sum_{s^p=0}^{S^p} \int_{Y_{mt}^{-1}(\alpha^p_{s^p+1})}^{Y_{mt}^{-1}(\alpha^p_{s^p})} \left(c^p_{mts^p} w_{mt} + d^p_{mts^p}\right) + \\ \displaystyle\int_{w^u_{mt}-\hat{w}_{mt}}^{Y_{mt}^{-1}(\alpha^p_{S^p+1})} \left(c^p_{mt(S^p+1)} w_{mt} + d^p_{mt(S^p+1)}\right) \end{pmatrix} \left(w_{mt} - w^u_{mt}\right) + g_t^p \begin{pmatrix} \displaystyle\sum_{s^n=0}^{S^n} \int_{Y_{mt}^{-1}(\alpha^n_{s^n})}^{Y_{mt}^{-1}(\alpha^n_{s^n+1})} \left(c^n_{mts^n} w_{mt} + d^n_{mts^n}\right) + \\ \displaystyle\int_{Y_{mt}^{-1}(\alpha^n_{S^n+1})}^{w^l_{mt}-\hat{w}_{mt}} \left(c^n_{mt(S^n+1)} w_{mt} + d^n_{mt(S^n+1)}\right) \end{pmatrix} \left(w^l_{mt} - w_{mt}\right) \right) d\delta_{mt}$$

- Observation of the convexity of risk terms
  - Pdf of the uncertainty parameter is unimodal
  - Each term in the integral is strictly monotonic, derivable, and convex on the domain
  - Then the integral is a convex function on the domain



$$Risk = \min_{Q^p_{mt}, Q^n_{mt}} \sum_{t=1}^{T}\sum_{m=1}^{M}\left(Q^p_{mt} + Q^n_{mt}\right)$$

s.t.

$$Q^p_{mt} \geq a^p_{mtsz} w^u_{mt} + b^p_{mtsz} \quad \forall m,t,s,z.$$

$$Q^n_{mt} \geq a^n_{mtsz} w^l_{mt} + b^n_{mtsz} \quad \forall m,t,s,z.$$

Linear Convex Model

Mathematical Formulation

- **First stage (cost minimization)**

$$\min_{z,u,\hat{p},w^u,w^l,Q^p,Q^n} \sum_{t=1}^{T}\sum_{g=1}^{G}\left(S_g z_{gt} + c_g u_{gt} + C_g(\hat{p}_{gt})\right)$$

UC

$$\text{s.t.} \quad -u_{g(t-1)} + u_{gt} - u_{gk} \le 0, \quad \forall g,t,k = t,\cdots,t+T_g^{on}-1.$$

$$u_{g(t-1)} - u_{gt} + u_{gk} \le 1, \quad \forall g,t,k = t,\cdots,t+T_g^{off}-1.$$

$$-u_{g(t-1)} + u_{gt} - z_{gt} \le 0, \quad \forall g,\forall t$$

Capacity

$$u_{gt}P_{\min}^g \le \hat{p}_{gt} \le u_{gt}P_{\max}^g \quad \forall g,\forall t$$

Ramping

$$\hat{p}_{gt} - \hat{p}_{g(t+1)} \le u_{g(t+1)}R_-^g + (1-u_{g(t+1)})P_{\max}^g \quad \forall g, \ \forall t$$

$$\hat{p}_{g(t+1)} - \hat{p}_{gt} \le u_{gt}R_+^g + (1-u_{gt})P_{\max}^g \quad \forall g, \ \forall t$$

Power balance

$$\sum_{g=1}^{G}\hat{p}_{gt} + \sum_{m=1}^{M}\hat{w}_{mt} = \sum_{j=1}^{J}D_{jt} \quad \forall t$$

Transmission

$$-F_l \le \sum_{g=1}^{G}\pi_{gt}\hat{p}_{gt} + \sum_{m=1}^{M}\pi_{mt}\hat{w}_{mt} - \sum_{j=1}^{J}\pi_{jt}D_{jt} \le F_l \quad \forall t$$

$$\min_{Q_{mt}^p,Q_{mt}^n} \sum_{t=1}^{T}\sum_{m=1}^{M}\left(Q_{mt}^p + Q_{mt}^n\right) \le Risk_{\text{dh}}$$

$$Q_{mt}^p \ge a_{mtsz}^p w_{mt}^u + b_{mtsz}^p \quad \forall m,\forall t, s = 0,1\ldots,S, z = 0,1,\ldots,Z-1.$$

$$Q_{mt}^n \ge a_{mtsz}^n w_{mt}^l + b_{mtsz}^n \quad \forall m,\forall t, s = 0,1\ldots,S, z = 0,1,\ldots,Z-1.$$

$$0 \le w_{mt}^l \le \hat{w}_{mt} \quad \forall m, \forall t$$

$$\hat{w}_{mt} \le w_{mt}^u \le w_m^{\max} \quad \forall m, \forall t$$

$$u_{gt}, w_{mt}^u, w_{mt}^l \in \Omega$$

- **Operational risk constraints**
- **Variable uncertainty set**

- **Second stage (feasibility check)**

$$\Omega := \left\{ \max_{v^u,v^l} \min_{p,\Delta w,\Delta D} \sum_{t=1}^{T}\left(\sum_{m=1}^{M}\Delta w_{mt} + \sum_{j=1}^{J}\Delta D_{jt}\right) = 0\right.$$

$$\text{s.t.} \quad u_{gt}P_{\min}^g \le p_{gt} \le u_{gt}P_{\max}^g \quad \forall g,\forall t$$

$$p_{gt} - p_{g(t+1)} \le u_{g(t+1)}R_-^g + (1-u_{g(t+1)})P_{\max}^g \quad \forall g,\forall t$$

$$p_{g(t+1)} - p_{gt} \le u_{gt}R_+^g + (1-u_{gt})P_{\max}^g \quad \forall g,\forall t$$

$$\sum_{g=1}^{G}p_{gt} + \sum_{m=1}^{M}\left(w_{mt} - \Delta w_{mt}\right) = \sum_{j=1}^{J}\left(D_{jt} - \Delta D_{jt}\right)$$

$$0 \le \Delta D_{jt} \le D_{jt} \quad \forall j, \forall t$$

$$0 \le \Delta w_{mt} \le w_{mt} \quad \forall m,\forall t$$

$$-F_l \le \sum_{g=1}^{G}\pi_{gt}p_{gt} + \sum_{m=1}^{M}\pi_{mt}\left(w_{mt} - \Delta w_{mt}\right) - \cdots$$

$$-\sum_{j=1}^{J}\pi_{jt}\left(D_{jt} - \Delta D_{jt}\right) \le F_l \quad \forall t$$

$$w_{mt} = (w_{mt}^u - \hat{w}_{mt})v_{mt}^u + (w_{mt}^l - \hat{w}_{mt})v_{mt}^l + \hat{w}_{mt}$$

$$\sum_{t=1}^{T}\left(v_{mt}^u + v_{mt}^l\right) \le \Gamma^T \quad \forall m$$

$$\sum_{m=1}^{M}\left(v_{mt}^u + v_{mt}^l\right) \le \Gamma^S \quad \forall t$$

$$v_{mt}^u + v_{mt}^l \le 1 \quad \forall m,\forall t$$

$$\left. v_{mt}^u, v_{mt}^l \in \{0,1\} \right\}$$

Mathematical Formulation-Cont'd

- An equivalent formulation

$$\min_{z,u,\hat{p},w^u,w^l,Q^p,Q^n} \sum_{t=1}^{T}\sum_{g=1}^{G}\left(S_g z_{gt} + c_g u_{gt} + C_g(\hat{p}_{gt})\right)$$

$$s.t.\ \min_{Q^p_{mt},Q^n_{mt}} \sum_{t=1}^{T}\sum_{m=1}^{M}(Q^p_{mt} + Q^n_{mt}) \le Risk_{\text{dh}}$$

Other constraints

$\Rightarrow$

$$\min_{z,u,\hat{p},w^u,w^l,Q^p,Q^n} \sum_{t=1}^{T}\sum_{g=1}^{G}\left(S_g z_{gt} + c_g u_{gt} + C_g(\hat{p}_{gt})\right)$$

$$s.t.\ \sum_{t=1}^{T}\sum_{m=1}^{M}(Q^p_{mt} + Q^n_{mt}) \le Risk_{\text{dh}}$$

Other constraints

- Minimize risk terms

$$\min_{z,u,\hat{p},w^u,w^l,Q^p,Q^n} \sum_{t=1}^{T}\sum_{g=1}^{G}\left(S_g z_{gt} + c_g u_{gt} + C_g(\hat{p}_{gt})\right) + K\cdot\sum_{t=1}^{T}\sum_{m=1}^{M}(Q^p_{mt} + Q^n_{mt})$$

$$s.t.\ \sum_{t=1}^{T}\sum_{m=1}^{M}(Q^p_{mt} + Q^n_{mt}) \le Risk_{\text{dh}}$$

Other constraints

Mathematical Formulation-Cont'd

- Comparison with existing models
  - The uncertainty set is a design variable
  - Operational risks are considered and controlled

- Compact Model

$$\min_{x\in X, z\in Z}(c^T x + b^T z) + \max_{w\in W(z)}\min_{y\in Y(x,w)}(d^T w + f^T y)$$

x: pre-dispatch variables (UC, ED)

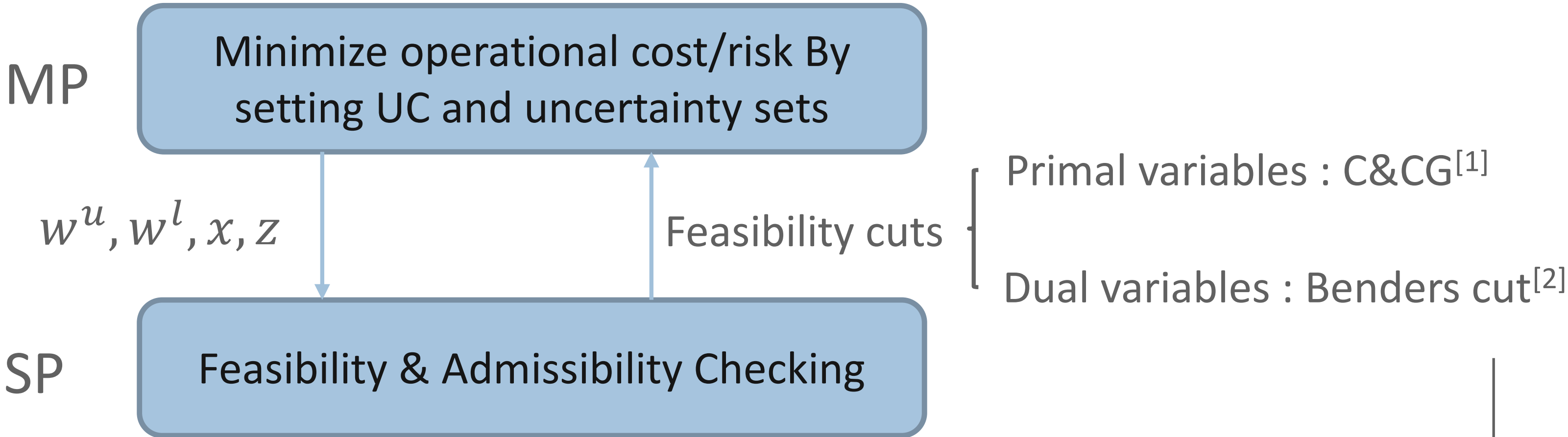
z : boundary of uncertainty set

w : realization of uncertainty

y: re-dispatch variable (reserve)

A standard two-stage robust model

Solution Methodologies



- Generalized bi-level treatment
  - KKT condition based method[3]
  - Strong duality theory based method[4]
  - C&CG[5]
- Max-min problem treatment
  - <u>Dual treatment & Big-M method</u>[2]
  - Dual treatment & Outer approximation (OA)[2]

MILP

[1] B. Zeng, and L. Zhao, "Solving two-stage robust optimization problems using a column-and-constraint generation method," Oper. Res. Lett.
[2] W. Wei, F. Liu, and S. Mei, "Two-level unit commitment and reserve level adjustment considering large-scale wind power integration," Int. Trans. Electr. Energ. Syst.
[3] S. J. Kazempour, A. J. Conejo, and C. Ruiz, "Strategic Generation Investment Using a Complementarity Approach," IEEE Trans. Power Syst.
[4] J. M. Arroyo, "Bilevel programming applied to power system vulnerability analysis under multiple contingencies," IET Gener. Transm. Dis
[5] Y. An, and B. Zeng, "Exploring the Modeling Capacity of Two-Stage Robust Optimization: Variants of Robust Unit Commitment Model," IEEE Trans. Power Syst.

Solution Methodology-Cont'd

- Solving the max-min problem
  - Bilinear terms: production of binaries and unbounded continuous variables
  - Linearization: auxiliary variables and Big-M constraints are introduced
  - Solved as an MILP
- Acceleration
  - Auxiliary binary variables and Big-M constraints reduction
  - Using nodal balance equation to replace the whole grid equation

Table Computational scale comparison

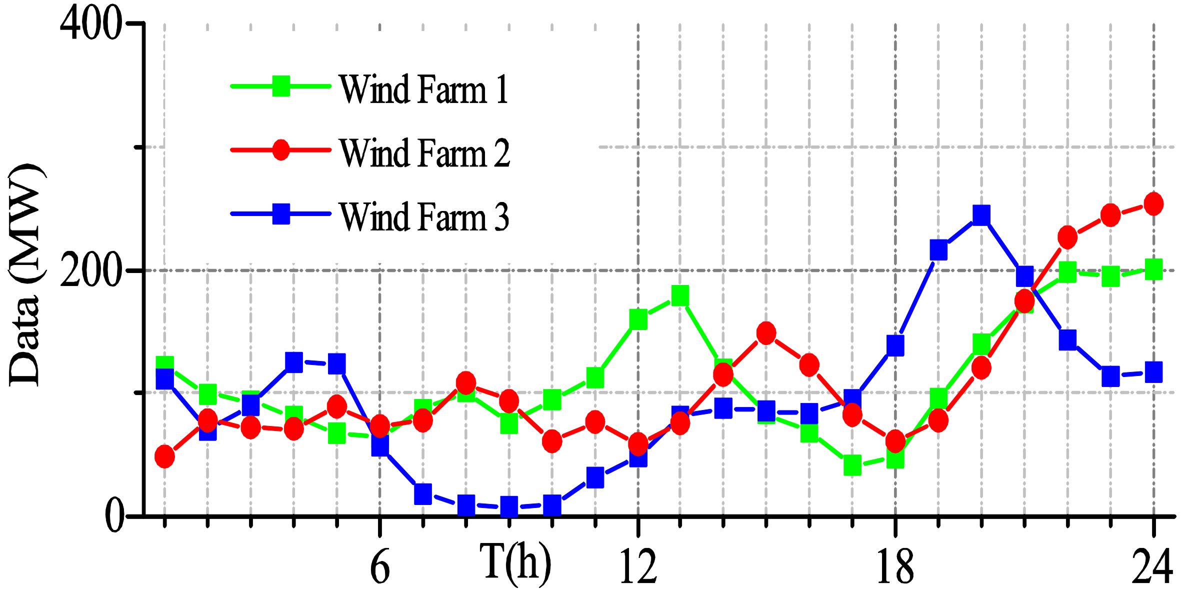
	Model 1-whole	Model 2-nodal
Binary Variables	$\mathbf{v}$: $2MT$	$\mathbf{v}$: $2MT$
Continuous Variables	$\boldsymbol{\lambda}$: $(3G+2L+2J+2M)T$	$\boldsymbol{\eta}$: $(3G+2L+2J+2M+3N+1)T$
Auxiliary Variable	$\boldsymbol{\gamma}$: $4(L+1)MT$	$\boldsymbol{\mu}$: $4MT$
Regular Constraints	$\boldsymbol{\lambda}, \mathbf{v}$: $(G+M+L)T$	$\boldsymbol{\eta}, \mathbf{v}$: $(G+M+L+N)T$
Regular Constraints	$\boldsymbol{\lambda}, \boldsymbol{\gamma}$: $8(L+1)MT$	$\boldsymbol{\lambda}, \boldsymbol{\mu}$: $8MT$
Big-M Constraints	$\boldsymbol{\lambda}, \mathbf{v}, \boldsymbol{\gamma}$: $8(L+1)MT$	$\boldsymbol{\eta}, \mathbf{v}, \boldsymbol{\mu}$: $8MT$

T : periods
M: wind farm
G : generator
J : load
N : node
L : line

Computational benefit increases with the number of wind farms

An Illustrative Example

- The modified IEEE- 118 test system[1]
  - ✓ 54 generators, 186 transmission lines
  - ✓ 3 wind farms
  - ✓ 0.1% MILP gap



[1] [Online] available: http://motor.ece.iit.edu/data/JEAS_IEEE118.doc

An Illustrative Example-Cont'd

- Comparison with Other UC models

  - ✓ DUC, deterministic UC, fixed spinning reserve rate 10%
  - ✓ SUC, stochastic UC, scenarios: 200->20
  - ✓ RUC, robust UC, boundary of uncertainty set: 95% confidence level
  - ✓ RRUC, robust risk-constrained UC, $Risk_{dh}$ is the same with RUC

Table Cost and risk under different UC models

	Total Cost ($)	UC Cost ($)	ED Cost ($)	Risk($)
DUC	1.287×10^6	1.90×10^4	1.262×10^6	2.67×10^4
SUC	1.304×10^6	2.79×10^4	1.276×10^6	9.86×10^3
RUC	1.312×10^6	3.29×10^4	1.283×10^6	7.23×10^3
RRUC	1.307×10^6	2.87×10^4	1.278×10^6	6.64×10^3

- ➢ RRUC outperforms RUC in both operational cost and risk
- ➢ The sum of operational cost and risk of RRUC is the lowest among four UC models

An Illustrative Example-Cont'd

- Comparison with Other UC models-Cont'd
  - ✓ Rare events: wind generation scenario being partly or fully out of the prescribed uncertainty set in RUC
  - ✓ 10,000 rare events

WGC : wind generation curtailment
LS : load shedding

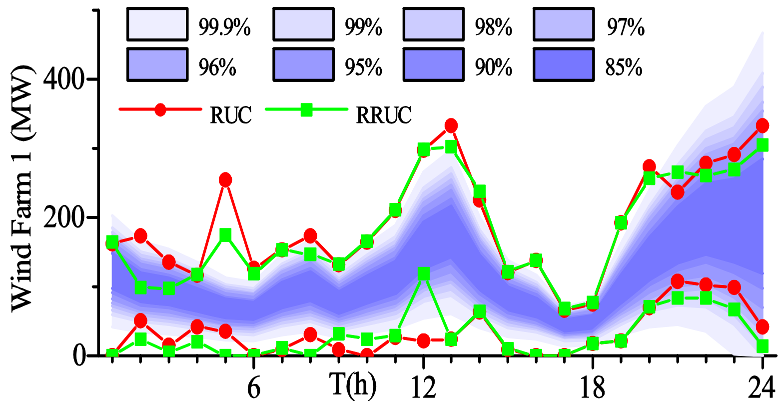
Table Operational loss of different UCs under rare event

	Average Operational Loss ($)		
	Total	WGC	LS
DUC	1.017×10^{6}	2.172×10^{5}	8.010×10^{5}
SUC	5.094×10^{5}	1.365×10^{5}	3.720×10^{5}
RUC	4.050×10^{5}	1.209×10^{5}	2.841×10^{5}
RRUC	3.357×10^{5}	1.317×10^{5}	2.043×10^{5}

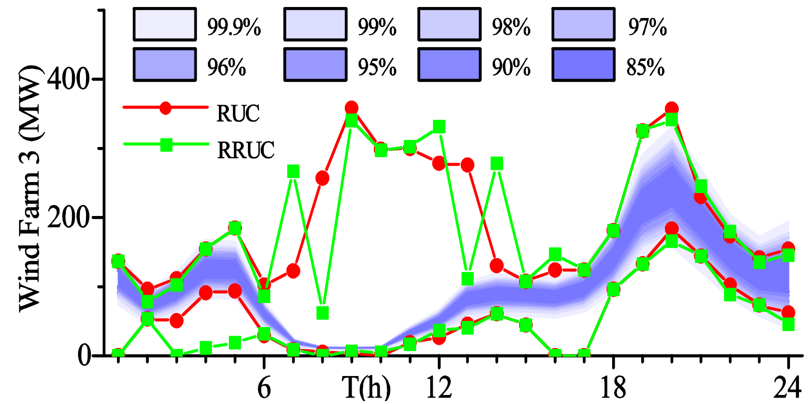
- ➢ The average of operational loss of RRUC is the lowest
- ➢ More realistic than operational risk

An Illustrative Example-Cont'd

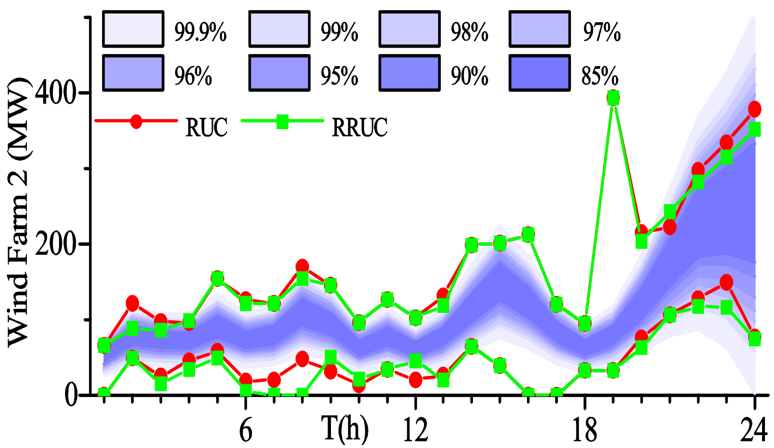
- Uncertainty Set and Admissibility Region



Wind Farm 1



Wind Farm 3



Wind Farm 2

- ✓ *W:* uncertainty set, *R*: admissibility region
- ✓ $R^{RRUC} \neq R^{RUC}$
- ✓ $W^{RUC} \subseteq R^{RUC}$
- ✓ $W^{RRUC} = R^{RRUC}$

An Illustrative Example-Cont'd

- Computational Efficiency
  - ✓ A1: whole network balance equation
  - ✓ A2: nodal balance equation

Table Computational efficiency under different cases and algorithms

		Total (s)	MP (s)	SP (s)	Iteration
A1	Γ^{T}=8	9775	4614	5161	12
	Γ^{T}=16	3447	1813	1634	7
	Γ^{T}=24	1365	602	763	4
A2	Γ^{T}=8	5399	4587	812	12
	Γ^{T}=16	2183	1811	372	7
	Γ^{T}=24	691	590	101	4

- ➢ The solution time for SP has a 500% reduction on average
- ➢ The solution time increases as the uncertainty budget decreases

An Illustrative Example-Cont'd

- Impacts of Uncertainty Budgets

Table Simulation results under different uncertainty budget.

		Total Cost ($)	Risk ($)
$\Gamma^S=1$	$\Gamma^T=8$	1.291×10^6	7.01×10^3
	$\Gamma^T=16$	1.302×10^6	6.39×10^3
	$\Gamma^T=24$	1.316×10^6	6.92×10^3
$\Gamma^S=2$	$\Gamma^T=8$	1.307×10^6	6.64×10^3
	$\Gamma^T=16$	1.320×10^6	7.12×10^3
	$\Gamma^T=24$	1.335×10^6	6.58×10^3
$\Gamma^S=3$	$\Gamma^T=8$	1.337×10^6	6.77×10^3
	$\Gamma^T=16$	1.354×10^6	6.61×10^3
	$\Gamma^T=24$	1.362×10^6	7.19×10^3

- As Γ^T and Γ^S increase, total operational cost increases
- As Γ^T and Γ^S increase, operational risk doesn't have a fixed pattern

An Illustrative Example-Cont'd

- Impacts of risk threshold

✓ The admissibility regions (AR) under different risk thresholds are different

✓ The AR with larger risk threshold is not the subset of the AR with a lower risk level

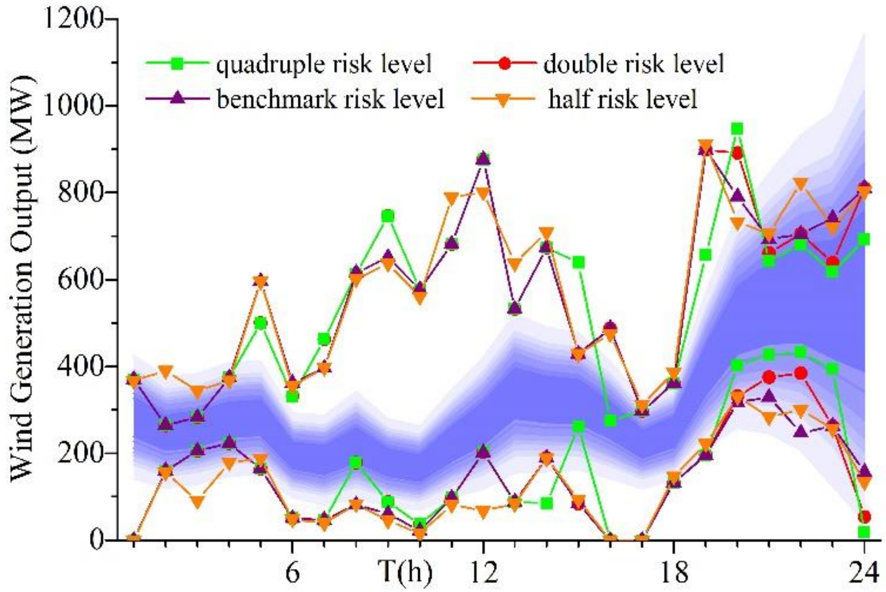


Fig. Optimal wind generation uncertainty set boundary under different risk level.

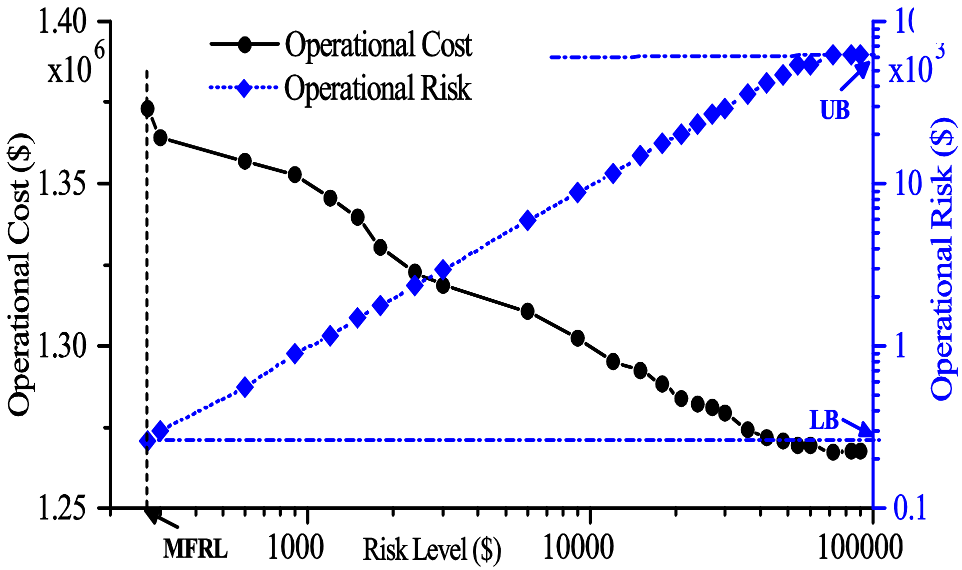


Fig. Operational cost of RRUC under different operational risk levels.

✓ Operational risks are not strictly linear with risk level

✓ The operational risks have upper and lower bounds

✓ The risk threshold has a lower bound

An Illustrative Example-Cont'd

- Impact of the Number of Wind Farms

  ✓ Temporal uncertainty budget is fixed, $\Gamma^T = 24$

Table Computational performance under different numbers of wind farm

Wind Farm	Budget	Total (s)	MP (s)	SP(s)	Iteration
6	Γ^S=4	748	601	147	4
9	Γ^S=6	876	627	249	4
12	Γ^S=8	1736	891	845	5

➢ The solution time increases rapidly, especially the solution time of SP

➢ The numbers of big-M constraints and auxiliary variables are proportional to the number of wind farms

Summary

- Robust Risk-Constrained Unit Commitment
  - Variable uncertainty set
  - Operational risk levels can be controlled
  - Outperforms RUC in both operational costs and risks
  - The computational time reduction by formulation selection can also be applied to RUC and other robust models.

- Possible future research
  - Other forms of uncertainty sets besides upper and lower bounds
  - More efficient computational methods

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Thanks!

Comments?

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