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Convex Hull Pricing

Rigorous Analysis and Implementation Challenges

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Motivation

- No one (including us) completely understands Convex Hull Pricing
- However, participants commonly suggest that ISO New England switch to Convex Hull Pricing (or MISO's ELMP method)
- Poorly understood pricing methods can have unexpected consequences

Goals of presentation

- Provide an overview of Convex Hull Pricing
- Clearly describe important Convex Hull Pricing properties
- Discuss foreseeable implementation challenges

- **This presentation is meant to call attention to the implications of Convex Hull Pricing**

ISO processes

- Three important ISO processes
  - Commitment
    What is the most efficient combination of units?
  - Dispatch
    What is the most efficient clearing of online units?
  - **Pricing**
    What uniform prices are appropriate given the cleared bids?

Pricing principles

- Pricing methods such as marginal cost pricing (i.e., prices based on the marginal cost of load) may not be satisfactory because of "nonconvexities" such as
  - Fixed costs
  - Minimum output levels
  - MW-dependent ramp rates
- Consequently, there is no "perfect price" (i.e., price that simultaneously satisfies every participant)
- To ensure that participants are satisfied with the market clearing, **side-payments** must be made

Side-payments

- Side-payment: a payment that is not associated with a uniform market clearing price
- Purpose: eliminate participant incentives to deviate from ISO-cleared quantities
- Types of side-payments
  - **Make-Whole Payments**
    Ensure that each participant receives at least its cleared bid-in cost
  - **Lost Opportunity Costs**
    Ensure that each participant receives its maximum possible profit given prices and its bid-in constraints
  - **Product Revenue Shortfall** *(specific to Convex Hull Pricing)*
    Ensure that ISO operations are "revenue adequate" for each system constraint/product

Convex Hull Pricing

- Convex Hull Pricing has *one and only one* purpose:

Identify uniform prices that

minimize certain side-payments

This is **NOT** "uplift" as traditionally defined!

Formulation of Convex Hull Pricing

- The Commitment problem can generally be formulated as

$$\min_{c,x} \quad \sum_t \sum_i c_{it}$$

Total production cost

$$\text{s.t.} \quad \sum_i A_{it} x_{it} \geq b_t \qquad \forall t$$

System-wide constraints
(e.g., reserves, transmission)

$$(c_i, x_i) \in \bigvee_{j \in J_i} X_i^j \qquad \forall i.$$

Private constraints
(e.g., output limits, ramping)

Formulation of Convex Hull Pricing

- In the Commitment problem,
  - the objective function is linear (nonlinear cost functions can be moved to the constraint set)
  - the system-wide constraints are linear
  - Private constraint sets are “disjunctive”
    - Each X_i^j reflects a specific commitment sequence possibility
    - Each X_i^j is assumed to be compact but not necessarily convex
    - For each i, it is assumed that $\vee_{j \in J_i} X_i^j \neq \emptyset$

The Convex Hull Pricing problem

- The corresponding [primal] Convex Hull Pricing problem is

$$\min_{c,x} \quad \sum_t \sum_i c_{it}$$

$$\text{s.t.} \quad \sum_i \mathrm{A}_{it} x_{it} \geq \mathrm{b}_t \qquad (\lambda_t) \quad \forall t$$

$$(c_i, x_i) \in \mathrm{conv}(\vee_{j \in \mathrm{J}_i} \mathrm{X}_i^j) \qquad \forall i.$$

Convex hull

- The initial work on Convex Hull Pricing used a Lagrangian dual formulation

Basic observations

- Locational Marginal Prices (LMPs) are derived from the optimal Lagrange multipliers λ^* of the system-wide constraints
- Convex Hull Pricing is based on the Commitment problem so it is inherently multi-interval for electricity markets
- Explicit convex hull formulations are required

Properties

- Convex Hull Pricing has several interesting properties
- Three important properties are presented here

Property 1. Side-payment minimization

- Convex Hull Pricing minimizes certain side-payments over its time horizon
- Relevant side-payments
  - **Lost Opportunity Costs (LOCs)**
  - **Product Revenue Shortfall (upcoming Property 2)**
  - Make-whole payments are NOT considered!
- Minimized side-payments ≠ Zero side-payments

Property 1. Proof

- Assume that Slater's condition holds for the Convex Hull Pricing problem
- The Lagrangian dual problem obtained by relaxing the system-wide constraints is

$$\max_{\lambda \geq 0} \left\{ \begin{array}{lll} \min_{c,x} & \sum_t \sum_i c_{it} - \sum_t \lambda_t \left(\sum_i \mathrm{A}_{it} x_{it} - \mathrm{b}_t \right) & \\ \text{s.t.} & (c_i, x_i) \in \text{conv}\left(\bigvee_{j \in \mathrm{J}_i} \mathrm{X}_i^j \right) & \forall i \end{array} \right\}.$$

Property 1. Proof

- Rearranging and adding/subtracting terms incorporating the cleared quantity solution,

$$-\min_{\lambda \geq 0} \left\{ \begin{array}{l} \sum_i \left(\begin{array}{ll} \max_{c_i, x_i} & -\sum_t c_{it} + \sum_t \lambda_t \mathrm{A}_{it} x_{it} \\ \text{s.t.} & (c_i, x_i) \in \operatorname{conv}\left(\bigvee_{j \in \mathrm{J}_i} \mathrm{X}_i^j \right) \end{array} \right) - \left(-\sum_t \sum_i c_{it}^{\text{Cleared}} + \sum_t \sum_i \lambda_t \mathrm{A}_{it} x_{it}^{\text{Cleared}} \right) \\ \qquad + \left(\sum_t \sum_i \lambda_t \mathrm{A}_{it} x_{it}^{\text{Cleared}} - \sum_t \lambda_t \mathrm{b}_t \right) \end{array} \right\}$$

Property 1. Proof

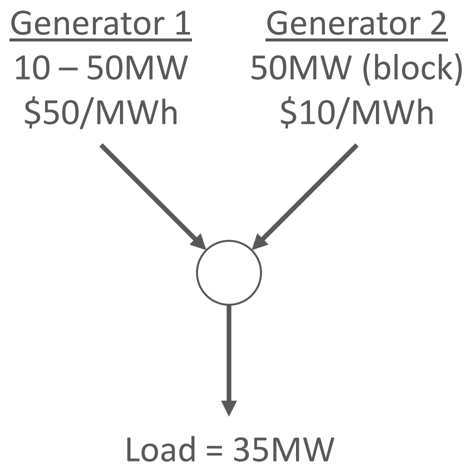
- A final simplification leads to

$$-\min_{\lambda\geq 0}\left\{\begin{array}{l}\sum_i\left[\begin{pmatrix}\max_{c_i, x_i} & -\sum_t c_{it}+\sum_t \lambda_t \mathrm{A}_{it}x_{it}\\ \text{s.t.} & (c_i, x_i)\in \bigvee_{j\in \mathrm{J}_i} \mathrm{X}_i^j\end{pmatrix}+\sum_t c_{it}^{\text{Cleared}}-\sum_t \lambda_t \mathrm{A}_{it}x_{it}^{\text{Cleared}}\right]\\ \qquad +\left(\sum_t\sum_i \lambda_t \mathrm{A}_{it}x_{it}^{\text{Cleared}}-\sum_t \lambda_t \mathrm{b}_t\right)\end{array}\right\}$$

Max possible profit — Cleared quantity profit — **Product Revenue Shortfall**

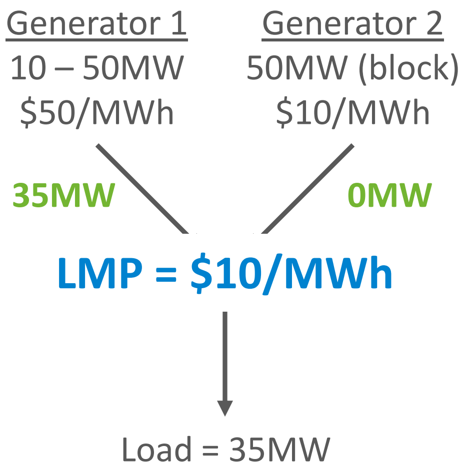
Lost Opportunity Costs

Property 1. Example



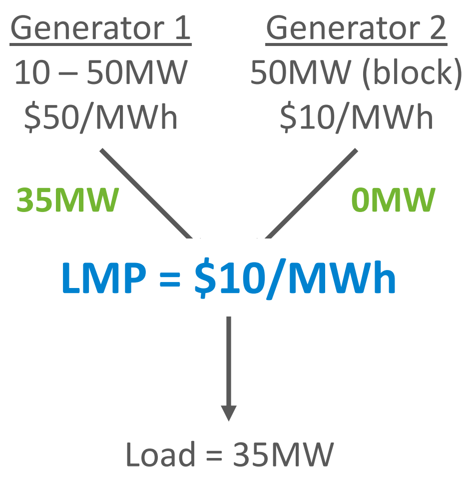
- Generator 1 is online
- Generator 2 is a fast, available unit for which a commitment decision must be made

Property 1. Example



- From the Commitment and Dispatch problems, the optimal outputs are
  - Generator 1: 35MW
  - Generator 2: 0MW
- From the Convex Hull Pricing problem, the LMP is **$10/MWh**
- What does this price mean?

Property 1. Example

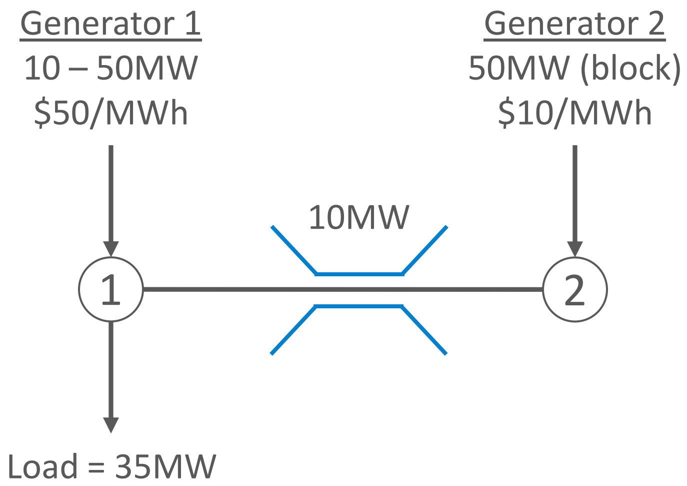


- Given the output levels and the LMP,
  - Generator 1 requires a $1000 **LOC** (max profit from 10MW)
  - Generator 2 does not require a side-payment (indifferent between online/offline)
- $1000 is the minimum side-payment
  - Easily observed via marginal LMP changes

Property 2. Positive prices for non-binding system-wide constraints

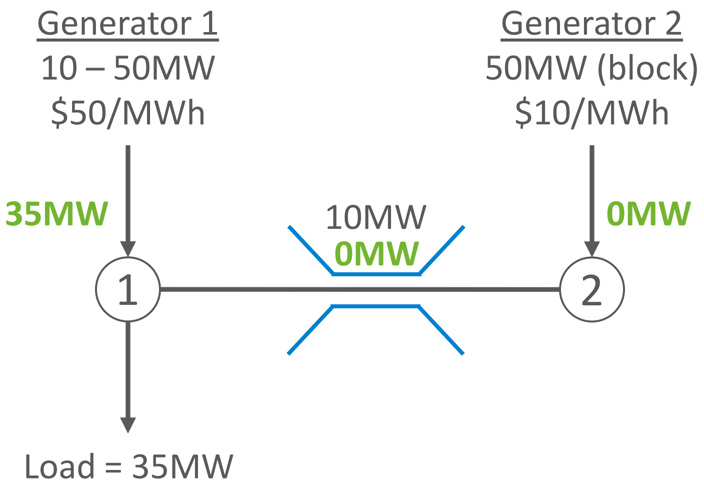
- Convex Hull Pricing can result in positive prices for non-binding system-wide constraints
  - Transmission constraints
  - Reserve constraints
- This behavior results in **Product Revenue Shortfall** (specific to Convex Hull Pricing)
- A “physical” explanation of this property is not obvious

Property 2. Example



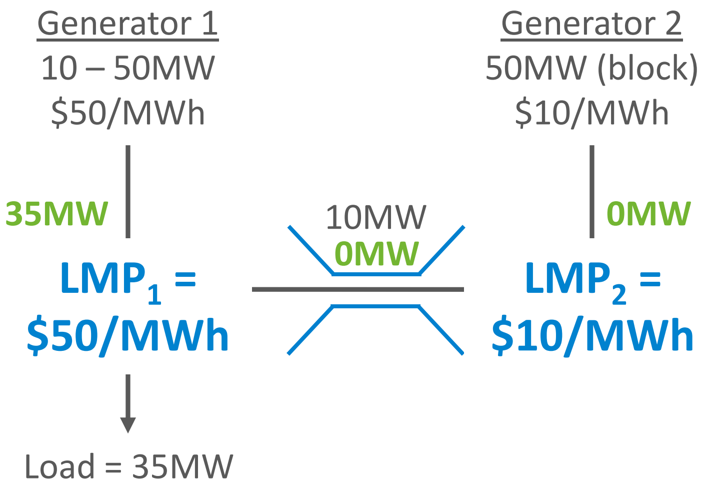
- The two units are now placed at different locations that are connected by a transmission line

Property 2. Example



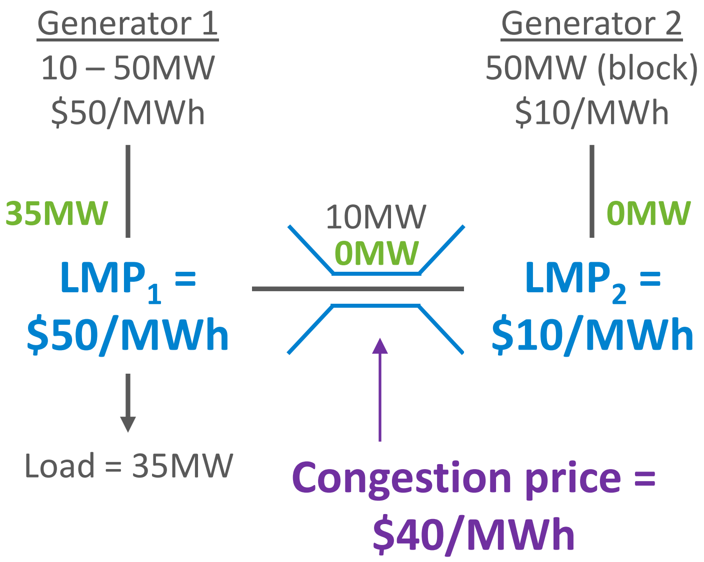
- From the Commitment and Dispatch problems, the optimal outputs are
  - Generator 1: 35MW
  - Generator 2: 0MW
- There is no flow along the transmission line

Property 2. Example



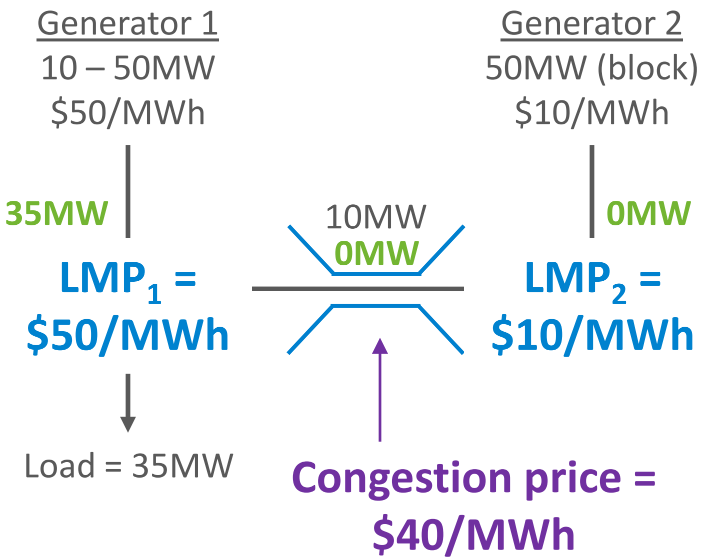
- From the Convex Hull Pricing problem, the LMPs are
  - Location 1: **$50/MWh**
  - Location 2: **$10/MWh**
- What do these prices mean?

Property 2. Example



- From the Convex Hull Pricing problem, the congestion price for the transmission line is **$40/MWh**
- What does this price mean?

Property 2. Example



- There is a revenue mismatch!
  - **$0** is collected from actual flow along the transmission line
  - If 10MW of financial transmission rights (FTRs) were sold in the FTR auction, FTR holders need **$400** more than what the ISO collects
- This $400 is the **Product Revenue Shortfall**

Property 2. Example

- Mathematically, the **Product Revenue Shortfall** term is

$$\sum_t \sum_i \lambda_t \mathrm{A}_{it} x_{it}^{\mathrm{Cleared}} - \sum_t \lambda_t \mathrm{b}_t$$

- The associated side-payment value can be shifted between **LOC** and **Product Revenue Shortfall** (allocation depends on clearing rules) but cannot be eliminated

Property 3. Convex Hull Pricing is all-or-nothing

- Convex Hull Pricing is based on a rigorous mathematical proof
- The proof will **NOT** hold if the Convex Hull Pricing problem is altered
- Therefore, Convex Hull Pricing is all-or-nothing
  - Either it is implemented in its entirety and all of its properties are realized, or
  - It is changed, loses its important properties, and can no longer rightly be called “Convex Hull Pricing”
- There is no such thing as “approximate Convex Hull Pricing”

Property review

1. Convex Hull Pricing minimizes certain side-payments (Lost Opportunity Costs + Product Revenue Shortfall) over its time horizon
2. Convex Hull Pricing can result in positive prices for non-binding system-wide constraints
3. Convex Hull Pricing is all-or-nothing

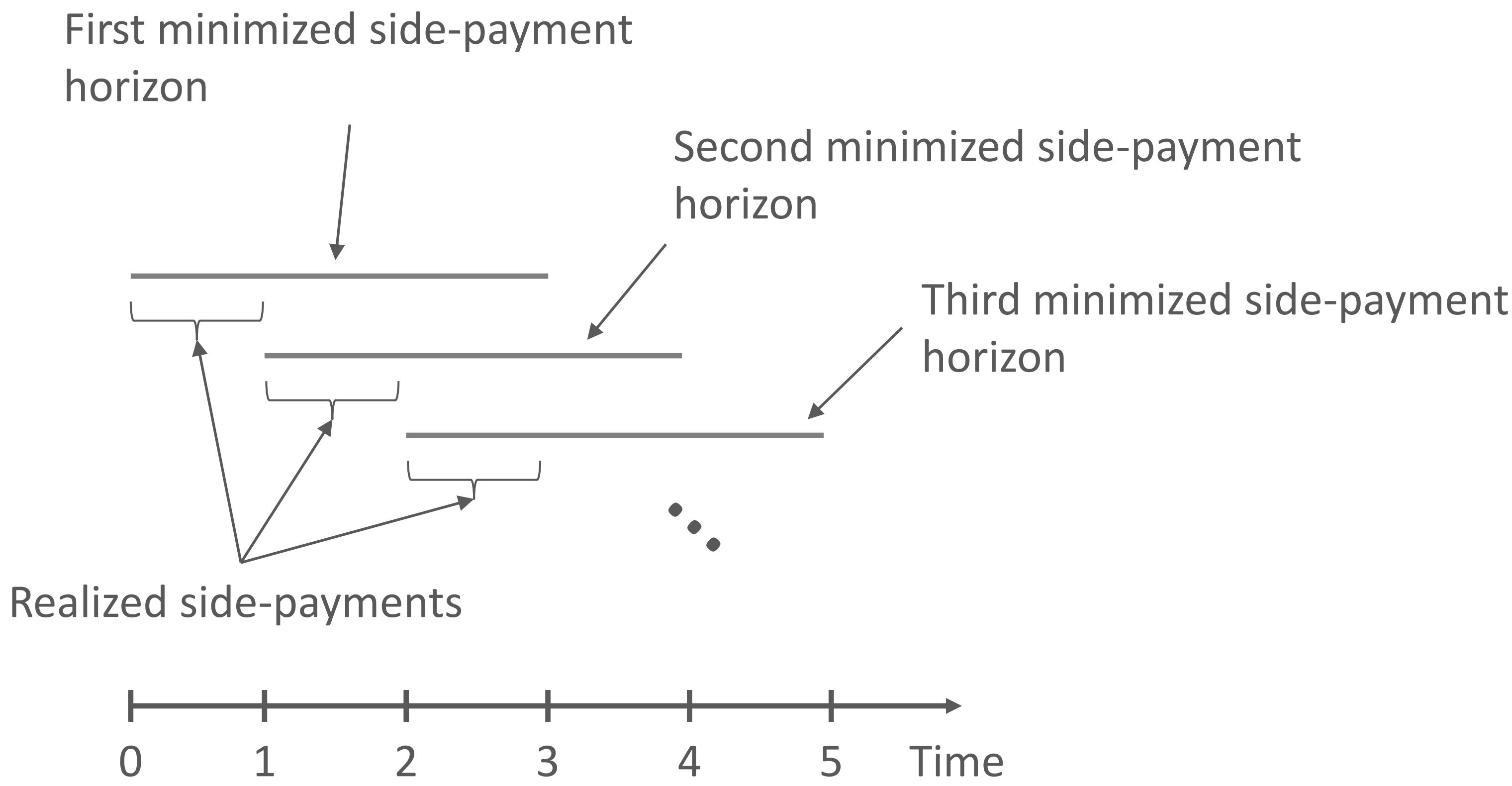
Implementation challenges

- Convex Hull Pricing has several implementation challenges
- Three foreseeable challenges are presented here

Challenge 1. Multi-interval method

- Convex Hull Pricing is inherently multi-interval for electricity markets
- In a real-time setting, a rolling time horizon implementation would be necessary
  - Prices are determined for the entire time horizon: how should this be factored into settlement?
  - Given that no forecast is perfect, the minimized side-payment cannot be realized (next slide)

Side-payment realization



Challenge 2. Product Revenue Shortfall

- Convex Hull Pricing can create a **Product Revenue Shortfall** due to Property 2 (positive prices for nonbinding system-wide constraints)
- This creates a revenue adequacy problem for the ISO
- The side-payment must be borne by participants
  - A variety of cost allocation schemes exist, but no scheme is acceptable to every participant simultaneously

Challenge 3. Computation

- Convex Hull Pricing requires explicit convex hulls (surprise!)
- If each X_i^j is polyhedral, an explicit formulation is available
- What happens if explicit convex hull formulations are not available?

Challenge review

1. Pricing and side-payment questions arise from the multi-interval nature of Convex Hull Pricing
2. Product Revenue Shortfall introduces cost allocation questions
3. Identifying convex hulls is not trivial

Conclusion

- Convex Hull Pricing
  - Is theoretically rigorous
  - Minimizes certain side-payments
    (Lost Opportunity Costs + Product Revenue Shortfall)
  - Can result in counterintuitive prices
  - Has implementation challenges
- **More research is needed before an informed judgment can be made regarding the pros and cons of Convex Hull Pricing**

References

- Market-Clearing Electricity Prices and Energy Uplift
  - Authors: Gribik, Hogan, Pope
  - http://www.hks.harvard.edu/fs/whogan/Gribik_Hogan_Pope_Price_Uplift_123107.pdf

- Convex Hull Pricing in Electricity Markets: Formulation, Analysis, and Implementation Challenges
  - Authors: Schiro, Zheng, Zhao, Litvinov
  - http://www.optimization-online.org/DB_HTML/2015/03/4830.html

Questions