Progress Report on Formulations for Convex Hull Pricing

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2020 FERC Conference

Joint work with Tong Zhang, Yanan Yu and Yonghong Chen

Overview

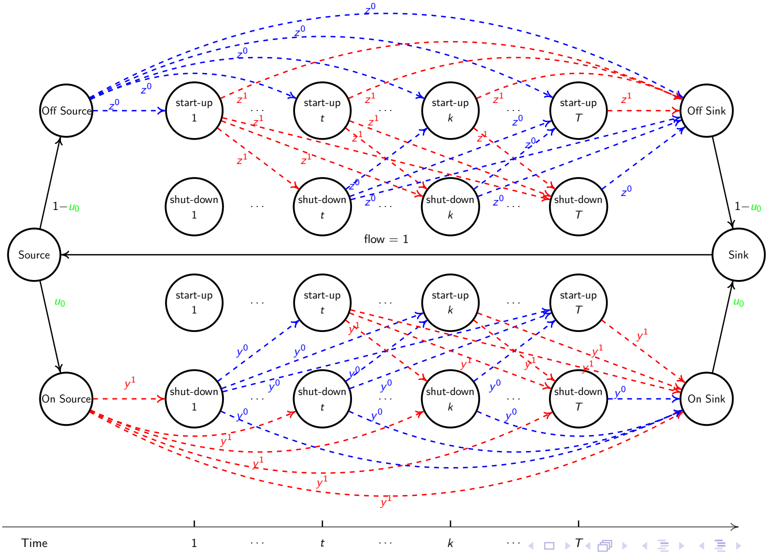
Overview

Unit Commitment (UC)

- Two types of decisions to make:
  - Which generators to turn on/off and when.
  - Generation amount of those "on" generators.
- Constraints to respect:
  - Generator characteristics (e.g., min-up/-down time, ramping rate capacity).
  - System-wise restrictions (e.g., transmission line capacity, system spinning reserve).
- Settings: system operators, market participants, minimize the total cost, maximize the revenue, cost functions, and time dependent start-up costs.
- Existing studies for extended formulations:

[Frangioni and Gentile, 2015], [Guan et al., 2018], [Knueven et al., 2018].

The Network Flow Representation for General Initial Conditions ([Zhang et al., 2020])



The Network Flow Representation for General Initial Conditions

$$
\begin{aligned}
\min \quad & \sum_{t=0}^{T-1} S(t+s_0)y_{0t}^1 + \sum_{tk \in \mathcal{TK}^1, t \neq 0} S(k-t+1)y_{tk}^1 \\
& + \sum_{kt \in \mathcal{KT}^1} S'(t-k-1)y_{kt}^0 + \sum_{tk \in \mathcal{TK}^1} \sum_{s=t}^{k} \phi_{tk}^{1s} \\
& + \sum_{t=1}^{T} S'(-s_0+t-1)z_{0t}^0 + \sum_{tk \in \mathcal{TK}^0} S(k-t+1)z_{tk}^1 \\
& + \sum_{kt \in \mathcal{KT}^0} S'(t-k-1)z_{kt}^0 + \sum_{tk \in \mathcal{TK}^0} \sum_{s=t}^{k} \phi_{tk}^{0s}
\end{aligned}
$$

The Network Flow Representation for General Initial Conditions

$$
\begin{aligned}
\text{s.t. } & \sum_{t=0}^{T} y_{0t}^{1} = u_0, \\
& \sum_{t=0}^{T} z_{0t}^{0} = 1 - u_0, \\
& -y_{0t}^{1} + \sum_{k=t+\ell+1}^{T} y_{tk}^{0} - \sum_{k=\ell+1}^{t-L+1} y_{kt}^{1} + y_{t}^{0} = 0, \forall t \in [0, T-1]_{\mathbb{Z}}, \\
& \sum_{k=\min\{t+L-1,T\}}^{T} y_{tk}^{1} - \sum_{k=0}^{t-\ell-1} y_{kt}^{0} = 0, \forall t \in [\ell+1, T]_{\mathbb{Z}}, \\
& -\sum_{k=1}^{t-L+1} z_{kt}^{1} + \sum_{k=t+\ell+1}^{T} z_{tk}^{0} + z_{t}^{0} = 0, \forall t \in [L, T-1]_{\mathbb{Z}}, \\
& -z_{0t}^{0} + \sum_{k=\min\{t+L-1,T\}}^{T} z_{tk}^{1} - \sum_{k=L}^{t-\ell-1} z_{kt}^{0} = 0, \ \forall t \in [1, T]_{\mathbb{Z}},
\end{aligned}
$$

The Network Flow Representation for General Initial Conditions

$$
\begin{aligned}
&\underline{C}y_{tk}^1 \leq q_{tk}^{1s} \leq \overline{C}y_{tk}^1, \forall s \in [t,k]_{\mathbb{Z}}, \forall tk \in \mathcal{TK}^1,\\
&q_{tk}^{1t} \leq \overline{V}y_{tk}^1, \forall tk \in \mathcal{TK}^1, t \neq 0, q_{tk}^{1k} \leq \overline{V}y_{tk}^1, \forall tk \in \mathcal{TK}^1, k \leq T-1,\\
&q_{tk}^{1(s-1)} - q_{tk}^{1s} \leq Vy_{tk}^1, \forall s \in [t+1,k]_{\mathbb{Z}}, \forall tk \in \mathcal{TK}^1,\\
&q_{tk}^{1s} - q_{tk}^{1(s-1)} \leq Vy_{tk}^1, \forall s \in [t+1,k]_{\mathbb{Z}}, \forall tk \in \mathcal{TK}^1,\\
&\phi_{tk}^{1s} - a_j q_{tk}^{1s} \geq b_j y_{tk}^1, \forall j \in [1,N]_{\mathbb{Z}}, \forall s \in [t,k]_{\mathbb{Z}}, \forall tk \in \mathcal{TK}^1,\\
&\underline{C}z_{tk}^1 \leq q_{tk}^{0s} \leq \overline{C}z_{tk}^1, \forall s \in [t,k]_{\mathbb{Z}}, \forall tk \in \mathcal{TK}^0,\\
&q_{tk}^{0t} \leq \overline{V}z_{tk}^1, \forall tk \in \mathcal{TK}^0, q_{tk}^{0k} \leq \overline{V}z_{tk}^1, \forall tk \in \mathcal{TK}^0, k \leq T-1,\\
&q_{tk}^{0(s-1)} - q_{tk}^{0s} \leq Vz_{tk}^1, \forall s \in [t+1,k]_{\mathbb{Z}}, \forall tk \in \mathcal{TK}^0,\\
&q_{tk}^{0s} - q_{tk}^{0(s-1)} \leq Vz_{tk}^1, \forall s \in [t+1,k]_{\mathbb{Z}}, \forall tk \in \mathcal{TK}^0,\\
&\phi_{tk}^{0s} - a_j q_{tk}^{0s} \geq b_j z_{tk}^1, \forall j \in [1,N]_{\mathbb{Z}}, \forall s \in [t,k]_{\mathbb{Z}}, \forall tk \in \mathcal{TK}^0,\\
&y, z \geq 0,
\end{aligned}
$$

The Network Flow Representation for General Initial Conditions

$$
\begin{aligned}
x_s &= \sum_{tk \in \mathcal{TK}^0, t \le s \le k} q_{tk}^{0s} + \sum_{tk \in \mathcal{TK}^1, t \le s \le k} q_{tk}^{1s}, \forall s \in [1, T]_{\mathbb{Z}}, \\
u_s &= \sum_{tk \in \mathcal{TK}^0, t \le s \le k} z_{tk}^{1} + \sum_{tk \in \mathcal{TK}^1, t \le s \le k} y_{tk}^{1}, \forall s \in [1, T]_{\mathbb{Z}}, \\
v_s &= z_{0s}^{0} + \sum_{kt \in \mathcal{KT}^0, t = s} z_{kt}^{0} + \sum_{kt \in \mathcal{KT}^1, t = s} y_{kt}^{0}, \forall s \in [1, T]_{\mathbb{Z}}.
\end{aligned}
$$

Remark: $\mathcal{O}(T^2)$ binary decision variables and $\mathcal{O}(T)$ constraints for the network-flow constraints part.

Overview

Convex Hull Pricing

System Optimization Model (UCED problem)

$$
\begin{aligned}
Z^*_{\text{QIP}} = \quad & \min_{x_g, y_g, u_g, \forall g \in \mathcal{G}} \quad \sum_{g \in \mathcal{G}} f_g(x_g, y_g, u_g) \\
& \text{s.t.} \quad \sum_{g \in \mathcal{G}} x_g = d, \\
& \qquad (x_g, y_g, u_g) \in \mathcal{X}_g, \forall g \in \mathcal{G}.
\end{aligned}
$$

Remark:

1. Transmission constraints can be incorporated similarly.
2. $\mathcal{X}_g$ represents the feasible region of commitment and dispatch decisions for generator g.

See [Gribik et al., 2007], [Schiro et al., 2016], [Wang et al., 2016], [Hua and Baldick, 2017], [Yu et al., 2020], among others.

Convex Hull Pricing

Profit Maximization for Each Participant

$$\begin{aligned} v_g(\lambda) = \quad & \max_{x_g, y_g, u_g, \forall g \in \mathcal{G}} && \lambda^T x_g - f_g(x_g, y_g, u_g) \\ & \text{s.t.} && (x_g, y_g, u_g) \in \mathcal{X}_g, \forall g \in \mathcal{G}. \end{aligned}$$

Uplift Payment

$$U_g(\lambda, x_g^*, y_g^*, u_g^*) = v_g(\lambda) - (\lambda^T x_g^* - f_g(x_g^*, y_g^*, u_g^*)),$$

where (x_g^*, y_g^*, u_g^*) is the optimal solution corresponding to Z_{QIP}^*.

See [Gribik et al., 2007], [Schiro et al., 2016], [Wang et al., 2016], [Hua and Baldick, 2017], [Yu et al., 2020], among others.

Convex Hull Pricing

Lagrangian Relaxation and Convex Hull Pricing

$$\max_{\lambda} \quad D(\lambda) = \sum_{g \in \mathcal{G}} \left(\min_{(x_g, y_g, u_g) \in \mathcal{X}_g} f_g(x_g, y_g, u_g) - \lambda^T x_g \right) + \lambda^T d.$$

Remark: An optimal Lagrangian multiplier is an exact convex hull price.

See [Gribik et al., 2007], [Schiro et al., 2016], [Wang et al., 2016], [Hua and Baldick, 2017], [Yu et al., 2020], among others.

Convex Hull Pricing

Theorem 1:[Hua and Baldick, 2017] Assuming $f_{g,\mathcal{X}_g}(x_g, y_g, u_g)$ is the convex envelope of $f_g(\cdot)$ taken over $\mathcal{X}_g, \forall g \in \mathcal{G}$, the optimal dual vector corresponding to constraint (5b) in the following CHP-Primal formulation is an exact convex hull price.

$$Z = \min_{x_g, y_g, u_g, \forall g \in \mathcal{G}} \sum_{g \in \mathcal{G}} f_{g,\mathcal{X}_g}(x_g, y_g, u_g) \tag{5a}$$

$$\text{s.t.} \quad \sum_{g \in \mathcal{G}} x_g = d, \tag{5b}$$

$$(x_g, y_g, u_g) \in \text{conv}(\mathcal{X}_g), \forall g \in \mathcal{G}. \tag{5c}$$

Remark: When the objective function is piecewise linear, the algorithm proposed in [Yu et al., 2020] can efficiently solve the above formulation by gradually adding the network-flow based integral formulation for some generators.

Overview

Practical Challenges

Max-up time limit: For some generators in the MISO market, there are restrictions on maximum time periods that the generator can stay online because of machine deterioration.

$$\sum_{j=t+1}^{t+\overline{L}_i} v_j \geq u_{(t+\overline{L}_i)}, \forall t \in \mathcal{T}.$$

For the extended integral formulation: We can set the maximum length of each "ON" arc in the network flow graph to accommodate this.

Practical Challenges

Flexible min-up/-down time limit: For example, participants may submit a must-run offer for a generator for hours $1-5$ and $10-24$ with the min-down time limit as 6 hours. This will force UCED to commit this generator between 6 and 9 even if it is costly. So the min-down time limit is relaxed to be 1 between hours 6 and 9 so that the generator will not be committed if it is costly.

$$\sum_{j=t-L+1}^{t} \kappa_t v_j \leq u_t, \forall t \in [L+1, |\mathcal{T}|]_{\mathbb{Z}}.$$

$$\sum_{j=t-\ell+1}^{|\mathcal{T}|} \varpi_t v_j \leq 1 - u_{t-\ell}, \forall t \in [\ell+1, |\mathcal{T}|]_{\mathbb{Z}}.$$

For the extended integral formulation: We can create the “ON” and “OFF” interval arcs in terms of arc length to accommodate these.

Practical Challenges

Time-variant parameters: In MISO, market participants are allowed to offer capacity and ramp rates varying by the hour.

For the extended integral formulation: Make the parameters dynamic instead of static in the model.

Case Studies without Transmission Constraints

Table: Test results for MISO without transmission constraints

Case	Model	Solution ($)	Uplift Payment ($)	Time (s)	Save ($)	Optimal	Diff ($/MWh)
C10	MIP	59,195,531	-	-	-	-	0.68
	LMP	-	11,613	36	-	N	
	TLP	59,193,235	1,899	13	9,714	N	
	IA1	59,194,229	1,302	108	+597	Y	
	IA2	59,194,229	1,302	115	+597	Y	
	IAC1	59,194,229	1,302	$\diamond(+0)$	+597	Y	
	IAC2	59,194,229	1,302	$\diamond(+0)$	+597	Y	
	OPT	59,194,229	1,302	9,584	+597	$\star$	
C11	MIP	49,628,808	-	-	-	-	0.38
	LMP	-	9,918	38	-	N	
	TLP	49,620,385	1,448	17	8,470	N	
	IA1	49,627,991	817	372	+631	Y	
	IA2	49,627,991	817	115	+631	Y	
	IAC1	49,627,991	817	$\diamond(+0)$	+631	Y	
	IAC2	49,627,991	817	$\diamond(+0)$	+631	Y	
	OPT	49,627,991	817	16,269	+631	$\star$	

Case Studies with Transmission Constraints

Table: Test results for MISO with transmission constraints

Case	Model	Solution ($)	Uplift Payment ($)	Time (s)	Save ($)	Optimal	Diff ($/MWh)
C10(T)	MIP	61,717,153	-	584	-	-	3.49
	LMP	-	1,667,967	68	-	N	
	TLP	61,596,521	92,541	69	1,575,426	N	
	IA1	61,602,290	87,824	1,182	+4,717	Y	
	IA2	61,602,290	87,824	1,240	+4,717	Y	
	IAC1	61,602,290	87,824	$\diamond(+0)$	+4,717	Y	
	IAC2	61,602,290	87,824	$\diamond(+0)$	+4,717	Y	
	OPT	61,602,290	87,824	81,630	+4,717	$\star$	
C11(T)	MIP	50,071,094	-	271	-	-	2.19
	LMP	-	476,190	58	-	N	
	TLP	50,020,529	24,538	41	451,652	N	
	IA1	50,030,415	23,498	512	+1,041	N	
	IA2	50,030,417	23,495	626	+1,044	Y	
	IAC1	50,030,417	23,495	$\diamond(+39)$	+1,044	Y	
	IAC2	50,030,417	23,495	$\diamond(+0)$	+1,044	Y	
	OPT	50,030,417	23,495	31,857	+1,044	$\star$	

Overview

Maximum Daily Starts

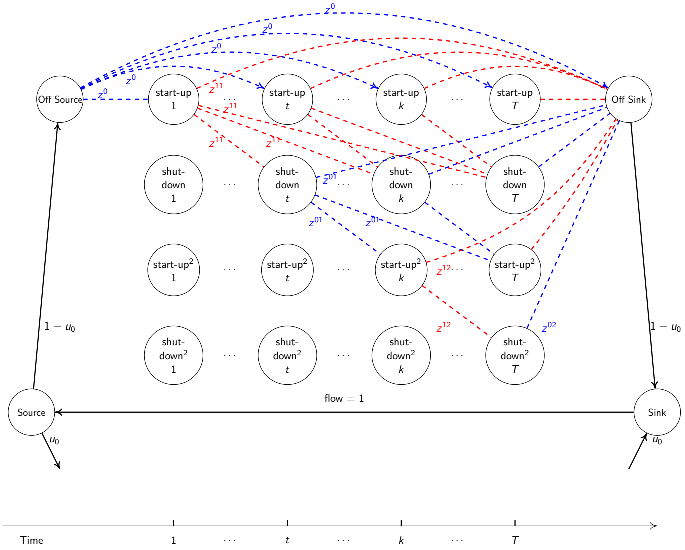


Figure: The Network Flow Graph with Maximum Daily Starts being Two

Convex Envelope

- The convex envelope $f_c(x)$ of a function $f(x)$ is defined as the largest convex function that is below $f(x)$ for all x in the convex set S.
- Convex envelope of a piecewise linear cost function of N pieces [Hua and Baldick, 2017]:

$$P_N = \begin{cases} \max\limits_{n\in\{1,..,N\}} a_n\rho + b_n, & \beta = 1, \\ \max\limits_{n\in\{1,..,N\}} a_n\rho + b_n\beta, & 0 < \beta < 1, \\ 0, & \beta = 0. \end{cases}$$

- Convex envelope of a quadratic cost function [Hua and Baldick, 2017]:

$$Q = \begin{cases} a\rho^2 + b\rho + c, & \beta = 1, \\ a\rho^2/\beta + b\rho + c\beta, & 0 < \beta < 1, \\ 0, & \beta = 0. \end{cases}$$

Convergence of Convex Envelope

Theorem 2: [Zhang et al., 2020] Suppose P_N is a convex piecewise linear cost function with N pieces that is used for approximating the convex quadratic cost function Q when $\beta = 1$. As the number of pieces $N \to \infty$, the optimal objective value of the CHP-Primal problem (5) with the piecewise linear convex envelope P_N converges to that of (5) with the quadratic convex envelope Q.

Remark: The convergence of convex envelope is equivalent to the convergence of the corresponding uplift payment.

Overview

IEEE 118-bus Case Studies

- 54 generators and 118 buses
- Operation periods: 24 hours
- Run on a PC with Intel Core i7-6500U CPU at 2.50GHz and 8GB memory
- Optimizer: Gurobi 8.0.1
- The required relative MIP gap is set to be 1e-3
- $$\text{Gap} = \frac{|\text{Obj\_Quad} - \text{Obj\_PWL}|}{\text{Obj\_Quad}} \times 100\%$$

Case Studies on Convergence Rate

Table: Numerical Results on Piecewise Linear Approximation for Cases without Transmission Constraints

Gap \ Case / Piece N	Case1	Case2	Case3	Case4	Case5
1	0.96%	0.95%	0.96%	0.97%	0.95%
3	0.17%	0.15%	0.16%	0.16%	0.15%
5	0.12%	0.10%	0.11%	0.11%	0.10%
8	0.09%	0.08%	0.09%	0.09%	0.08%
Obj_Quad ($)	1,447,671	1,450,365	1,447,701	1,444,700	1,447,508

Case Studies on Convergence Rate

Table: Numerical Results on Piecewise Linear Approximation for Cases with Transmission Constraints

Gap \ Case / Piece N	Case1(T)	Case2(T)	Case3(T)	Case4(T)	Case5(T)
1	1.21%	1.20%	1.17%	1.20%	1.19%
3	0.20%	0.21%	0.16%	0.19%	0.19%
5	0.12%	0.13%	0.09%	0.11%	0.11%
8	0.09%	0.10%	0.06%	0.09%	0.08%
Obj_Quad ($)	1,458,611	1,461,483	1,458,312	1,455,994	1,458,493

Case Studies on MIP Performance

Table: Numerical Results on UCED problems without Transmission Constraints

	Quad		PWL (8)		Gap
	Obj($)	Time(sec)	Obj($)	Time(sec)	
Case1	1,446,482	7.019	1,446,317	0.969	0.01%
Case2	1,449,478	4.248	1,449,231	1.077	0.02%
Case3	1,446,982	1.286	1,446,463	1.048	0.04%
Case4	1,443,585	3.613	1,443,424	0.946	0.01%
Case5	1,446,669	4.706	1,446,444	1.525	0.02%
Case6	1,229,302	2.226	1,229,152	1.103	0.01%
Case7	1,445,314	3.806	1,445,164	1.207	0.01%
Case8	1,444,865	11.63	1,444,714	0.946	0.01%
Case9	1,446,019	3.853	1,445,834	1.097	0.01%
Case10	1,442,819	6.203	1,442,607	0.897	0.01%

Case Studies on MIP Performance

Table: Numerical Results on UCED problems with Transmission Constraints

	Quad		PWL (8)		Gap
	Obj($)	Time(sec)	Obj($)	Time(sec)	
Case1(T)	1,457,809	57.593	1,457,560	6.316	0.02%
Case2(T)	1,460,905	45.685	1,460,350	5.413	0.04%
Case3(T)	1,458,470	12.522	1,458,222	5.691	0.02%
Case4(T)	1,455,879	46.088	1,456,119	4.454	0.02%
Case5(T)	1,457,868	63.008	1,458,417	6.687	0.04%
Case6(T)	1,237,245	8.201	1,237,112	5.028	0.01%
Case7(T)	1,457,670	11.506	1,457,513	7.247	0.01%
Case8(T)	1,457,254	6.676	1,456,949	5.448	0.02%
Case9(T)	1,457,860	12.257	1,457,896	5.419	0.00%
Case10(T)	1,454,085	36.421	1,454,490	4.278	0.03%

Other Ongoing Topics

- Incorporating maximum energy
- Incorporating combined-cycles

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