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Study of Line Switching Under Contingencies: Formulations and Algorithms

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Structure

- Background and Motivation
- Robust UC and Column-and-Constraint Generation Method
- Mathematical Formulations and Results
- Algorithm Improvement
- Conclusions

I – Background and Motivation

Background and Motivation

- Transmission lines were treated static facilities
- Post-contingency corrective operations --- line switching to handle contingencies
- Recent initiative on co-optimization between dispatch+ line switching ---- dynamic topology
- Dispatch cost can be saved up to 25% (Hedman et al.2009)
- Including line switching in preventive analysis and operations considering contingencies – before contingencies

Background and Motivation

- Vulnerability analysis with line switching -- what is the most destructive N-k contingency
- Grid upgrading with line switching -- how to make use of line switching in system design
- Robust Unit Commitment with N-k consideration and line switching -- what are daily benefit and cost reduction (on-going)

- II – Robust UC & Column-and-Constraint Generation Method

A Revisit of Robust Unit Commitment

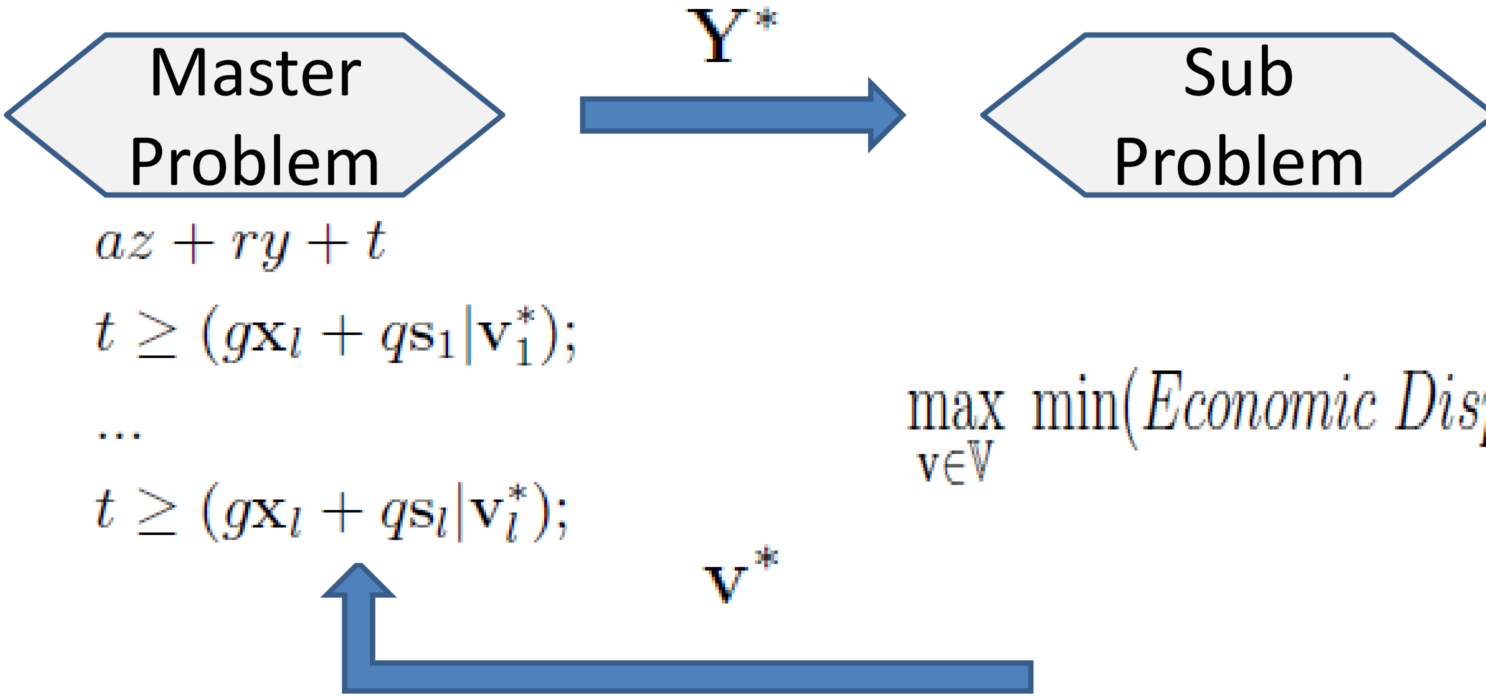
- Two-stage Robust Unit Commitment

$$\overbrace{\min_{\mathbf{y},\mathbf{z}}(\mathbf{az}+\mathbf{ry})}^{\text{Day ahead}}+\max_{\mathbf{v}\in\mathbb{V}}\ \min_{\mathbf{x},\mathbf{s}\in\Omega(\mathbf{y},\mathbf{z},\mathbf{v})}\overbrace{(\mathbf{gx}+\mathbf{qs})}^{\text{Economic Dispatch}}$$

$$\begin{aligned}\text{s.t.}\quad & \mathbf{Dy}+\mathbf{Fz}\geq\mathbf{f};\ \ \mathbf{y},\mathbf{z}\ \mathit{binary},\\ & \Omega(\mathbf{y},\mathbf{z},\mathbf{v})=\{(\mathbf{x},\mathbf{s}):\mathbf{Ex}\leq\mathbf{e},\\ & \mathbf{Ax}\leq\mathbf{L}-\mathbf{Gy}-\mathbf{Pz}-\mathbf{Rv},\mathbf{Ix}+\mathbf{Hs}=\mathbf{d}-\mathbf{Tv}\}\end{aligned}$$

- Computing methods:
  - Benders decomposition (Bertsimas et al., Jiang et al.)
  - Column-and-Constraint Generation algorithm (Zeng and Zhao)

Column-and-Constraint Generation



$$
\begin{aligned}
\min \quad & az + ry + t \\
\text{s.t.} \quad & t \geq (g\mathbf{x}_l + q\mathbf{s}_1 | \mathbf{v}_1^*); \\
& \cdots \\
& t \geq (g\mathbf{x}_l + q\mathbf{s}_l | \mathbf{v}_l^*);
\end{aligned}
$$

$$\max_{\mathrm{v} \in \mathbb{V}} \min\left(\mathit{Economic\ Dispatch} \middle| \mathbb{Y}^* \right)$$

- Iteratively solve master and subproblems until LB=UB
- Initially, called “primal cut alg.” Formal name: Column-and-constraint generation method (Zeng and Zhao: *Operations Research letters 2013*)

A Revisit of Robust Unit Commitment

- BA – Benders algorithm
- C & CG – Column and Constraint Generation method

		BA			C&CG	
cases	profit	time(s)	iterations	profit	time(s)	iterations
case1	596669	2239	80	594674	50	3
case2	589931	4619	70	589478	243	3
case3	581290*	>20000*	120*	583293	803	3
case4	578362	12670	59	575876	324	2
case5	572166*	>20000*	70*	571181	27	2

- drastically faster than Benders: >20 times
  - ISO-NE adopted, time reduced 5 hours to 5 mins
- A general method for *min-max -min* problem

III – Math Formulations & Results

Vulnerability Analysis with Switching

- z_l : binary variable for switching (0 open, 1 closed)
- w_l: binary variable for forced outage

$$\max_{\mathbf{w}\in\mathcal{A}} \min \sum_j (load\ shed)_j$$

$$st.\ z_l w_l(\theta_m - \theta_n - x_l f_l^{mn}) = 0, \forall l$$ DC flow

$$-F_l z_l w_l \le f_l^{mn} \le z_l w_l F_l, \forall l$$ Flow bounds

$$p_i^n \le P_i^{max}, \forall i$$

$$d_j^n \le D_j^n, \forall j$$

$$\sum_l f_l^{\cdot n} + p_i^n = \sum_l f_l^{n\cdot} + d_j^n, \forall n$$ Node balance

$$p_i^n \ge 0, d_j^n \ge 0, f_l^{mn}, \theta_n \text{ free}, z_l \in \{0,1\}$$

$$\mathcal{A} = \{w_l \in \{0,1\}, \textstyle\sum_l (1 - w_l) \le K\}$$ N-k conting.

Vulnerability Analysis with Switching

- An open problem (A. Delgadillo et al. TPWRS 2010)
- The inner problem is an Mixed Integer Program
  - No valid KKT conditions or strong duality
  - No effective algorithm available
- Bi-level to Tri-level

$$\max_{\mathbf{w}\in\mathcal{A}} - \min_{\mathbf{z},\mathbf{x}} \quad \Rightarrow \quad \max_{\mathbf{w}\in\mathcal{A}} - \min_{\mathbf{z}} - \min \sum_j (load\ shed)_j$$

$$\max_{\mathbf{w}\in\mathcal{A}} - \min_{\mathbf{z}} - \max\, dual\ (Economic\ dispatch)$$

Vulnerability Analysis with Switching

- Benchmark with A. Delgadillo et al.
- MSBD: multi-start Benders decomposition
- C&CG: column-and-constraint generation
- *Inexact solutions in MSBD

Table: Algorithm Performance Comparison for IEEE One-Area RTS-96 Systems

K	Time (s) of MSBD	Time (s) of C&CG	Iterations of C&CG
1	18.48	23.48	7
2	293.31*	22.01	6
3	2261.59*	20.51	6
4	2180.67*	19.42	4
5	1610.35*	17.92	2
6	1520.47*	17.04	2
7	0.89	16.24	1
8	1312.42*	15.65	3
9	1155.64*	14.77	2
10	1029.01*	14.28	2
11	0.85	13.48	1
12	0.88	12.85	1
Average Time	**948.71**	**17.30**	

* indicating the solution quality is unknown in MSBD.

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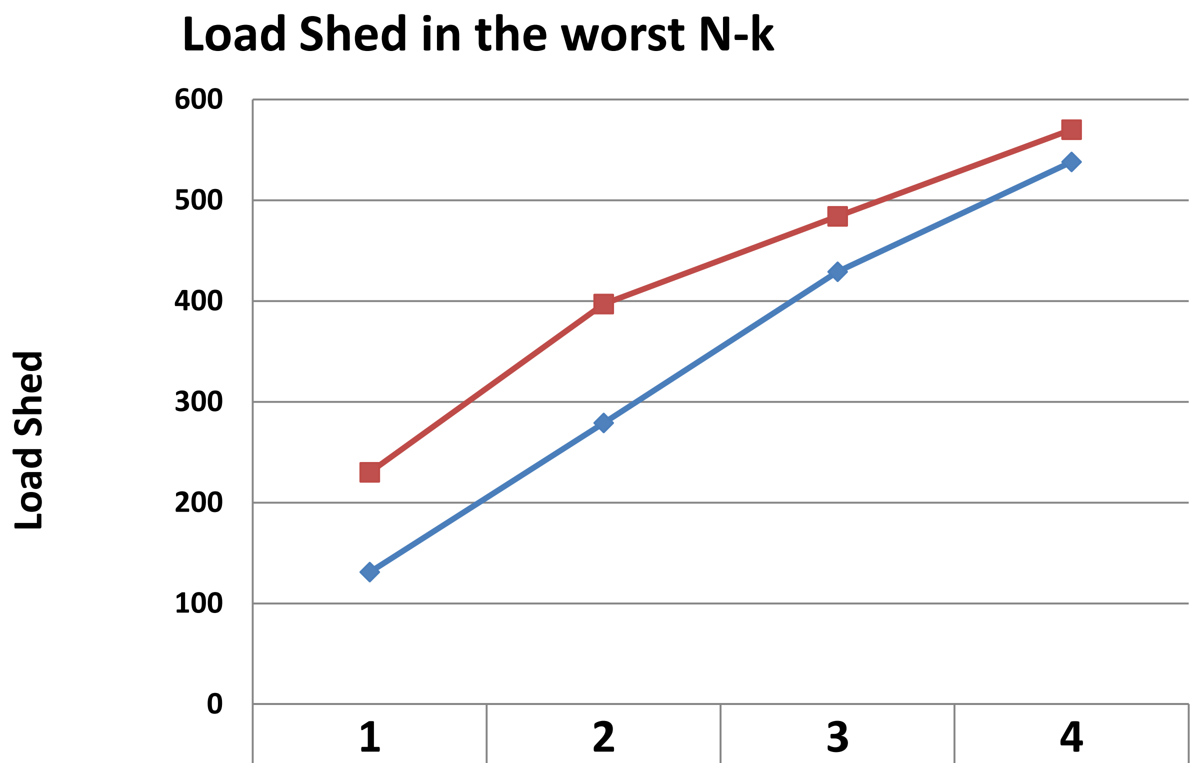
Benefits of Line Switching



	1	2	3	4
with switching	131	279	429	538
without switching	230	397	484	570

- Very useful when k is small
- On average: 22.4% load reduction in N-k contingencies

Grid Expansion with Line Switching

Design Stage

Contingency

Economic Dispatch
+ Line Switching

$$\min_{z} \left\{ \underbrace{\sum_{g\in \mathbf{G}} c_g z_g + \sum_{e\in \mathbf{E}} c_e z_e}_{\text{Design Stage}} + \sigma \max_{v} \underbrace{\min_{w,p,d,\theta} \sum_{n\in \mathbf{N}} c_n d_n}_{\text{Economic Dispatch + Line Switching}} \right\}$$

$$s.t. \quad p_l x_l = w_l v_l [\theta_{O(l)} - \theta_{D(l)}], \quad \forall l$$

$$p_e x_e = w_e v_e z_e [\theta_{O(e)} - \theta_{D(e)}], \quad \forall e$$

DC flow on new line

$$\sum_{j\in \mathbf{J_n}} p_j + \sum_{g\in \mathbf{G_n}} p_g - \sum_{l|O(l)=n} p_l + \sum_{l|D(l)=n} p_l - \sum_{e|O(e)=n} p_e + \sum_{e|D(e)=n} p_e + d_n = D_n, \forall n$$

$$-p_l^{max} \le p_l \le p_l^{max}, \quad \forall l$$

$$-p_e^{max} \le p_e \le p_e^{max}, \quad \forall e$$

Power from new gen. cap.

$$0 \le p_j \le p_j^{max}, \quad \forall j$$

$$0 \le p_g \le z_g p_g^{max}, \quad \forall g$$

$$0 \le d_n \le D_n, \quad \forall n$$

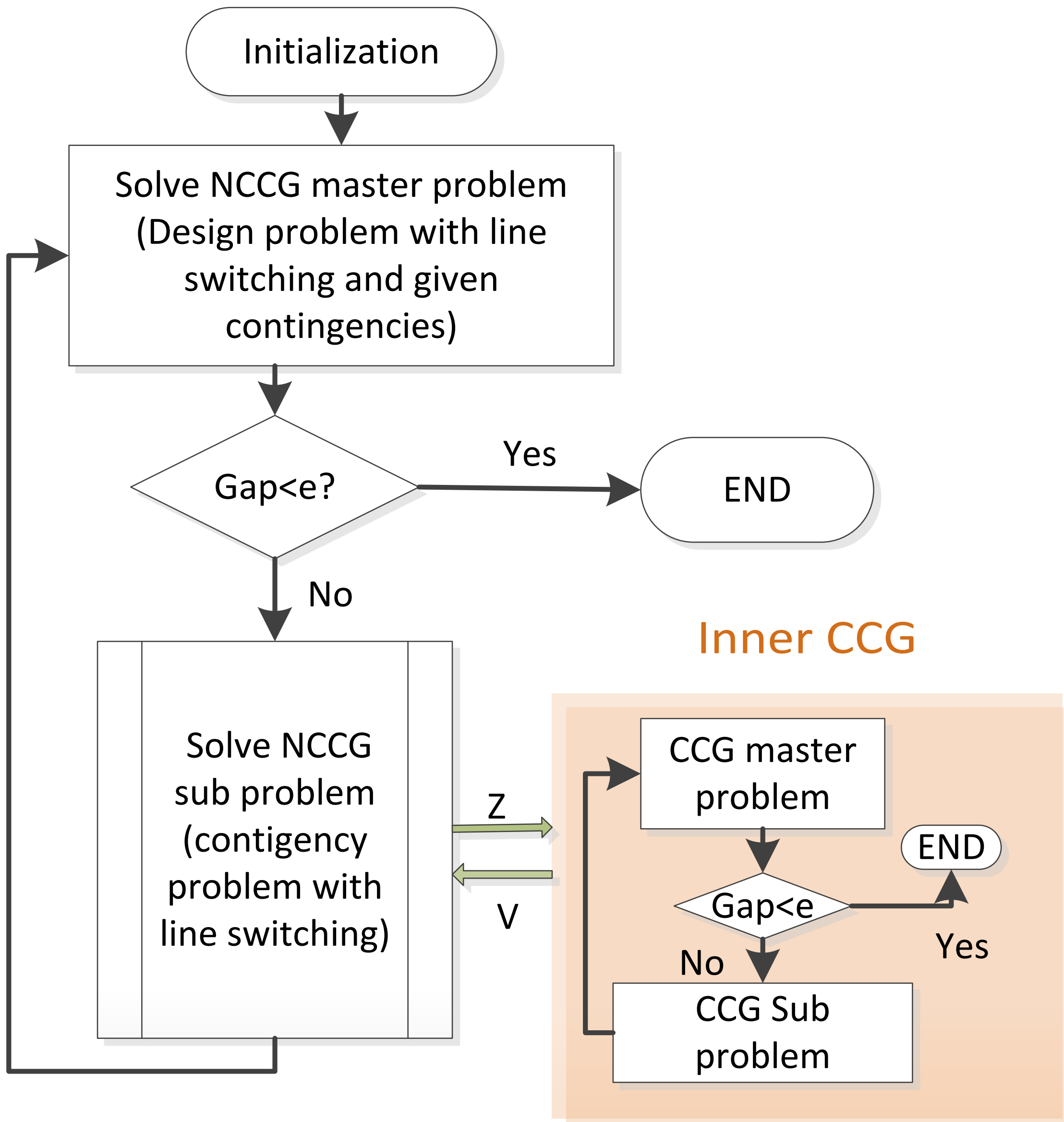
$$\sum_{l\in \mathbf{L}} (1 - v_l) + \sum_{e\in \mathbf{E}} (1 - v_e) = k$$

N-k contingency

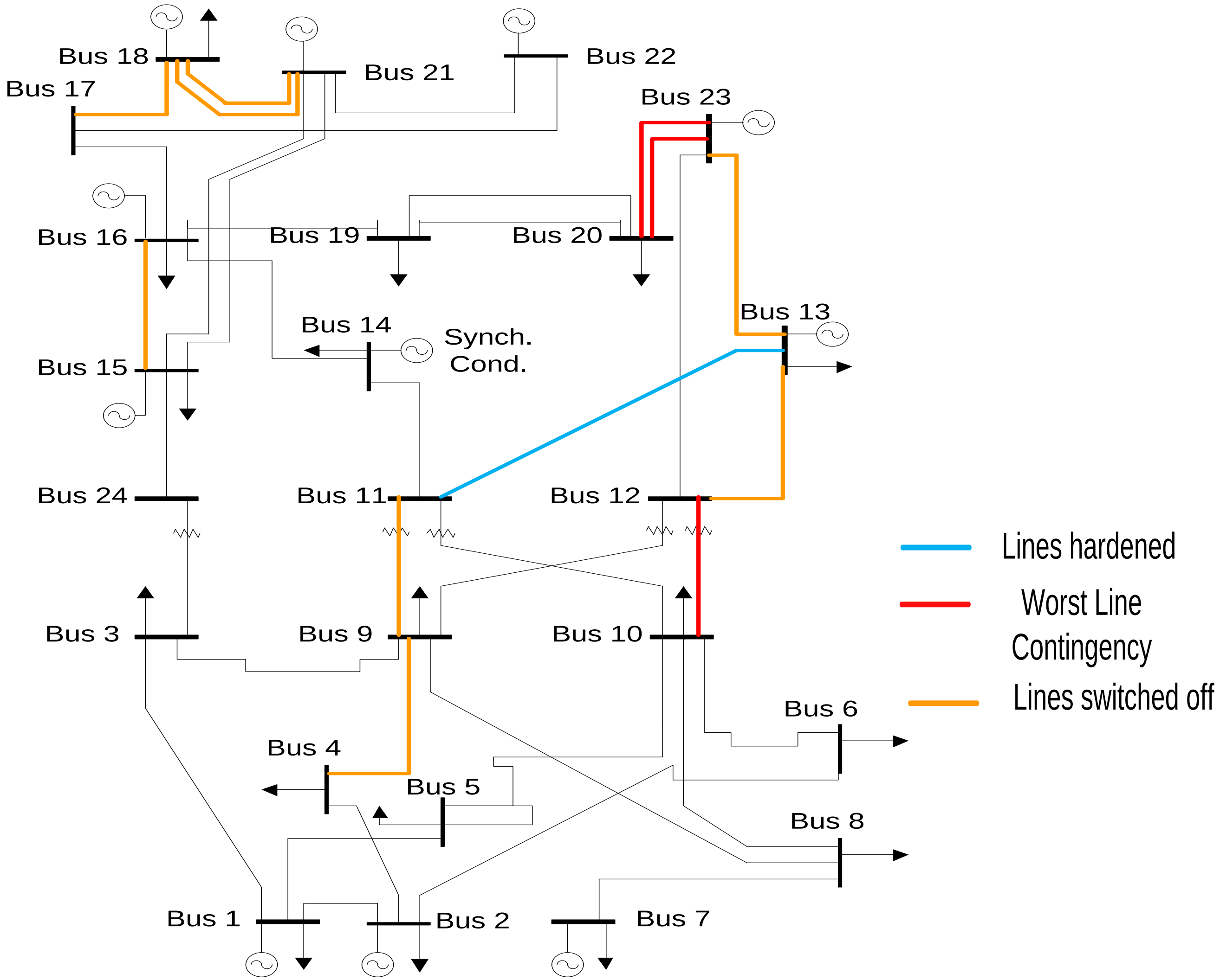
$$z_e, z_g, v_l, v_e, w_l, w_e \in \{0,1\}, \quad \forall e, g, l, e$$

Grid Expansion with Line Switching

- **Column-and-constraint implemented in a nested fashion (NCCG)**
- **May need a larger number of iterations in outer+inner loops**
- **Algorithm improvement is important**



A Hardening Illustration (no new cap.)



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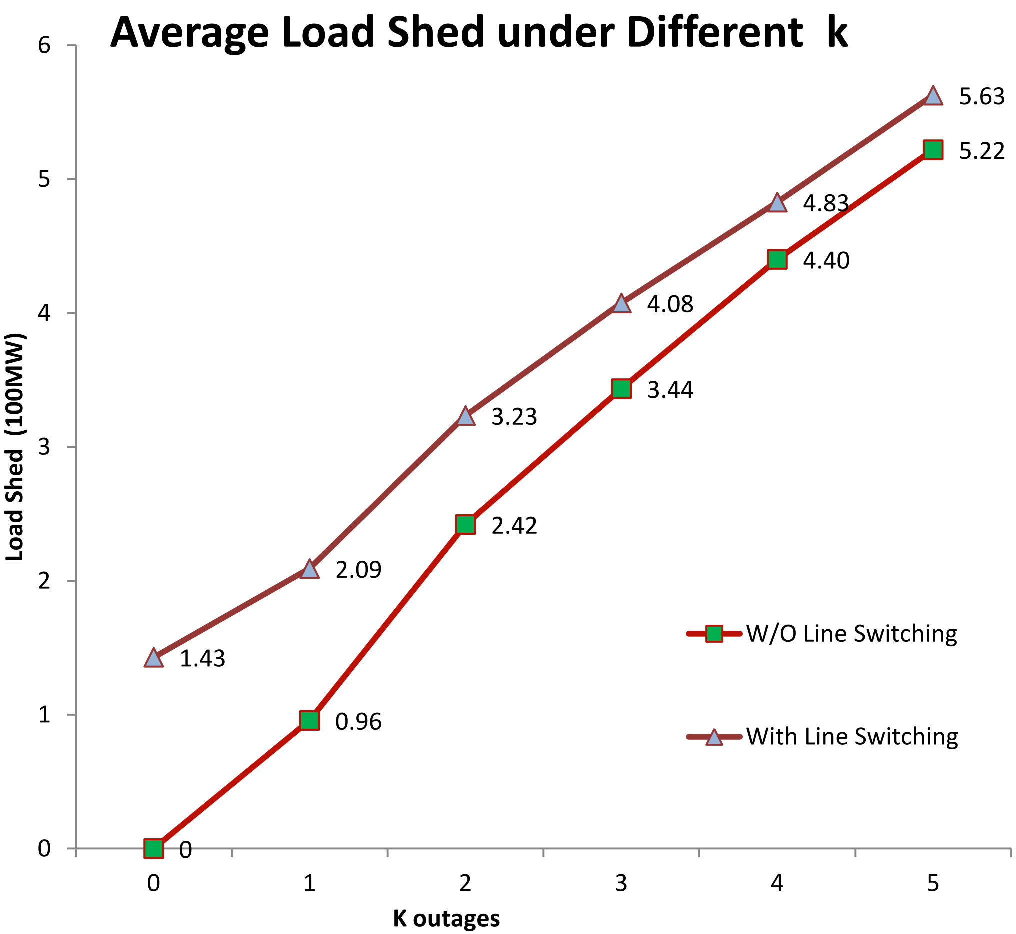
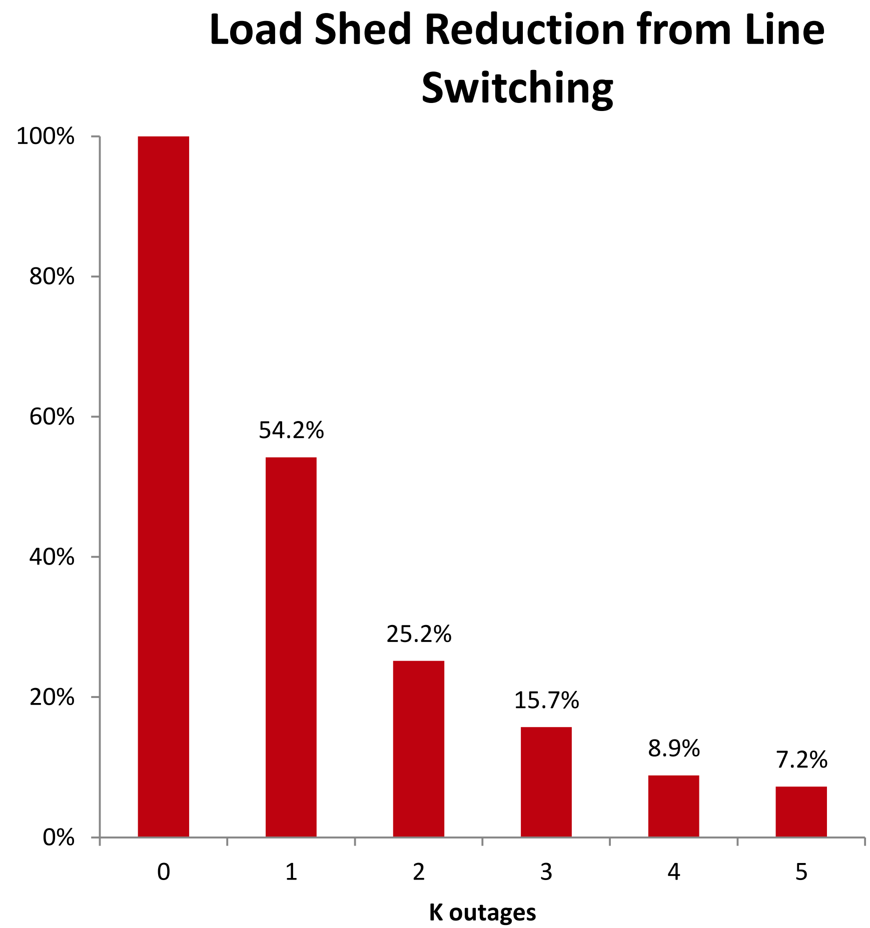
Benefits of Line Switching in Hardening





22.8 % reduction in load shed through line switching

IV – Algorithm Improvement

Algorithm Improvements

- Initialization with your intelligence
  - Analyzing problem structure to identify "worst case" scenarios
  - Analyzing problem structure to identify "most promising" line switching
- Make use of multi-optimal solutions from solvers
- Do you need to solve the problem to optimality before termination? – No. Adjust your opt. tolerance..
- More tricks – welcome to discuss

Example of CPU Time Reduction

On a random data set of vulnerability analysis problem on IEEE-96 RTS

Our basic method	with Initial switching
1.2	**2.4**
4.9	**15.4**
9.3	**12.8**
20.9	**41.6**
185.9	**6.1**
613	**2623.34**
137.2	**16.5**
1220	**18.8**
6949.1	**19.6**
453.6	**10**
99.8	**3.8**
10.701	**3.3**
808.8000833	231.1366667

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V – Conclusions

Conclusions

- Column-and-constraint generation algorithm is far better than Benders decomposition
- Nested Column-and-constraint generation can deal with mixed integer recourse problem inside two-stage robust grid expansion/unit commitment problem
- Line switching can be very effective in load shed reduction
- More advanced algorithm improvements are needed

Thank you!

- Questions?

- Contact:

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