Tight and Compact MILP Formulations for Unit Commitment Problems

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INSTITUTO DE INVESTIGACIÓN TECNOLÓGICA

Outline

UC and MIP

- Significant breakthroughs in Mixed-Integer Programming (MIP)
  - Solving MIP 100 million times faster than 20 years ago[1]

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  - Solving algorithms (e.g., solvers, decomposition techniques)
  - Improving the MIP-Based UC formulation ⇒ ↓ solving times

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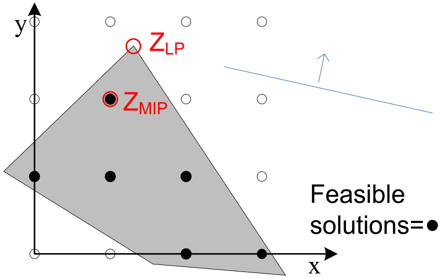
Outline

Solving MIP Through The Powerful LP

Shaping the linear feasible region to arrive from vertex Z_{LP} to Z_{MIP}



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- Branch and bound (divide and conquer)

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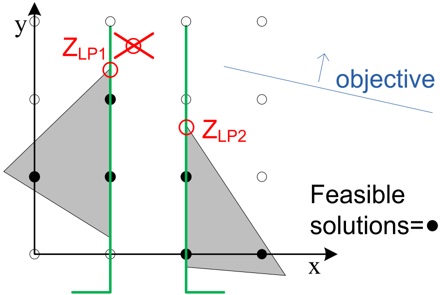

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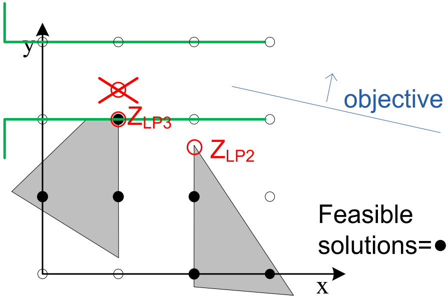

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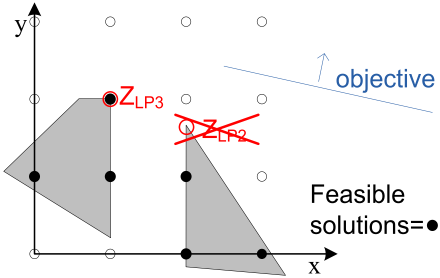

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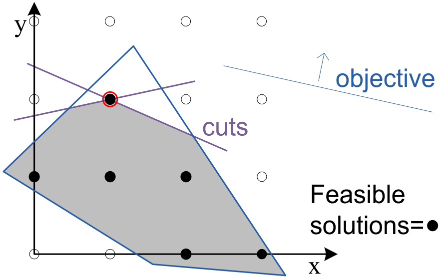
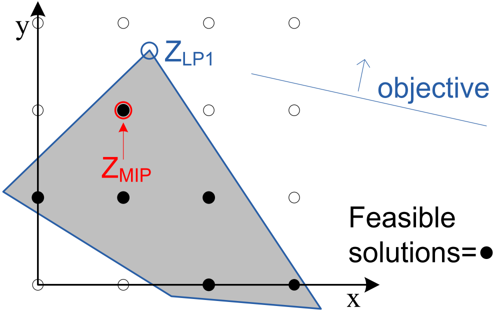


To prove optimality Z_{MIP} must become a vertex by:

- Branch and bound (divide and conquer)
- and/or by adding cuts

An MIP Has Infinite LP Formulations



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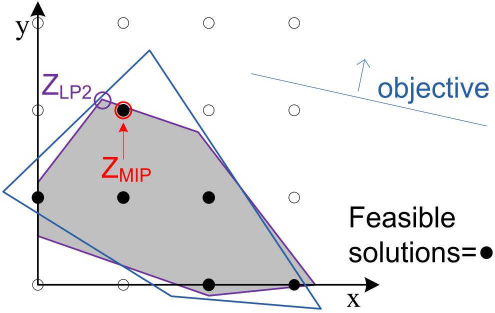

LP1 and LP2 represent the same MIP problem

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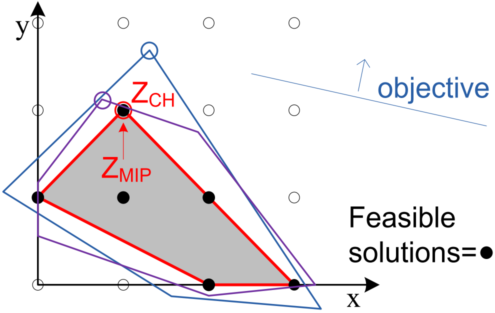

LP1, LP2 and CH represent the same MIP problem

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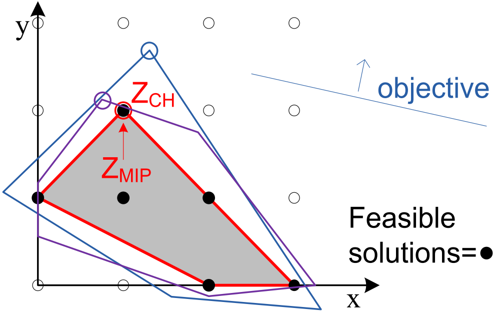


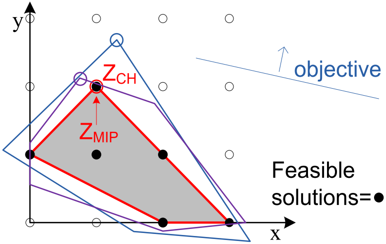
LP1, LP2 and CH represent the same MIP problem

which one to choose?

Convex Hull: The Tightest Formulation

Convex Hull (CH)

Smallest convex feasible region containing all the feasible integer points[2]



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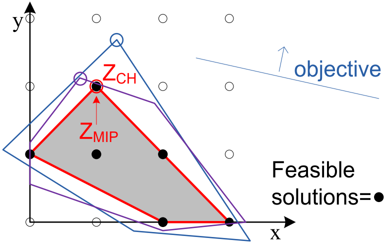

- The *convex hull* problem solves an MIP as an LP
  - Each vertex satisfies the integrality constraints
- Unfortunately...

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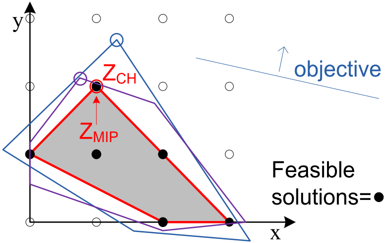

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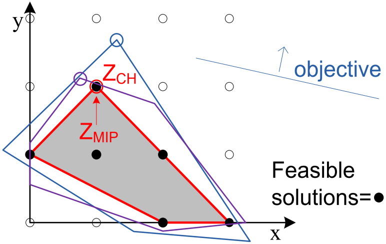

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- Unfortunately, the *convex hull* is typically too difficult to obtain[2,3]
  - An enormous (exponential) number of inequalities is needed

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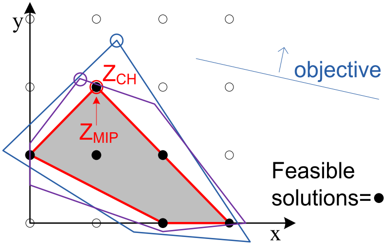

- The *convex hull* problem solves an MIP as an LP
  - Each vertex satisfies the integrality constraints
  - So an LP optimum is also an MIP optimum
- Unfortunately, the *convex hull* is typically too difficult to obtain[2,3]
  - An enormous (exponential) number of inequalities is needed
  - To solve an MIP is usually easier than trying to find its *convex hull*

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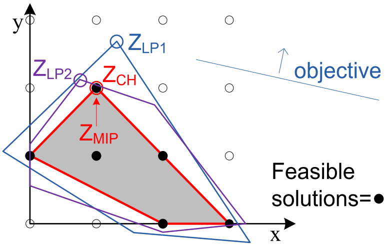
Outline

Choosing The Best Formulation

Measuring The Tightness

Integrality Gap (IGap)

Relative distance between MIP and LP optima



$$\mathrm{IGap}_{LP1} = \frac{Z_{MIP} - Z_{LP1}}{Z_{MIP}}$$

Choosing The Best Formulation

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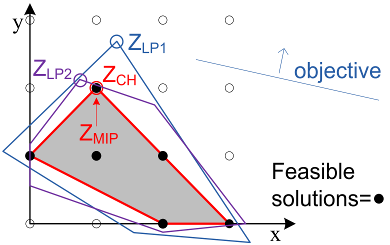

$$\mathrm{IGap}_{\mathrm{LP1}} = \frac{Z_{\mathrm{MIP}} - Z_{\mathrm{LP1}}}{Z_{\mathrm{MIP}}} > \mathrm{IGap}_{\mathrm{LP2}} = \frac{Z_{\mathrm{MIP}} - Z_{\mathrm{LP2}}}{Z_{\mathrm{MIP}}}$$

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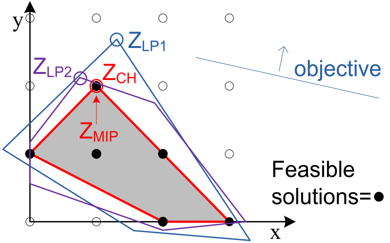

$$\mathrm{IGap}_{\mathrm{LP1}} = \frac{Z_{\mathrm{MIP}} - Z_{\mathrm{LP1}}}{Z_{\mathrm{MIP}}} > \mathrm{IGap}_{\mathrm{LP2}} = \frac{Z_{\mathrm{MIP}} - Z_{\mathrm{LP2}}}{Z_{\mathrm{MIP}}} > \mathrm{IGap}_{\mathrm{CH}} = \frac{Z_{\mathrm{MIP}} - Z_{\mathrm{CH}}}{Z_{\mathrm{MIP}}} = 0$$

$$\Downarrow$$

As an MIP problem:

LP2 is expected to be solved faster than LP1

Choosing The Best Formulation

Measuring The Tightness

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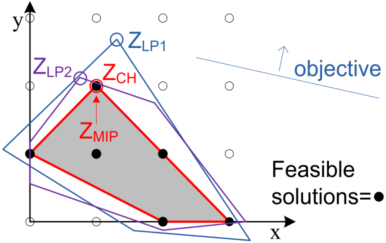

$$\mathrm{IGap}_{LP1} = \frac{Z_{MIP} - Z_{LP1}}{Z_{MIP}} > \mathrm{IGap}_{LP2} = \frac{Z_{MIP} - Z_{LP2}}{Z_{MIP}} > \mathrm{IGap}_{CH} = \frac{Z_{MIP} - Z_{CH}}{Z_{MIP}} = 0$$

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As an MIP problem:

LP2 is expected to be solved faster than LP1

CH will be solved way faster than LP2

Concepts: Tightness and Compactness

- Tightness: defines the search space (relaxed feasible region) that the solver needs to explore to find the solution
- Compactness (problem size): defines the searching speed (data to process) that the solver takes to find the solution

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- Tightness: defines the search space (relaxed feasible region) that the solver needs to explore to find the solution
- Compactness (problem size): defines the searching speed (data to process) that the solver takes to find the solution
- *Convex hull*: The tightest formulation ⇒ MIP solved as LP

Tightening an MIP Formulation

- The most common strategy is adding cuts
  - In fact, this is the most effective strategy of current MIP solvers[4]
  - This may add a huge number of inequalities $\Rightarrow\uparrow$ Time

[4] R. Bixby and E. Rothberg, "Progress in computational mixed integer programming—A look back from the other side of the tipping point," *Annals of Operations Research*, vol. 149, no. 1, pp. 37–41, Jan. 2007

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  - Trade-off: Tightness vs. Compactness
- Improving the MIP formulation
  - Provide the *convex hull* for some set of constraints
  - If available, use the *convex hull* for some set of constraints

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Tight and Compact (TC) Formulation

- The whole formulation can be found in the paper TC-UC[5]

[5] G. Morales-Espana, J. M. Latorre, and A. Ramos, "Tight and compact MILP formulation for the thermal unit commitment problem," *IEEE Transactions on Power Systems*, 2013, Special Section on Analysis and Simulation of Very Large Power Systems. In Press

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- Let's focus on the core of UC formulations:
  - Min/max outputs
  - SU & SD capabilities
  - Minimum up/down (TU/TD) times

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- Let's focus on the core of UC formulations:
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  - SU & SD capabilities
  - Minimum up/down (TU/TD) times, *convex hull* already available[6]

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[6] D. Rajan and S. Takriti, "Minimum Up/Down polytopes of the unit commitment problem with start-up costs," IBM, Research Report RC23628, Jun. 2005

Formulation (II)

- Generation limits taking into account: maximum $\overline{P}$ and minimum $\underline{P}$ output, as well as maximum SU and SD capabilities:

$$p_t \le \left(\overline{P} - \underline{P}\right) u_t - \left(\overline{P} - SD\right) w_{t+1} - \max\left(SD - SU, 0\right) v_t \quad \forall t \quad (1)$$

$$p_t \le \left(\overline{P} - \underline{P}\right) u_t - \left(\overline{P} - SU\right) v_t - \max\left(SU - SD, 0\right) w_{t+1} \quad \forall t \quad (2)$$

Total generation $= \underline{P} \cdot u_t + p_t$.

Formulation (III)

- Logical relationship: commitment u_t, startup v_t and shutdown w_t:

$$u_t - u_{t-1} = v_t - w_t \quad \forall t \tag{3}$$

$$v_t \leq u_t \quad \forall t \tag{4}$$

$$w_t \leq 1 - u_t \quad \forall t \tag{5}$$

  where (4) and (5) avoid the simultaneous startup and shutdown.

- Variable bounds

$$p_t \geq 0 \quad \forall t \tag{6}$$

$$0 \leq u_t, v_t, w_t \leq 1 \quad \forall t \tag{7}$$

Tightness of the Formulation

Let's study the polytope (1)-(7) using PORTA[7]:

- PORTA enumerates all vertices of a convex feasible region

[7] T. Christof and A. Löbel, "PORTA: POlyhedron representation transformation algorithm, version 1.4.1," *Konrad-Zuse-Zentrum für Informationstechnik Berlin, Germany*, 2009

Tightness of the Formulation

Let's study the polytope (1)-(7) using PORTA[7]:

- PORTA enumerates all vertices of a convex feasible region
- Example: 3 periods and $\overline{P} = 200, \underline{P} = SU = SD = 100$ for:
  - Case 1: $TU = TD = 1$
  - Case 2: $TU = TD = 2$
- For the complete and detailed formulation and a comprehensive study, see paper Tight LP-UC[8]

[7] T. Christof and A. Löbel, "PORTA: POlyhedron representation transformation algorithm, version 1.4.1," *Konrad-Zuse-Zentrum für Informationstechnik Berlin, Germany*, 2009

[8] G. Morales España, C. Gentile, and A. Ramos, "Tight LP formulation of the unit commitment problem presenting integer solutions," *IEEE Transactions on Power Systems*, 2013, Under Review

Case 1: Providing The Convex Hull

Formulation:

$$p_t \le \left(\overline{P} - \underline{P}\right) u_t - \left(\overline{P} - SD\right) w_{t+1} - \max\left(SD - SU, 0\right) v_t \quad (1)$$

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PORTA results for $(TU = TD = 1)$

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PORTA results for $(TU = TD = 1)$
$u_1, u_2, u_3, v_2, v_3, w_2, w_3, p_1, p_2, p_3$:

```
DIM = 10

CONV_SECTION
(  1) 0 0 0 0 0 0 0   0   0   0
(  2) 0 0 1 0 1 0 0   0   0   0
(  3) 1 0 0 0 0 1 0   0   0   0
(  4) 0 1 0 1 0 0 1   0   0   0
(  5) 0 1 1 1 0 0 0   0   0   0
(  6) 0 1 1 1 0 0 0   0   0 100
(  7) 1 1 0 0 0 0 1   0   0   0
(  8) 1 1 0 0 0 0 1 100   0   0
(  9) 1 1 1 0 0 0 0   0   0   0
( 10) 1 1 1 0 0 0 0   0   0 100
( 11) 1 1 1 0 0 0 0   0 100   0
( 12) 1 1 1 0 0 0 0   0 100 100
( 13) 1 1 1 0 0 0 0 100   0   0
( 14) 1 1 1 0 0 0 0 100   0 100
( 15) 1 1 1 0 0 0 0 100 100   0
( 16) 1 1 1 0 0 0 0 100 100 100
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END
```

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$$u_t - u_{t-1} = v_t - w_t \quad (3)$$

$$v_t \le u_t \quad (4)$$

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All vertices are integer

$\Downarrow$

Convex Hull

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(  1) 0 0 0 0 0 0 0   0   0   0
(  2) 0 0 1 0 1 0 0   0   0   0
(  3) 1 0 0 0 0 1 0   0   0   0
(  4) 0 1 0 1 0 0 1   0   0   0
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( 17) 1 0 1 0 1 1 0   0   0   0
END
```

Case 2: Providing and Using Convex Hulls (I)

Formulation + *TU/TD Convex hull*:

$$p_t \leq \left(\overline{P} - \underline{P}\right) u_t - \left(\overline{P} - SD\right) w_{t+1} - \max\left(SD{-}SU, 0\right) v_t \quad (1)$$

$$p_t \leq \left(\overline{P} - \underline{P}\right) u_t - \left(\overline{P} - SU\right) v_t - \max\left(SU{-}SD, 0\right) w_{t+1} \quad (2)$$

$$u_t - u_{t-1} = v_t - w_t \quad (3)$$

$$\sum_{i=t-TU+1}^{t} v_i \leq u_t \quad (4)$$

$$\sum_{i=t-TD+1}^{t} w_i \leq 1 - u_t \quad (5)$$

PORTA results for $(TU{=}TD{=}2)$

Case 2: Providing and Using Convex Hulls (I)

Formulation + *TU/TD Convex hull*:

$$p_t \leq \left(\overline{P}-\underline{P}\right)u_t - \left(\overline{P}-SD\right)w_{t+1} - \max\left(SD-SU,0\right)v_t \qquad (1)$$

$$p_t \leq \left(\overline{P}-\underline{P}\right)u_t - \left(\overline{P}-SU\right)v_t - \max\left(SU-SD,0\right)w_{t+1} \qquad (2)$$

$$u_t - u_{t-1} = v_t - w_t \qquad (3)$$

$$\sum_{i=t-TU+1}^{t} v_i \leq u_t \qquad (4)$$

$$\sum_{i=t-TD+1}^{t} w_i \leq 1 - u_t \qquad (5)$$

How to remove the fractional vertices?

PORTA results for $(TU = TD = 2)$
$u_1, u_2, u_3, v_2, v_3, w_2, w_3, p_1, p_2, p_3$:

```
DIM = 10

CONV_SECTION
(  1)   0 0   0   0 0 0   0   0   0   0
(  2) 1/2 1 1/2 1/2 0 0 1/2   0  50   0
(  3) 1/2 1 1/2 1/2 0 0 1/2   0  50  50
(  4) 1/2 1 1/2 1/2 0 0 1/2  50  50   0
(  5) 1/2 1 1/2 1/2 0 0 1/2  50  50  50
(  6)   0 0   1   0 1 0   0   0   0   0
(  7)   1 0   0   0 0 1   0   0   0   0
(  8)   0 1   1   1 0 0   0   0   0   0
(  9)   0 1   1   1 0 0   0   0   0 100
( 10)   1 1   0   0 0 0   1   0   0   0
( 11)   1 1   0   0 0 0   1 100   0   0
( 12)   1 1   1   0 0 0   0   0   0   0
( 13)   1 1   1   0 0 0   0   0   0 100
( 14)   1 1   1   0 0 0   0   0 100   0
( 15)   1 1   1   0 0 0   0   0 100 100
( 16)   1 1   1   0 0 0   0 100   0   0
( 17)   1 1   1   0 0 0   0 100   0 100
( 18)   1 1   1   0 0 0   0 100 100   0
( 19)   1 1   1   0 0 0   0 100 100 100
END
```

Case 2: Providing and Using Convex Hulls (II)

Reformulating (1) and (2) for $TU \geq 2$:

$$\cancel{p_t \leq \left(\overline{P} - \underline{P}\right) u_t - \left(\overline{P} - SD\right) w_{t+1} - \max\left(SD - SU, 0\right) v_t} \quad (1)$$

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$$p_t \leq \left(\overline{P} - \underline{P}\right) u_t - \left(\overline{P} - SU\right) v_t - \left(\overline{P} - SD\right) w_{t+1} \quad (8)$$

$$u_t - u_{t-1} = v_t - w_t \quad (3)$$

$$\sum_{i=t-TU+1}^{t} v_i \leq u_t \quad (4)$$

$$\sum_{i=t-TD+1}^{t} w_i \leq 1 - u_t \quad (5)$$

PORTA results for $(TU = TD = 2)$
$u_1, u_2, u_3, v_2, v_3, w_2, w_3, p_1, p_2, p_3$:

```
DIM = 10

CONV_SECTION
(  1) 0 0 0 0 0 0 0   0   0   0
(  2) 0 0 1 0 1 0 0   0   0   0
(  3) 1 0 0 0 0 1 0   0   0   0
(  4) 0 1 1 1 0 0 0   0   0   0
(  5) 0 1 1 1 0 0 0   0   0 100
(  6) 1 1 0 0 0 0 1   0   0   0
(  7) 1 1 0 0 0 0 1 100   0   0
(  8) 1 1 1 0 0 0 0   0   0   0
(  9) 1 1 1 0 0 0 0   0   0 100
( 10) 1 1 1 0 0 0 0   0 100   0
( 11) 1 1 1 0 0 0 0   0 100 100
( 12) 1 1 1 0 0 0 0 100   0   0
( 13) 1 1 1 0 0 0 0 100   0 100
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END
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$\Rightarrow$ *Convex Hull*

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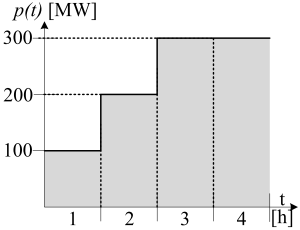
```
DIM = 10

CONV_SECTION
(  1)  0 0 0 0 0 0 0    0    0    0
(  2)  0 0 1 0 1 0 0    0    0    0
(  3)  1 0 0 0 0 1 0    0    0    0
(  4)  0 1 1 1 0 0 0    0    0    0
(  5)  0 1 1 1 0 0 0    0    0  100
(  6)  1 1 0 0 0 0 1    0    0    0
(  7)  1 1 0 0 0 0 1  100    0    0
(  8)  1 1 1 0 0 0 0    0    0    0
(  9)  1 1 1 0 0 0 0    0    0  100
( 10)  1 1 1 0 0 0 0    0  100    0
( 11)  1 1 1 0 0 0 0    0  100  100
( 12)  1 1 1 0 0 0 0  100    0    0
( 13)  1 1 1 0 0 0 0  100    0  100
( 14)  1 1 1 0 0 0 0  100  100    0
( 15)  1 1 1 0 0 0 0  100  100  100
END
```

Outline

Energy vs Power

Generation levels are usually considered as energy blocks.
Example: $\overline{P} = 300$MW; $\underline{P} = 100$MW; Up/Down ramp rate: 100 MW/h

Traditional UC



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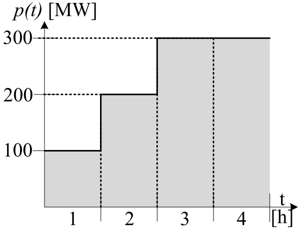
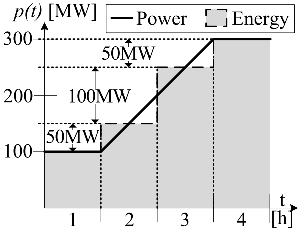


Feasible energy profile



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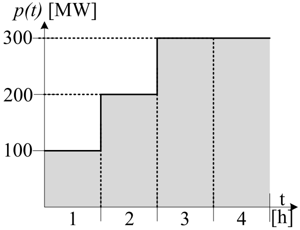
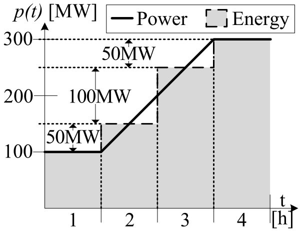


Infeasible energy delivery[9]

Feasible energy profile



[9] X. Guan, F. Gao, and A. Svoboda, "Energy delivery capacity and generation scheduling in the deregulated electric power market," *IEEE Transactions on Power Systems*, vol. 15, no. 4, pp. 1275–1280, Nov. 2000

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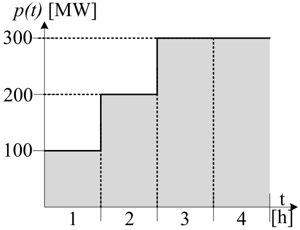
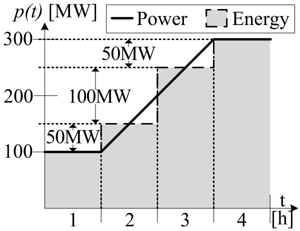


Infeasible energy delivery[9]

Overestimated ramp availability

Feasible energy profile



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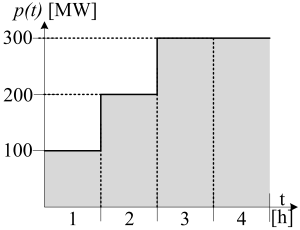
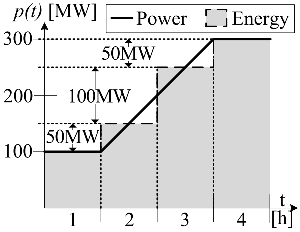


Infeasible energy delivery[9]
Overestimated ramp availability

Feasible energy profile



A clear difference between power and energy is required in an UC

[9] X. Guan, F. Gao, and A. Svoboda, "Energy delivery capacity and generation scheduling in the deregulated electric power market," *IEEE Transactions on Power Systems*, vol. 15, no. 4, pp. 1275–1280, Nov. 2000

TC P-Based UC: Providing The Convex Hull

Power-Based UC formulation for $SU = SD = \underline{P}$:

$$p'_t \leq \left(\overline{P} - \underline{P}\right)\left(u_t - w_{t+1}\right) \quad (9)$$

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PORTA results for $(TU = TD = 2)$

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PORTA results for $(TU = TD = 2)$
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```
DIM = 10

CONV_SECTION
(  1) 0 0 0 0 0 0 0   0   0   0
(  2) 0 0 1 0 1 0 0   0   0   0
(  3) 0 0 1 0 1 0 0   0   0 100
(  4) 1 0 0 0 0 1 0   0   0   0
(  5) 0 1 1 1 0 0 0   0   0   0
(  6) 0 1 1 1 0 0 0   0   0 100
(  7) 0 1 1 1 0 0 0   0 100   0
(  8) 0 1 1 1 0 0 0   0 100 100
(  9) 1 1 0 0 0 0 1   0   0   0
( 10) 1 1 0 0 0 0 1 100   0   0
( 11) 1 1 1 0 0 0 0   0   0   0
( 12) 1 1 1 0 0 0 0   0   0 100
( 13) 1 1 1 0 0 0 0   0 100   0
( 14) 1 1 1 0 0 0 0   0 100 100
( 15) 1 1 1 0 0 0 0 100   0   0
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END
```

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All vertices are integer

$\Downarrow$

Convex Hull

PORTA results for $(TU = TD = 2)$
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DIM = 10

CONV_SECTION

(1)	0	0	0	0	0	0	0	0	0	0
(2)	0	0	1	0	1	0	0	0	0	0
(3)	0	0	1	0	1	0	0	0	0	100
(4)	1	0	0	0	0	1	0	0	0	0
(5)	0	1	1	1	0	0	0	0	0	0
(6)	0	1	1	1	0	0	0	0	0	100
(7)	0	1	1	1	0	0	0	0	100	0
(8)	0	1	1	1	0	0	0	0	100	100
(9)	1	1	0	0	0	0	1	0	0	0
(10)	1	1	0	0	0	0	1	100	0	0
(11)	1	1	1	0	0	0	0	0	0	0
(12)	1	1	1	0	0	0	0	0	0	100
(13)	1	1	1	0	0	0	0	0	100	0
(14)	1	1	1	0	0	0	0	0	100	100
(15)	1	1	1	0	0	0	0	100	0	0
(16)	1	1	1	0	0	0	0	100	0	100
(17)	1	1	1	0	0	0	0	100	100	0
(18)	1	1	1	0	0	0	0	100	100	100

END

Outline

Ignoring SU and SD Power Trajectories

Generation output below minimum output is usually ignored in UCs

- SU & SD ramps are deterministic events in day-ahead UCs

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Generation output below minimum output is usually ignored in UCs

- SU & SD ramps are deterministic events in day-ahead UCs
- Ignoring them change commitment decisions and increase costs[10,11]
- This energy must be optimally allocated by day-ahead UCs

[10] G. Morales-Espana, J. M. Latorre, and A. Ramos, "Tight and compact MILP formulation of start-up and shut-down ramping in unit commitment," *IEEE Transactions on Power Systems*, vol. 28, no. 2, pp. 1288–1296, 2013

[11] G. Morales-Espana, A. Ramos, and J. Garcia-Gonzalez, "An MIP formulation for joint market-clearing of energy and reserves based on ramp scheduling," *IEEE Transactions on Power Systems*, 2013, Special Section on Electricity Markets Operation. In Press

Modeling SU and SD Power Trajectories

Total unit's production including SU & SD power trajectories[12]:

$$\widehat{p}'_t = \underbrace{\sum_{i=1}^{SU^{\mathrm{D}}} P_i^{\mathrm{SU}} v_{\left(t-i+SU^{\mathrm{D}}+2\right)}}_{\text{SU trajectory}} + \underbrace{\sum_{i=2}^{SD^{\mathrm{D}}+1} P_i^{\mathrm{SD}} w_{(t-i+2)}}_{\text{SD trajectory}} + \underbrace{\underline{P}\left(u_t + v_{t+1}\right) + p_t}_{\text{Traditional Output}}$$

- Key aspects to not destroy the *convex hull*:
  - Output above and below $\underline{P}$ are managed independently
  - Overlapping is avoided by using the min up/down constraints

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- Different SU ramps can be easily included in a similar way, for either energy- or power-based formulations[12]

[12] G. Morales-Espana, J. M. Latorre, and A. Ramos, "Tight and compact MILP formulation of start-up and shut-down ramping in unit commitment," *IEEE Transactions on Power Systems*, vol. 28, no. 2, pp. 1288–1296, 2013

Outline

Case Studies

- Formulations tested –modeling the same MIP problem:
  - *TC*[13]: Proposed Tight & Compact
  - *1bin*[14]: 1-binary variable (u)
  - *3bin*[15]: 3-binary variable version (u,v,w) + TU/TD *convex hull*

---

[13] G. Morales-Espana, J. M. Latorre, and A. Ramos, "Tight and compact MILP formulation for the thermal unit commitment problem," *IEEE Transactions on Power Systems*, 2013, Special Section on Analysis and Simulation of Very Large Power Systems. In Press

[14] M. Carrion and J. Arroyo, "A computationally efficient mixed-integer linear formulation for the thermal unit commitment problem," *IEEE Transactions on Power Systems*, vol. 21, no. 3, pp. 1371–1378, 2006

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All results are expressed as percentages of *1bin* results

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Case Study A: Self-UC (I)

Results presented as percentages of *1bin:*

	3bin (%)	*TC* (%)
Constraints	<78	<48
Nonzeros	89	72
Real Vars	33.3	33.3
Bin Vars	=300	=300

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	3bin (%)	*TC* (%)
Constraints	<78	<48
Nonzeros	89	72
Real Vars	33.3	33.3
Bin Vars	=300	=300

$\Downarrow$

TC is more Compact

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Constraints	$<$78	$<$48
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Real Vars	33.3	33.3
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Integrality Gap	34	$=$0

$\Downarrow$

TC is Tighter **and Simultaneously** more Compact

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Constraints	<78	<48
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Real Vars	33.3	33.3
Bin Vars	$=300$	$=300$
Integrality Gap	34	$=0$
MIP Sol.Time	6.3	0.192
MIP Sol.Time (best-worst)	0.46 - 87	0.001 - 3.3

$\Downarrow$

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MIP Sol.Time (best-worst)	0.46 - 87	0.001 - 3.3
LP Sol.Time	80	49.8

$\Downarrow$

TC is Tighter **and Simultaneously** more Compact

Case Study A: Self-UC (II)

Performance of the Energy-Based formulations:

	3bin (%)	*TC* (%)	*TC SU&SD*
Constraints	$<$78	$<$48	$<$59
Nonzeros	89	72	95
Real Vars	33.3	33.3	66.7
Bin Vars	=300	=300	=300
Integrality Gap	34	=0	=0
MIP Sol.Time	6.3	0.192	0.196
MIP Sol.Time (best-worst)	0.46 - 87	0.001 - 3.3	0.001 - 3.3
LP Sol.Time	80	49.8	54.1

Case Study A: Self-UC (II)

Performance of the Energy-Based formulations:

	3bin (%)	*TC* (%)	*TC SU&SD*
Constraints	<78	<48	<59
Nonzeros	89	72	95
Real Vars	33.3	33.3	66.7
Bin Vars	=300	=300	=300
Integrality Gap	34	=0	=0
MIP Sol.Time	6.3	0.192	0.196
MIP Sol.Time (best-worst)	0.46 - 87	0.001 - 3.3	0.001 - 3.3
LP Sol.Time	80	49.8	54.1

$\Downarrow$

The Energy-Based *TC* formulations describe the *convex hull*
then solving MIP (non-convex) as LP (convex)

Case Study A: Self-UC (II)

Performance of the Power-Based formulations *P-TC:*

	3bin (%)	*P-TC* (%)	*P-TC SU&SD*
Constraints	<78	<45	<56
Nonzeros	89	67	92
Real Vars	33.3	33.3	66.7
Bin Vars	=300	=300	=300
Integrality Gap	34	=0	=0
MIP Sol.Time	6.3	0.18	0.191
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Case Study B: UC for 40 power system mixes

Results presented as percentages of *1bin:*

	3bin (%)	*TC* (%)
Constraints	$<$99	$<$40
Nonzeros	$\sim$100	$<$35
Real Vars	75	50
Bin Vars	$=$300	$<$500
Integrality Gap	72	40

Case Study B: UC for 40 power system mixes

Results presented as percentages of *1bin:*

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TC is Tighter **and Simultaneously** more Compact

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Total Average Sol.Time	71	7
Sol.Time (best-worst)	11 – 269	2 – 57

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Sol.Time (best-worst)	11 – 269	2 – 57
Sol.Time Small Cases	67	11
Sol.Time Large Cases	77	4.5

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Stochastic UC: Case Study

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- 10 to 200 scenarios in demand

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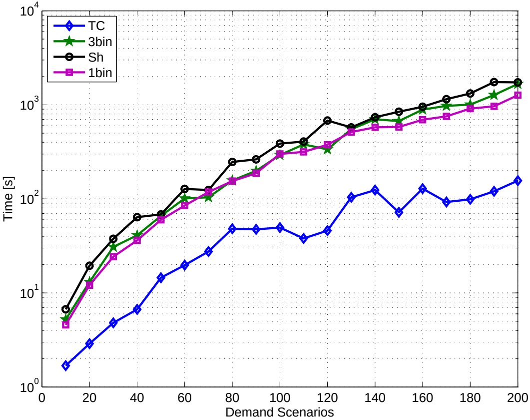
Stochastic: Cplex



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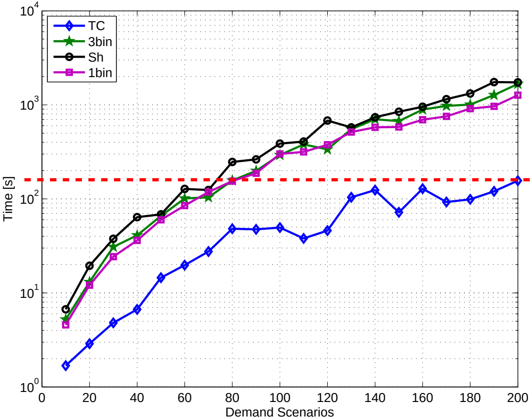
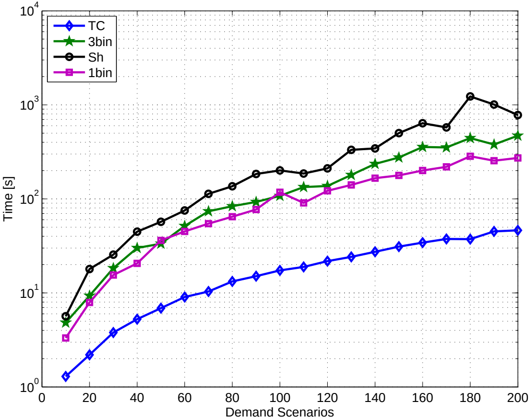


TC deals with 200 scenarios within the time that others deal with 80

Stochastic: Gurobi



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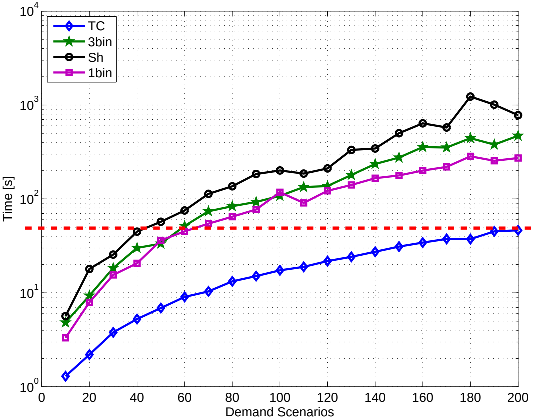
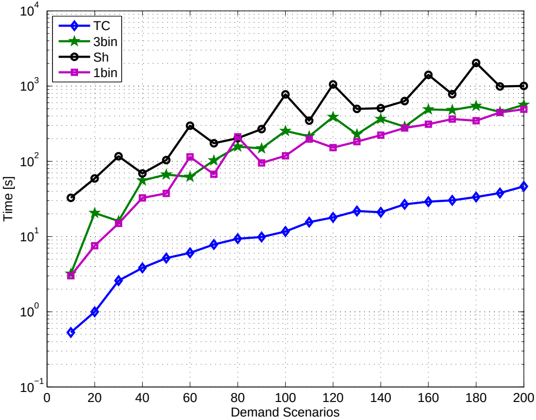


TC deals with 200 scenarios within the time that others deal with 60

Stochastic: XPRESS



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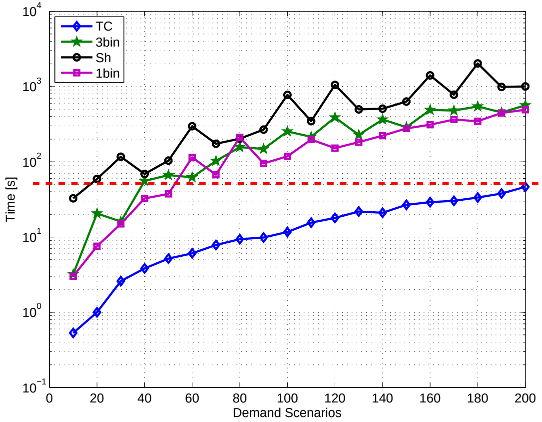

TC deals with 200 scenarios within the time that others deal with 50

Outline

Ramp-Based Scheduling Approach

- Some drawbacks of current UC formulations were identified[17]
- Then the TC-UC was reformulated for better scheduling (↓ costs)[18], mainly by introducing new features, e.g.,
  - Linear piece-wise power scheduling
  - SU & SD power trajectories

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  - SU & SD power trajectories was the main challenge
  - So, a Tight & Compact MIP formulation for SU & SD trajectories was proposed[19]

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Some Details per Unit

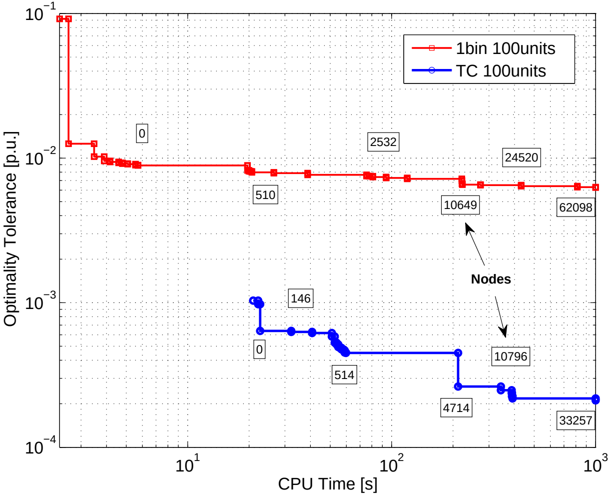
	1bin	*Ramp-Based*
Co-optimization	No	Yes
SU costs	3 types	3 types
SU ramps	–	3 types
Operating Ramps	2 types	6 types
Online reserves	1	4
Offline reserves	–	2

Problem Size

100 units for a time span of 24 hours

	1bin	*Ramp-Based*	Increase (%)
Constraints	40449	43271	6.97
Nonzeros	208445	217661	4.42
Real Vars	9624	15840	64.6
Bin Vars	2400	13650	468.8
Integrality Gap (%)	1.76	0.333	-81.1

Convergence Evolution



Conclusions

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- Better UC core $\Rightarrow$ critical solving time reductions in further UC extensions

For Further Reading

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