AUCTION DESIGN
FOR
WHOLESALE ELECTRICITY MARKETS

by
Haso Peljto
EMCon² - Energy Market Consulting, Inc.
www.EMCon2.com

FERC June 25, 2012

(763) 245-8150 haso.peljto@emcon2.com
Auction Design for Wholesale Electricity Markets

Outline

- Electricity Market Design
  - Structure/concepts/objectives

- Market Commodity Clearing
  - Economic efficiency
  - Fenchel decomposition
  - Coordination process and convergence criterion

- Market Commodity Pricing
  - Vickrey payments and participation tickets
  - Incentive compatibility
  - Voluntary participation
  - Market budget balancing

- Summary
Electricity Market Design

Proposal History

- **1986:** Economic Reform of Electricity Industry
  - Generators, loads and network as atomic economic entities

- **1990:** Existence and Optimality of Market Equilibrium
  - Walras model and Pareto optimality

- **1995:** Fenchel Duality and Decomposition
  - Convex and continuous market model formulation

- **1999:** Standard Market Design
  - Pricing scheme: worse of ex-ante LMP or ex-post as-bid price

- **2005:** MIP Co-Optimization Engine
  - Industry leading performances and functionality

- **2010:** Auction Design
  - Efficient, incentive compatible, rational and budget balanced
Electricity Market Design
*Theoretical Concepts*

- **Microeconomics**
  - Mathematical modeling instead of “economic lab”
  - Existence and optimality of competitive equilibrium

- **Game Theory**
  - Information availability and game strategy concepts
  - Existence and value of core of game

- **Auction theory**
  - Incentive compatibility and volunteer participation
  - Allocation efficiency and budget balance

- **Convex Analysis**
  - Duality theory and decomposition
  - Optimization software technology

Note: All these concepts converged into Algorithmic Mechanism Design
Electricity Market Design

Auction Design Principles

1. Economic Efficiency
   - Maximal overall market benefits; *optimal equilibrium*

2. Incentive Compatibility
   - Truthful submission of operating costs; *no market power*

3. Volunteer Participation
   - No negative market outcomes; *no need for uplift*

4. Market Balance
   - Market commodity and budget balance; *no market subsidiary*

- **Impossibility Theorem**
  
  *It is not possible to comply with all four principles under general conditions with three or more market participants*
Electricity Market Design

LMP Market vs. Design Principles

**LMP-Based Market**

- Market efficiency maximization
  - Clearing non-truthful bids/offers

- Market power always exists
  - Market power mitigation is necessary

- No guarantied profitability
  - Uplift payments are necessary

- Market budget deficit
  - Due to out-of-market payments

**Design Principles**

1. Economic Efficiency
   - Maximize total market profit

2. Incentive Compatibility
   - Truthful submissions

3. Volunteer Participation
   - Non-negative market outcomes

4. Market Balance
   - Commodity and budget balance

---

Needed: Payment System Improvements

- Incentive compatible pricing scheme
- Energy block pricing
Maximize economic efficiency of overall electricity market

\[
\max_{x^G, x^L} \left\{ \sum_L f^L(x^L) - \sum_G f^G(x^G) \mid (x^G, x^L) \in (X^G \times X^L) \cap X^N \right\}
\]

\(f^L(x^L) = -\text{Load Curtailment Costs} + \text{Min En Demand Block Benefits} + \text{Inc En Demand Benefits} - \text{AS Costs}\)

\(f^G(x^G) = \text{Generation Startup Costs} + \text{Min En Supply Block Benefits} + \text{Inc En Supply Benefits} + \text{AS Costs}\)
Market Commodity Clearing

Technology Sets

- **Supply Technology Set**

\[
X^G = \text{conv} \begin{cases}
\text{Power Output Limits : } x^G_{\min} \leq x^G \leq x^G_{\max} \\
\text{Ramping Limits : } R^G_{dn} \leq \Delta x^G \leq R^G_{up} \\
\text{Minimum ON / OFF Time} \\
\text{Max Number of Starts}
\end{cases}
\]

- **Demand Technology Set**

\[
X^L = \text{conv} \begin{cases}
\text{Power Demand Limits : } x^L_{\min} \leq x^L \leq x^L_{\max} \\
\text{Ramping Limits : } R^L_{dn} \leq \Delta x^L \leq R^L_{up} \\
\text{Maximum Curtailment Time} \\
\text{Max Number of Curtailments}
\end{cases}
\]
Market Commodity Clearing

Technology Sets cont.

Network Technology Set

\[
X^N = \text{conv}\left\{
\begin{align*}
\text{Active Power Balance: } & \quad P^G_i - P^L_i = \sum_{j \to i} P^{ij}(V^i, V^j, \theta^{ij}) \\
\text{Reactive Power Balance: } & \quad Q^G_i - Q^L_i = \sum_{j \to i} Q^{ij}(V^i, V^j, \theta^{ij}) \\
\text{Transmission Constraints: } & \quad |S^{ij}| = \sqrt{P^{ij}^2 + Q^{ij}^2} \leq S^\text{max}_{ij} \\
\text{Voltage Limits: } & \quad V^i_{\text{min}} \leq V^i \leq V^i_{\text{max}}
\end{align*}
\right\}
\]

• Note

The supply, demand and transmission network technology sets are extended to form convex hulls
Market Commodity Clearing

Fenchel Decomposition

\[
\begin{align*}
\max_{x^G, x^L} f(x^G, x^L) &= \max_{x^G, x^L} \left\{ \sum_L f^L(x^L) - \sum_G f^G(x^G) \middle| (x^G, x^L) \in (X^G \times X^L) \cap X^N \right\} = \\
\min_{c^G, c^L} \left\{ \sum_G \phi^G(c^G) + \sum_{G,L} \phi^N(c^G, c^L) + \sum_L \phi^L(c^L) \right\} &= \min_{c^G, c^L} \{ \phi(c^G, c^L) \}
\end{align*}
\]

\[
\begin{align*}
\phi^G(c^G) &= \max_{x^G} \{ c^G \cdot x^G - f^G(x^G) \middle| x^G \in X^G \} \\
\phi^N(c^G, c^L) &= \max_{x^G, x^L} \{ c^L \cdot x^L - c^G \cdot x^G \middle| (x^G, x^L) \in X^N \} \\
\phi^L(c^L) &= \max_{x^L} \{ f^L(x^L) - c^L \cdot x^L \middle| x^L \in X^L \}
\end{align*}
\]

Supply Profit + Network Profit + Demand Profit = Overall Market Profit

• Invisible Hand

If individual profit for each supply, demand and transmission entity is maximal then the overall market profit is maximal. (A. Smith)
Market Commodity Clearing

... and Coordination

SUPPLY OPTIMIZATION
Supply Schedule
Profit maximization
Supply constraints

DEMAND OPTIMIZATION
Demand Schedule
Demand benefit maximization
Demand constraints

TRANSMISSION OPTIMIZATION
Transmission Schedule
Congestion revenue maximization
Transmission constraints

MARKET COORDINATION
Price adjustment

SUPPLY OPTIMIZATION
Supply Prices
$G^x$

DEMAND OPTIMIZATION
Demand Prices
$L^x$

SUPPLY OPTIMIZATION
Supply Schedule
$G^x$

DEMAND OPTIMIZATION
Demand Schedule
$L^x$

TRANSMISSION OPTIMIZATION
Transmission Schedule

$G^c$

$G^x$

$L^c$

$L^x$
Market Commodity Clearing

Market Equilibrium

- **Optimality Condition**

\[
\begin{bmatrix}
  x_g^G - x_n^G \\
  x_n^L - x_l^L
\end{bmatrix}
= 0 \in \partial \varphi(c^G, c^L) = \partial \left\{ \sum_G \phi^G(c^G) + \sum_{G,L} \phi^N(c^G, c^L) + \sum_L \phi^L(c^L) \right\}
\]

- **Price Adjustment**

\[
\begin{bmatrix}
  \Delta c^G_{k+1} \\
  \Delta c^L_{k+1}
\end{bmatrix}
= \begin{bmatrix} c^G \\ c^L \end{bmatrix}_{k+1} - \begin{bmatrix} c^G \\ c^L \end{bmatrix}_k
= \begin{bmatrix} \alpha^G \\ \alpha^L \end{bmatrix} \cdot \begin{bmatrix} x_g^G - x_n^G \\
  x_n^L - x_l^L
\end{bmatrix}
\]

- **Supply/demand low**
Market Commodity Clearing

Supply Optimization

- Profit Maximization Objective
  \[ \phi^G (c^G) = \max_{x^G} \left\{ c^G \cdot x^G - f^G (x^G) \right\} \quad \text{for} \quad x^G \in X^G \]

- Supply Technology Set \( X^G \)
  - Power output limits
  - Ramping limits
  - Inter-temporal constraints

- Optimality Condition
  \[ c^G \in \partial f^G (x^G) \]
Market Commodity Clearing

Demand Optimization

- Benefit Maximization Objective
  \[ \phi^L(c^L) = \max_{x^L} \left\{ f^L(x^L) - c^L \cdot x^L \mid x^L \in X^L \right\} \]

- Demand Technology Set \( X^L \)
  - Consumption limits
  - Ramping limits
  - Inter-temporal constraints

- Optimality Condition
  \[ -c^L \in \partial(-f^L(x^L)) \]
Market Commodity Clearing

Transmission Network Optimization

- Maximization of Congestion Revenue

\[ \varphi^N (c^G, c^L) = \max_{x^G, x^L} \left\{ c^L \cdot x^L - c^G \cdot x^G \mid (x^G, x^L) \in X^N \right\} \]

\[ = \max_{x^G, x^L} \left\{ c^L \cdot x^L - c^G \cdot x^G - \delta(X^N) \right\} \]

\[ \delta(X^N) = \begin{cases} 0 & (x^G, x^L) \in X^N \\ \infty & (x^G, x^L) \notin X^N \end{cases} \]

- Optimality Condition

\[ (-c^G, c^L) \in \partial \delta(X^N) \]

- Congestion Revenue

\[ \varphi^N_{\text{max}} = c^L \cdot x^L_{\text{opt}} - c^G \cdot x^G_{\text{opt}} \geq 0 \]
Market Commodity Pricing

**Market Power Existence**

- **Supply Monopoly**
  \[
  \max_{x^G} \left\{ c^G(x^G) \cdot x^G - f^G(x^G) \right\} \quad x^G \in X^G
  \]
  \[
  c^G(x^G) + x^G \cdot \frac{dc^G(x^G)}{dx^G} = \frac{df^G(x^G)}{dx^G}
  \]

- **Demand Monopsony**
  \[
  \max_{x^L} \left\{ f^L(x^L) - c^L(x^L) \cdot x^L \right\} \quad x^L \in X^L
  \]
  \[
  \frac{df^L(x^L)}{dx^L} = c^L(x^L) + x^L \cdot \frac{dc^L(x^L)}{dx^L}
  \]
Market Commodity Pricing

**Vickrey Payments**
- Suppliers paid all created profit

**Vickrey Charges**
- Consumers charge only for incurred costs

**Budget Deficit**
- Both supply and demand entities contribute to budget deficit
Market Commodity Pricing

*Incentive Compatibility*

- **Supply Incentive Compatibility**

- **Demand Incentive Compatibility**

**Diagram**

- **Price** [$/MWh]
- **Supply Profit**
- **Supply Payment**
- **Supply Price Curve**
- **Demand Profit**
- **Demand Charge**
- **Demand Price Curve**
Market Commodity Pricing

**Participation Tickets**

- **Supply Ticket Prices**
  - Added to offers to sell
- **Demand Ticket Prices**
  - Subtracted from bids to buy
- **Budget Balance**
  - Deficit reduced
  - Remaining deficit distributed to price takers
- **Economic Efficiency**
  - Slightly degraded

![Graph showing market commodity pricing with supply and demand ticket prices, and budget balance.]
Market Commodity Pricing

*Budget Deficit Distribution*

- **Supply and Demand Price Takers**
  - Do not submit bid and offer prices; no market power
  - Self-committed, self-scheduled and self-dispatched market commodities
  - Price non-sensitive market participants
  - Out-of-market entities

- **Price Taker Charges**
  - Pro-rata distribution of market budget deficit after each market run; always balanced budget
  - If price takers market share is significant than participation tickets are not needed
Market Commodity Pricing

Pricing Engine

For Each Market Participant

Exclude Bids/Offers for Selected Market Participant

MARKET COMMODITY CLEARING ENGINE

Calculate Vickrey Payments and Charges

Market Commodity Awards

Calculate Ticket Charges

Distribute Budget Deficits

Submitted Bids/Offer
Electricity Market Design

Summary of Results

- Efficient Auction Design
  - Economic efficiency for overall electricity market using competitive trading platform
  - Separate market commodity clearing and market commodity pricing

- Market Commodity Clearing
  - Decomposition into atomic supply/demand/transmission optimization sub-problems
  - Supply and demand optimizations are convexified MIP models
  - Transmission optimization is convexified MIP model with linear objective and AC power flow equations
  - Optimization software can be utilized on multi-processor platforms
Electricity Market Design

Summary of Results cont...

- **Market Commodity Pricing**
  - Continuous and discrete commodity pricing
  - Incentive compatible strategies
  - Voluntary participation with free market exit
  - Budget balance at every market run

<table>
<thead>
<tr>
<th>Market profit reduction</th>
<th>1. Economic Efficiency</th>
</tr>
</thead>
<tbody>
<tr>
<td>• Due to market participation tickets</td>
<td>• Maximize total market profit</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Market power does not exist</th>
<th>2. Incentive Compatibility</th>
</tr>
</thead>
<tbody>
<tr>
<td>• Due to Vickrey payments/charges</td>
<td>• Truthful submissions</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Guarantied profitability</th>
<th>3. Volunteer Participation</th>
</tr>
</thead>
<tbody>
<tr>
<td>• Due to Vickrey payments/charges</td>
<td>• Non-negative market outcomes</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Market budget balanced</th>
<th>4. Market Balance</th>
</tr>
</thead>
<tbody>
<tr>
<td>• Due to deficit distribution</td>
<td>• Commodity and budget balance</td>
</tr>
</tbody>
</table>
Electricity Market Design

Next Steps

- Prototype MIP Decomposition Engine
  - Specify into details MIP models for supply, demand and transmission optimization problems
  - Develop Fenchel coordination mechanism
  - Evaluate performances and solution quality

- Prototype Pricing Engine
  - Calculate Vickrey payments and charges
  - Distribute budget deficit
  - Evaluate data volumes and calculation performances

- Proof the Concept of Auction Design
Electricity Market Design

References

Electricity Market Design

Discussion

THANK YOU!