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Linear-Programming Approximations of AC Power Flows

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Australian Government
Department of Broadband, Communications and the Digital Economy
Australian Research Council

NICTA Funding and Supporting Members and Partners
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Outline

- Motivation
- The LPAC Models
- Experimental Results
  - LDC versus LPAC versus AC solutions
  - LPAC Variants
- Capacitor Placement Problem
- Power Restoration

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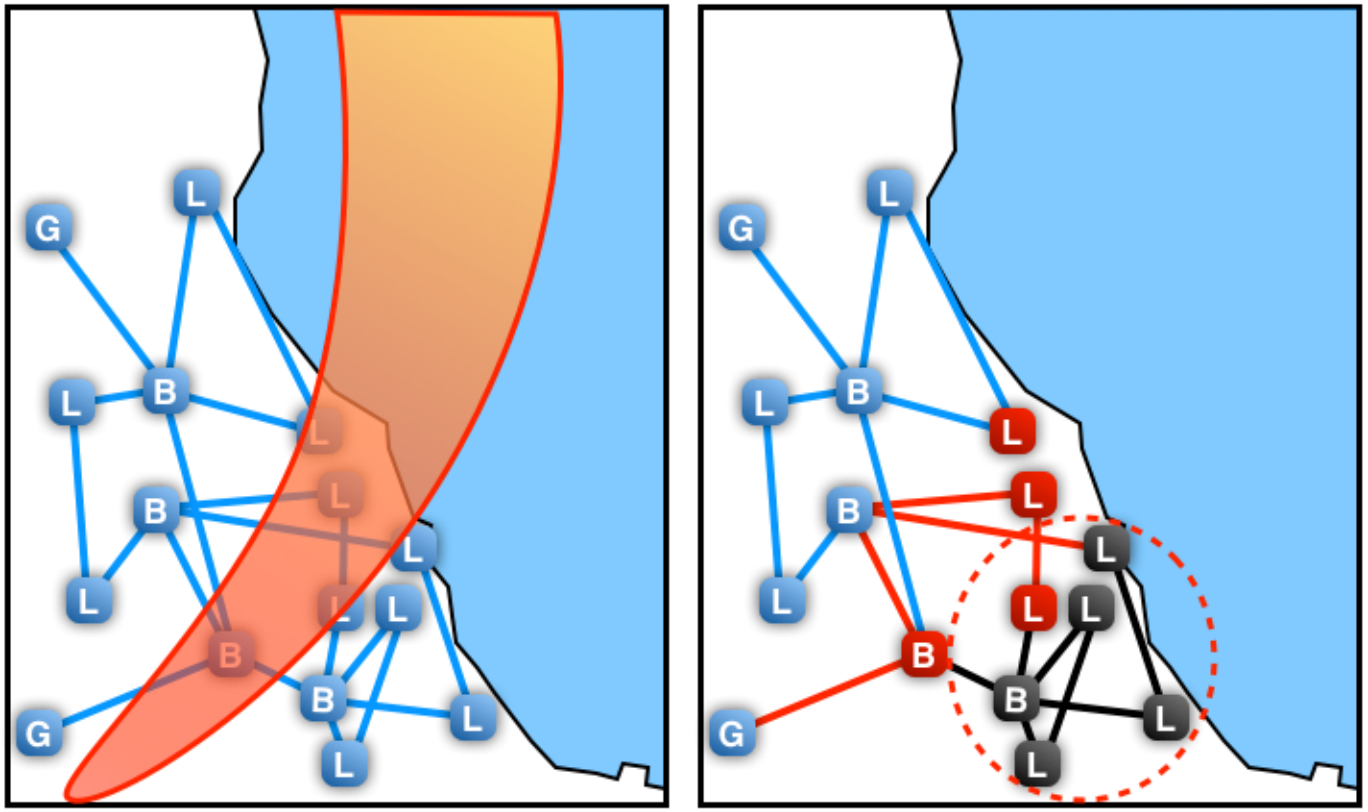
Motivation

Collaboration with LANL

Motivation



Power Restoration

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- One challenge (PSCC'11)
  - Schedule a fleet of repair crews to repair the grid and minimize the overall size of the blackout after a disaster
- Two fundamental aspects
  - Scheduling the repairs
  - Scheduling the power restoration
  - Both are challenging in their own right
- Assumptions for Last-Mile Restoration
  - Steady state behavior of the power grid
  - Ability to shed load and generation continuously
  - Transient/configuration aspects in a second step

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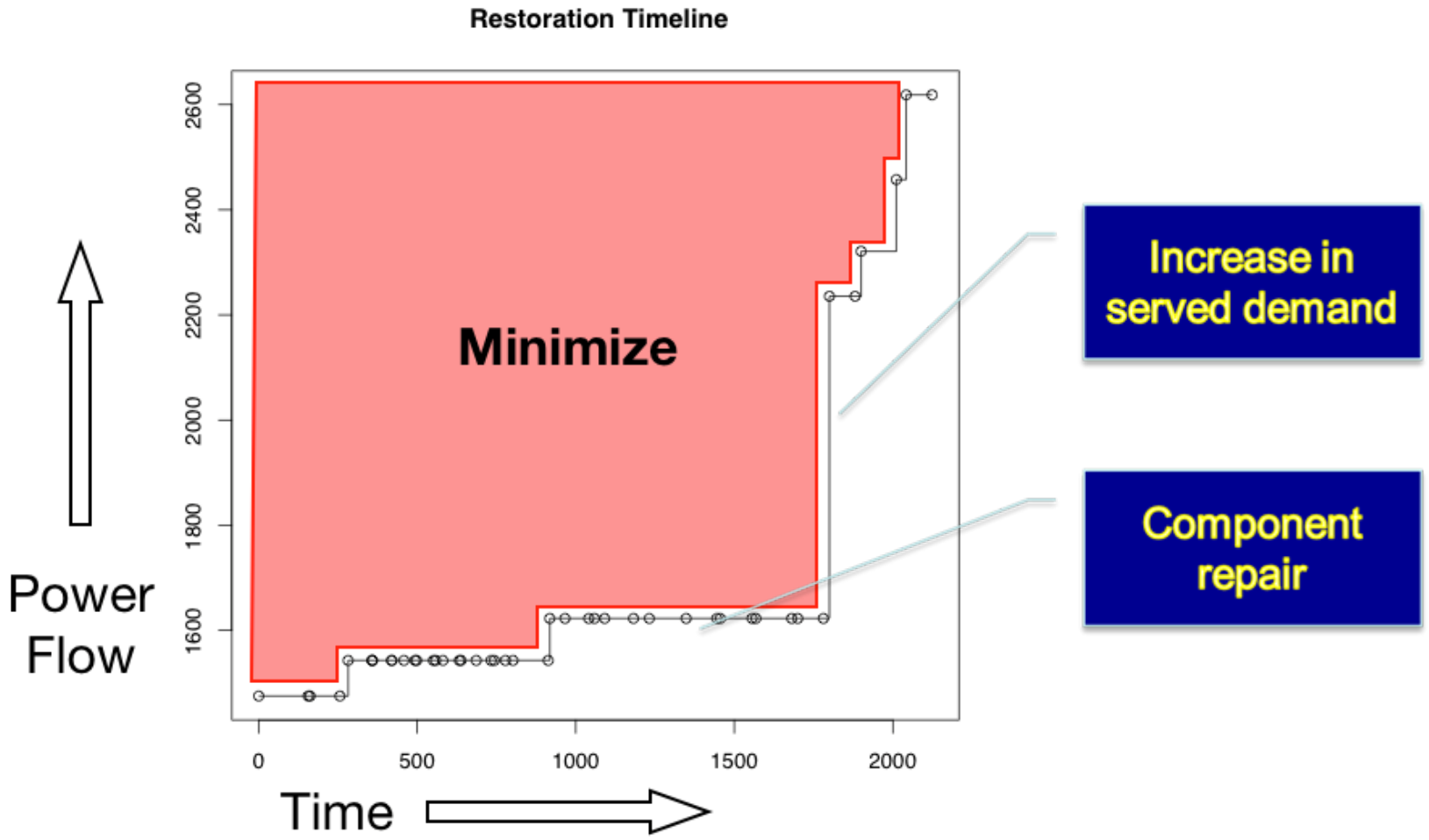
Power Restoration



Power Restoration

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- Optimal Activation Problem
  - Generalized optimal line switching [Fisher et al, 98]
- Approximate the power flows equations
  - Linear DC Model
- Discrete optimization over the LDC model
  - MIP solver
- Solutions to large benchmarks [CPAIOR'12]
  - 4000 components, a third of which were damaged
  - Using hybrid optimization (MIP + CP + LNS)

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Optimal Activation

- find which items to activate
- find how much power to produce and consume
- find the phase angles at buses
- to maximize the served load

- Generalized optimal line switching [Fisher et al, 98]

Inputs:

$\mathcal{PN} = \langle N, L \rangle$	the power network
D	the set of damaged items
R	the set of repaired items

Variables:

$y_i \in \{0, 1\}$	- item i is activated
$z_i \in \{0, 1\}$	- item i is operational
$P_i^l \in (-\hat{P}_i^l, \hat{P}_i^l)$	- power flow on line i
$P_i^v \in (0, \hat{P}_i^v)$	- power flow on node i
$\theta_i \in (-\frac{\pi}{6}, \frac{\pi}{6})$	- phase angle on bus i

Maximize

$$\sum_{b \in N^b} \sum_{i \in N_b^l} P_i^v \quad (1)$$

Subject to:

$$y_i = 1 \quad \forall i \in N \setminus D \quad (2)$$

$$y_i = 0 \quad \forall i \in D \setminus R \quad (3)$$

$$z_i = y_i \quad \forall i \in N^b \quad (4)$$

$$z_i = y_i \wedge y_j \quad \forall j \in N^b, \forall i \in N_j^g \cup N_j^l \quad (5)$$

$$z_i = y_i \wedge y_{L_i^+} \wedge y_{L_i^-} \quad \forall i \in L \quad (6)$$

$$\sum_{j \in N_i^l} P_j^v = \sum_{j \in N_i^g} P_j^v + \sum_{j \in LI_i} P_j^l - \sum_{j \in LO_i} P_j^l \quad \forall i \in N^b \quad (7)$$

$$0 \leq P_i^v \leq \hat{P}_i^v * z_i \quad \forall i \in N_j^g \cup N_j^l \quad (8)$$

$$-\hat{P}_i^l * z_i \leq P_i^l \leq \hat{P}_i^l * z_i \quad \forall i \in L \quad (9)$$

$$P_i^l \geq B_i * (\theta_{L_i^+} - \theta_{L_i^-}) + M * (\neg z_i) \quad \forall i \in L \quad (10)$$

$$P_i^l \leq B_i * (\theta_{L_i^+} - \theta_{L_i^-}) - M * (\neg z_i) \quad \forall i \in L \quad (11)$$

Figure 1: A MIP Model for the Unserved Load.

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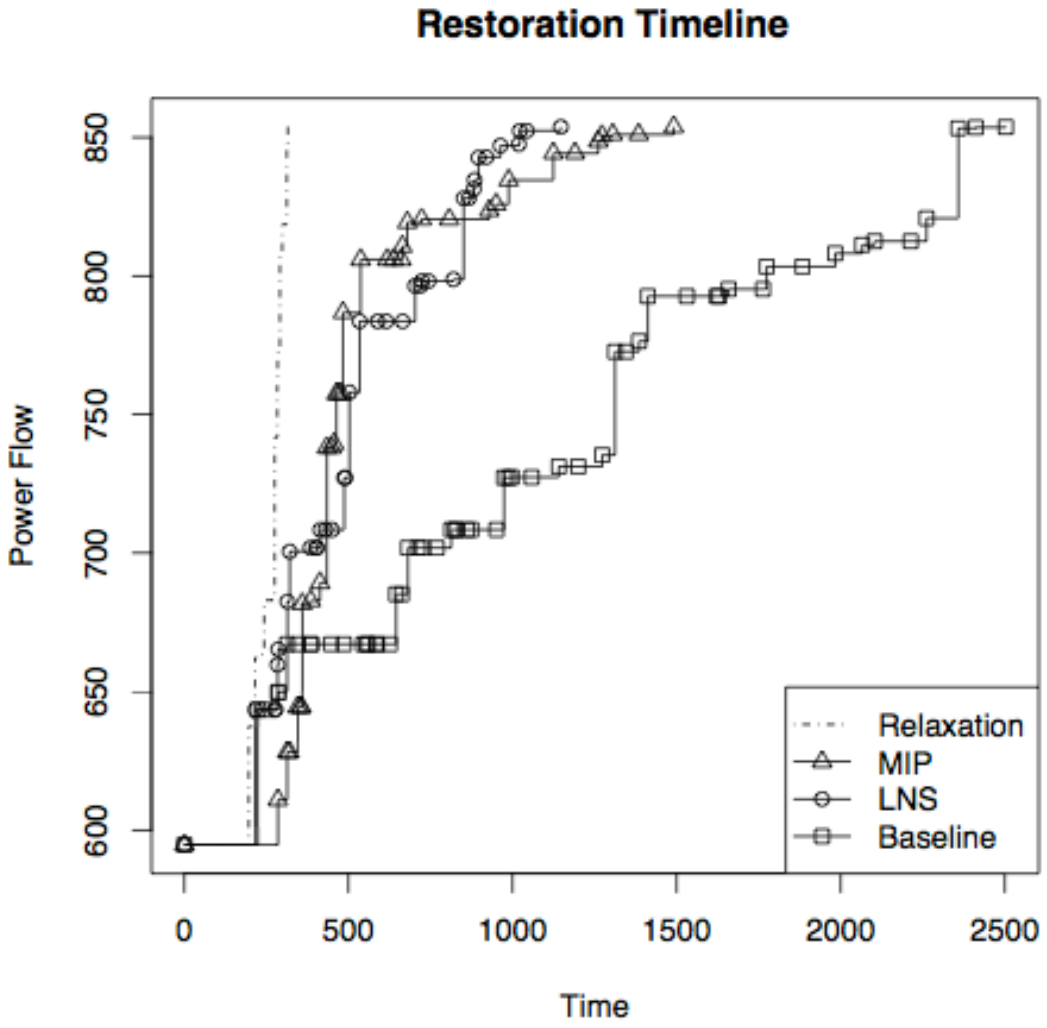
Power Restoration



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Power Restoration

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- A fundamental open question
  - Is this “optimal” restoration plan “feasible” operationally?

- These are not normal operating conditions
  - “Maddeningly difficult” to find an AC solution in cold start contexts [Overbye et al, 2004]
- The network is stressed
  - Does the LDC model “overfit”?
- How accurate is the LDC model?
  - Can the LDC solution be turned into an AC solution?

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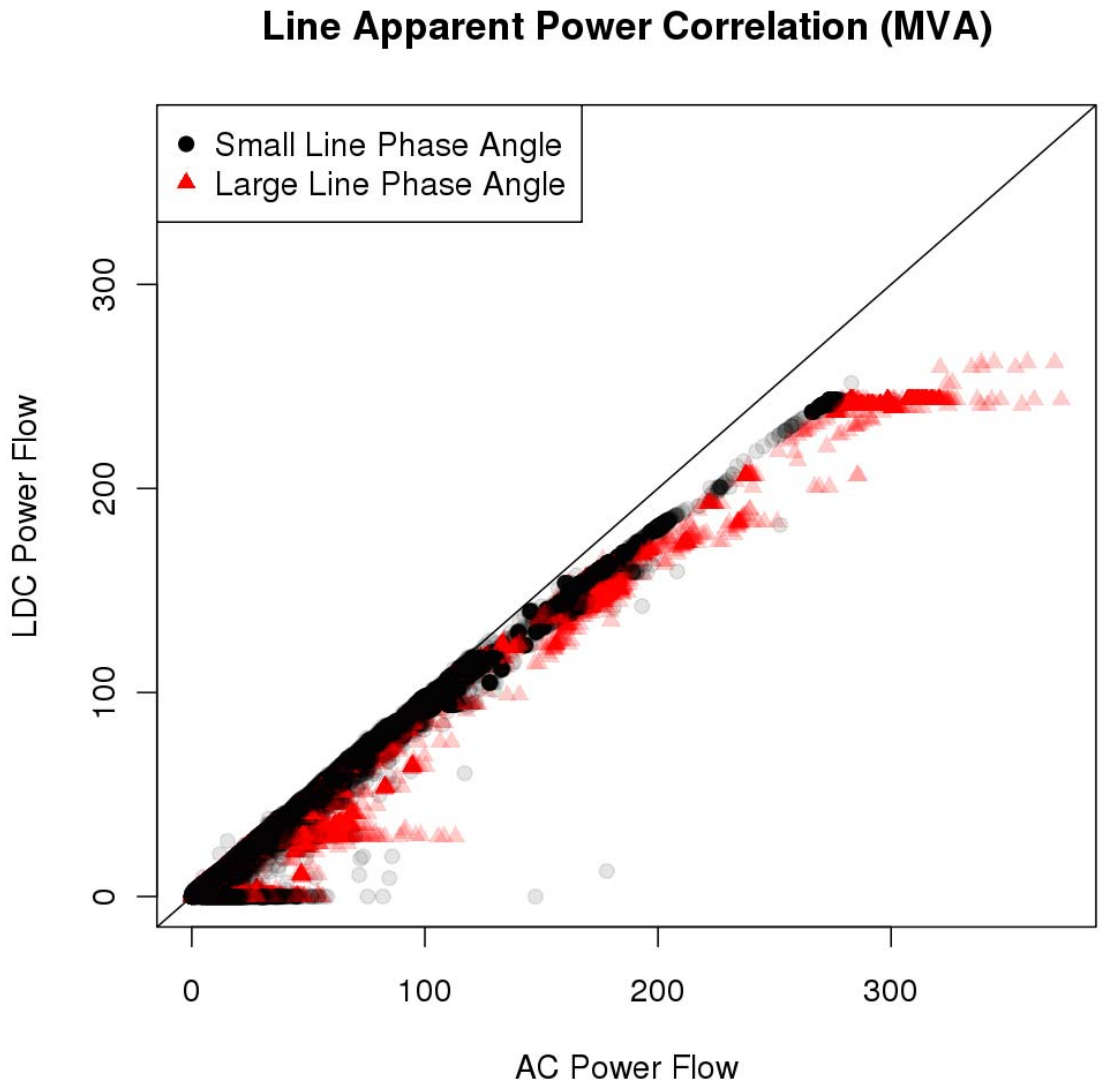
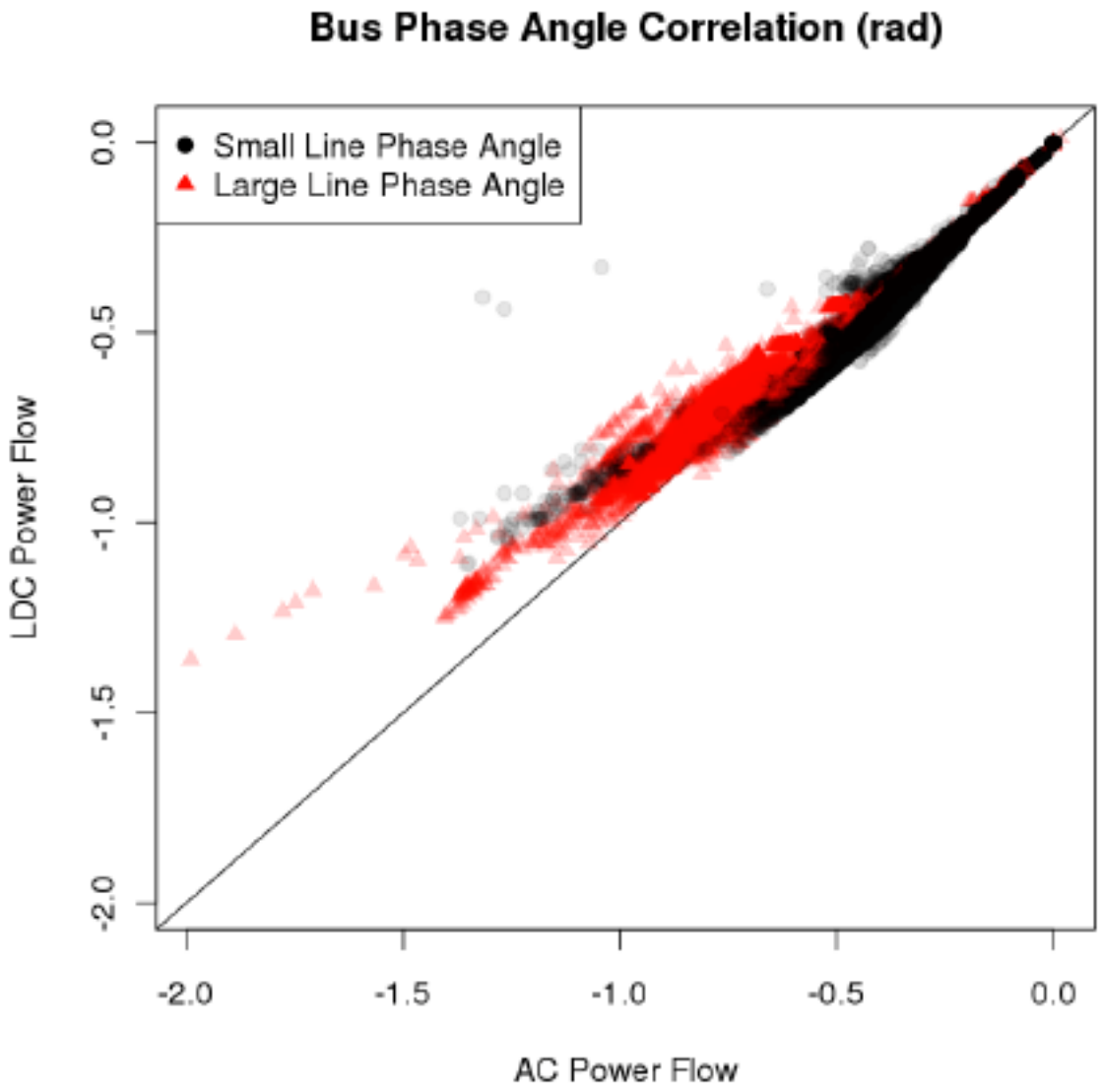
N-3 Contingencies (IEEE-30)



IEEE PES'12

N-3 Contingencies



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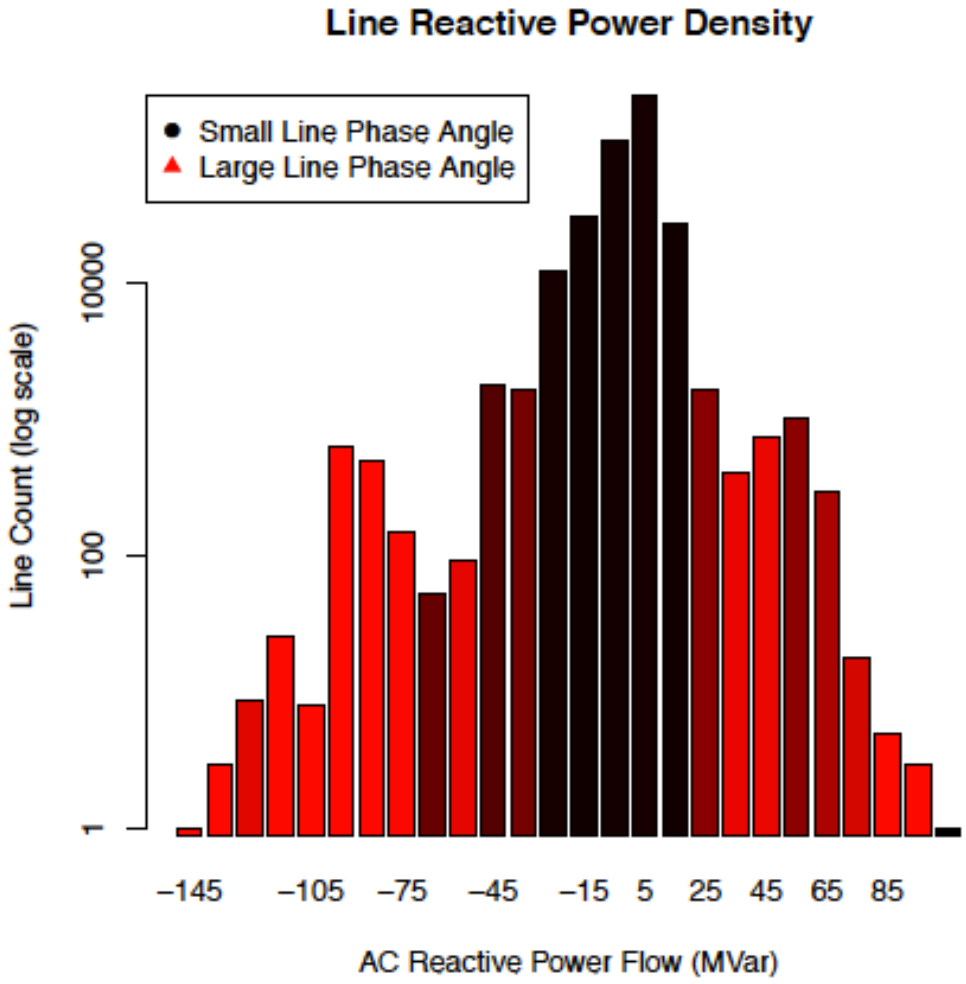
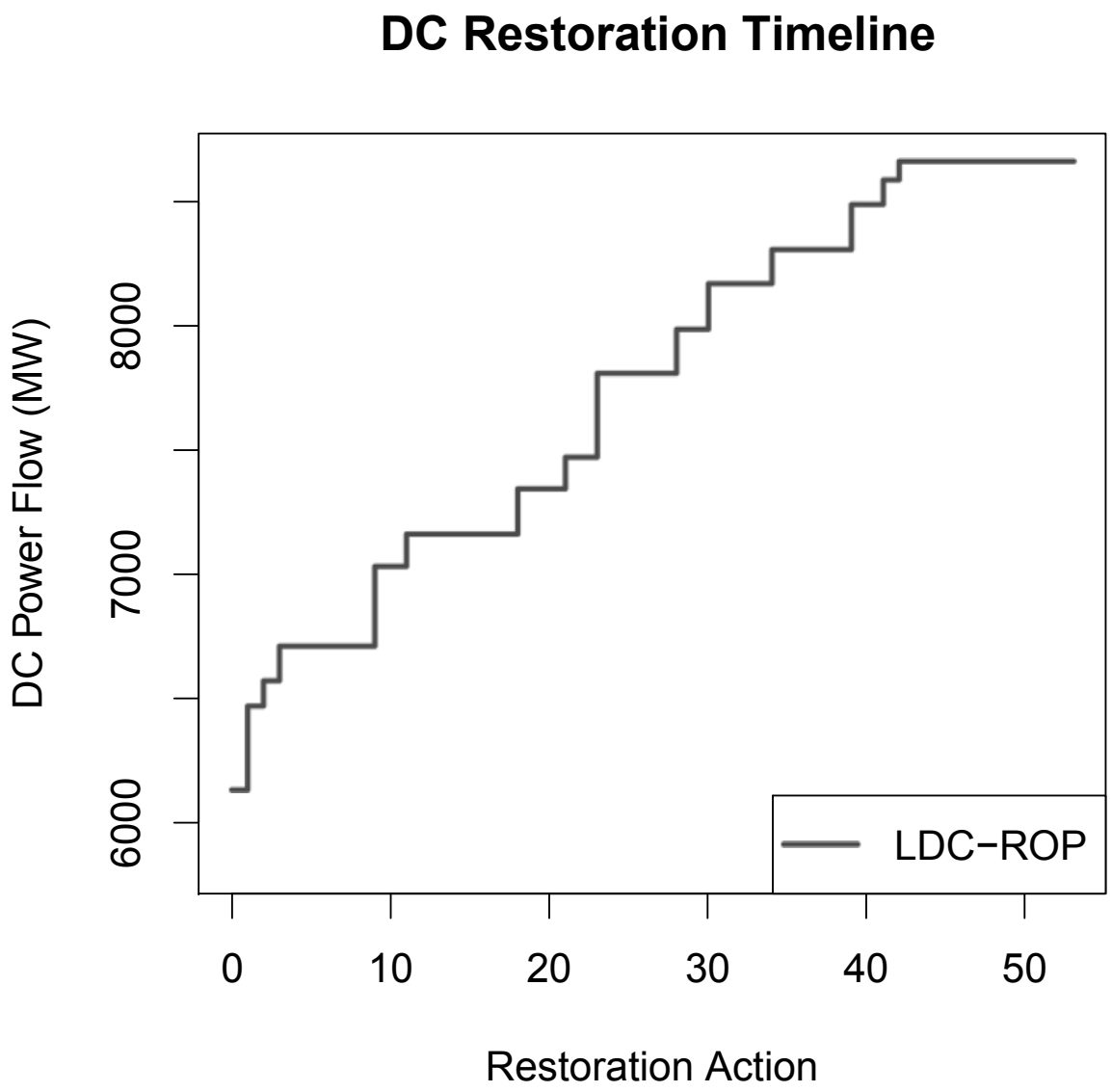
N-3 Contingencies



Power Restoration



Power Restoration

Research Agenda

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- Find an approximation of AC power flows that
  - is more accurate than the LDC model
  - is useful outside normal operating conditions
  - reasons about voltage magnitudes and reactive power
  - can be embedded in discrete optimization solvers
    - mixed integer programming solvers
- Applications
  - Power restoration, vulnerability analysis, capacitor placement, expansion planning, …

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AC Power Flow

$$
\begin{aligned}
p_n &= \sum_{m}^{n \neq m} p_{nm} \\
q_n &= \sum_{m}^{n \neq m} q_{nm} \\
p_{nm} &= |\widetilde{V}_n|^2 g_{nm} - |\widetilde{V}_n||\widetilde{V}_m| g_{nm} \cos(\theta_n^\circ - \theta_m^\circ) - |\widetilde{V}_n||\widetilde{V}_m| b_{nm} \sin(\theta_n^\circ - \theta_m^\circ) \\
q_{nm} &= -|\widetilde{V}_n|^2 b_{nm} + |\widetilde{V}_n||\widetilde{V}_m| b_{nm} \cos(\theta_n^\circ - \theta_m^\circ) - |\widetilde{V}_n||\widetilde{V}_m| g_{nm} \sin(\theta_n^\circ - \theta_m^\circ)
\end{aligned}
$$

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Linear Programming Approximations

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- Hot-Start Context
  - An AC base-point solution is available
- Warm-Start Context
  - Target voltage magnitudes are available and “useful”
  - E.g., from normal operating conditions
- Cold-Start Context
  - No useful information is available on voltage magnitudes

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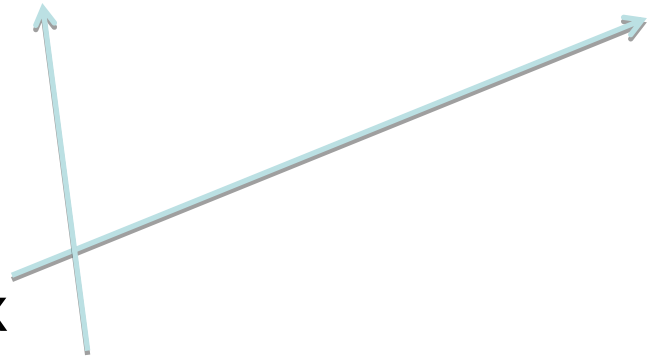
Hot-Start LP Approximation

$$\hat{p}^h_{nm} = |\widetilde{V}^h_n|^2 g_{nm} - |\widetilde{V}^h_n||\widetilde{V}^h_m| g_{nm} \widehat{\cos}(\theta^\circ_n - \theta^\circ_m) - |\widetilde{V}^h_n||\widetilde{V}^h_m| b_{nm} (\theta^\circ_n - \theta^\circ_m)$$

$$\hat{q}^h_{nm} = -|\widetilde{V}^h_n|^2 b_{nm} + |\widetilde{V}^h_n||\widetilde{V}^h_m| b_{nm} \widehat{\cos}(\theta^\circ_n - \theta^\circ_m) - |\widetilde{V}^h_n||\widetilde{V}^h_m| g_{nm} (\theta^\circ_n - \theta^\circ_m)$$



- Two approximations
  - sin(x) is approximated by x
  - piecewise approximation of cos(x)

Hot-Start LP Approximation

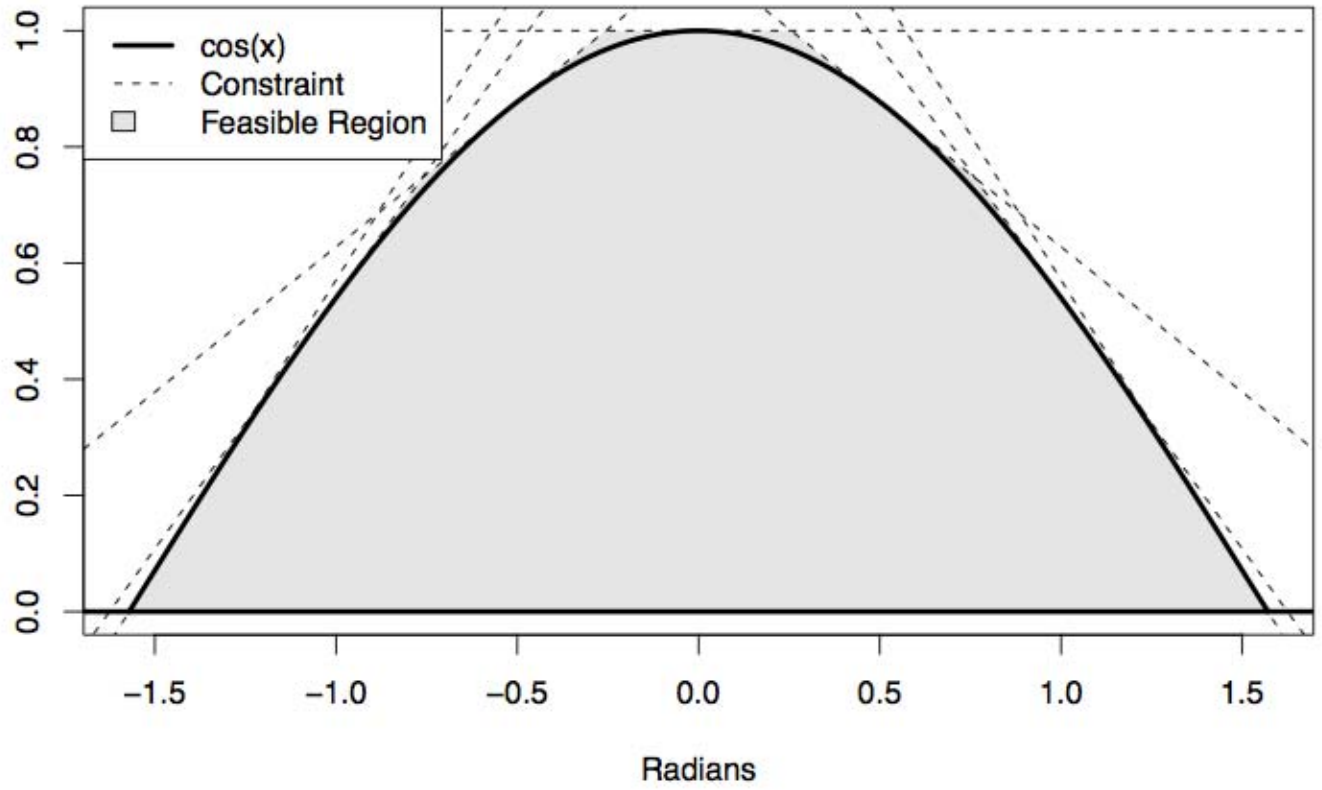


Fig. 1. A Piecewise-Linear Approximation of Cosine using 7 Inequalities.

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Warm-Start LP Approximation

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- Understanding power flows [Grainger, 94]
  - Phase angle differences determine active power
  - Voltage magnitude differences determine reactive power

- Experiments
  - Per unit system
  - Look at how the equations behave when
    - g = 0.2 and b = 1.0

$$|\widetilde{V}_n| = 1.0, |\widetilde{V}_m| \in (1.2, 0.8), \theta_n^\circ - \theta_m^\circ \in (-\pi/6, \pi/6)$$

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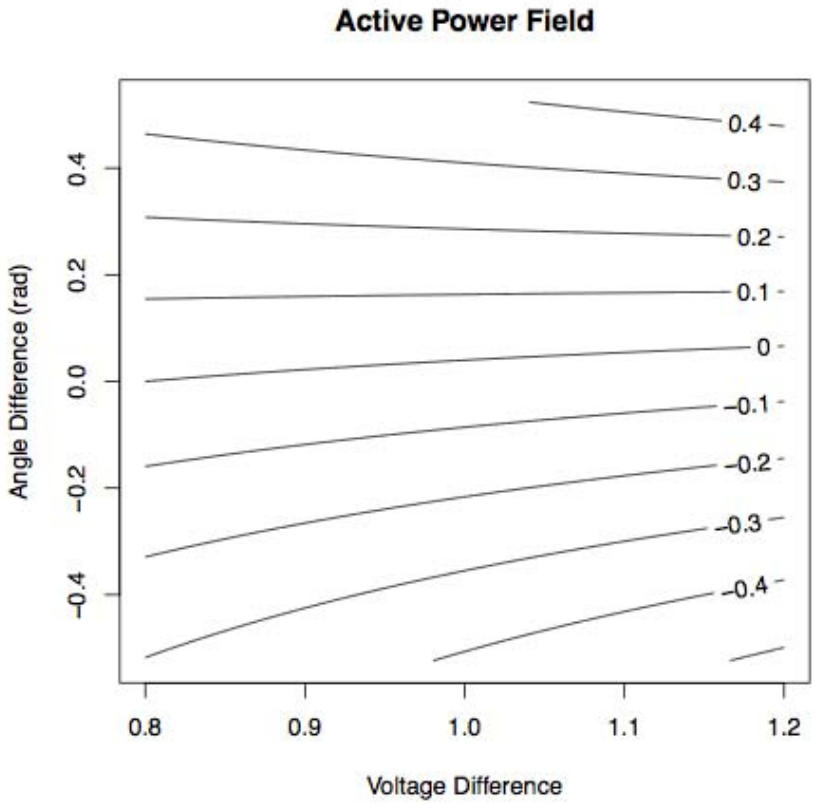
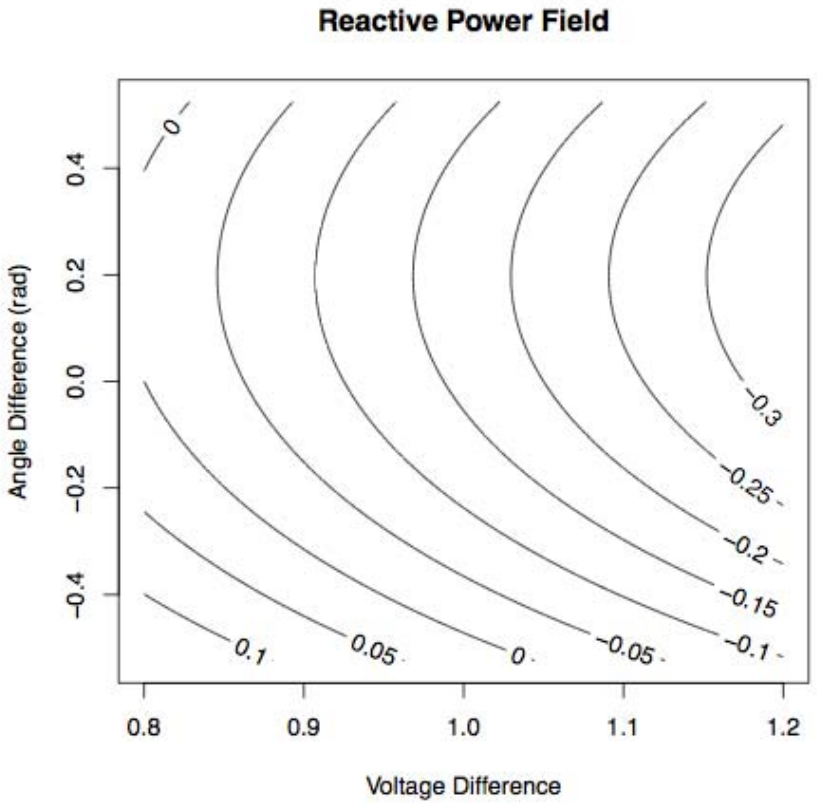
Warm-Start LP Approximation





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Warm-Start LP Approximation

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- Assumptions
  - We have target voltages
- Basic approach
  - Active power as in the hot-start model
  - Reactive power should capture voltage magnitudes and phase angles
- Key idea
  - Substitute $|\widetilde{V}| = |\widetilde{V}^t| + \phi$ into the power flow equations

Warm-Start LP Approximation

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- Reactive power

$$q_{nm} = q^t_{nm} + q^{\Delta}_{nm}$$

- Target part

$$q^t_{nm} = -|\widetilde{V}^t_n|^2 b_{nm} + |\widetilde{V}^t_n||\widetilde{V}^t_m| b_{nm} \cos(\theta^\circ_n - \theta^\circ_m) - |\widetilde{V}^t_n||\widetilde{V}^t_m| g_{nm} \sin(\theta^\circ_n - \theta^\circ_m)$$

- Delta part

$$q^{\Delta}_{nm} = -(2|\widetilde{V}^t_n|\phi_n + \phi^2_n) b_{nm} - (|\widetilde{V}^t_n|\phi_m + |\widetilde{V}^t_m|\phi_n + \phi_n\phi_m)(g_{nm} \sin(\theta^\circ_n - \theta^\circ_m) - b_{nm} \cos(\theta^\circ_n - \theta^\circ_m))$$

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Warm-Start LP Approximation

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- Target part approximation

$$\hat{q}_{nm}^{t} = -|\widetilde{V}_n^t|^2 b_{nm} + |\widetilde{V}_n^t||\widetilde{V}_m^t| b_{nm} \widehat{\cos}(\theta_n^\circ - \theta_m^\circ) - |\widetilde{V}_n^t||\widetilde{V}_m^t| g_{nm} (\theta_n^\circ - \theta_m^\circ)$$

- Delta part approximation

$$\hat{q}_{nm}^{\Delta} = -|\widetilde{V}_n^t| b_{nm} (\phi_n - \phi_m) - (|\widetilde{V}_n^t| - |\widetilde{V}_m^t|) b_{nm} \phi_n$$

Model 1 The Warm LPAC Model.

Inputs:

$\mathcal{PN} = \langle N, L, G, s \rangle$	- the power network
$\lvert\widetilde{V}^t\rvert$	- target voltage magnitudes
cs	- cosine approximation segment count

Variables:

$\theta^\circ_n \in (-\infty, \infty)$	- phase angle on bus n (radians)
$\phi_n \in (-\lvert V^t\rvert, \infty)$	- voltage change on bus n (Volts p.u.)
$\widehat{cos}_{nm} \in (0, 1)$	- Approximation of $\cos(\theta^\circ_n - \theta^\circ_m)$

Maximize:

$$\sum_{\langle n,m\rangle \in L} \widehat{cos}_{nm} \qquad \text{(M1.1)}$$

Subject to:

$$\theta^\circ_s = 0, \phi_s = 0 \qquad \text{(M1.2)}$$

$$\phi_i = 0 \;\; \forall i \in G \qquad \text{(M1.3)}$$

$$p_n = \sum_{\substack{m \in N \\ n \neq m}} \hat{p}^t_{nm} \;\; \forall n \in N \;\; n \neq s \qquad \text{(M1.4)}$$

$$q_n = \sum_{\substack{m \in N \\ n \neq m}} \hat{q}^t_{nm} + \hat{q}^\Delta_{nm} \;\; \forall n \in N \;\; n \neq s \;\; n \notin G \qquad \text{(M1.5)}$$

$$\forall \langle n, m\rangle, \langle m, n\rangle \in L$$

$$\hat{p}^t_{nm} = |\widetilde{V}^t_n|^2 g_{nm} - |\widetilde{V}^t_n||\widetilde{V}^t_m|(g_{nm}\widehat{\cos}_{nm} + b_{nm}(\theta^\circ_n - \theta^\circ_m)) \qquad \text{(M1.6)}$$

$$\hat{q}^t_{nm} = -|\widetilde{V}^t_n|^2 b_{nm} - |\widetilde{V}^t_n||\widetilde{V}^t_m|(g_{nm}(\theta^\circ_n - \theta^\circ_m) - b_{nm}\widehat{\cos}_{nm}) \qquad \text{(M1.7)}$$

$$\text{PWL}\langle COS\rangle(\widehat{\cos}_{nm}, (\theta^\circ_n - \theta^\circ_m), -\pi/3, \pi/3, cs) \qquad \text{(M1.8)}$$

$$\hat{q}^\Delta_{nm} = -|\widetilde{V}^t_n| b_{nm}(\phi_n - \phi_m) - (|\widetilde{V}^t_n| - |\widetilde{V}^t_m|) b_{nm}\phi_n \qquad \text{(M1.9)}$$

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Cold-Start LP Approximation

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- Simply use the warm-start model with
  - Target voltages at 1.0
  - Use an appropriate ϕ for voltage-controlled generators

- Note that the delta part of reactive power becomes

$$\hat{q}_{nm}^{\Delta} = -b_{nm}(\phi_n - \phi_m)$$

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Extensions of the LPAC Model

- Range for generators
  - Simply include a decision variable
- Removing the slack bus
  - No need for a slack bus in the LPAC model
- Shedding load
  - Simply use decision variables for loads
- Additional constraints
  - Voltages: $\underline{|V|} \leq |V_n^t| + \phi_n \quad \forall n \in N$
  - Apparent power: $(\hat{p}_{nm}^t)^2 + (\hat{q}_{nm}^t + \hat{q}_{nm}^\Delta)^2 \leq \overline{|S_{nm}|}^2$
  - Reactive power $\sum_{m \in N}^{n \neq m} \hat{q}_{nm}^t + \hat{q}_{nm}^\Delta \leq \overline{q_n} \quad \forall n \in G$

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Experimental Results

- Wide variety of IEEE and MATPOWER Benchmarks
  - ieee14,mp24,ieee30,mp30,mp39,ieee57,ieee118,ieedd17,mp300
  - Small benchmarks are easy in general
  - IEEE 118 is also easy
  - All LPAC models solved almost instantly (LPs)
- This talk
  - MP300 for scalability and brevity
- Comparison with an AC Solver
  - LDC and LPAC solutions versus an AC solution
- Comparison with alternative linearizations
  - Evaluating the importance of all components

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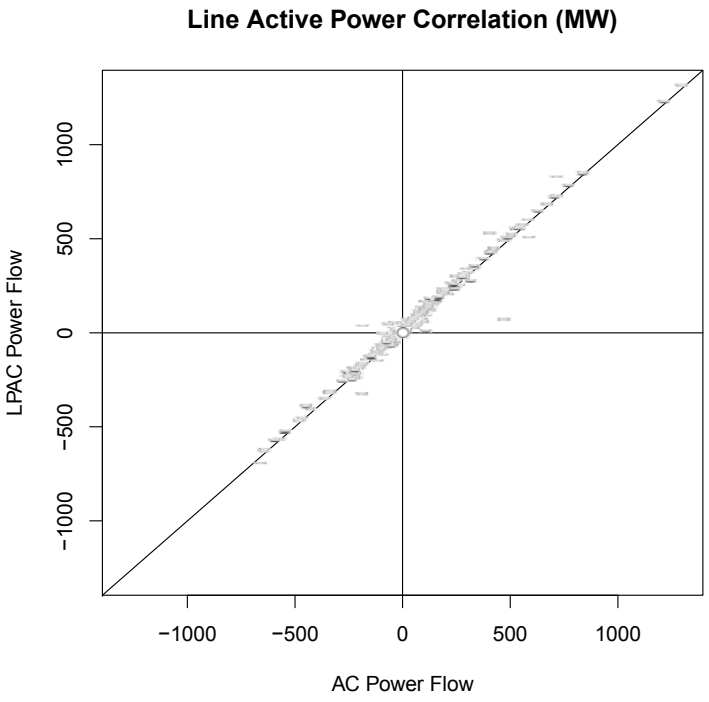
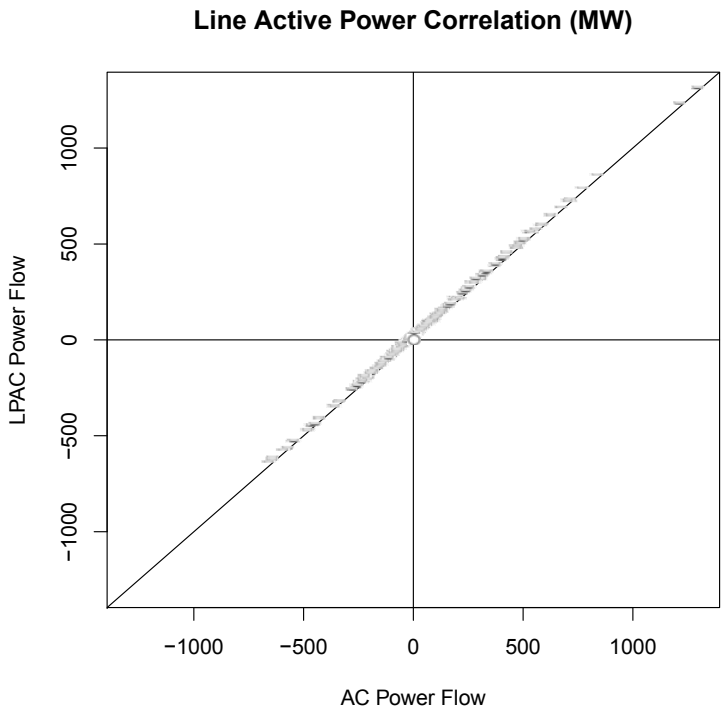
Line Active Power



LDC Model



Cold-start LPAC Model

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Line Active Power

Table 1: Active Power Flow Accuracy Comparison

Benchmark	Active Power (MW)							
	Corr	$\mu(\Delta)$	$\max(\Delta)$	$\delta(\arg\max(\Delta))$	$\mu(\delta)$	$\max(\delta)$	$\Delta(\arg\max(\delta))$	approx(%)
The LDC Model								
ieee14	0.9994	1.392	10.64	6.783	6.052	24.33	0.3927	65
mp24	0.9989	5.659	19.7	23.65	6.447	29.89	6.656	47.06
ieee30	0.9993	1.046	13.1	7.562	6.406	31.23	0.5646	80.49
mp30	0.9993	0.2964	2.108	19.36	3.086	19.36	2.108	82.93
mp39	0.9995	7.341	43.64	6.527	9.566	52.18	12.86	76.09
ieee57	0.9989	1.494	8.216	8.055	105.8	4193	0.9607	52.56
ieee118	0.9963	3.984	56.3	44.74	29.2	445.9	6.526	51.96
ieeedd17	0.9972	4.933	201.3	13.84	15.2	215	0.5265	50.71
ieeedd17m	0.9975	4.779	191.1	13.23	14.56	231.3	3.066	51.43
mp300	0.991	11.09	418.5	90.02	29.35	2859	46.14	67.73
The LPAC-Cold Model								
ieee14	0.9989	1.636	5.787	13.13	11.52	35.67	2.623	40
mp24	0.9999	1.884	6.159	2.933	3.871	17.23	3.837	41.18
ieee30	0.9998	0.5475	2.213	2.523	5.751	31.33	0.5666	75.61
mp30	0.9995	0.2396	1.641	15.07	2.402	15.07	1.641	78.05
mp39	1	2.142	8.043	3.288	4.357	24.78	6.106	43.48
ieee57	0.9995	0.9235	4.674	9.728	110.1	4500	1.031	46.15
ieee118	1	0.622	3.708	2.038	5.318	99.61	0.5519	55.31
ieeedd17	0.9999	1.827	30.38	2.088	10.92	420.2	1.029	55.36
ieeedd17m	0.9999	1.475	20.21	1.399	7.766	144.5	2.547	56.79
mp300	0.9998	2.455	18	8.675	7.104	337.2	6.95	57.21

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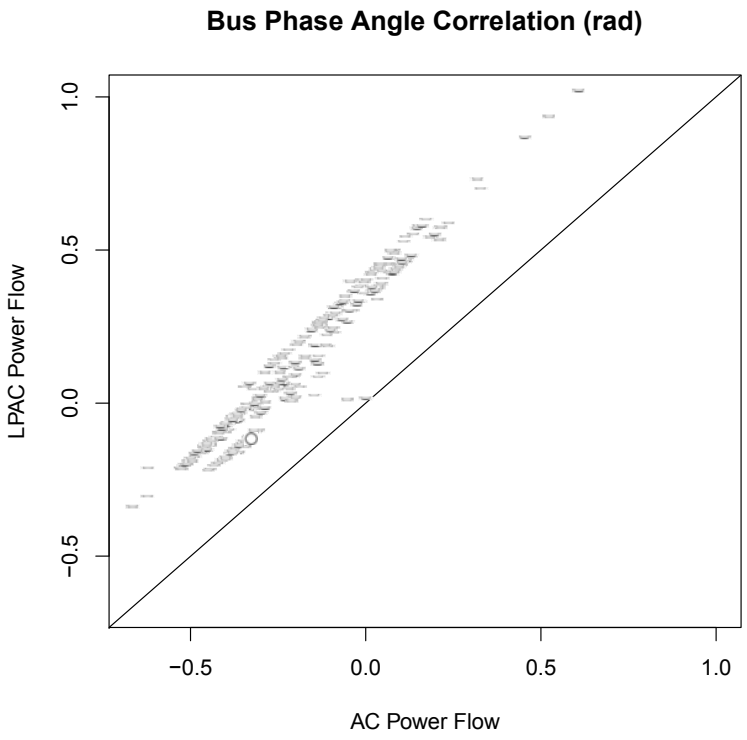
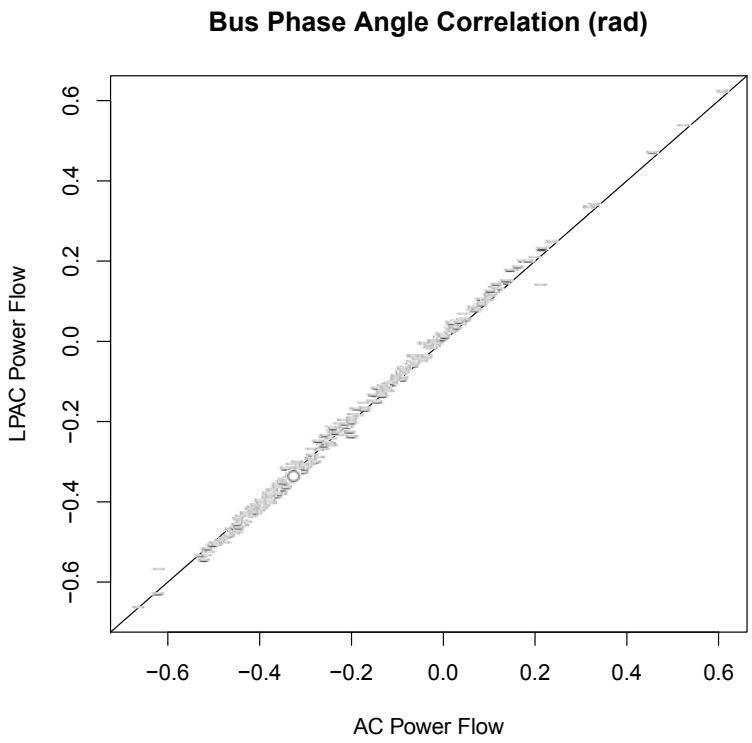
Bus Angles



LDC Model



Cold-Start LPAC Model

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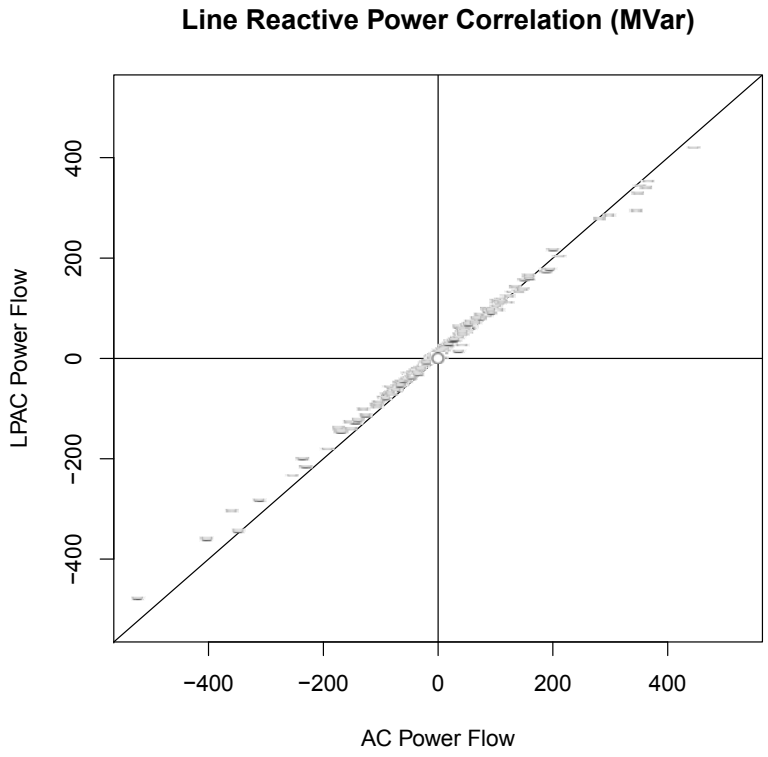
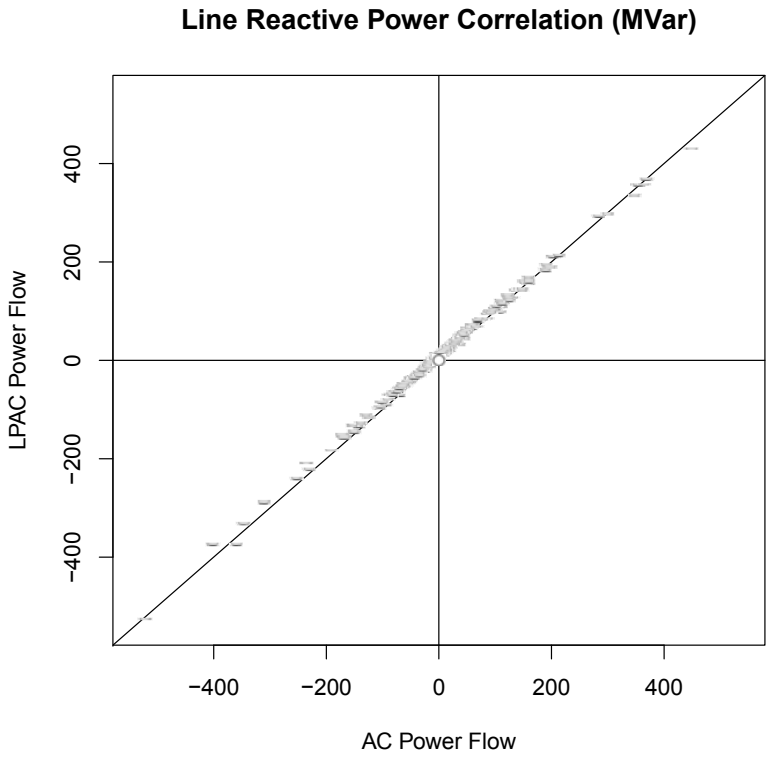
Line Reactive Power



Cold-Start LPAC Model



Warm-Start LPAC Model

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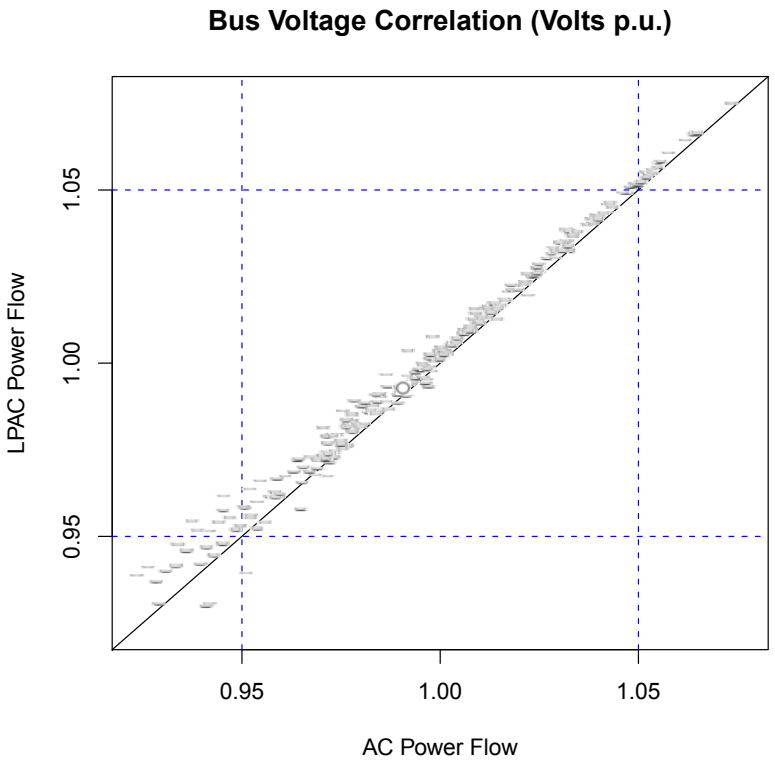
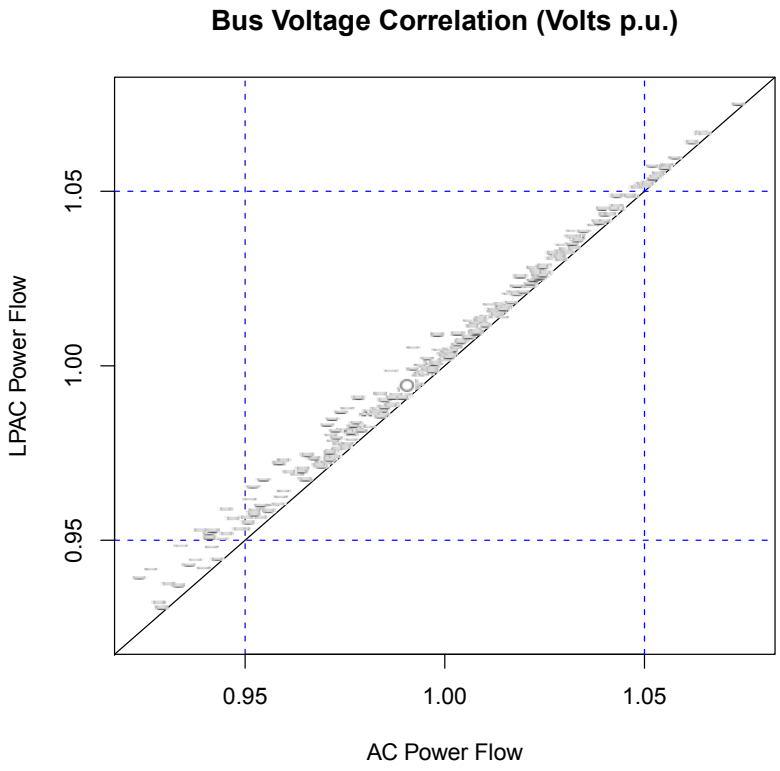
Bus Voltages



Cold-Start LPAC Model



Warm-Start LPAC Model

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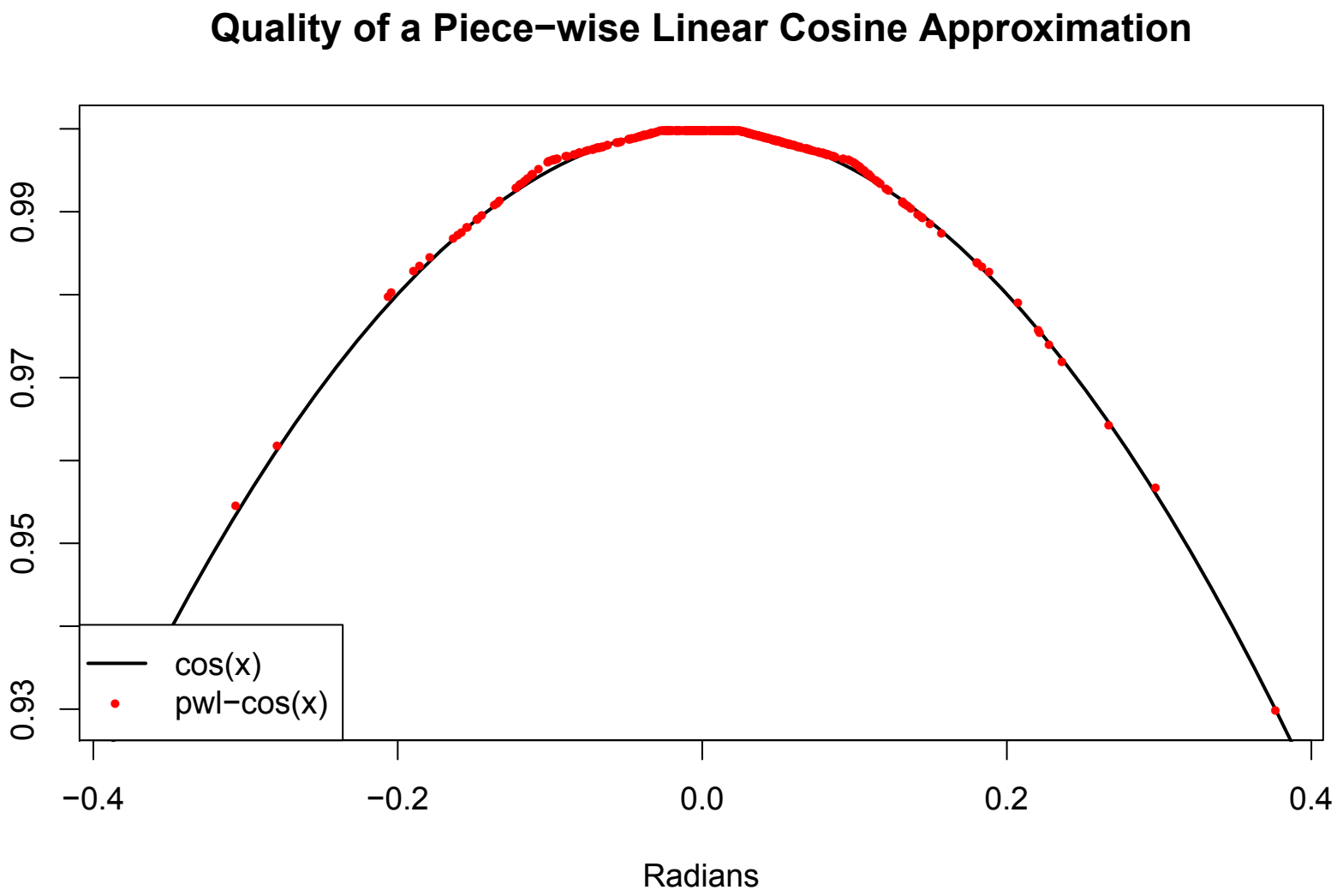
Cosine Approximation



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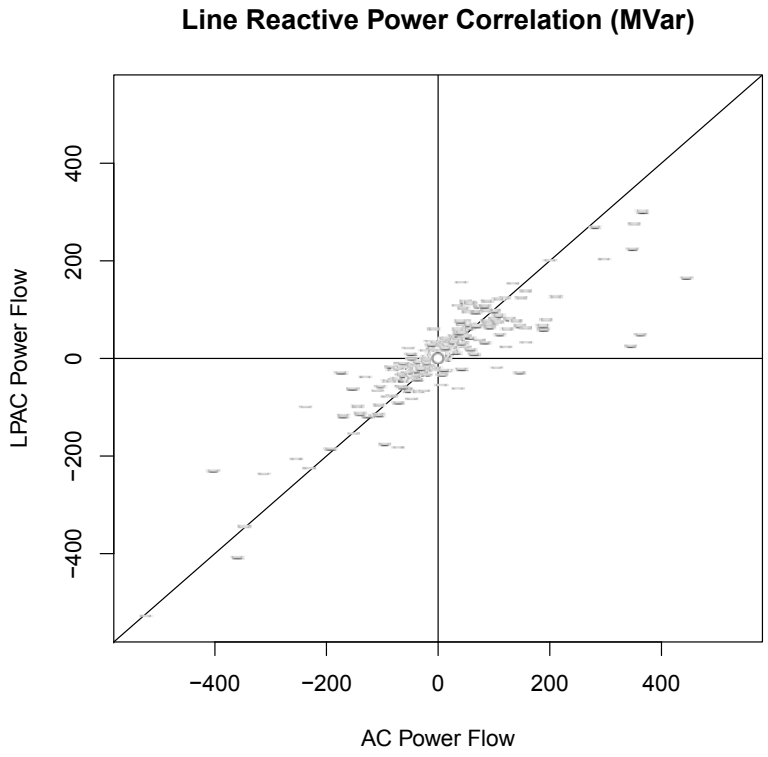
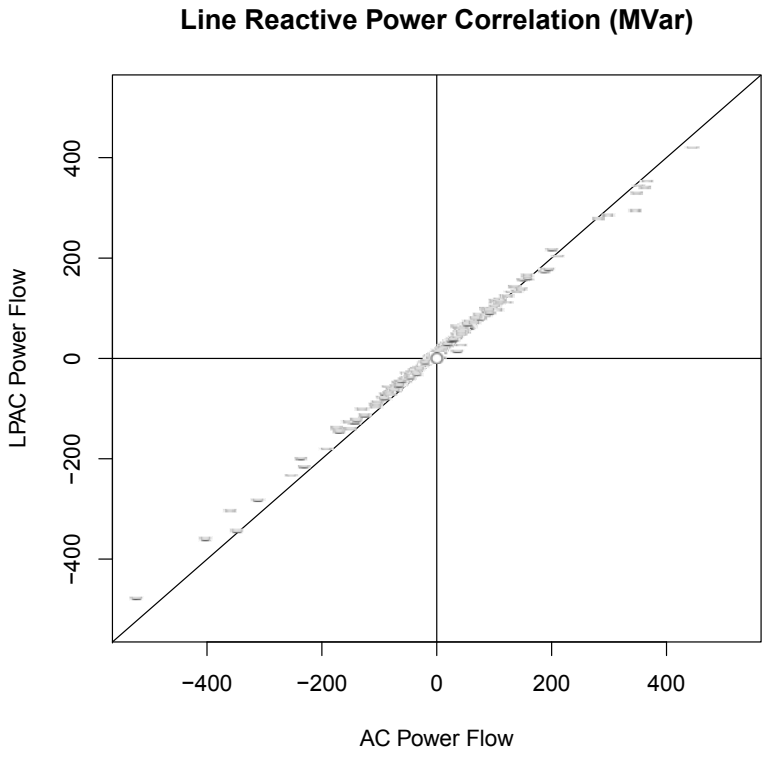
Importance of g: Reactive Power



Cold-Start LPAC Model (g=0)



Cold-Start LPAC Model

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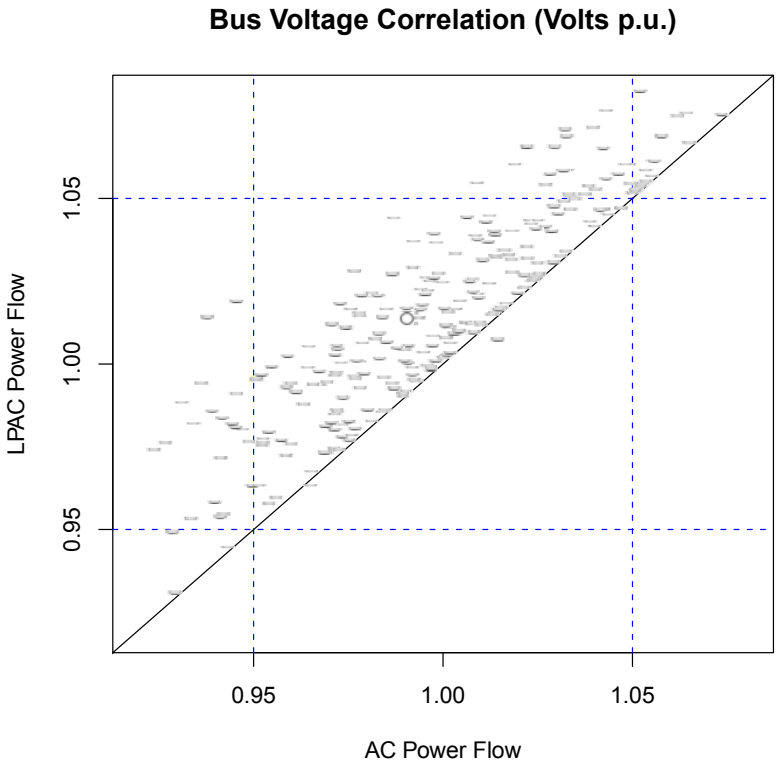
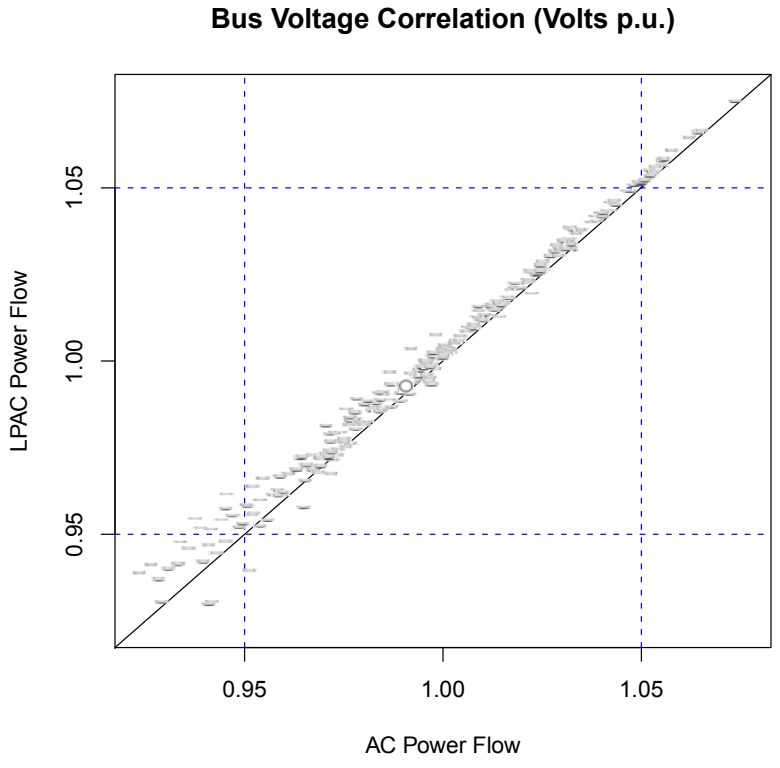
Importance of g: Bus Voltages



Cold-Start LPAC Model (g=0)



Cold-Start LPAC Model

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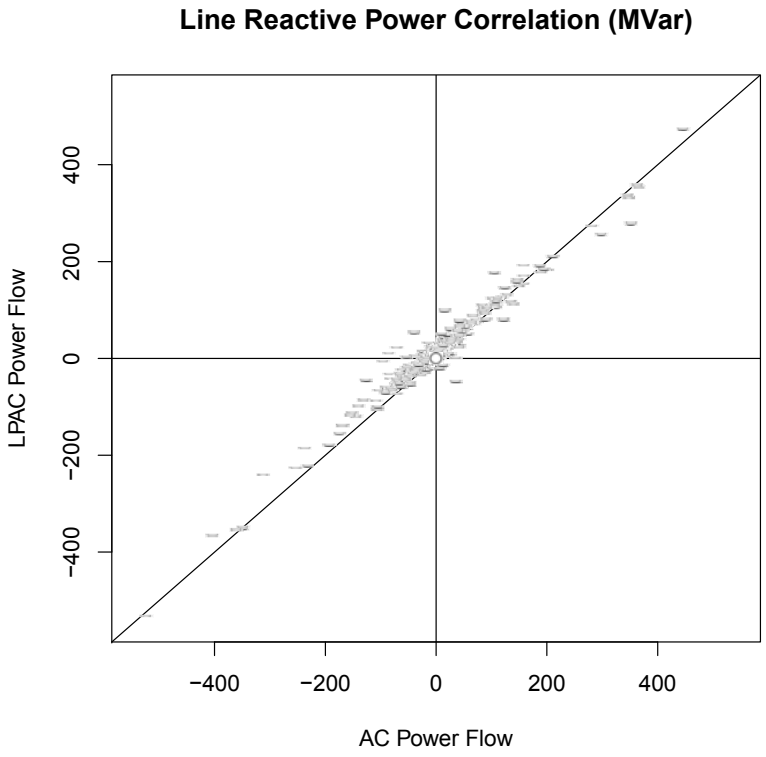
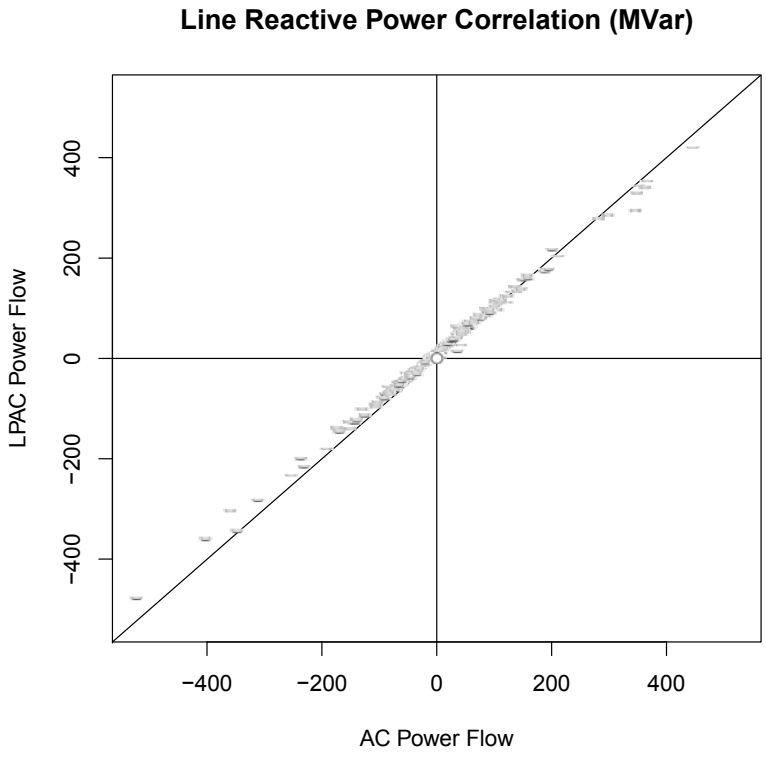
Importance of cos: Reactive Power



Cold-Start LPAC Model (cos=1)



Cold-Start LPAC Model

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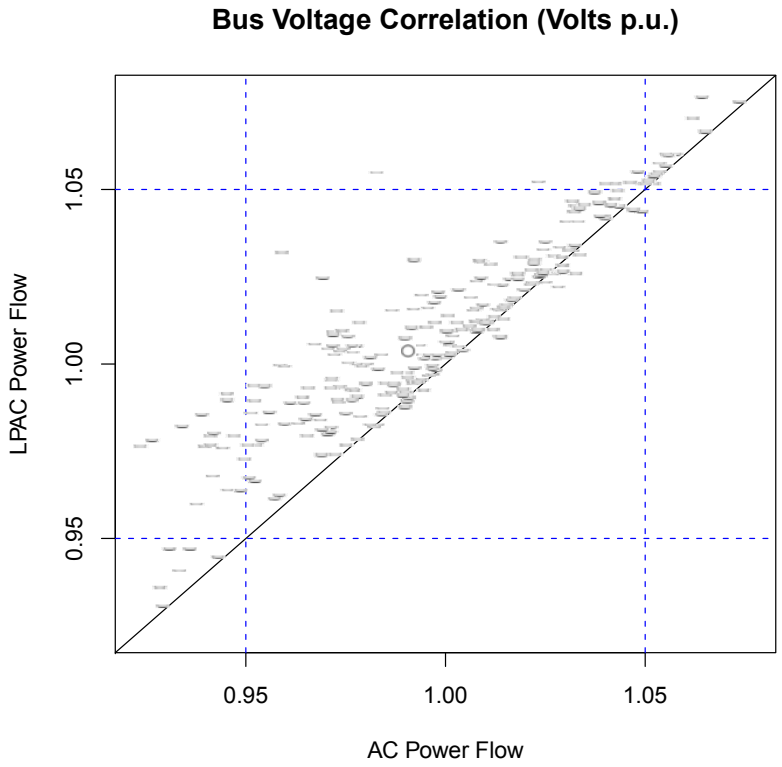
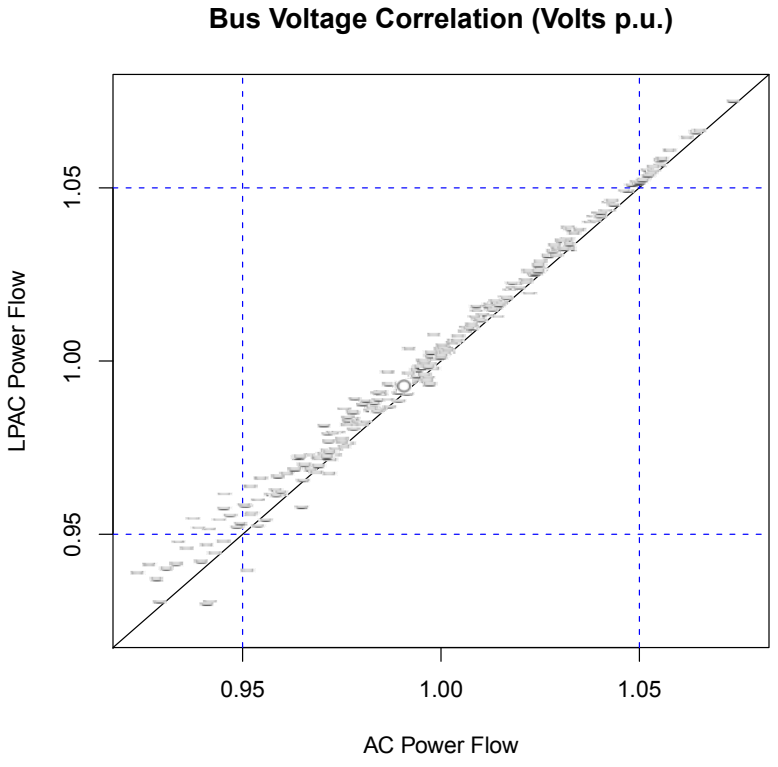
Importance of cos: Bus Voltages



Cold-Start LPAC Model (cos=1)



Cold-Start LPAC Model

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Outline

- Motivation
- The LPAC Model
- Experimental Results
  - LDC versus LPAC versus AC solutions
  - LPAC Variants
- Capacitor Placement Problem
  - Build on top of the cold LPAC model
- Power Restoration

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Capacitor Placement

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- The Problem
  - place capacitors in a power network to improve voltage stability

- Minimize the number of capacitors subject to
  - lower bounds on the voltages
  - upper bounds on reactive capacitor injection
  - upper bounds on reactive generation injection

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Capacitor Placement

Inputs:

$\overline{q_n^g}$ - injection bound for generator n

$\overline{q^c}$ - capacitor injection bound

$\underline{|\widetilde{V}|}$ - minimum desired voltage magnitude

Inputs from The Cold LPAC Model

Variables:

$q_n^c \in (0, \overline{q^c})$ - capacitor reactive injection

$c_n \in \{0, 1\}$ - capacitor placement indicator

Variables from The Cold LPAC Model

Minimize:

$$\sum_{n \in N} c_n$$

Subject to:

$$\underline{|\widetilde{V}|} \leq 1.0 + \phi_n \leq 1.05 \quad \forall n \in N$$

$$q_n^c \leq M c_n$$

$$q_n \leq \overline{q_n^g} \quad \forall n \in G$$

$$q_n = \sum_{m \in N}^{n \neq m} \hat{q}_{nm}^t + \hat{q}_{nm}^{\Delta} \quad \forall n \in G$$

$$q_n - q_n^c = \sum_{m \in N}^{n \neq m} \hat{q}_{nm}^t + \hat{q}_{nm}^{\Delta} \quad \forall n \in N : n \neq s \wedge n \notin G$$

Constraints from The Cold LPAC Model

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Experimental Results

- Modified IEEE 57 Benchmark
  - Remove the transformers
  - Remove the synchronous condensers
  - This induces severe voltage problems
  - Impose increasingly tighter voltage lower bounds
- The capacitor placement model
  - Meets all voltage requirements but is an approximation

How well does it do compared to the AC model?

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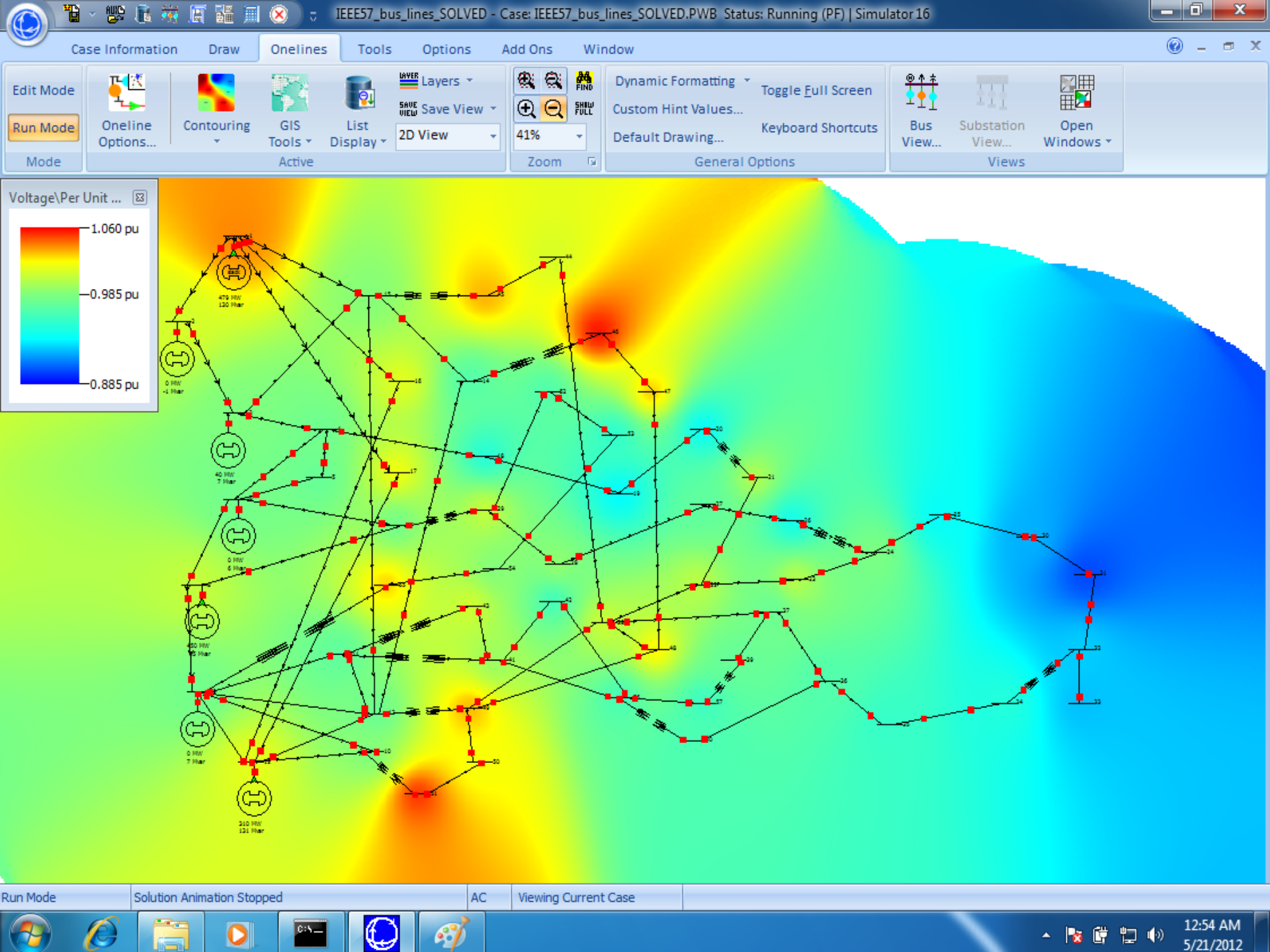
IEEE57_bus_lines_SOLVED - Case: IEEE57_bus_lines_SOLVED.PWB Status: Running (PF) | Simulator 16
Case Information
Draw
Onelines
Tools
Options
Add Ons
Window
Edit Mode
Run Mode
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GIS Tools
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Substation View...
Open Windows
Views
Voltage\Per Unit ...
1.060 pu
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0.885 pu
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Solution Animation Stopped
AC
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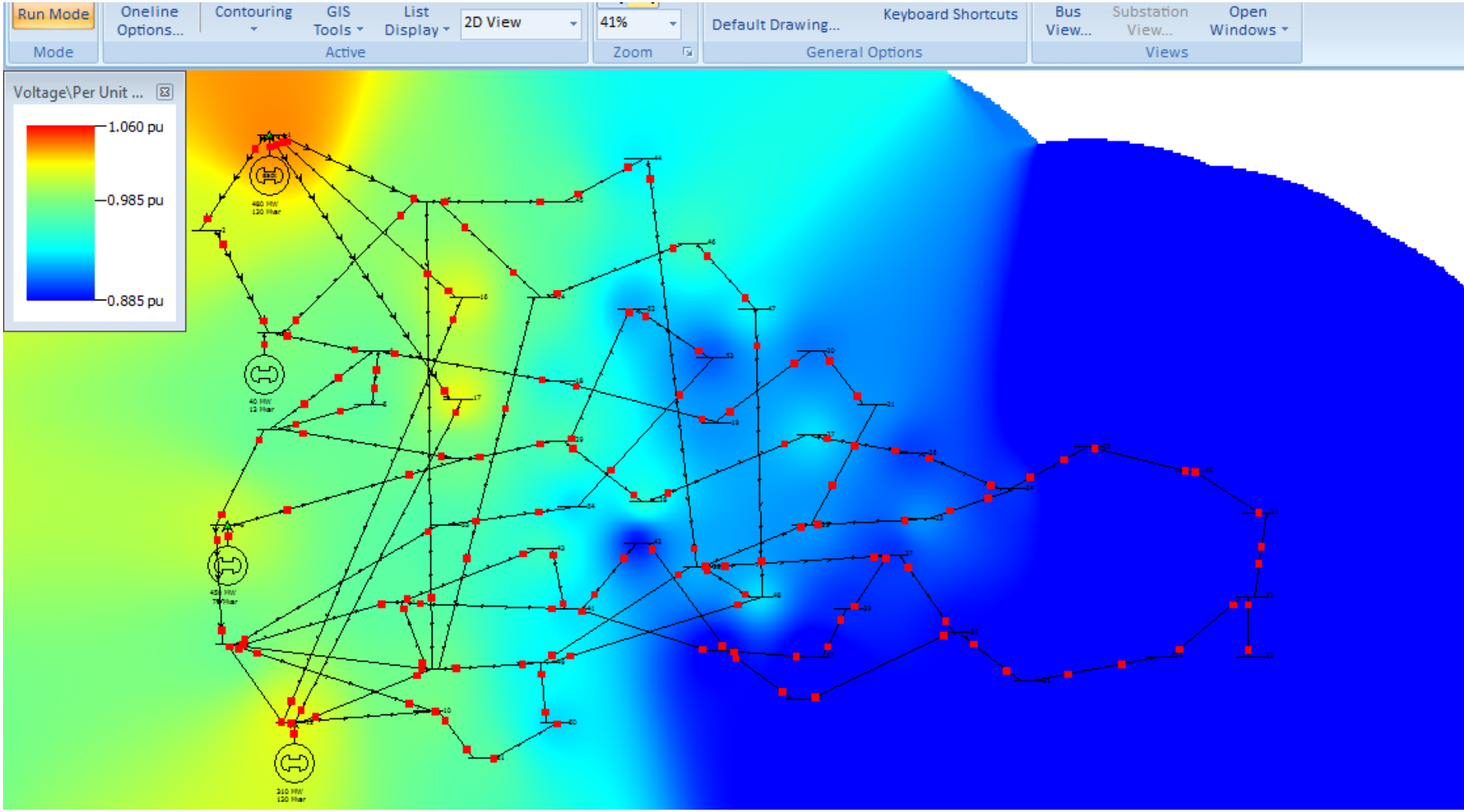
IEEE57: 0 Capacitor



Capacitor Placement

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Table 1: Capacitor Placement: Effects of $\underline{|\widetilde{V}|}$ on IEEE57-C, $\overline{q^c} = 30$ MVar

$\underline{\lvert\widetilde{V}\rvert}$	$\min(\lvert\widetilde{V}\rvert)$	$\max(\lvert\widetilde{V}\rvert)$	$\max(q_n)$	$\sum c_n$	Time (sec.)
0.8850	0.000000	0.0	0.0	1	1
0.9350	0.000000	0.0	0.0	3	8
0.9600	0.000000	0.0	0.0	5	156
0.9750	-0.000000	0.0	0.0	6	177
0.9775	-0.000000	0.0	0.0	6	139
0.9800	-0.000000	0.0	0.0	6	75
0.9840	-0.000802	0.0	0.0	7	340

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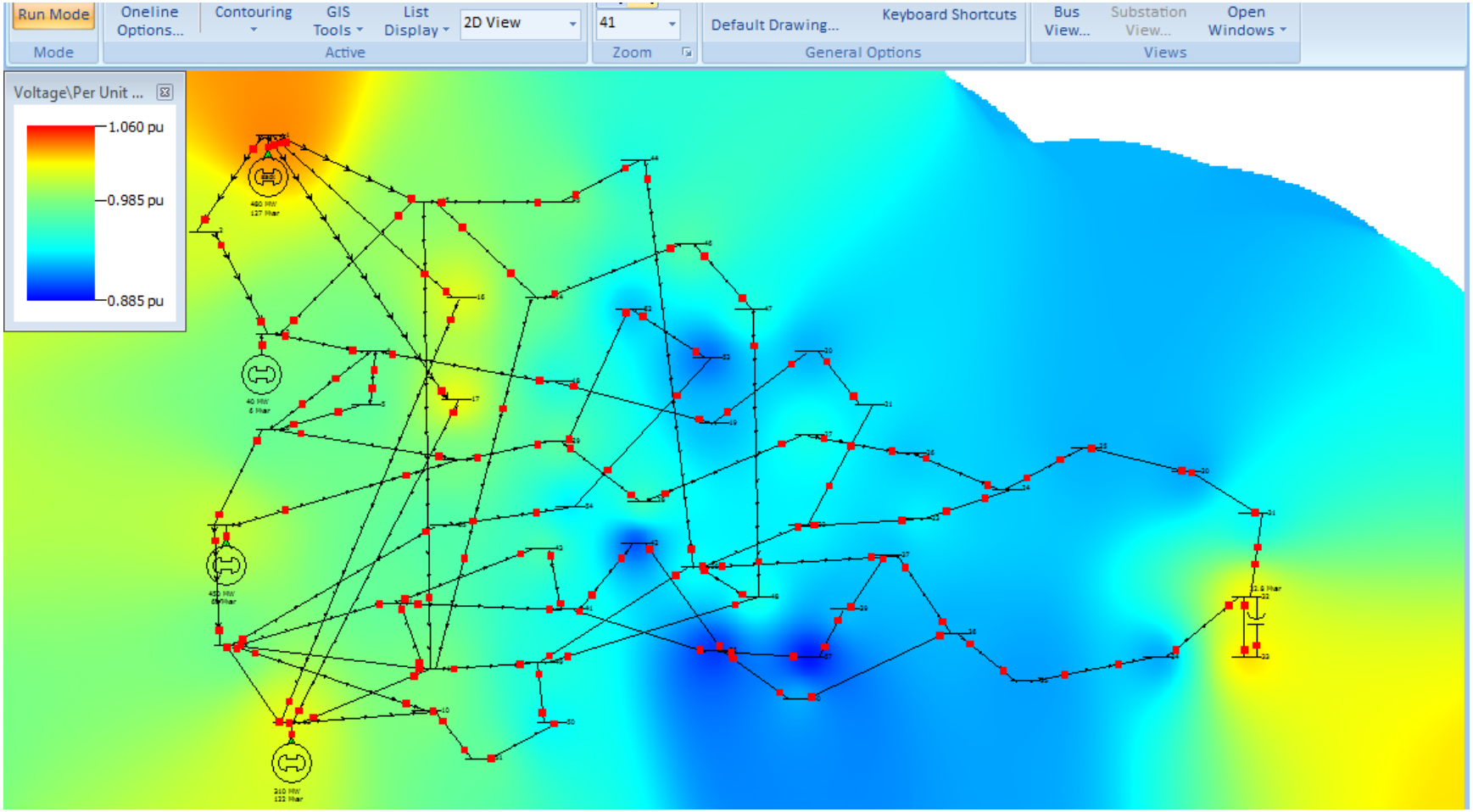
.8850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Keyboard Shortcuts
Bus View...
Substation View...
Open Windows
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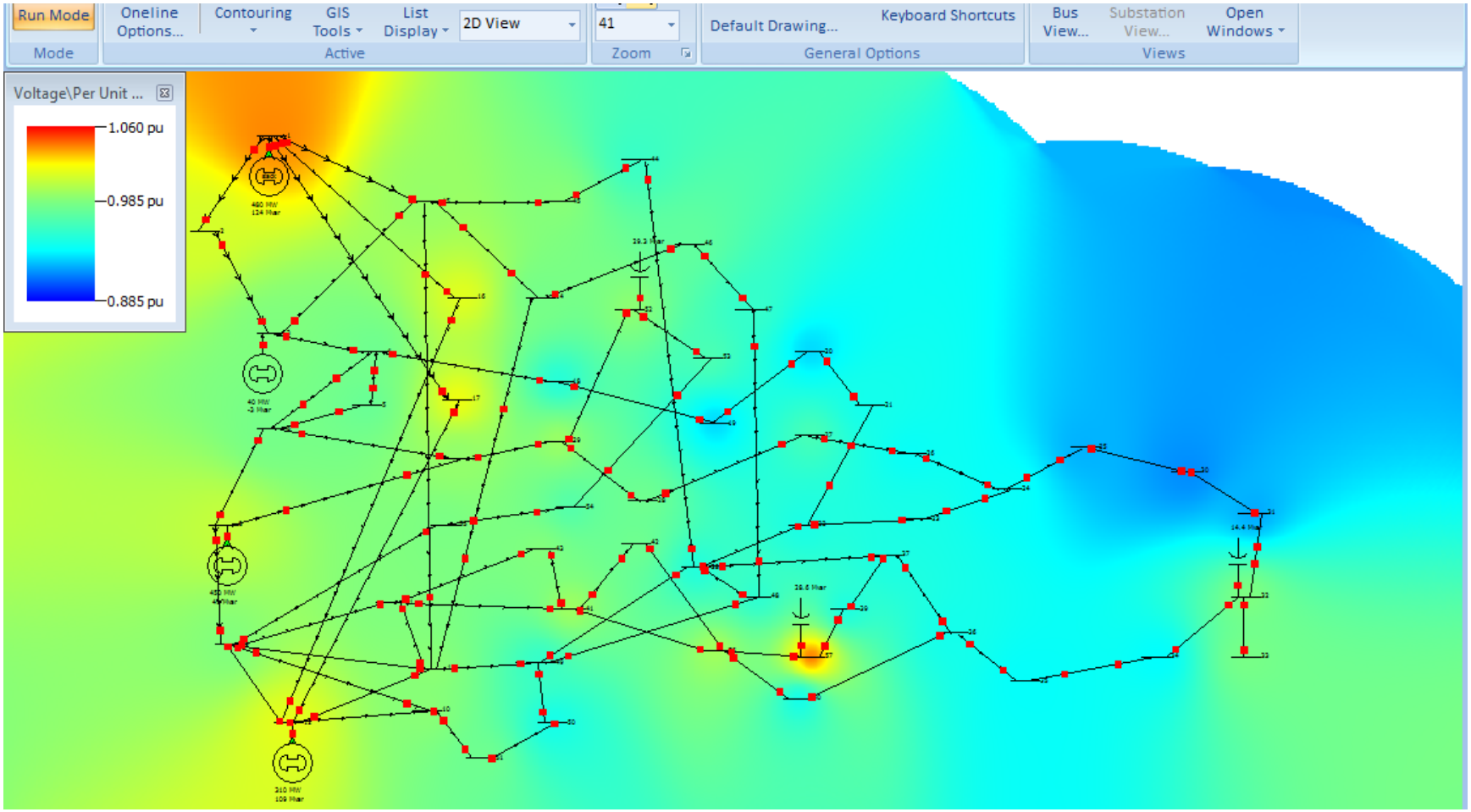
1.060 pu
0.985 pu
0.885 pu

.9350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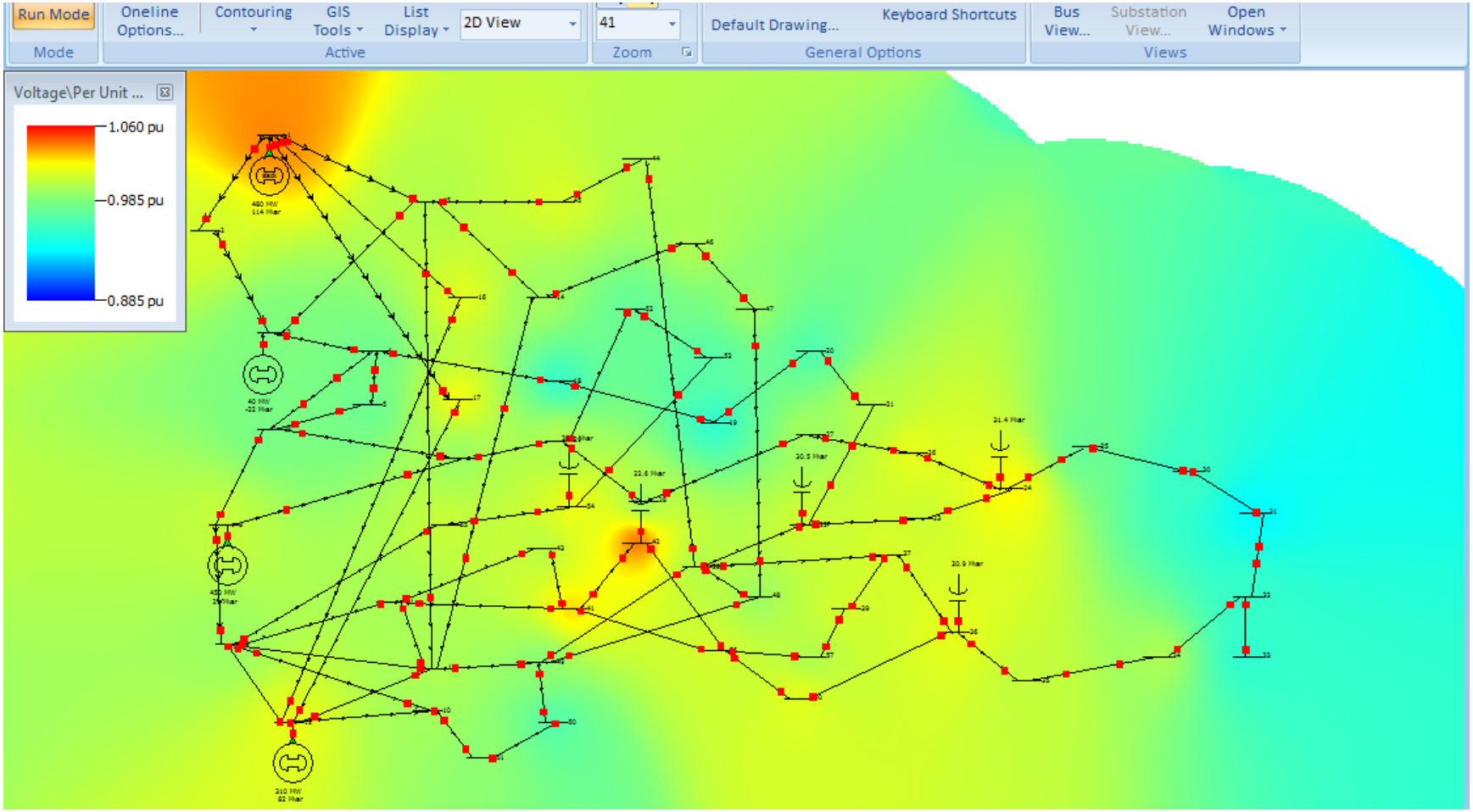
0.885 pu

.9600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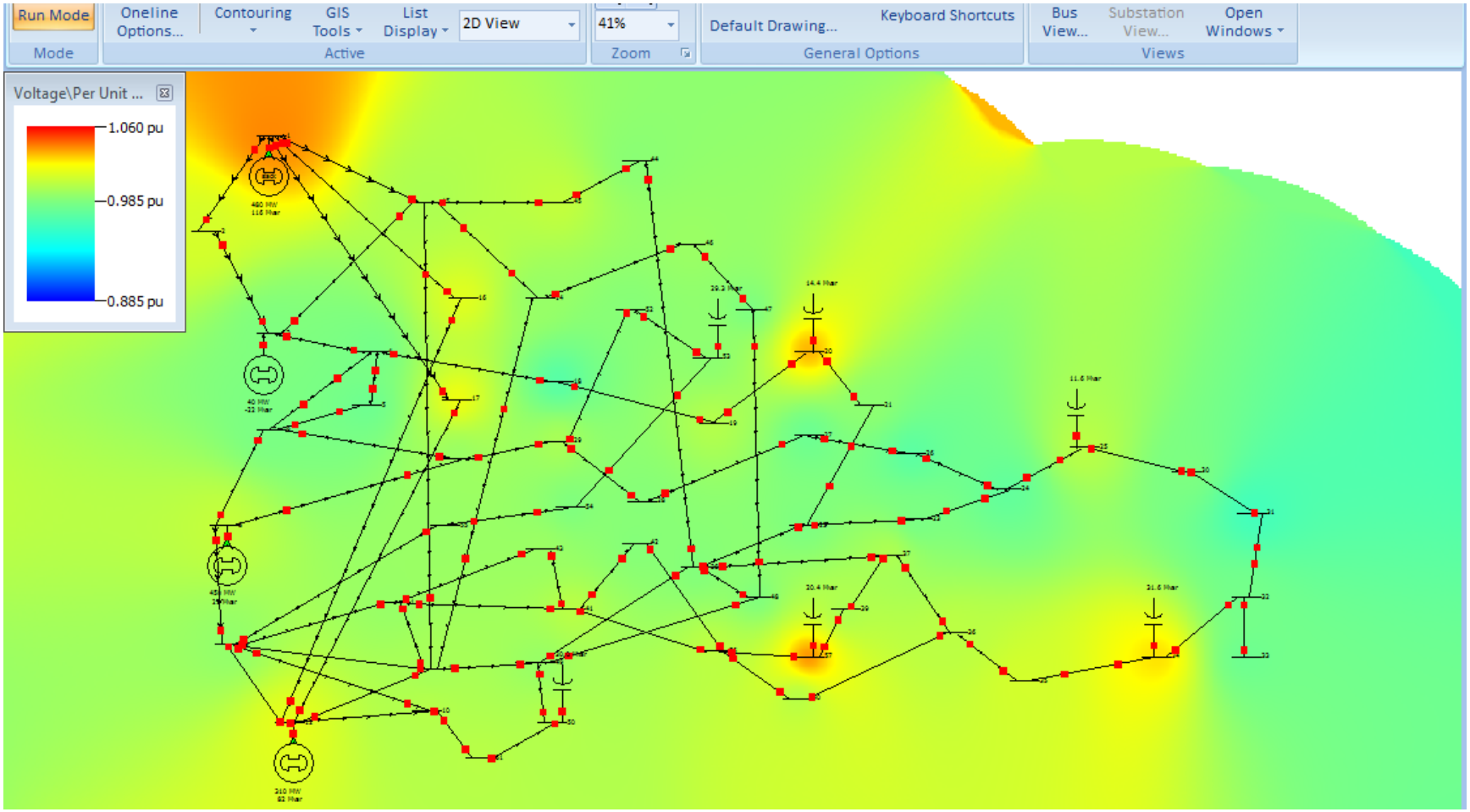
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Bus View...
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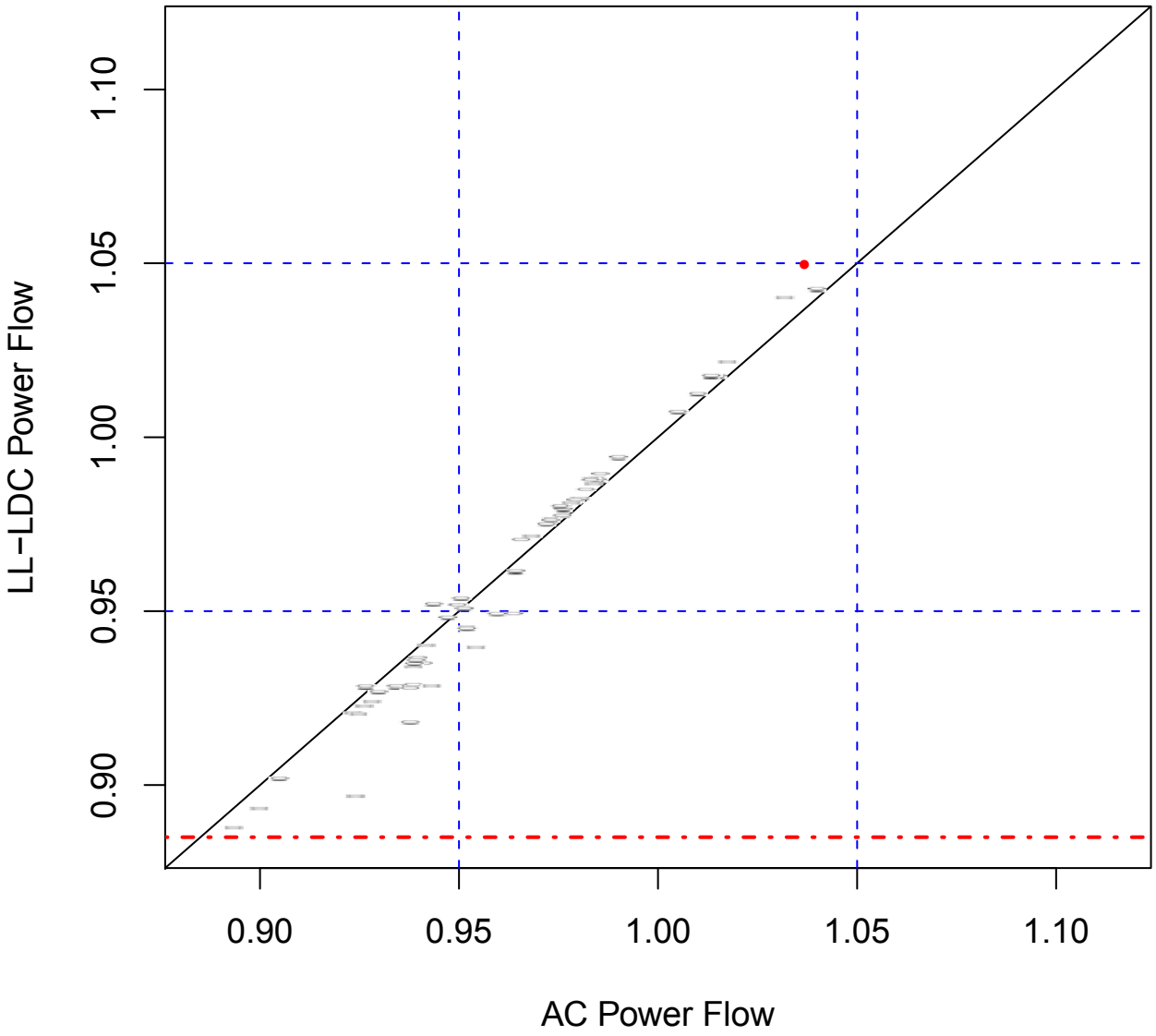
1.060 pu
0.985 pu
0.885 pu

Bus Voltage Correlation (Volts p.u.)



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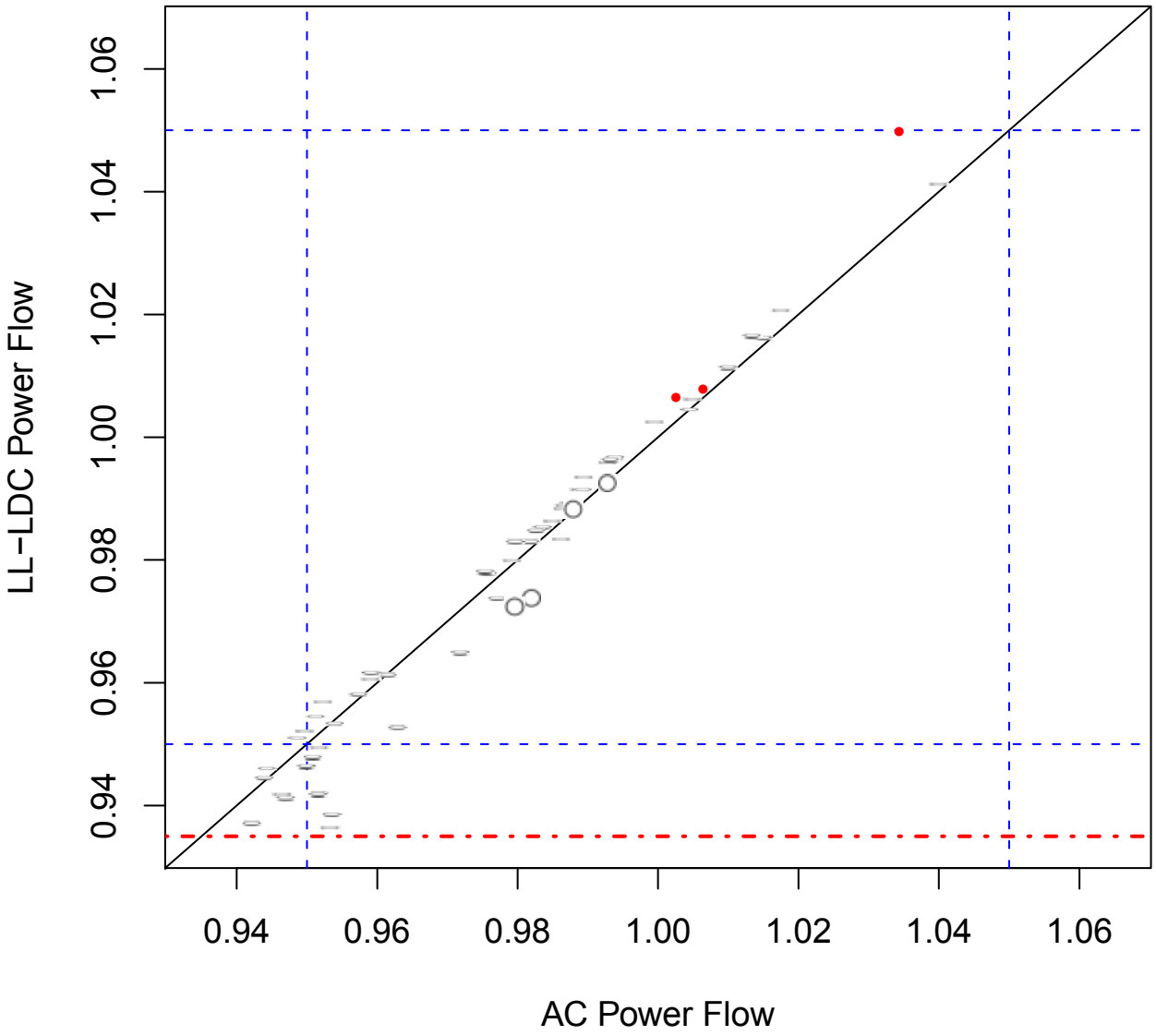
Bus Voltage Correlation (Volts p.u.)



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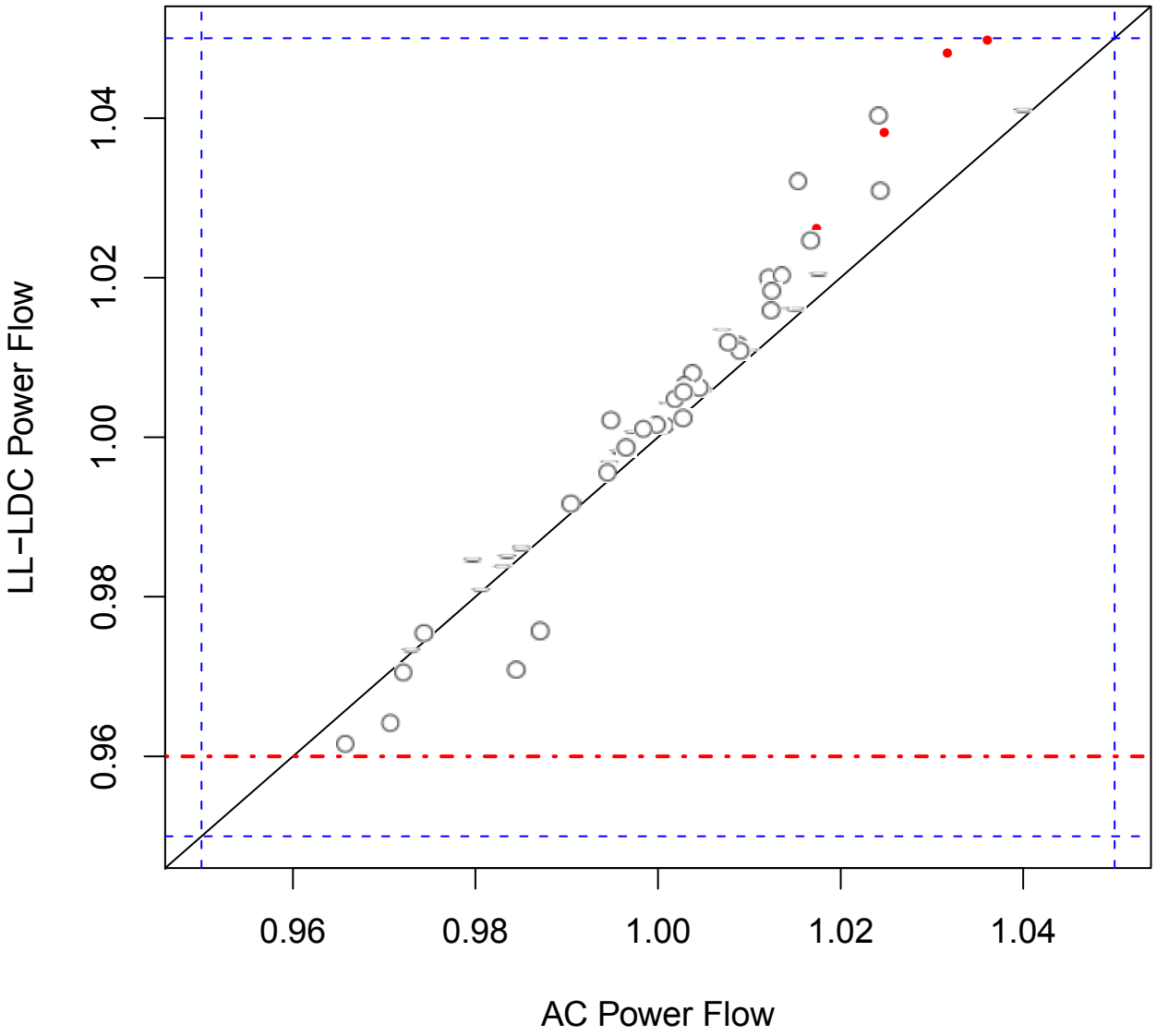
Bus Voltage Correlation (Volts p.u.)



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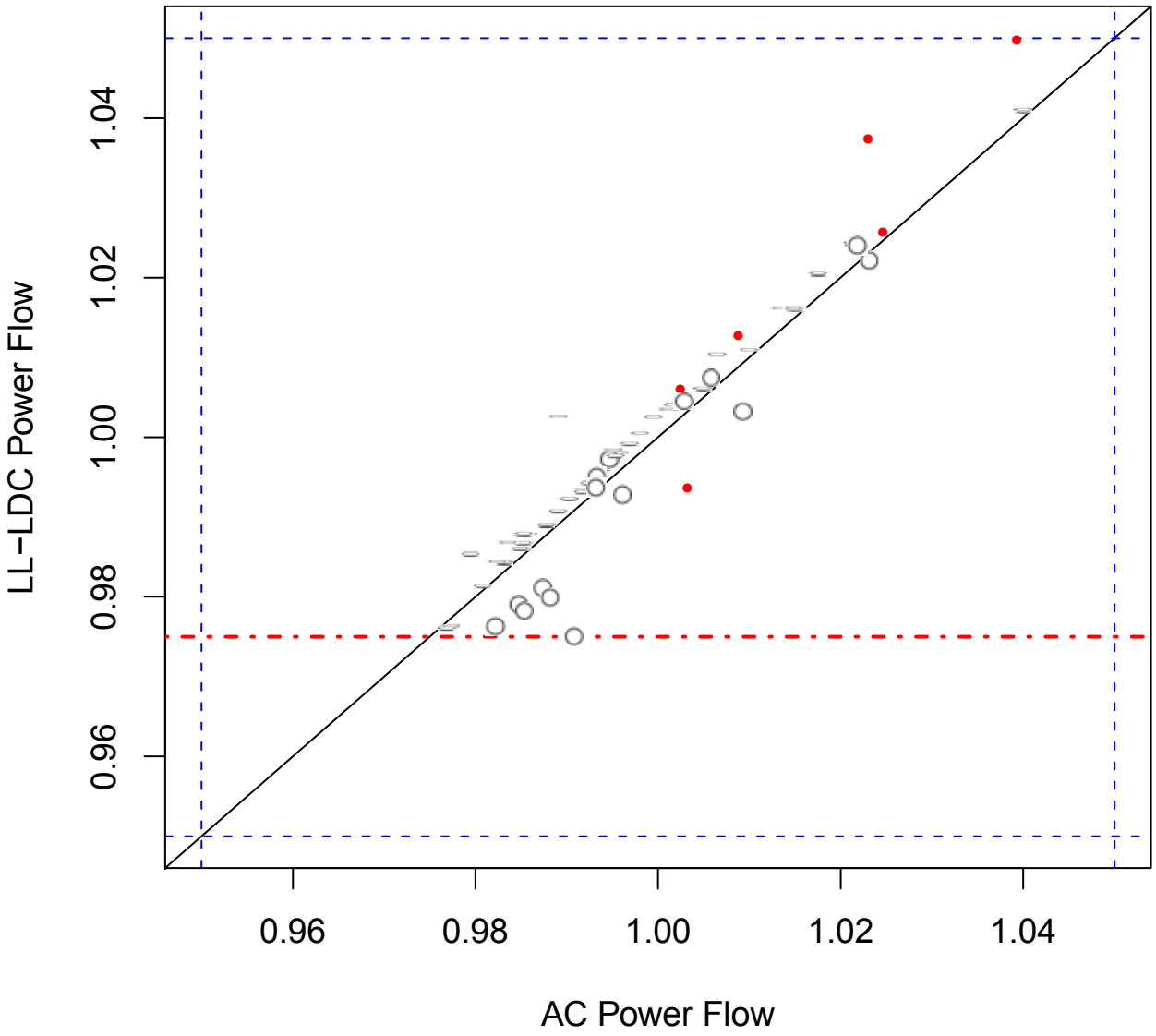
Bus Voltage Correlation (Volts p.u.)



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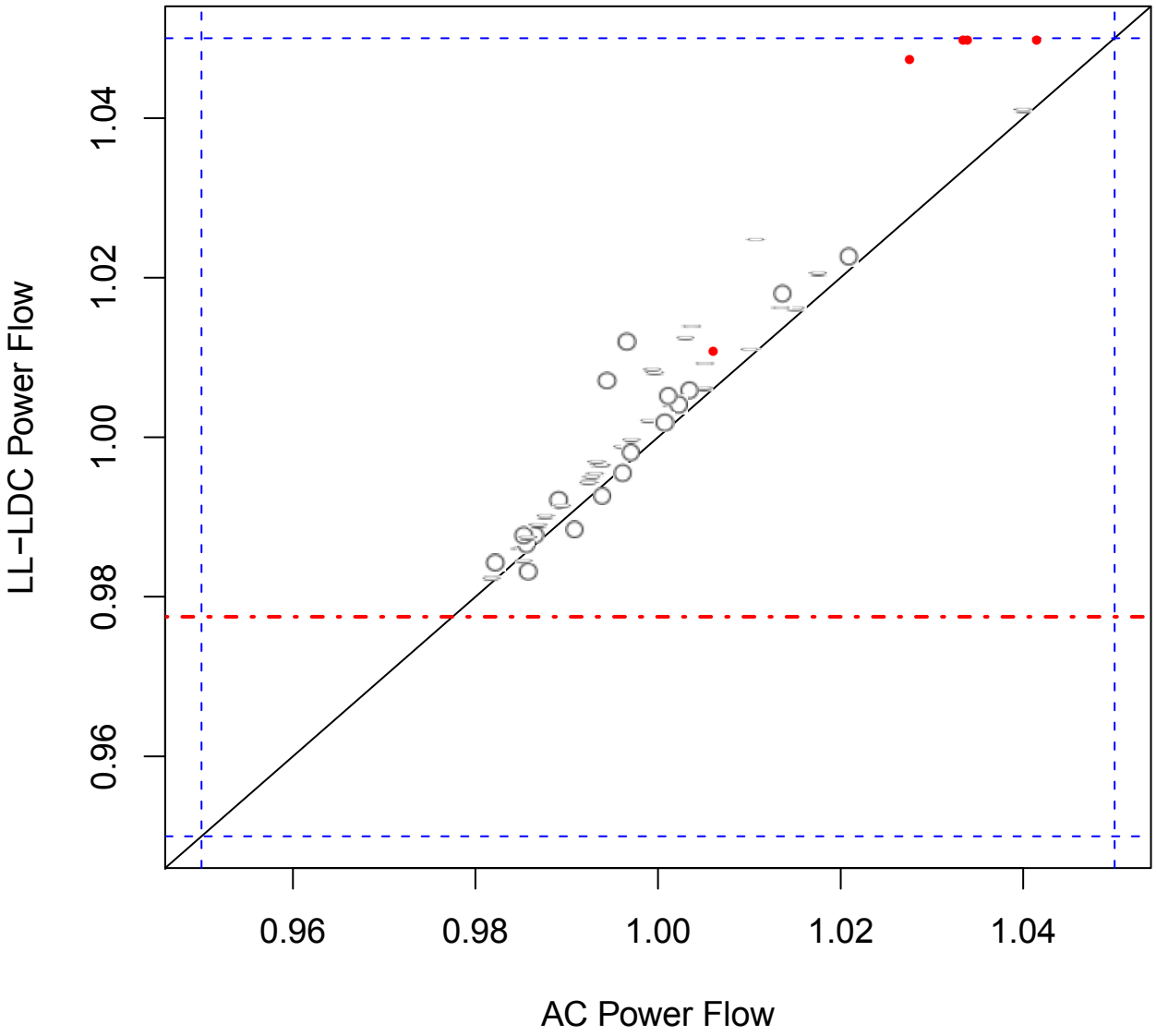
Bus Voltage Correlation (Volts p.u.)



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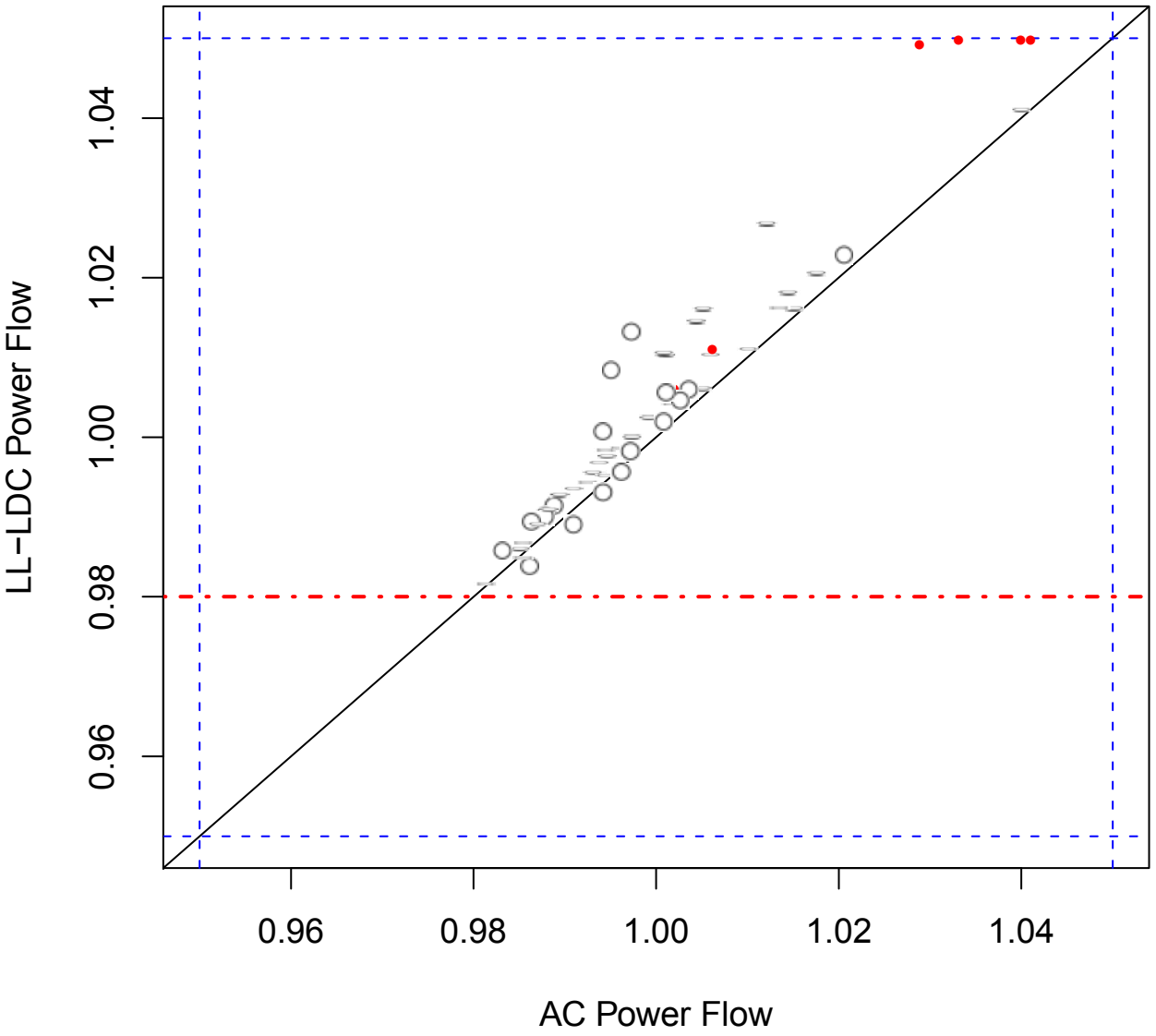
Bus Voltage Correlation (Volts p.u.)



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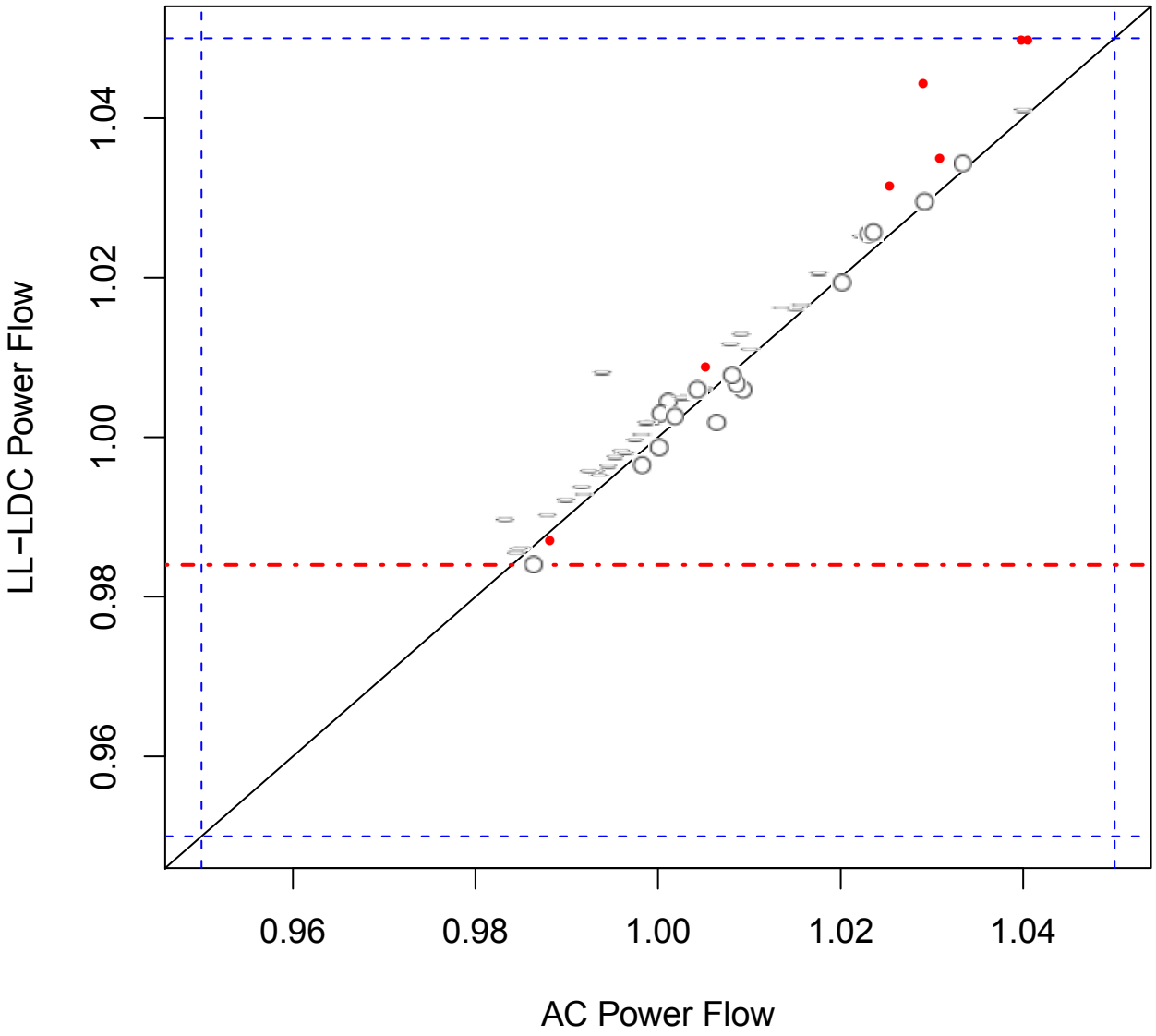
Bus Voltage Correlation (Volts p.u.)



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  - Build on top of the warm LPAC model

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Demand Maximization

Inputs:

$\overline{p_n^g}$ - maximum active injection for bus n

$\overline{p_n^l}$ - desired active load at bus n

$\overline{q_n^l}$ - desired reactive load at bus n

Inputs from the Warm LPAC Model

Variables:

$p_n^g \in (0, \overline{p_n^g})$ - active generation at bus n

$q_n^g \in (-\infty, \infty)$ - reactive generation at bus n

$l_n \in (0, 1)$ - percentage of load served at bus n

Variables from the Warm LPAC Model

Maximize:

$$\sum_{n \in N} l_n$$

Subject to:

$$p_n = -\overline{p_n^l} l_n + p_n^g \quad \forall n \in N$$

$$q_n = -\overline{q_n^l} l_n + q_n^g \quad \forall n \in N$$

$$q_n^g = 0 \quad \forall n \in N \setminus G$$

$$q_n = \sum_{\substack{m \in N \\ n \neq m}} \hat{q}_{nm}^t + \hat{q}_{nm}^\Delta \quad \forall n \in G$$

Constraints from the Warm LPAC Model

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IEEE-30 Contingencies

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	N-9	N-11	N-12	N-13	N-15	N-16	N-17	%
LDC	7436	6511	5344	6805	5931	7236	6877	66%
LPAC	9998	10000	9996	9981	9998	10000	9911	99.8%

	N-9	N-11	N-12	N-13	N-15	N-16	N-17	
LDC	14.2	20.14	57.77	73.67	44.54	64.58	67.88	
LPAC	35.96	30.38	57.74	62.83	57.49	66.69	64.73	

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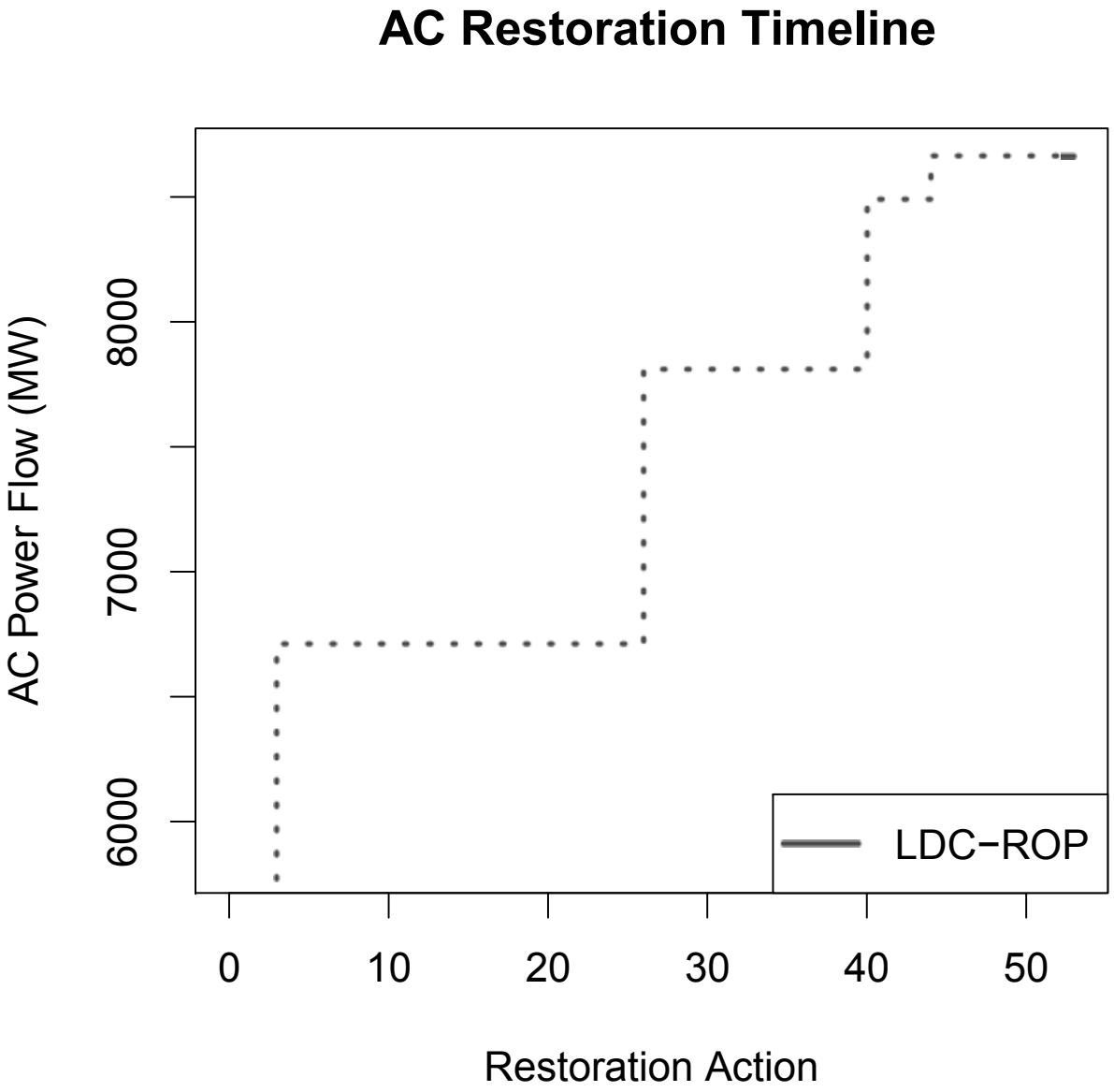
Power Restoration



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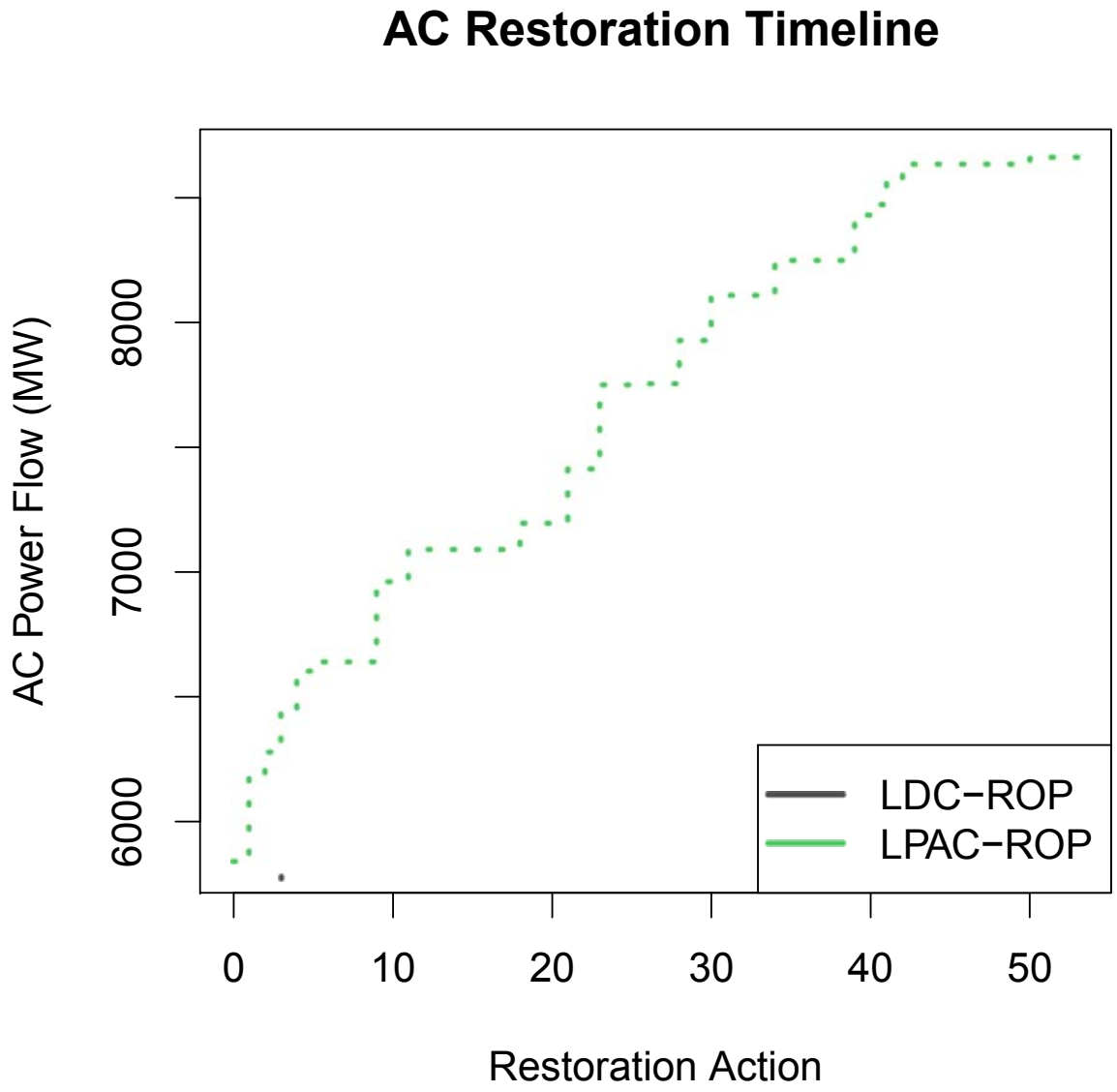
Power Restoration



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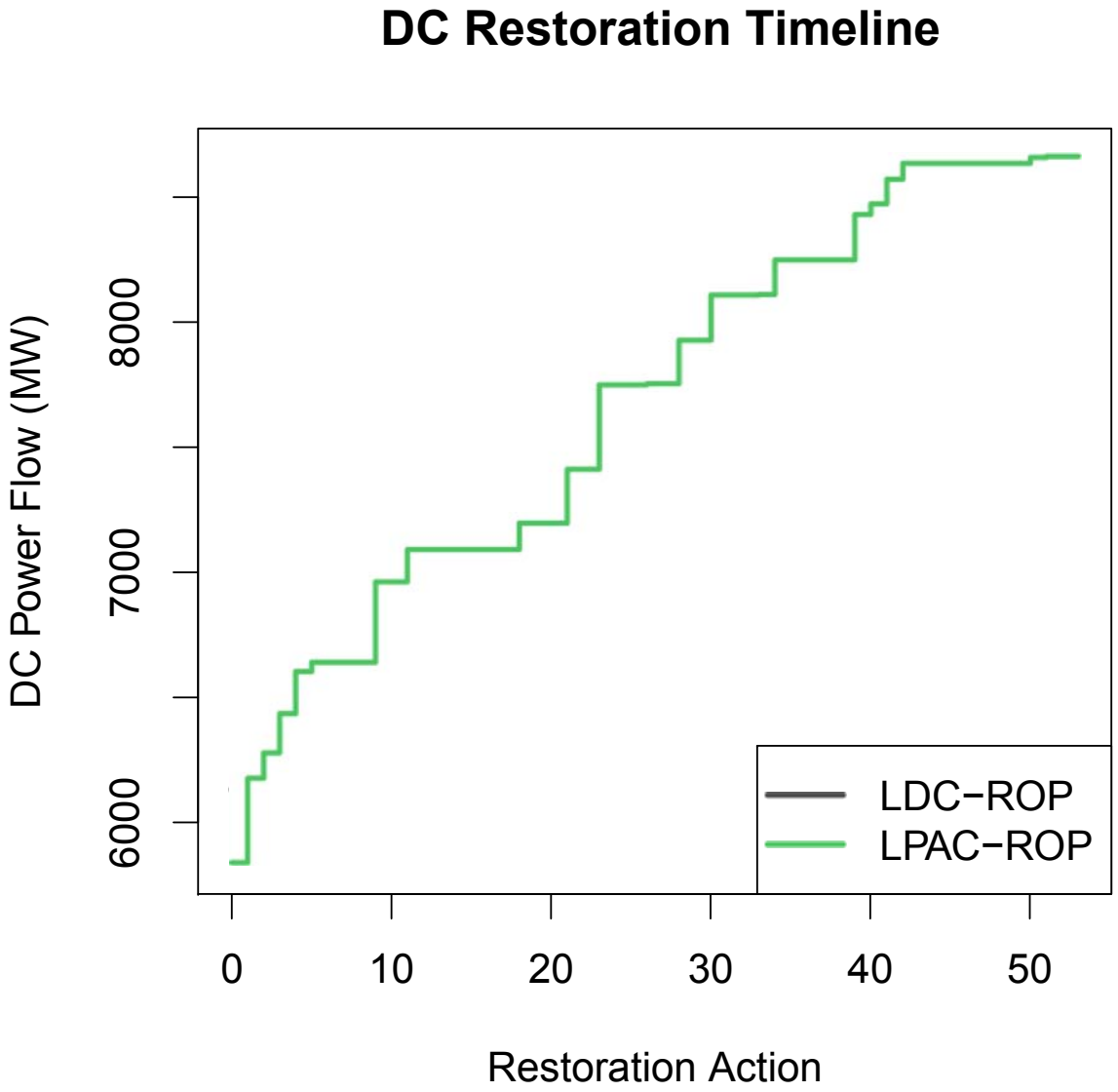
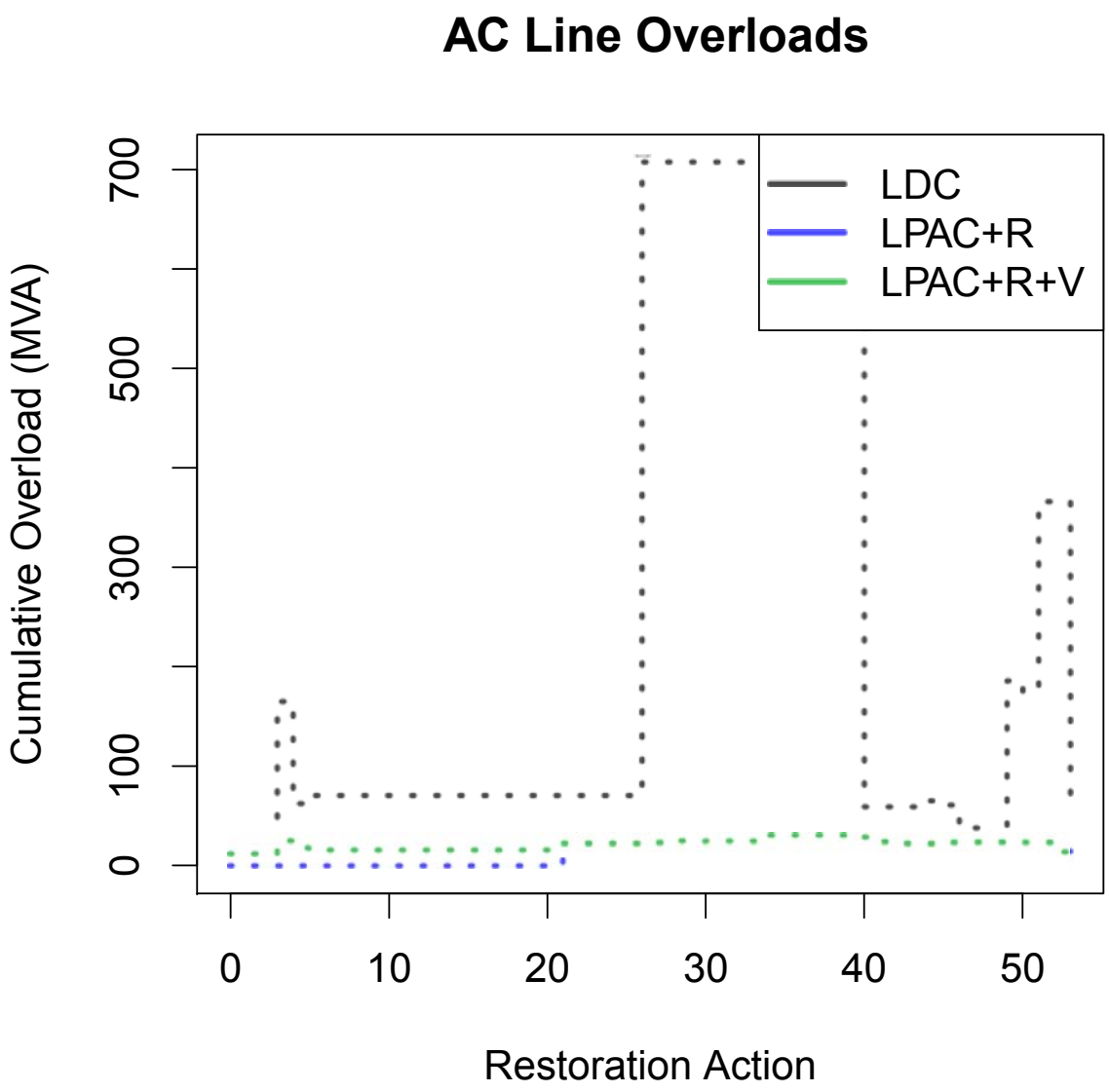
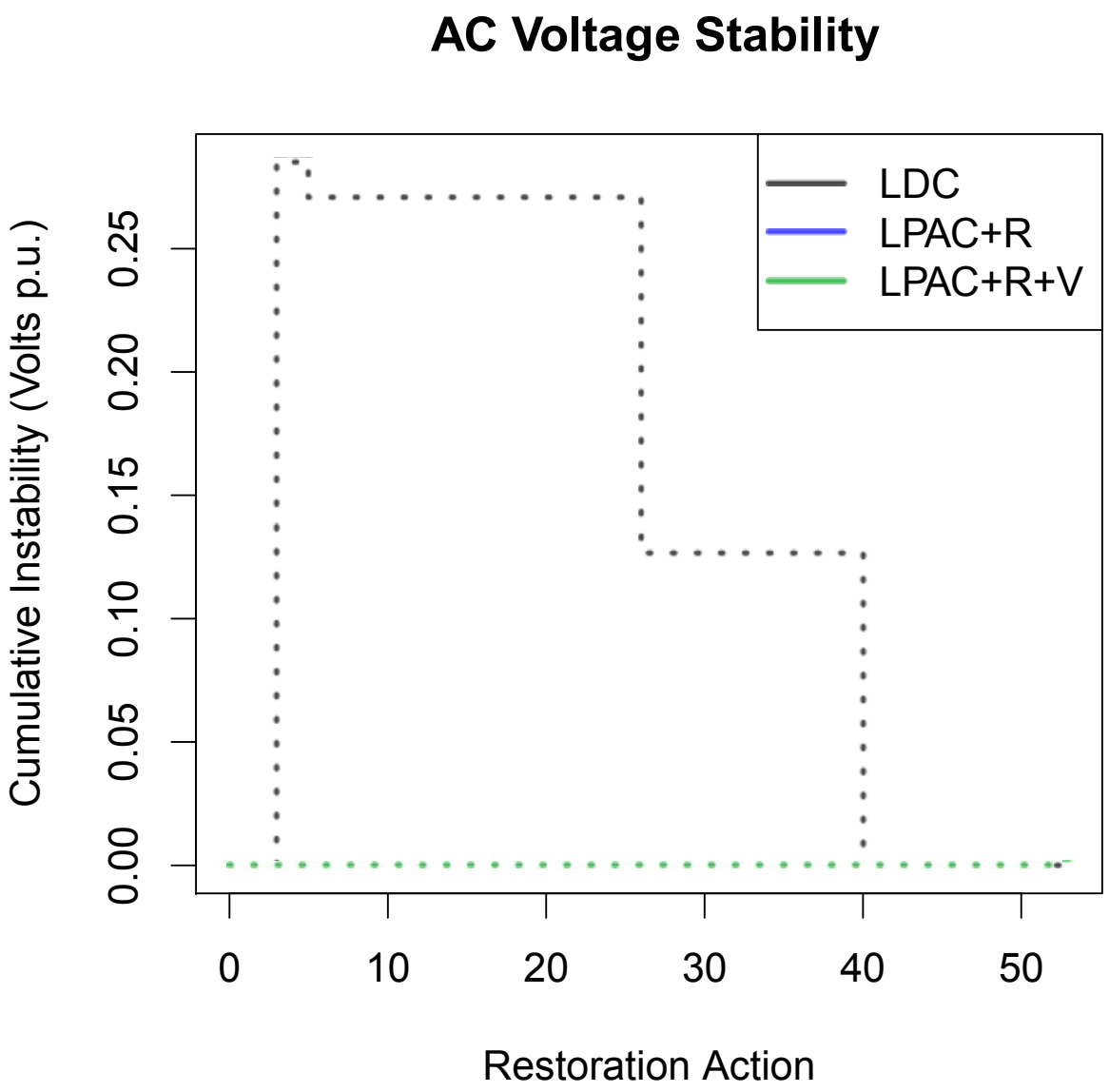
Power Restoration



Power Restoration



Power Restoration



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Conclusion

- LPAC Models: Linear-Programming approximations
  - Much more accurate than the LDC model
  - useful outside normal operating conditions
  - reason about voltage magnitudes and reactive power
  - can be embedded in MIP solvers
- Experimental results
  - Very high accuracy when compared to AC solutions
- Case studies
  - Capacitor placement problem
  - Power Restoration

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