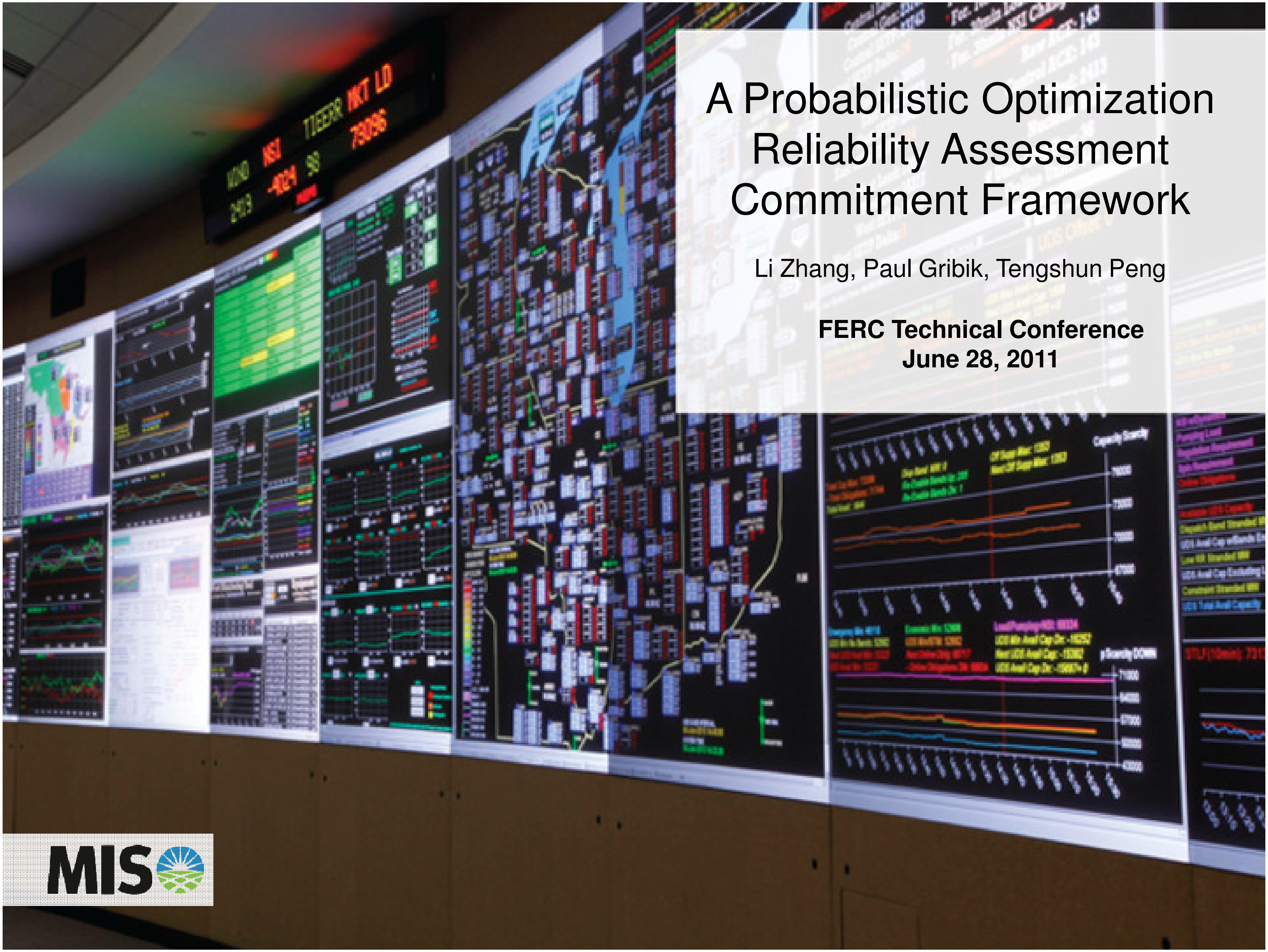

A Probabilistic Optimization Reliability Assessment Commitment Framework

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FERC Technical Conference
June 28, 2011

Outline of the presentation

- **Brief Review of Reliability Assessment Commitment**
- **Probabilistic Optimization Framework**
- **Simplified Probabilistic Optimization**
- **Robust Optimization**
  - An example of the simplified probabilistic optimization
  - Appropriate when accuracy of the probabilities is an issue
- **Numerical Examples**
- **Appendices: Mathematical Formulations**

Brief Review of Reliability Assessment Commitment

Brief Review of Reliability Assessment Commitment (RAC)

- **A commitment process after Day Ahead clearing and throughout the operating day**
- **The purpose of the RAC process is to ensure that sufficient capacity is available to meet Real-Time demand for energy and reserves.**
  - Forward RAC is run prior to the operating day for the entire day.
  - Intra-day RAC is run periodically during the day and covers a period from current hour to the end of the day.
- **RAC depends upon forecasts of Demand, Net Scheduled Interchange, Intermittent Resource Availability, etc.**
  - Considerable uncertainty can exist in the forecasts given that they cover periods that may be several hours in the future.

Dealing with Uncertainty

- **The current RAC formulation employs a deterministic unit commitment**
  - How to deal with uncertainty?
    - Allocating enough operating reserve
      - Usually to cover the worst case scenario, can be expensive
    - Operator's judgment and response to uncertainty
- **Characteristics of the resources for commitment can be quite different**
  - Slow start resources, long notification time and may require hours to come on-line
  - Fast start, can be on line within 10 or 30 minutes etc
    - Commitment of fast start resources can wait till real time, after uncertainty resolved.

Dealing with Uncertainty

- **Ideally, we should commit resources taking into account the uncertainties around future conditions (Demand, NSI, Intermittent Resource Availability, etc.) at the time.**
  - The state at time t will consist of the demand, NSI, intermittent resource availability, etc. at time t as well as the states (demand, NSI, intermittent resource availability) that were occupied in times 1 through t-1
  - As time moves forward, past states will be known and future states will be subject to uncertainty.
  - Initially, we will assume that we have an estimate of the probability that the system will be in a given state at a particular time given the states prior to that time.
- **Probabilistic optimization is a natural framework to deal with such a problem.**

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Probabilistic Optimization Framework

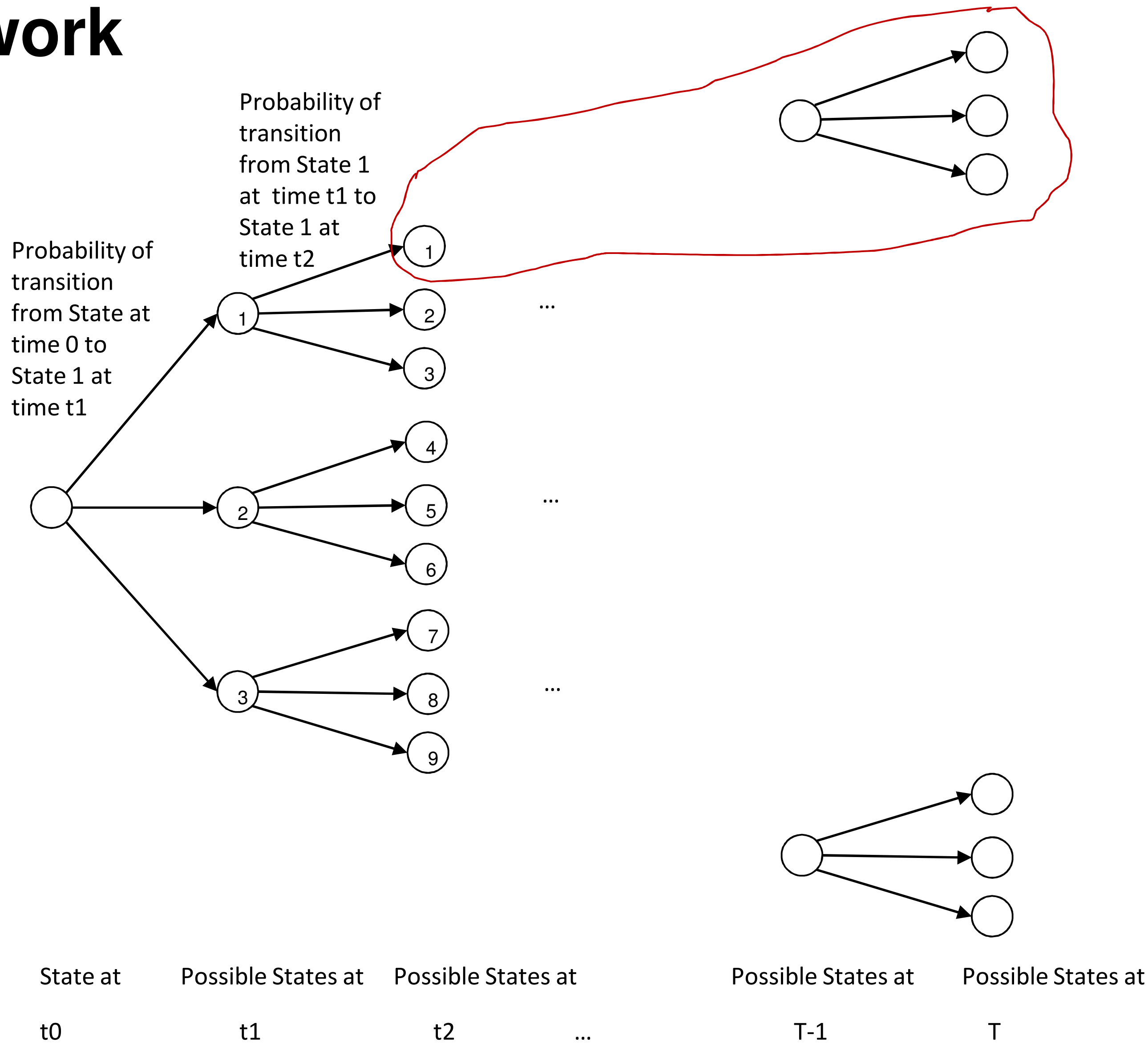
Framework

- **Assume that we will run commitment and dispatch problems at times 1, 2, … T.**
  - At time 0 (prior to the operating day) we will run a commitment problem only.
- **Suppose that we are at time t:**
  - Outcomes for all conditions at time = 1, 2, …, t are known.
  - Commitment and dispatch actions taken at time = 0, 1, … t-1 are fixed.
  - Given the state at time t, we have estimates of the probability distribution for states at time t+1.
    - Similarly, for each state at time t+1, we have probability distributions for states at time t+2; etc.

Framework

- **The next slide shows the tree of possible future states starting at time 0.**
- **At time t, we know the state of the system. Pruning the tree to start at this state and moving to times t+1,…, T shows the possible future states and their probabilities.**
  – The part circled in red, shows the tree of possible future states starting from time 2 assuming that we are in state 1 at time 2.

Framework



Probability of transition from State 1 at time t1 to State 1 at time t2
Probability of transition from State at time 0 to State 1 at time t1
1
2
3
1
2
3
4
5
6
7
8
9
...
...
...
State at t0
Possible States at t1
Possible States at t2
...
Possible States at T-1
Possible States at T

Framework

- **At time t, we want to determine:**
  - Resources to which we should send start signals at time t
  - Dispatch instructions to resources on line at time t to meet requirements at time t
- **We want to minimize:**
  - The cost of commitment and dispatch actions taken at time t

  plus
  - The expected costs of commitment and dispatch actions that we will take at times t+1,…, T to meet requirements in the future.
- **We minimize expected production cost from time t through T given the state at time t.**

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Probabilistic Optimization Framework

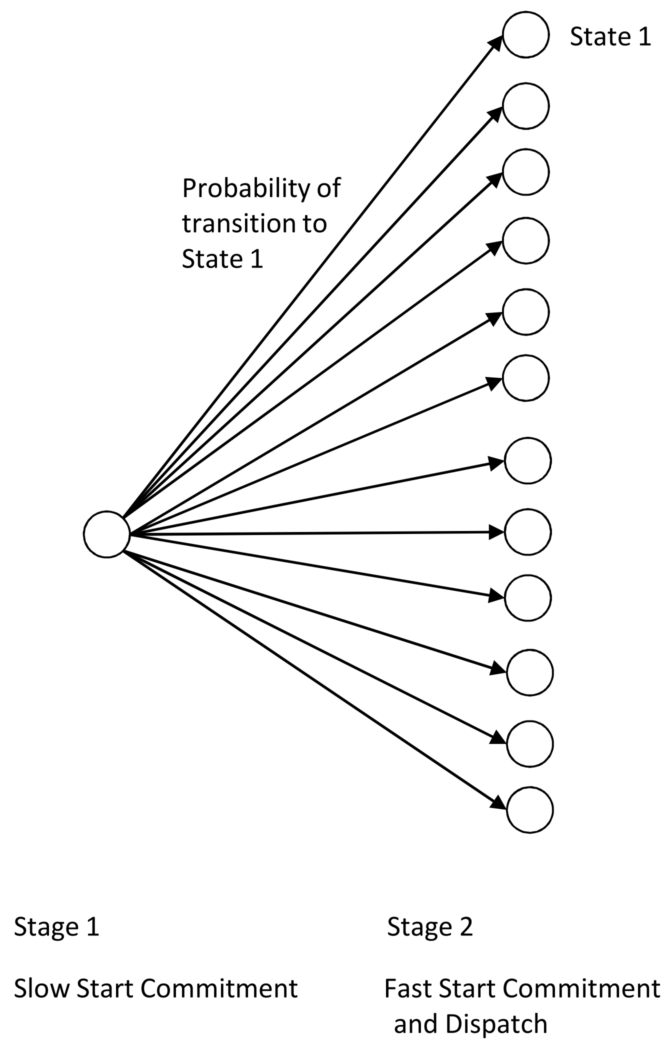
- **The previous probabilistic optimization framework results in a very large optimization problem.**
  - Problem size grows exponentially in number of states and decision variables as number of stages grows (time steps).
  - Not practical to solve with existing commercial software.
- **We can seek to reduce the number of stages in the optimization.**
  - Formulate a simplified problem as a two stage probabilistic optimization.

Simplified Probabilistic Optimization

Simplified Probabilistic Optimization

- **Two stage probabilistic optimization:**
  - Stage 1: Commitment decisions for slow start units for remainder of the day must be made at the start of the optimization before uncertainty is resolved.
  - Stage 2: Commitment decisions for fast start resources and dispatch decisions for all committed resources are made after uncertainty is resolved.
    - In reality, these commitment and dispatch decisions will be made for a few hours at a time.
    - Will fit in the future look ahead commitment (LAC) and look ahead dispatch (LAD) framework
- **System states in stage 2 will be uncertain when decisions are made in the first stage.**
  - Index system states by l.
  - Let p_l be the probability of state l occurring in the second stage.

Simplified Tree



Simplified Formulation

- **This formulation minimizes:**
  - The cost of committing slow start resources before uncertainty is resolved

  Plus
  - The expected cost of committing fast start resources and dispatching all committed resources to meet requirements after uncertainty is resolved in all possible states.
- **Problem size is significantly reduced and can be solved using commercial solvers as long as we keep the number of states small.**

Problems with Simplified Formulation

- **Problems remain with the simplified formulation.**
  - It can be difficult to set realistic probabilities on the states after reducing the complex tree to the simple two step tree.
  - This can produce results that do not actually minimize expected production costs in the more realistic model.
- **We can address these issues by choosing to minimize costs to ensure the ability to operate reliably in all states.**
  - In the first stage, commit slow start resources that will enable the RTO meet requirements in the second stage by committing fast start resources and dispatch.
  - Ignore costs in the second stage and focus only on first stage costs to meet reliability goals.
  - We can call this a “robust optimization” formulation.

Robust Optimization

Robust Optimization

- **Robust optimization can be viewed as setting the probabilities in the simplified formulation to zero.**
  - The expected costs of committing fast start resources and dispatching committed resources are ignored in the optimization.
    - The argument is: managing the economic impact of uncertainty should be left to the participants and not taken on by the RTO.
  - Only the feasibility constraints in the second stage are considered.
- **Extension to Robust optimization formulation exists, e.g.,**
  - The costs of committing fast start resources for a pre-defined scenario is considered in the optimization.
    - This can help RTO to achieve additional goals
  - The feasibility constraints in the second stage are considered.

Numerical Examples

A Six Generators Example

- **Generator information**

Generator #	Unit Type	EconMin (MW)	EconMax (MW)	StartUpCost ($)	NoloadCost ($)	Incremental Cost ($/MWh)
1	Slow start	5500	6500	1000	0	5
2	Slow start	50	580	0	580	50
3	Slow start	150	350	500	350	20
4	Slow start	300	400	500	100	19
5	Fast start	50	60	0	3500	120
6	Fast start	50	50	0	0	160

- **Assuming energy only clearing, no Ancillary Service requirement, also assuming Generator #1 is combination of several smaller units with similar cost structure.**
- **Expected Load at 7051MW**
- **Possible high and low load are 7351 and 6801MW**

Re deterministic model – minimum commitment cost

- **Commit against expected Load 7051MW**
- **Unit 1 and 2 will be on, and the dispatch results are:**

unit	on/off	Dispatch MW (MW)	Commitment Cost ($)	Production Cost ($)	LMP ($/MWh)	Revenue ($)	RSG ($)
1	1	6500	28500	33500	50	325000	0
2	1	551	3080	28130	50	27550	580
3	0	0	0	0	50	0	0
4	0	0	0	0	50	0	0
5	0		0	0	50	0	0
6	0		0	0	50	0	0

- **What will happen if actual load is higher than the expected load, for example, load goes up to 7351MW?**
  - Fast start unit 5 & 6 will be called on
  - Slow start units 3 & 4 can not be on due to time limitation
    - Total available capacity is 7190MW, short of 161MW.
    - Scarcity!

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Two Stage Probabilistic Model

- **2 stage stochastic model**
- **Assuming three scenarios with different load**

	Scenario1	Scenario2	Scenario3
Probability	0.3	0.6	0.1
Demand (MW)	7351	7051	6801

- **Commitment results**

Slow start unit commitment result			
unit	on/off		
1	1		
2	1		
3	1		
4	1		
Fast start unit	scenario 1	scenario 2	scenario 3
5	0	0	0
6	0	0	0

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Two Stage Probabilistic Model

- **Dispatch results under different scenarios:**

	Scenario1 (demand = 7351 MW)						Scenario2 (demand = 7051 MW)						Scenario3 (demand = 6081 MW)					
Unit #	Dispatch MW (MW)	Commitment Cost ($)	Production Cost ($)	LMP ($/MWh)	Revenue ($)	RSG ($)	Dispatch MW (MW)	Commitment Cost ($)	Production Cost ($)	LMP ($/MWh)	Revenue ($)	RSG ($)	Dispatch MW (MW)	Commitment Cost ($)	Production Cost ($)	LMP ($/MWh)	Revenue ($)	RSG ($)
1	6500	28500	33500	50	325000	0	6500	28500	33500	19	123500	0	6301	28500	32505	5	31505	1000
2	101	3080	5630	50	5050	580	50	3080	3080	19	950	2130	50	3080	3080	5	250	2830
3	350	3850	7850	50	17500	0	150	3850	3850	19	2850	1000	150	3850	3850	5	750	3100
4	400	6300	8200	50	20000	0	351	6300	7269	19	6669	600	300	6300	6300	5	1500	4800
5	0	0	0	50	0	0		0	0	19	0	0		0	0	5	0	0
6	0	0	0	50	0	0		0	0	19	0	0		0	0	5	0	0
Total	7351	41730	55180			580	7051	41730	47699			3730	6801	41730	45735			11730

- **Expected production cost: $49,746.9**
- **Commitment results rely on probabilities of scenarios**

  Note: RSG is the uplift payment needed to cover the cost

Two Stage Robust Model

- **Still assume same three load scenarios:**
  - High load: 7351MW
  - Middle load: 7050MW
  - Low load: 6801MW

- **Commitment results:**

Slow start unit commitment result			
unit	on/off		
1	1		
2	1		
3	1		
4	0		
Fast start unit	scenario 1	scenario 2	scenario 3
5	1	1	0
6	1	1	1

Two Stage Robust Model

- **Dispatch results under different scenarios:**

	Scenario1 (demand = 7351 MW)						Scenario2 (demand = 7051 MW)						Scenario3 (demand = 6081 MW)					
unit	Dispatch MW (MW)	Commitment Cost ($)	Production Cost ($)	LMP ($/MWh)	Revenue ($)	RSG ($)	Dispatch MW (MW)	Commitment Cost ($)	Production Cost ($)	LMP ($/MWh)	Revenue ($)	RSG ($)	Dispatch MW (MW)	Commitment Cost ($)	Production Cost ($)	LMP ($/MWh)	Revenue ($)	RSG ($)
1	6500	28500	33500	50	325000	0	6500	28500	33500	50	325000	0	6500	28500	33500	20	130000	0
2	501	3080	25630	50	25050	580	201	3080	10630	50	10050	580	50	3080	3080	20	1000	2080
3	350	3850	7850	50	17500	0	350	3850	7850	50	17500	0	251	3850	5870	20	5020	850
4		0	0	50	0	0		0	0	50	0	0		0	0	20	0	0
5		0	0	50	0	0		0	0	50	0	0		0	0	20	0	0
6		0	0	50	0	0		0	0	50	0	0		0	0	20	0	0
Total	7351	35430	66980			580	7051	35430	51980			580	6801	35430	42450			2930

The accuracy of the probabilities for each scenario is not important
The reach-ability of the scenarios is still maintained.

Conclusion

- **A simplified probabilistic optimization RAC framework is presented**
  - To deal with more uncertainties in RTO's business
  - Application to RTO's operation seems achievable
- **Variations for actual applications can be made to achieve different goals**
- **Benefits:**
  - Improved reliability with capability to cover different scenarios
  - More economic if RTO can reduce reserve procurement

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Appendices: Mathematical Formulations

Mathematical Formulation for Probabilistic Optimization Framework

Probabilistic Optimization Framework

- **Index possible system states at time t by l_t**
  - We will assume that only demand depends upon state
    - Let the vector of nodal demands be given by $\mathbf{d}_t^{l_t}$
  - Let the probability of transitioning to state l_t depend upon state at time t-1: $p(l_t | l_{t-1})$
- **Index resources by i**
  - Resource characteristics for resource i:

m_{it} = minimum output from unit i in period t if unit is on

M_{it} = maximum output from unit i in period t if unit is on

$ramp_{it}$ = maximum ramp from unit i between period t-1 and period t

$StartCost_{it}$ = Cost to start unit i in period t

$NoLoad_{it}$ = No load cost for unit i in period t if unit is on

$GenCost_{it}(\cdot)$ = Production cost above No Load Cost to produce energy from unit i in period t

$notify_i$ = notification time needed from receiving start signal to being on-line

Probabilistic Optimization Framework

- **Decision variables for resource i:**

$$start_{it}^{l_t} = \begin{cases} 0 \text{ if the decision in period t is not to start unit i given the state in period t} \\ 1 \text{ if the decision in period t is to start unit i given the state in period t} \end{cases}$$

$$on_{it}^{l_t} = \begin{cases} 0 \text{ if unit i is off in period t given the state in period t} \\ 1 \text{ if unit i is on in period t given the state in period t} \end{cases}$$

$$g_{it}^{l_t} = \text{output of unit i in period t given the state in periods t}$$

  - At time 0, we only will have commitment decisions since it is before the operating day.
  - We will use the constant on_{i0} to indicate that a unit was on at the end of the last day and does not require a start decision to be on-line.

Formulation for Energy Only

$$\min_{\mathrm{g,on,start}} \left[\begin{array}{l} \sum_{t=1}^{T} \left[\sum_{l_t} p(l_t | l_{t-1}) \cdot \left[\sum_i \left(StartCost_{i,t+notify_i} \cdot start_{it}^{l_t} + NoLoad_{it} \cdot on_{it}^{l_t} \right) + \sum_i GenCost_{it}\left(g_{it}^{l_t}\right) \right] \right] \\ + \sum_i \left(StartCost_{i,notify_i} \cdot start_{i0} \right) \end{array} \right]$$

subject to

$$m_{it} \cdot on_{it}^{l_t} \le g_{it}^{l_t} \le M_{it} \cdot on_{it}^{l_t} \qquad \forall i, l_t, t \ge 1$$

$$-ramp_{it} \le g_{it}^{l_t} - g_{i,t-1}^{l_{t-1}} \le ramp_{it} \cdot on_{it}^{l_t} \qquad \forall i, l_t, t \ge 1$$

$$\left. \begin{array}{ll} 0 \le on_{it}^{l_t} \le on_{i,t-1}^{l_{t-1}} & \text{if } t - notify_i < 0 \\ start_{i,0} \le on_{it}^{l_t} \le start_{i,0} + on_{i,t-1}^{l_{t-1}} & \text{if } t - notify_i = 0 \\ start_{i,t-notify_i}^{l_1 \cdots l_{t-notify_i}} \le on_{it}^{l_1 \cdots l_t} \le start_{i,t-notify_i}^{l_1 \cdots l_{t-notify_i}} + on_{i,t-1}^{l_1 \cdots l_{t-1}} & \text{otherwise} \end{array} \right\} \qquad \forall i, l_t, t \ge 1$$

$$start_{i0} = 0 \text{ or } 1 \qquad \forall i$$

$$start_{it}^{l_t} = 0 \text{ or } 1 \qquad \forall i, l_t, t \ge 1$$

$$on_{it}^{l_t} = 0 \text{ or } 1 \qquad \forall i, l_t, t \ge 1$$

$$\mathbf{e}^T \mathbf{g}_t^{l_t} + \left(\mathbf{LossSen}_t^{l_t}\right)^T \mathbf{g}_t^{l_t} = \left(\mathbf{e} + \mathbf{LossSen}_t^{l_t}\right)^T \mathbf{d}_t^{l_t} - OffSet_t^{l_t} \qquad \forall l_t, t \ge 1$$

$$\left(\nabla Flow_{kt}^{l_t}\right)^T \mathbf{g}_t^{l_t} \le F_{kt}^{max,l_t} + \left(\nabla Flow_{kt}^{l_t}\right)^T \mathbf{d}_t^{l_t} \qquad \forall k, l_t, t \ge 1$$

Formulation for Energy Only Expanded to Show Nested Nature of Decisions

$$
\min_{\mathrm{start}_0}\left[\begin{array}{l}
\sum_i \left(StartCost_{i,notify_i}\cdot start_{i0}\right)\\
+\left[\sum_{l_1} p(l_1)\cdot\left[\begin{array}{l}
\min\limits_{\mathbf{g}_1^{l_1},\mathbf{on}_1^{l_1},\mathbf{start}_1^{l_1}} \sum_i \left(StartCost_{i,1+notify_i}\cdot start_{i,1}^{l_1} + NoLoad_{i,1}\cdot on_{i,1}^{l_1}\right)+\sum_i GenCost_{i,1}\left(g_{i,1}^{l_1}\right)\\
+\sum_{l_2} p\left(l_2 \middle| l_1\right)\cdot\left[\begin{array}{l}
\min\limits_{\mathbf{g}_2^{l_2},\mathbf{on}_2^{l_2},\mathbf{start}_2^{l_2}} \sum_i \left(StartCost_{i,2+notify_i}\cdot start_{i,2}^{l_2} + NoLoad_{i,2}\cdot on_{i,2}^{l_2}\right)+\sum_i GenCost_{i,2}\left(g_{i,2}^{l_2}\right)\\
+\ldots\\
\text{Subject to}\\
\text{Operating constraints at time 2}
\end{array}\right]\\
\text{Subject to}\\
\text{Operating constraints at time 1}
\end{array}\right]\right]
\end{array}\right]
$$

Subject to

$$start_{i0} = 0,1 \quad \forall i$$

Operating Constraints in Formulation

- **Operating constraints at time t for state l_t**

$$m_{it} \cdot on_{it}^{l_t} \le g_{it}^{l_t} \le M_{it} \cdot on_{it}^{l_t} \qquad \forall i$$

$$-ramp_{it} \le g_{it}^{l_t} - g_{i,t-1}^{l_{t-1}} \le ramp_{it} \cdot on_{it}^{l_t} \qquad \forall i$$

$$\left.\begin{array}{ll} 0 \le on_{it}^{l_t} \le on_{i,t-1}^{l_{t-1}} & \text{if } t - notify_i < 0 \\ start_{i,0} \le on_{it}^{l_t} \le start_{i,0} + on_{i,t-1}^{l_{t-1}} & \text{if } t - notify_i = 0 \\ start_{i,t-notify_i}^{l_{t-notify_i}} \le on_{it}^{l_t} \le start_{i,t-notify_i}^{l_{t-notify_i}} + on_{i,t-1}^{l_{t-1}} & \text{otherwise} \end{array}\right\} \qquad \forall i$$

$$start_{it}^{l_t} = 0 \text{ or } 1 \qquad \forall i$$

$$on_{it}^{l_t} = 0 \text{ or } 1 \qquad \forall i$$

$$\mathbf{e}^T \mathbf{g}_t^{l_t} + \left(\mathbf{LossSen}_t^{l_t}\right)^T \mathbf{g}_t^{l_t} = \left(\mathbf{e} + \mathbf{LossSen}_t^{l_t}\right)^T \mathbf{d}_t^{l_t} - OffSet_t^{l_t}$$

$$\left(\nabla Flow_{kt}^{l_t}\right)^T \mathbf{g}_t^{l_t} \le F_{kt}^{max,l_t} + \left(\nabla Flow_{kt}^{l_t}\right)^T \mathbf{d}_t^{l_t} \qquad \forall k$$

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Mathematical Formulation for Simplified Probabilistic Optimization Framework

Formulation

- **We will change the start up decision variable to reflect the decision to have a resource on-line at time t instead of the time the start-up signal is sent.**

$$start_{it} = \text{the decision variable to start slow start resoruce i to be on - line at time t}$$

$$start_{it}^{l} = \begin{pmatrix} \text{the decision variable to start a fast start resoruce i to be on - line at time t} \\ \text{when the system is in state } l \end{pmatrix}$$

Simplified Formulation

$$
\min_{\substack{\text{on,start for}\\ \text{Slow Start}}}
\left(
\begin{array}{l}
\displaystyle\sum_{t=1}^{T} \sum_{i\in SlowStart} \left(StartCost_{it}\cdot start_{it} + NoLoad_{it}\cdot on_{it} + GenCost_{it}\left(on_{it}\cdot m_{it}\right)\right) \\
\displaystyle\sum_{l} p_l \cdot
\left[
\begin{array}{ll}
\displaystyle\min_{\substack{\text{on,start for}\\ \text{Fast Start}\\ \text{g for all}}} \sum_{t=1}^{T}
\left[
\begin{array}{l}
\displaystyle\sum_{i\in FastStart} \left(StartCost_{it}\cdot start_{it}^{l} + NoLoad_{it}\cdot on_{it}^{l}\right) \\
\displaystyle + \sum_{i\in FastStart} GenCost_{it}\left(g_{it}^{l}\right) + \sum_{i\in SlowStart} GenCost_{it}\left(g_{it}^{l}\right) - \sum_{i\in SlowStart} GenCost_{it}\left(on_{it}\cdot m_{it}\right)
\end{array}
\right] & \\
\text{subject to} & \\
m_{it}\cdot on_{it}^{l} \le g_{it}^{l} \le M_{it}\cdot on_{it}^{l} & \forall i\in FastStart, t \\
-ramp_{it} \le g_{it}^{l} - g_{i,t-1}^{l} \le ramp_{it}\cdot on_{it}^{l} & \forall i\in FastStart, t \\
start_{it}^{l} \le on_{it}^{l} \le start_{it}^{l} + on_{i,t-1}^{l} & \forall i\in FastStart, t \\
start_{it}^{l} = 0 \text{ or } 1 & \forall i\in FastStart, t \\
on_{it}^{l} = 0 \text{ or } 1 & \forall i\in FastStart, t \\
m_{it}\cdot on_{it} \le g_{it}^{l} \le M_{it}\cdot on_{it} & \forall i\in SlowStart, t \\
-ramp_{it} \le g_{it}^{l} - g_{i,t-1}^{l} \le ramp_{it}\cdot on_{it} & \forall i\in SlowStart, t \\
\mathbf{e}^{T}\mathbf{g}_{t}^{l} + \left(\mathbf{LossSen}_{t}^{l}\right)^{T}\mathbf{g}_{t}^{l} = \left(\mathbf{e} + \mathbf{LossSen}_{t}^{l}\right)^{T}\mathbf{d}_{t}^{l} - OffSet_{t}^{l} & \forall t \\
\left(\nabla Flow_{kt}^{l}\right)^{T}\mathbf{g}_{t}^{l} \le F_{kt}^{max,l} + \left(\nabla Flow_{kt}^{l}\right)^{T}\mathbf{d}_{t}^{l} & \forall k, t
\end{array}
\right]
\end{array}
\right)
$$

subject to

$$
\begin{array}{ll}
start_{it} \le on_{it} \le start_{it} + on_{i,t-1} & \forall i\in SlowStart, t \\
start_{it} = 0 \text{ or } 1 & \forall i\in SlowStart, t \\
on_{it} = 0 \text{ or } 1 & \forall i\in SlowStart, t
\end{array}
$$

Mathematical Formulation for Robust Optimization Framework

Robust Optimization Framework

$$\min_{\substack{\text{g, on,start for} \\ \text{All Units}}} \sum_{t=1}^{T} \sum_{i \in SlowStart} \left(StartCost_{it} \cdot start_{it} + NoLoad_{it} \cdot on_{it} + GenCost_{it}\left(on_{it} \cdot m_{it}\right)\right)$$

subject to

$$m_{it} \cdot on_{it} \le g_{it}^{l} \le M_{it} \cdot on_{it} \qquad \forall i \in SlowStart, l, t$$

$$-ramp_{it} \le g_{it}^{l} - g_{i,t-1}^{l} \le ramp_{it} \cdot on_{it} \qquad \forall i \in SlowStart, l_{t}, t$$

$$start_{it} \le on_{it} \le start_{it} + on_{i,t-1} \qquad \forall i \in SlowStart, t$$

$$start_{it} = 0 \text{ or } 1 \qquad \forall i \in SlowStart, t$$

$$on_{it} = 0 \text{ or } 1 \qquad \forall i \in SlowStart, t$$

$$m_{it} \cdot on_{it}^{l} \le g_{it}^{l} \le M_{it} \cdot on_{it}^{l} \qquad \forall i \in FastStart, l, t$$

$$-ramp_{it} \le g_{it}^{l} - g_{i,t-1}^{l} \le ramp_{it} \cdot on_{it}^{l} \qquad \forall i \in FastStart, l, t$$

$$start_{it}^{l} \le on_{it}^{l} \le start_{it}^{l} + on_{i,t-1}^{l} \qquad \forall i \in FastStart, l, t$$

$$start_{it}^{l} = 0 \text{ or } 1 \qquad \forall i \in FastStart, l, t$$

$$on_{it}^{l} = 0 \text{ or } 1 \qquad \forall i \in FastStart, l, t$$

$$\mathbf{e}^{T}\mathbf{g}_{t}^{l} + \left(\mathbf{LossSen}_{t}^{l}\right)^{T}\mathbf{g}_{t}^{l} = \left(\mathbf{e} + \mathbf{LossSen}_{t}^{l}\right)^{T}\mathbf{d}_{t}^{l} - OffSet_{t}^{l} \qquad \forall l, t$$

$$\left(\nabla Flow_{kt}^{l}\right)^{T}\mathbf{g}_{t}^{l} \le F_{kt}^{max,l} + \left(\nabla Flow_{kt}^{l}\right)^{T}\mathbf{d}_{t}^{l} \qquad \forall k, l, t$$

MISO