Topological and Impedance Element Ranking (TIER) of the Bulk-Power System

Bernie Lesieutre   Chris DeMarco   Dan Schwarting
University of Wisconsin-Madison

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Bulk-Power System vs. Distribution

FERC/FPA: The term 'bulk-power system' means: a) facilities and operating systems necessary for operating an interconnected electric energy transmission network (or any portion thereof); and b) electric energy from generation facilities needed to maintain transmission system reliability. The term does not include facilities used in the local distribution of electric energy. (Federal Power Act, Section 215a.)

NERC: Bulk Electric System: As defined by the Regional Reliability Organization, the electrical generation resources, transmission lines, interconnections with neighboring systems, and associated equipment, generally operated at voltages of 100 kV or higher. Radial transmission facilities serving only load with one transmission source are generally not included in this definition.

Goal

Our Task: An approach to distinguish those facilities that should/should not be considered part of the Bulk-Power System based on the network layout (“topology”) and electrical properties of connections.

Approach

Our Approach: Rank/Classify components by their potential to impact capacity resource dispatch.

Optimization Formulation: Used to uniquely relate and measure the components potential impact.

Background

Optimization Terminology

- Objective: minimize some "Objective" or "Cost" function:

$$\min_{x} \boxed{C(x)}$$

"Cost" Function

Examples:
Production Costs,
Energy Loss …

Decision variables – adjusted to meet optimum

Example: Production

Background

Optimization Terminology

- Constraints: Additional conditions that must be satisfied.

$$f(x,y)=0$$

"**Equality Constraints**" often represent fundamental laws.

Examples:
Conservation of Energy,
Kirchhoff's Laws …

Decision and dependent variables

$$g(x,y)\le 0$$

"**Inequality Constraints**" often represent practical limits.

Examples: Thermal, Voltage, stability limits (often by proxy)

Background

Optimization Terminology

- **Lagrange Multipliers**: Traditional method for Constrained Optimization

$$\min_{x,y,\lambda} L(x,y,\lambda) = C(x) + \lambda f(x,y)$$

The constraint is included in an augmented "cost" function, multiplied by a "lagrange multiplier, λ

The Optimal solution is obtained using basic calculus: partial derivatives with respect to variables are set equal to zero.

Background

Optimization Terminology

- **Lagrange Multipliers**: Traditional method for Constrained Optimization

$$\min_{x,y,\lambda} L(x,y,\lambda) = C(x) + \boxed{\lambda} f(x,y)$$

The lagrange multiplier is interpreted as a sensitivity measure of the constraint's impact on "cost".

$\lambda = 0$ No Impact

$\lambda \neq 0$ Impact

Examples:
LMP
Flowgate "shadow prices"

Background

In terms of an optimization problem

- We are interested in how a branch constraint impacts a bus constraint through the network.

  branch flow constraint $\longrightarrow$ generator dispatch

- We compare the corresponding lagrange multipliers.
- We are NOT interested in cost.

Background

- Assume power system operated optimally, but DO NOT assume knowledge of objective functions being optimized.

- *Could* use $/hr operating cost as familiar objective (many other possibilities exist).

  BUT KEY POINT:
  $ COSTS DO NOT IMPACT METHOD.

Background

- Q: How can cost to be optimized *not matter?*

- A: Use properties that arise purely out of network constraints inherent in any power system optimization problem.

  for the mathematicians…

  Optimization identifies *pattern* of Lagrange multipliers associated with each element constraint – these patterns yield rankings.

Background: Optimization

DC Optimal Power Flow – optimize while constraining one facility at a time:

$$\min_{P_g,\theta} \; Cost(P_g)$$

Cost (production $/hr typical - but again, choice here does not matter!)

$$\text{subject to} \quad P_{injected} = ABA^T\theta$$

Network Constraints (power balance at each bus)

$$\text{and} \quad P_{line} = b_{line}A_{line}^T\theta = P_{\text{limit}}$$

Single Facility Constraint (flow limit on a line or transformer)

Background: Optimization

Incorporate the constraints into an augmented "Lagrangian" cost function:

$$\min_{P_g,\theta} \; L(P_g,\theta) \; = \; Cost(P_g) + \lambda^T \left(P_{injected} - ABA^T\theta\right) + \mu_{line}\left(P_{\text{limit}} - b_{line}A_{line}^T\theta\right)$$

First-year calculus tells us conditions that must be satisfied at *any* optimal solution …

Background: Optimization

Network Property of Optimal Solution

$$\min_{P_g,\theta} \; L(P_g,\theta) \; = \; Cost(P_g) + \lambda^T \left(P_{injected} - ABA^T\theta\right) + \mu_{line}\left(P_{\text{limit}} - b_{line}A_{line}^T\theta\right)$$

(calculus)

$$\frac{\partial L(P_g,\theta)}{\partial\theta} = \boxed{-A^T B^T A\lambda - A_{line} b_{line}\mu_{line} = 0}$$

These are (part of) necessary conditions for optimal solution - those that are independent of cost

<u>Bus constraints</u>
Lagrange multipliers at buses
In familiar cases these are
Bus LMPs in $'s/MWh

<u>Line constraint</u>
Lagrange multiplier
of constrained element
Redispatch $'s/MWh

Background: Optimization

$$\frac{\partial L(P_g,\theta)}{\partial \theta} = -A^T B^T A\lambda - A_{line} b_{line} \mu_{line} = 0$$

This couples constrained element (via scalar μ), to allowable patterns in the λ's

Topology information in matrix "*A*"

Components properties in matrix "*B*" and scalar "*b*"

Here λ is a vector of quantities, related to dispatch at every connection point (λ's are bus LMPs in market case). μ is a single scalar quantity, associated with a single line or transformer to be ranked.

So Far…

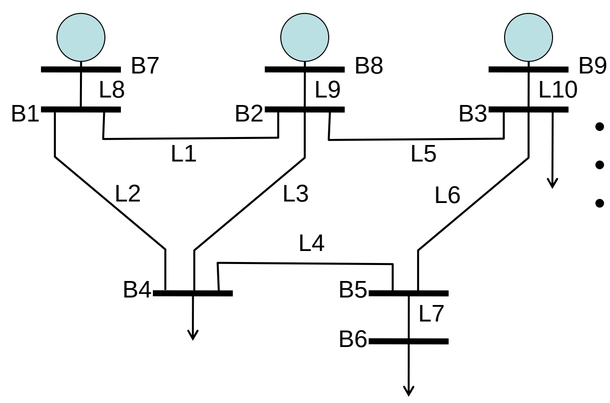
- Set up representation for optimal operation of the power system.
- Extract those conditions that have to do **ONLY** with network constraints.
- Important to emphasize –DEPENDENCE ON COST/OBJECTIVE FUNCTION WILL "DROP OUT" IN RESULT OF METHOD.

Next Step: Rankings

- With no line or transformer constrained, only possible pattern is all λ's exactly equal (case of uniform LMPs in familiar market case)
- Constraints purely in distribution system can't alter these equal λ's at generators.
- However, constraints in the transmission network ***can*** yield unequal λ's at generators.

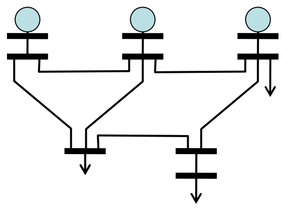
Rank each component by degree to which it moves generator λ's (e.g., LMP profiles) away from uniform, all equal pattern.

Small Example



- 9 buses
- 10 lines
- 3 generators

Small Example



The bus Lagrange multipliers can be expressed in ratio to the network Lagrange multipliers.

$$\begin{bmatrix}\lambda_1\\\lambda_2\\\lambda_3\\\lambda_4\\\lambda_5\\\lambda_6\\\lambda_7\\\lambda_8\\\lambda_9\end{bmatrix} = \mu_0\begin{bmatrix}1\\1\\1\\1\\1\\1\\1\\1\\1\end{bmatrix} + \mu_1\begin{bmatrix}0.40\\-0.23\\-0.14\\0.04\\-0.05\\-0.05\\0.40\\-0.23\\-0.14\end{bmatrix} + \cdots + \mu_{10}\begin{bmatrix}0.11\\0.11\\0.11\\0.11\\0.11\\0.11\\0.11\\0.11\\-0.89\end{bmatrix}$$

LMP

“Energy Component”

“Congestion Component”

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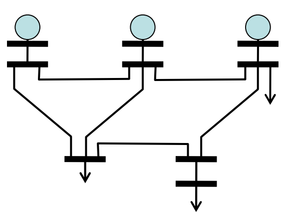
Small Example



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Focus on how each column shapes the pattern at dispatchable buses (i.e., at generators).

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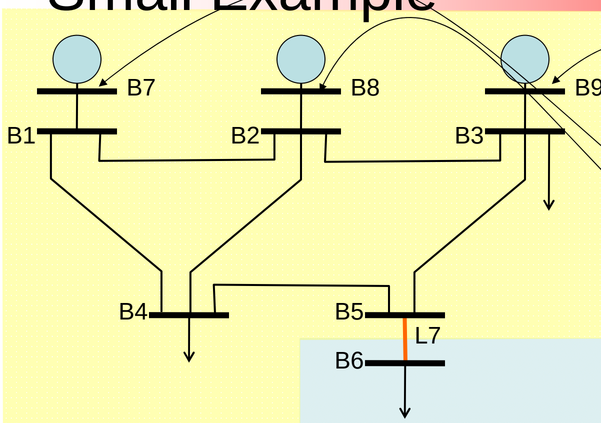
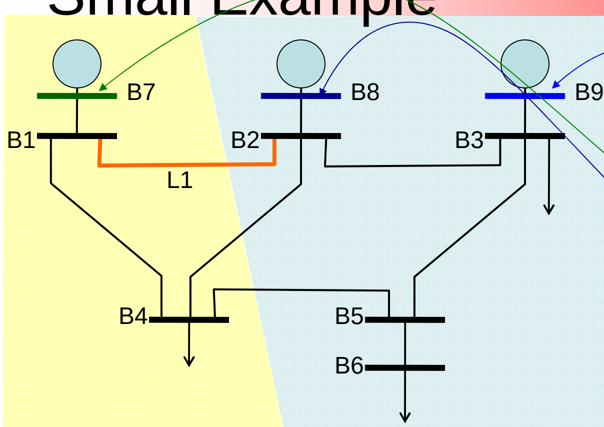
Small Example



$$\begin{bmatrix}\lambda_1\\\lambda_2\\\lambda_3\\\lambda_4\\\lambda_5\\\lambda_6\\\lambda_7\\\lambda_8\\\lambda_9\end{bmatrix} = \mu_0\begin{bmatrix}1\\1\\1\\1\\1\\1\\1\\1\\1\end{bmatrix} + \mu_7\begin{bmatrix}-0.11\\-0.11\\-0.11\\-0.11\\-0.11\\0.89\\-0.11\\-0.11\\-0.11\end{bmatrix}$$

Examine the effect of line L7 (radial load).
Yields no impact on dispatchable network resources (i.e., values are all uniform at generator buses).

Small Example



$$\begin{bmatrix}\lambda_1\\\lambda_2\\\lambda_3\\\lambda_4\\\lambda_5\\\lambda_6\\\lambda_7\\\lambda_8\\\lambda_9\end{bmatrix} = \mu_0\begin{bmatrix}1\\1\\1\\1\\1\\1\\1\\1\\1\end{bmatrix} + \mu_1\begin{bmatrix}0.40\\-0.23\\-0.14\\0.04\\-0.05\\-0.05\\0.40\\-0.23\\-0.14\end{bmatrix}$$

In contrast, line L1 ***does*** affect values at dispatchable network resources - no longer all equal.

So Far…

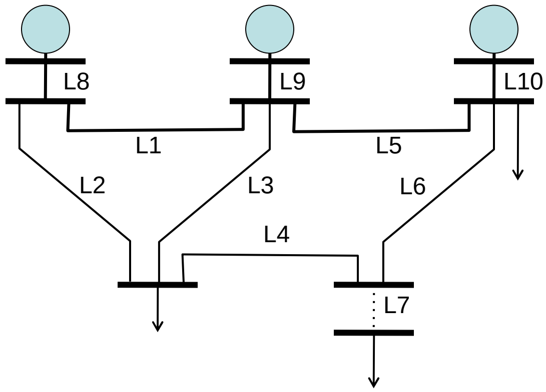
- Introduced an element ranking approach based on the degree to which it moves generator λ's away from uniform, all equal pattern.
- Mathematically expressed as a unit-less sensitivity between bus and network lagrange multipliers – independent of cost function.
- Illustrated with a small example.

Small Example

Q: How to measure (with single numeric indicator) degree to which λ's for dispatchable resources deviate from the all-equal case? (larger deviation indicates larger and more "network-wide" impact).

A: Choose the standard deviation of λ profile as single numeric measure of departure from the unconstrained, all-equal case.

Small Example



LINE	METRIC
L1	0.34
L2	0.24
L3	0.14
L4	0.19
L5	0.40
L6	0.19
L7	0
L8	0.58
L9	0.58
L10	0.58

Lines and their computed metrics: standard deviation of generator bus Lagrange multipliers, due to line constraint.

Results: PJM Model

- ~8000 buses
- ~9000 lines
- Detailed representation of lower-voltage, subtransmission, and distribution buses
- 875 Generators
- Detailed representation outside of PJM
  - 7000 additional buses
  - 8000 additional lines

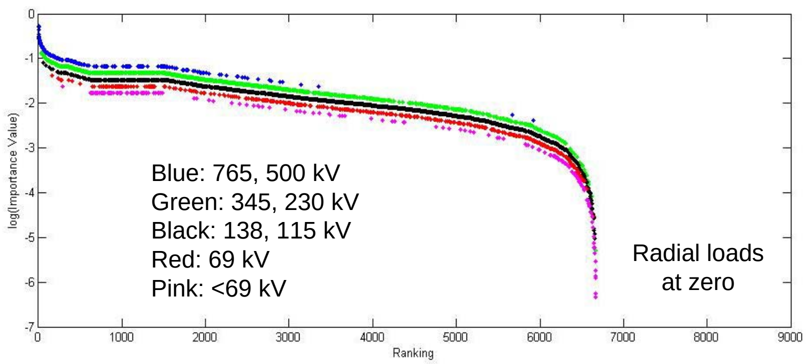
Computation time for Lagrange sensitivities to all 9000 lines: 6 minutes.

Source: PJM website

30	765 kV buses
100	500 kV
210	345 kV
800	230 kV
2550	138 kV
750	115 kV
930	69 kV
2240	<69 kV

Results: PJM Model

- Distribution of TIER Values
  Logarithmic plot, voltages color-coded



Results: PJM System

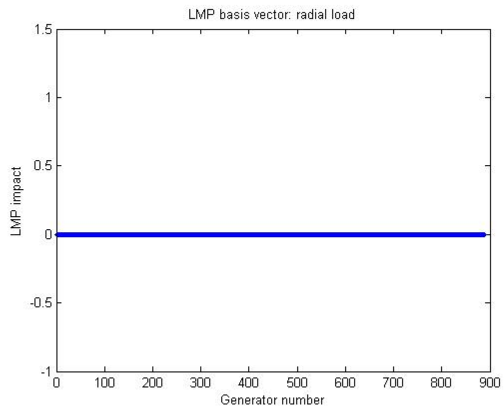
- Typical TIER Values

  (excluding lines to radial loads, radial generators, and ties to non-PJM areas)

Voltage	Low	Average	High
765 kV	0.0220	0.072	0.263
500 kV	0.0021	0.059	0.217
345 kV	0.00013	0.023	0.093
230 kV	3.5×10^{-6}	0.021	0.095
138/115 kV	9.4×10^{-6}	0.010	0.082
69 kV	4.2×10^{-5}	0.0072	0.050
<69 kV	9.1×10^{-7}	0.0045	0.026

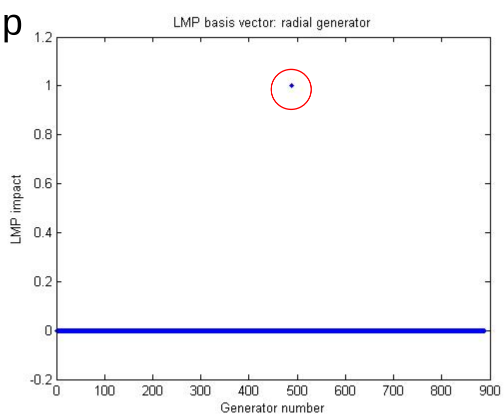
Results: PJM System

- Sample Sensitivity Plots
  - Radial load (no variation in λ, implies value = 0)



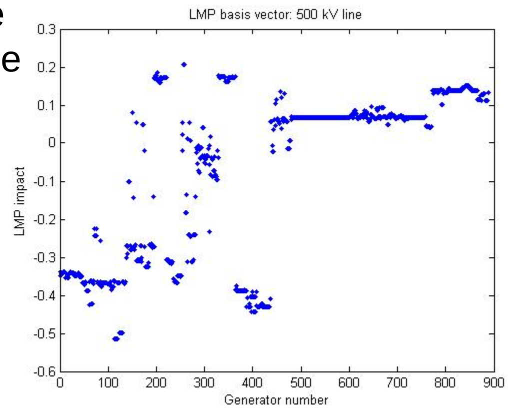
Results: PJM System

- Sample Sensitivity Plots
  - Generator step-up transformer (value = 0.0336)



Results: PJM System

- Sample Sensitivity Plots
  - Highest importance value for 500 kV line



Results: PJM System

- Importance values produced here were reviewed by FERC staff and PJM engineers, very familiar with system.
- Overall pattern of rankings confirmed as reasonable.
- Moreover, seeming anomalies examined, and confirmed as consistent with the specifics of the given topology.

Discussion

- The highest EHV level components, when networked, have a high TIER.
- Radial loads have zero TIER.
- The degree to which elements are networked, i.e., the topology, has major impact on the TIER results.

Discussion

Eastern Interconnect TIER (US only)

60000 branches, 6000 generators