Tight MIP Formulation of Transition Trajectories of Combined-Cycle Units

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Outline

Start-Up and Shut-Down Trajectories of Power Plants

- For physical reasons, power plants have little ability to follow an exterior control signal during start-up (synchronization and ramping up to minimum output) and shut-down process, although the unit injects power into grid after synchronization.
- The plant's electrical output is reasonably predictable during start-up process [Anders et al., 2005].

Example from [Simoglou et al., 2010]

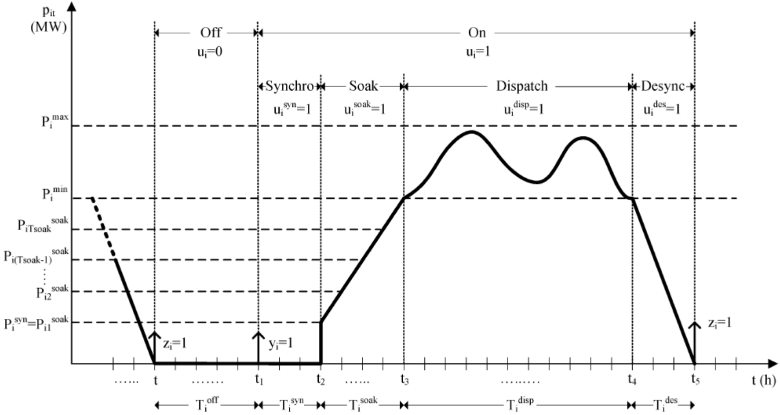


Figure: Start-up and shut-down trajectories of a simple-cycle unit.

Current solution

- In most unit commitment formulations, units are considered to start/end their production within one interval while the start-up and shut-down ramps are ignored.
- Enough lead time in day-ahead for units to start up.
- In the real-time dispatch, units in the starting up/shutting down process can be modeled as fixed injection whose value comes from SCADA.
- The commitment and dispatch decisions in day-ahead are sub-optimal compared to a model that considers such trajectories [Morales-Espana et al., 2013].

Combined-Cycle Generator Modeling

- In the UC formulation, we have assumed that at each time interval each generating unit may either be on or off.
- This assumption is only a rough approximation for **combined cycle generators (CCGs)**, a type of generator that consists of:
  - one or more combustion turbines (CTs),
  - each with a heat recovery steam generator (HRSG), and
  - one or more steam turbines (STs).
- Indispatchable start-up and shut-down processes also exist for combustion turbines and steam turbines in CCGs [Anders et al., 2005].
- During transitions between configuration, the total output of a CCG can be considered as a *fixed trajectory*.

Illustration of Starting Up Process

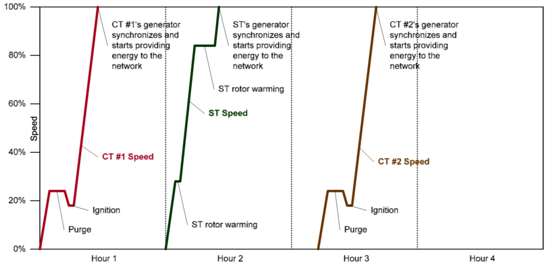


Figure 9 Component Speeds – Representative Cold Start

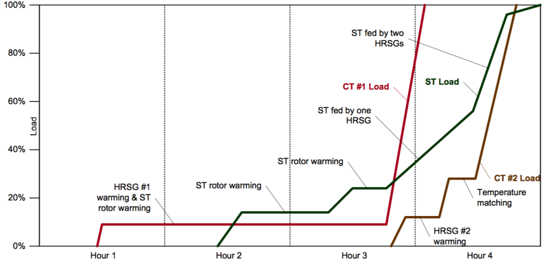


Figure 10 Component Loadings – Representative Cold Start

Look-Ahead Commitment

- MISO plans to implement more detailed modeling of CCGs in both day-ahead and real-time look-ahead commitment and dispatch.
- Since each interval in real time is five minutes, transitions might take multiple intervals.
- Modeling indispatchable CCGs in transition as dispatchable leads to efficiency loss.
  - The discrepancy between dispatch solutions and actual injections from CCGs may be soaked up by regulation.
  - In practice, CCGs during transition may submit *a ramp rate limit of 0.1 MW as a proxy of the fixed trajectory*.

Proposed Model

- Inspired by the work of [Morales-Espana et al., 2013], we propose a mixed-integer programming model for the transitions of CCGs.
- The power output of CCGs in transitions is a fixed trajectory.
- Our model is computationally efficient:
  - no new variables or constraints are introduced.
  - new terms added to constraints and the objective function.

Outline

Different Approaches

Modeling approaches for CCU:

- aggregated modeling,
- configuration-based modeling:
  - based on different combinations of CTs and STs, a CCU can be operated in one of several *configurations* as opposed to binary states;
  - one binary variable for each *configuration*.
- component-based modeling:
  - One binary variable for each *unit*.

Configuration-Based Modeling

- Configuration-based modeling is a popular approach for its simplicity.

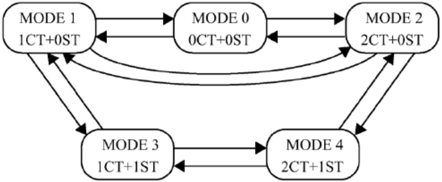


Figure: Configurations and transitions of a 2CT+1ST CCG.

- Implemented at ERCOT and SPP.

Decision Variables

- We only model the feasible set of a single combined-cycle unit.
- Let $y \in \mathcal{Y}$ be the set of configurations.
- The variables are:
  - u_t^y (binary): whether configuration y is *on* at t;
  - $v_t^{y,y'}$ (binary): indicator for transition from y to y' at t. These variables only exist for feasible transitions;
  - p_t^y (continuous): power output from configuration y at t;
  - p_t (continuous): power output of the CCG at t.

Configurations and Transitions

- The first constraint guarantees that the configurations are mutually exclusive:

$$\sum_{y \in \mathcal{Y}} u_t^y = 1, \; \forall t. \tag{1}$$

- The second constraint links configuration variables with transition variables:

$$u_t^y - u_{t-1}^y = \sum_{y' \in \mathcal{M}^{T,y}} v_t^{y'y} - \sum_{y' \in \mathcal{M}^{F,y}} v_t^{yy'}, \; \forall t, \forall y. \tag{2}$$

where $\mathcal{M}^{F,y}$ is the set of reachable configurations from y, and $\mathcal{M}^{T,y}$ is the set of reachable configurations to y.

Power Output

- Bounds on the power output of each configuration:

$$\underline{p}^y u_t^y \leq p_t^y \leq \overline{p}^y u_t^y, \forall t, \forall y. \quad (3)$$

- Total power output of CCG:

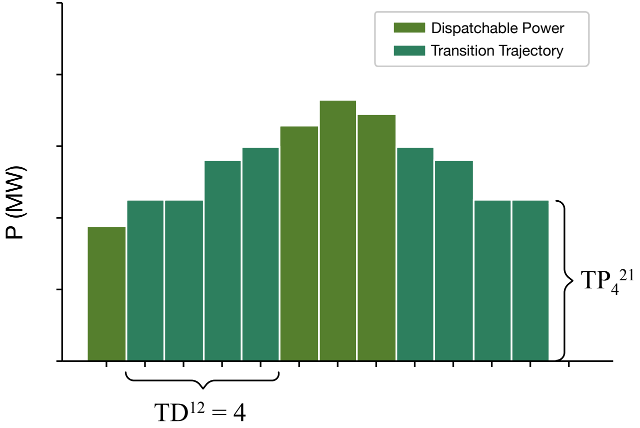
$$p_t = \sum_{y \in \mathcal{Y}} p_t^y, \forall t. \quad (4)$$

- Additional constrains including ramping and minimum up/down time of configuration/turbine.

Outline

Data for Transition Trajectory

- Let $TP_i^{yy'}$ be the *total power output from CCG in transition* at the end of the i-th interval of the transition process.
- Let $TD^{yy'}$ be the *duration* (number of intervals) of the transition process between y and y'

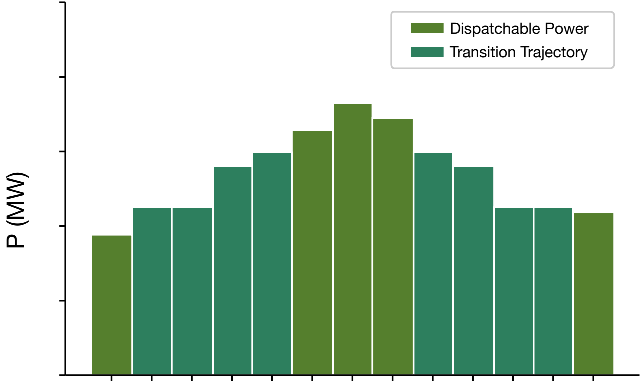


Decision Variables

- We only model the feasible set of a single combined-cycle unit.
- Let $y \in \mathcal{Y}$ be the set of configurations.
- Keep these variables:
  - u_t^y (binary),[1] $v_t^{y,y'}$ (binary), and p_t (continuous) remain;
  - p_t^y (continuous): power output *above minimum production* from configuration y at t;
- New variables (helpful for the sake of explanation but can be swapped out):
  - w_t^y (binary): whether configuration y is *dispatchable* at t;

[1] turns one when configuration y *becomes dispatchable*, and *stays zero* until a new configuration becomes dispatchable

Example



t	1	2	3	4	5	6	7	8	9	10	11	12	13
u_1	1	1	1	1	1	0	0	0	1	1	1	1	1
u_2	0	0	0	0	0	1	1	1	0	0	0	0	0
w_1	**1**	0	0	0	0	0	0	0	0	0	0	0	**1**
w_2	0	0	0	0	0	**1**	**1**	**1**	0	0	0	0	0

Figure: Transition of a CCG with 1ST+1CT.

Keep These Constraints

- Same constraint for mutually exclusive configurations:

$$\sum_{y \in \mathcal{Y}} u_t^y = 1, \; \forall t. \tag{5}$$

- Same constraint for transitions:

$$u_t^y - u_{t-1}^y = \sum_{y' \in \mathcal{M}^{T,y}} v_t^{y'y} - \sum_{y' \in \mathcal{M}^{F,y}} v_t^{yy'}, \; \forall t, \forall y. \tag{6}$$

.

- At most one transition per interval:

$$\sum_{yy' \in \mathcal{M}} v_t^{yy'} \leq 1, \; \forall t. \tag{7}$$

Modified Constraints for Power Output

- Bounds on the power output of each configuration:

$$0 \le p_t^y \le (\overline{p}^y - \underline{p}^y) w_t^y, \ \forall t, \forall y. \tag{8}$$

- Total power output of CCG:

$$\begin{aligned} p_t = & \sum_{y \in \mathcal{Y}} p_t^y + \sum_{y \in \mathcal{Y}} \underline{p}^y w_t^y + \sum_{yy' \in \mathcal{M}^U} \sum_{i=1}^{TD^{yy'}} TP_i^{yy'} v_{t-i+1+TD^{yy'}}^{yy'} \\ & + \sum_{yy' \in \mathcal{M}^D} \sum_{i=1}^{TD^{yy'}} TP_i^{yy'} v_{t-i+1}^{yy'}, \ \forall t. \end{aligned} \tag{9}$$

  - $\mathcal{M}^U$ and $\mathcal{M}^D$ are respectively the set of all upward and downward feasible transitions
  - last two terms represent the output from transition trajectory

New Binary Variable w

- Define w_t^y as:

$$w_t^y = u_t^y - \sum_{yy' \in \mathcal{M}^U} \sum_{i=1}^{TD^{yy'}} v_{t-i+1+TD^{yy'}}^{yy'} - \sum_{y'y \in \mathcal{M}^D} \sum_{i=1}^{TD^{y'y}} v_{t-i+1}^{y'y} \quad (10)$$

  - Last two terms force w_t^y to zero when in transition.
- w_t^y can be swapped out in the final formulation.

Ramping

- Intra-configuration and inter-configuration ramp rates are defined in existing models [Morales-Espana et al., 2016].
- Inter-configuration ramp rate can only be a rough proxy to the transition trajectories.

Ramping

- We define plant-wise ramping constraints:

$$p_t - p_{t-1} \leq \sum_{y \in \mathcal{Y}} R^y u_t^y, \; \forall t \tag{11}$$

- For some CCGs, CT has to reach its maximum output before committing ST. Plant-wise ramping constraints can take this into consideration.
- Plant-wise ramping constraints lead to less number of constraints compared to the existing formulations.

Tightness

- We can show: without ramping constraints, if we can describe the convex hull of the binary variables (u and v), then we have the convex hull of the whole feasible set defined on u, v, and p^y.
- Ramping constraints complicate the convex hull.
- Characterizing the convex hull of the binary variables (u and v) is itself difficult.
  - Easy for simple-cycle units with minimum up/down time constraints, but not for CCG.

Outline

Conclusions

- Accurate model for transitions of CCG.
- No new variables/constraints introduced.
- Further computational tests to be conducted.

Outline

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