Unit Commitment of Integrated Electric and Gas Systems with an Enhanced Second-Order Cone Gas Flow Model

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The following are our own views and not necessarily those of the Electricity Advisory Committee or the U.S. Department of Energy.
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1. Introduction and Background
2. Model Formulations
3. Case Study
   - Four-Node System
   - IEEE 118-Bus Test System
4. Concluding Remarks
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Future U.S. Power System

More natural gas and renewables $\Rightarrow$ increasingly interdependent electricity and gas systems

Figure: https://www.eia.gov/outlooks/aeo/
Operational Challenges
[Hibbard and Schatzki, 2012]

- Large-scale cascading electricity and gas service outage in southwest U.S. in February, 2011
- Gas-supply failures $\Rightarrow$ loss of gas-fired generation $\Rightarrow$ loss of electricity service $\Rightarrow$ loss of electrically driven compressors $\Rightarrow$ further loss of gas supply $\Rightarrow$ ...
- 1.3 million electricity and 50000 gas customers disrupted
Motivation

- It is important to coordinate the operations of increasingly interdependent electricity and gas networks.
- The dependence of the energy price of one system on the other system has not been fully investigated.
Contributions

- We propose a unit commitment model for the integrated electric and gas systems that incorporates an enhanced second order conic dynamic gas flow model.
- We enhance this model using convex envelopes of bilinear terms, resulting in a tight UC formulation.
- We investigate the impact of gas system congestion on electric LMPs (ELMPs) and the impact of power system congestion on gas LMPs (GLMPs).
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Gas System-Operation Constraints

\[ F_{S,m,t} - F_{L,m,t} - \sum_{k \in C(m)} \tau_{k,t} - \sum_{i \in G_p(m)} F_{G,i,t} = \]
\[ \sum_{n \in G(m)} F_{m,n,t} + \sum_{k \in C(m)} F_{C,k,t} \quad \forall m \in \Psi_G, t \in T \]

\[ \frac{\bar{F}_{m,n,t}^2}{C_{m,n}^2} = \pi_{m,t}^2 - \pi_{n,t}^2 \quad \forall m, n \in G_B, t \in T \]

\[ \bar{F}_{m,n,t} = \frac{1}{2} (F_{m,n,t} - F_{n,m,t}) \quad \forall m, n \in G_B, t \in T \]

\[ F_{m,n,t} + F_{n,m,t} = L_{m,n,t} - L_{m,n,t-1} \quad \forall m, n \in G_B, t \in T \]

\[ L_{m,n,t} = \frac{1}{2} K_{m,n} \cdot (\pi_{m,t} + \pi_{n,t}) \quad \forall m, n \in G_B, t \in T \]

\[ \tau_{k,t} = \theta_k F_{C,k,t} \quad \forall k \in G_C, t \in T \]

- **Inequalities:** Nodal pressure, gas production, gas compressor, and line-pack limits
Second-Order Conic Model

- Non-convex equality:
  \[
  \frac{\bar{F}_{m,n,t}^2}{C_{m,n}^2} = \pi_{m,t}^2 - \pi_{n,t}^2 \quad \forall m, n \in G_B, t \in T
  \]  

- Second-order conic relaxation:
  \[
  \frac{\bar{F}_{m,n,t}^2}{C_{m,n}^2} + \pi_{n,t}^2 \leq \pi_{m,t}^2 \quad \forall m, n \in G_B, t \in T
  \]  

- Enhanced second-order conic non-convex relaxation:
  \[
  \frac{\bar{F}_{m,n,t}^2}{C_{m,n}^2} \geq \pi_{m,t}^2 - \pi_{n,t}^2 \quad \forall m, n \in G_B, t \in T
  \]  

**NB:** (2) + (3) = (1)
Convexification

- To convexify (3):
  \[
  \frac{\bar{F}^2_{m,n,t}}{C^2_{m,n}} \geq \pi^2_{m,t} - \pi^2_{n,t} \quad \forall m, n \in G_B, t \in T
  \]

  replace the bilinear terms with their convex envelopes [McCormick, 1976]

- Convex envelope of \( \bar{F}^2_{m,n,t} \):
  \[
  \langle \bar{F}^2_{m,n,t} \rangle^M = \begin{cases} 
    \kappa_{m,n} \geq \bar{F}^2_{m,n,t} \\
    \kappa_{m,n} \leq (F^\max_{m,n,t} + F^\min_{m,n,t}) \bar{F}_{m,n,t} - F^\max_{m,n,t} F^\min_{m,n,t}
  \end{cases}
  \]

- Convexify \( \pi^2_{m,t} - \pi^2_{n,t} \) by defining \( a_{m,n,t} = \pi_{m,t} + \pi_{n,t} \), \( b_{m,n,t} = \pi_{m,t} - \pi_{n,t} \), and:
  \[
  \langle a_{m,n,t} b_{m,n,t} \rangle^M = \begin{cases} 
    \lambda_{m,n,t} \geq a^\min_{m,n,t} b_{m,n,t} + b^\min_{m,n,t} a_{m,n,t} - a^\min_{m,n,t} b^\min_{m,n,t} \\
    \lambda_{m,n,t} \geq a^\max_{m,n,t} b_{m,n,t} + b^\max_{m,n,t} a_{m,n,t} - a^\max_{m,n,t} b^\max_{m,n,t} \\
    \lambda_{m,n,t} \leq a^\min_{m,n,t} b_{m,n,t} + b^\max_{m,n,t} a_{m,n,t} - a^\min_{m,n,t} b^\max_{m,n,t} \\
    \lambda_{m,n,t} \leq a^\max_{m,n,t} b_{m,n,t} + b^\min_{m,n,t} a_{m,n,t} - a^\max_{m,n,t} b^\min_{m,n,t}
  \end{cases}
  \]

- (3) becomes: \( \kappa_{m,n}/C^2_{m,n} \geq \lambda_{m,n,t} \)
**Convexification**

**Figure**: Convex envelope of $F_{m,n,t}$

**Figure**: Convex envelope of $\pi_{m,t} - \pi_{n,t}$
Unit Commitment Models

- We compare three unit commitment models:
  1. UC with exact non-convex gas-flow constraints
  2. UC with SOC gas-flow constraints
  3. UC with enhanced SOC gas-flow constraints

- Tightness of the enhanced SOC gas-flow constraints depends on the choices of $F_{m,n,t}^{\text{max}}, F_{m,n,t}^{\text{min}}, a_{m,n,t}^{\text{max}}, a_{m,n,t}^{\text{min}}, b_{m,n,t}^{\text{max}},$ and $b_{m,n,t}^{\text{min}}$

- We update these iteratively when solving model 3 as:
  
  $F_{m,n,t}^{\text{max}} = (1 + \epsilon)F_{m,n,t}^{*}$
  $a_{m,n,t}^{\text{max}} = (1 + \epsilon)a_{m,n,t}^{*}$
  $b_{m,n,t}^{\text{max}} = (1 + \epsilon)b_{m,n,t}^{*}$
  
  $F_{m,n,t}^{\text{min}} = (1 - \epsilon)F_{m,n,t}^{*}$
  $a_{m,n,t}^{\text{min}} = (1 - \epsilon)a_{m,n,t}^{*}$
  $b_{m,n,t}^{\text{min}} = (1 - \epsilon)b_{m,n,t}^{*}$
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Test System

Figure: Four-Bus Power System with Four-Node Natural Gas System
Cases

1. Baseline electricity and natural gas demands
2. +10% natural gas demands
3. +20% natural gas demands
### Objective-Function Values

**Case 3: +20% Natural Gas Demands**

<table>
<thead>
<tr>
<th>Objective-Function Value [$ million]</th>
<th>Error [%]</th>
</tr>
</thead>
<tbody>
<tr>
<td>UC + Non-Convex</td>
<td>3.943</td>
</tr>
<tr>
<td>UC + SOC</td>
<td>3.907</td>
</tr>
<tr>
<td>UC + Enhanced SOC</td>
<td>3.913</td>
</tr>
<tr>
<td>(Starting Convexification)</td>
<td></td>
</tr>
<tr>
<td>UC + Enhanced SOC</td>
<td>3.928</td>
</tr>
<tr>
<td>(Updated Convexification)</td>
<td></td>
</tr>
</tbody>
</table>
Constraint Errors
Case 3: +20% Natural Gas Demands

Defined as:

\[ \sum_{m,n \in G_B} \frac{\pi^2_{m,t} - \bar{F}^2_{m,n,t}}{C^2_{m,n} - \pi^2_{n,t}} \]

![Graph showing constraint errors for different hours with data points for different scenarios: SOC, Enhanced SOC [Starting Convexification], and Enhanced SOC [Updated Convexification].]
Gas LMPs

Figure: Load-Weighted Gas LMPs

- Increase in hours 9–12 due to gas system congestion and unavoidable gas-demand curtailment
Electric LMPs

- High GLMPs yield high ELMPs
- 19 GW of gas-fired generation with baseline demand, reduced to 17 GW and 13 GW in other cases due to its higher relative cost.

*Figure*: Load-Weighted Electric LMPs
Test System

Figure: IEEE 118-Bus Test System with 48-Node Natural Gas System
Cases

1. Baseline
2. −20% capacity on all transmission lines
3. −40% capacity on all transmission lines
Electric LMPs

**Figure**: Load-Weighted Electric LMPs

- Reduced transmission capacity affects ELMPs in hours 8–24
Gas LMPs

- Increased GLMPs during hours 19–22 (peak-electric-demand periods)
- Power system congestion results in higher ELMPs and GLMPs simultaneously

**Figure**: Load-Weighted Gas LMPs
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Summary

- Our proposed convex UC model with an enhanced second-order conic gas flow model results in a tighter and more efficient UC solution compared with a simple SOC or non-convex gas flow models.
- Four-node case study shows the impact of gas system congestion on ELMPs.
- IEEE 118 bus test system case study shows the impact of power system congestion on GLMPs.
- Our proposed model could serve as an effective tool for analyzing interdependencies of electric and natural gas system.
Questions?