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Pricing Under Uncertainty: A Chance Constraint Approach to a Robust Competitive Equilibrium

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Overview: Why Chance Constraints?

- **Power grid operations have shifted to uncertainty-aware decision-making frameworks**
  - Scenario/interval-based optimization
  - Robust optimization
  - Chance constraints
  - Real-life OPF/UC applications: China, Switzerland, Russia (hydro+nucs)
- **Electricity markets are largely lagging far behind**
  - No consensus contract design
  - Uncertainty factors are not explicitly internalized the price formation process
  - No systematic framework to map uncertainty to a given network
  - "Stochasticity" concerns: What does the "stochastic" pay off actually mean? How to resolve the risk versus expectation dilemma? And how to explain it to a generation owners?
  - Lack of data or format dependencies on third-party providers (e.g. NOAA)
  - Scalability concerns

Chance constraints can be just the right framework to address the key issues

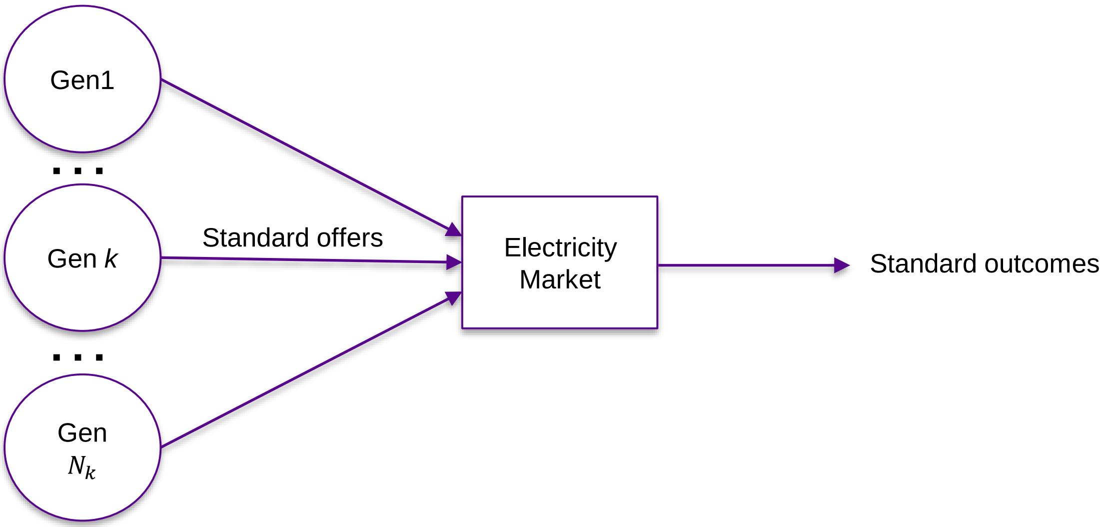
Overview: Why Chance Constraints?

- **Feasibility and effectiveness of chance constraints have been well-established**
  - Demonstration of cost-efficient and tractable reformulation (Bienstock et al, 2014) applied to a network with a 2,000+ nodes with location-specific treatment of uncertainty
  - Discriminatory treatment of small and large constraint violations (Roald et al, 2015; Dvorkin et al, 2017) for non-affine control policies and separating primary, secondary, and tertiary reserve needs
  - Scalable extensions to distributionally robust formulations, both algorithmically (Lubin, 2016) and via exact, or almost, convex reformulations (Xie et al, 2018)
  - Enable a "complete" electricity market design via a linearization of ac power flows (Lubin, 2018) or a convex relaxation (Halilbasic et al, 2018)
  - Support contingency-constrained formulations (Roald et al, 2016)
- **Can leverage existing results**
  - The exact SOCP reformulation is convex
  - Results obtained using the LP duality (deterministic markets) can be extended to a more general SOCP case (with some modifications)
  - SOCP duality ensures compatibility with legacy electricity market designs (important for the successful transition; Kuhn, 1962)

Chance constraints ensure high performance, modeling fidelity and compatibility with legacy market designs

This presentation

- **Contract design & market equilibrium with chance constraints**
  - Single node case
  - Contract design with chance constraints
  - Market equilibrium under chance constraints
- **Extensions to network-specific pricing with chance constraints**
  - How to enforce the chance-constrained apparent power flow limits?
  - Implications on pricing
  - Contract design feasibility: is possible with the single-node contract?
- **Not in this presentation**
  - Explicit treatment of non-convexities
  - Can be internalized using previous results for deterministic markets (using a connection between the LP – SOCP duality)



- **Contract design = {Standard offers, Standard outcomes}**
- **Standard offers include energy and reserve offers (capacity, price)**
- **Standard outcomes include cleared offers and prices**

- **Chance-constrained, single-node, single-period unit commitment problem**

$$\min \quad \mathbb{E}\left(\sum_k c_k(p_k - \alpha_k \boldsymbol{\Omega}) + f_k z_k\right)$$

Incremental cost

Fixed cost

Affine control with the power output p_k, participation factor α_k and system-wide uncertainty $\boldsymbol{\Omega}$

- Factors in the cost of real-time output: $\boldsymbol{p}_k = p_k - \alpha_k \boldsymbol{\Omega}$
- Real-time system-wide uncertainty: $\boldsymbol{\Omega} \sim N(0, \sigma^2)$
- Affine response and Gaussian, zero-mean assumptions are for the sake of convenience; can be revisited

- **Chance-constrained, single-node, single-period unit commitment problem**

$$\min \quad \mathbb{E}\left(\sum_k c_k(p_k - \alpha_k \boldsymbol{\Omega}) + f_k z_k\right)$$

$$\text{s.t.} \quad \sum_k p_k + W^f = D,$$

System-wide power balance constraint (deterministic)

$$p_k^{\min} z_k \le p_k \le p_k^{\max} z_k, \qquad \forall k,$$

Output limits on generators (deterministic)

$$\mathbb{P}\left(p_k - \alpha_k \boldsymbol{\Omega} \le p_k^{\max} z_k\right) \ge 1 - \epsilon, \qquad \forall k,$$

$$\mathbb{P}\left(p_k - \alpha_k \boldsymbol{\Omega} \ge p_k^{\min} z_k\right) \ge 1 - \epsilon, \qquad \forall k,$$

Chance constrained output limits on generators

$$\sum_k \alpha_k = 1,$$

Constraint on the system-wide response

$$\alpha_k \ge 0, z_k \in \{0, 1\}, \qquad \forall k,$$

- **Chance-constrained, single-node, single-period unit commitment problem**

$$
\begin{aligned}
\min \quad & \mathbb{E}\left(\sum_k c_k(p_k - \alpha_k \boldsymbol{\Omega}) + f_k z_k\right) \\
\text{s.t.} \quad & \sum_k p_k + W^f = D, \\
& p_k^{\min} z_k \le p_k \le p_k^{\max} z_k, && \forall k, \\
& \mathbb{P}\left(p_k - \alpha_k \boldsymbol{\Omega} \le p_k^{\max} z_k\right) \ge 1 - \epsilon, && \forall k, \qquad \text{(CCUCP)} \\
& \mathbb{P}\left(p_k - \alpha_k \boldsymbol{\Omega} \ge p_k^{\min} z_k\right) \ge 1 - \epsilon, && \forall k, \\
& \sum_k \alpha_k = 1, \\
& \alpha_k \ge 0, z_k \in \{0,1\}, && \forall k,
\end{aligned}
$$

This problem can be reduced to an LP (using the zero-mean assumption + fixing binary decisions)

- **Deterministic LP for the chance-constrained, single-node, single-period unit commitment problem**

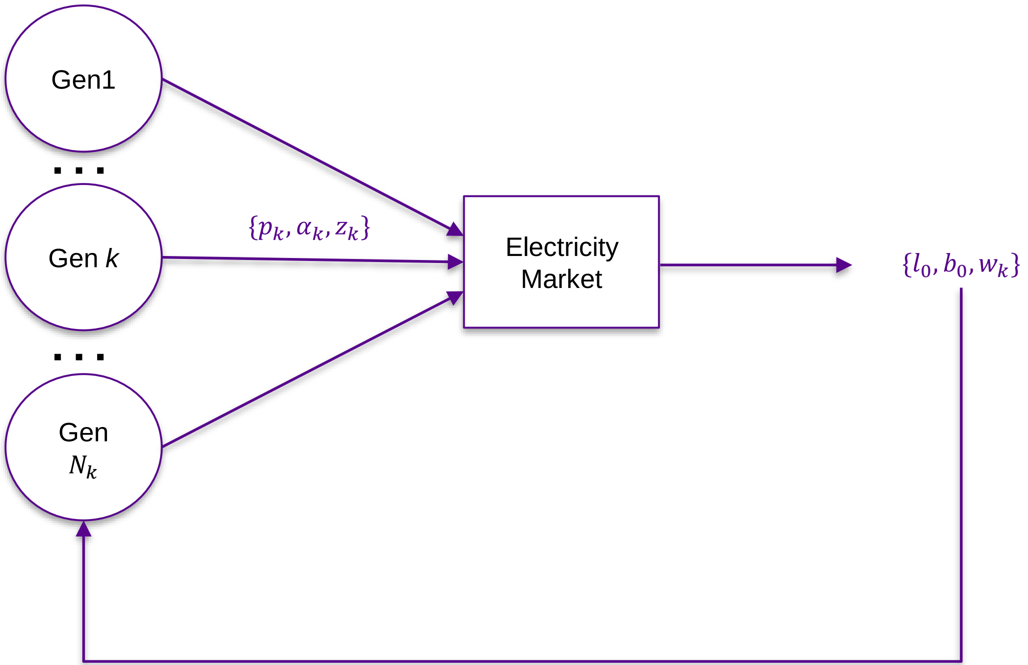
$$
\begin{aligned}
\min \quad & \sum_k c_k p_k + f_k z_k \\
\text{s.t.} \quad & \sum_k p_k + W^f = D, \\
& p_k^{\min} z_k - \Phi_\epsilon^{-1} \sigma \alpha_k \le p_k \le p_k^{\max} z_k + \Phi_\epsilon^{-1} \sigma \alpha_k, \quad \forall k, \\
& \sum_k \alpha_k = 1, \\
& \alpha_k \ge 0, \quad \forall k,
\end{aligned}
\qquad (\mathrm{CCUCP}_{\mathrm{IP}})
$$

Φ_ϵ^{-1} can be scaled to represent non-Gaussian distribution

- This LP can be then decomposed into "generators" problem (O'Neil, 2005)

$$
\begin{aligned}
\min \quad & c_k p_k + f_k z_k - l_0 p_k - b_0 \alpha_k - w_k z_k \\
\text{s.t.} \quad & p_k^{\min} z_k - \Phi_\epsilon^{-1} \sigma \alpha_k \le p_k \le p_k^{\max} z_k + \Phi_\epsilon^{-1} \sigma \alpha_k, \\
& \alpha_k \ge 0,
\end{aligned}
\qquad (\mathrm{CCUCP}k_{\mathrm{IP}})
$$

$\{l_0, b_0, w_k\}$ define the compensation of each generator for the power price, ramp power price, and commitment compensation

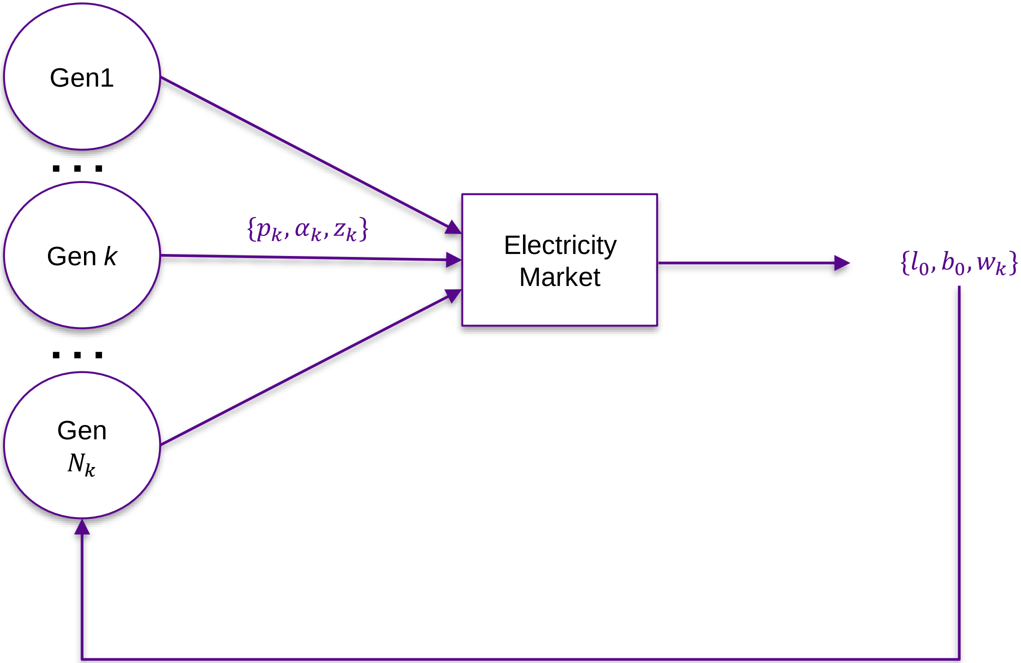


- Contract design

Let T_k be a contract between the power market operator and generator k with the following terms: (1) Generator's k decision is given by $\{p_k, \alpha_k, z_k\}$, and (2) Generator k receives an amount from the power market operator equal to the following payment function: $l_0 p_k + b_0 \alpha_k + w_k z_k$.

- This contract design leads to a stable market equilibrium

Market Equilibrium with Chance Constraints



- Market equilibrium must satisfy two conditions:

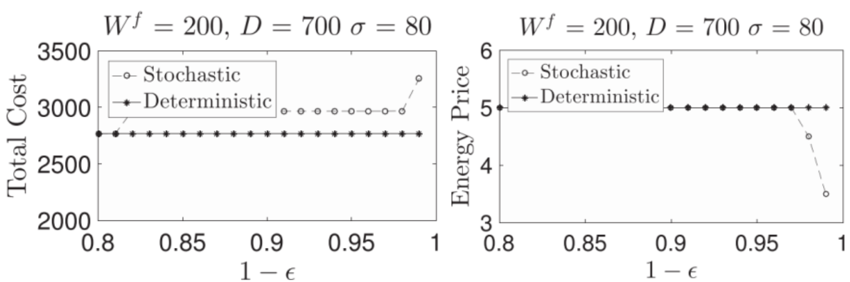
$$\sum_k p_k + W^f = D,$$

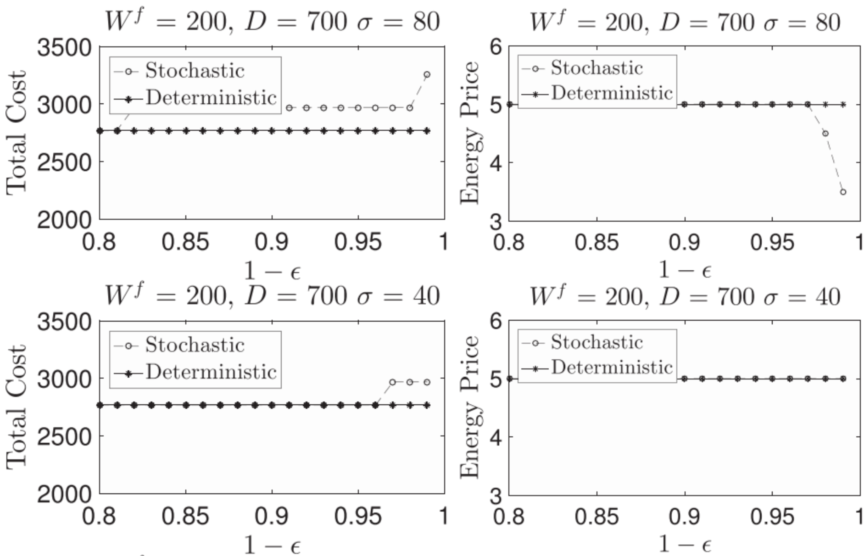
$$\sum_k \alpha_k = 1$$

Market Equilibrium with Chance Constraints

Theorem 1: Let $\{p_k^*, \alpha_k^*, z_k^*\}$ for all k be an optimal solution of (CCUCP) (or equivalently, of (CCUCP$_{\text{IP}}$)), and let $\{\lambda_0^*, \beta_0^*, \{\omega_k^* \, \forall k\}\}$ for all k be an optimal solution of the dual LP of (CCUCP$_{\text{LP}}(z^*)$). Then the prices $l_0 = \lambda_0^*, b_0 = \beta_0^*, w_k = \omega_k^*$ for all k, and the decisions $p_k' = p_k^*, \alpha_k' = \alpha_k^*$, and $z_k' = z_k^*$ for all k represent a robust competitive equilibrium.

- Our proof exploits LP duality (as in O'Neil, 2005)
- Still it works for a single-node case, transmission constraints need to be accounted for additionally
- See our proof in Kuang, 2018.





- The price formation process adequately reflects uncertainty (ϵ, σ)
- Externalities (ϵ, σ) can be related to power grid operations and have well-defined temporal and spatial interpretations (important for transmission-constrained extensions)
- Provides a high customization level for the assumptions on uncertainties, but does not increase computational complexity
- Has connections to the existing practice
  - One bid, no multiple bids for multiple scenarios
  - Easy interpretation + deterministic dc network constraints can be factored in straightforwardly

- **How to enforce power flow constraints?**
  - AC power flows (e.g. voltage + reactive power limits are accounted for)
  - Apparent power limits → no exact reformulation
  - Voltage limits → reformulated into linear deterministic constraints
- **A few modeling choices:**
  - Power flow linearization around an given operating point (an feasible AC power flow solution exists)
  - Affine response policies
  - Zero-mean, Gaussian uncertainty
  - Single-period optimization

Power Flow Linearization

- **AC power flow equations can be linearized or relaxed**

$$f_{ij}^{p}(v,\theta) = v_i v_j \left[G_{ij}\cos(\theta_i-\theta_j) + B_{ij}\sin(\theta_i-\theta_j)\right]$$
$$f_{ij}^{q}(v,\theta) = v_i v_j \left[G_{ij}\sin(\theta_i-\theta_j) - B_{ij}\cos(\theta_i-\theta_j)\right],$$

- **Linearization is based on the Taylor's approximation**
  - Can be solved sequentially to improve accuracy of the approximated solution
- **Even linearized AC power flow equations are difficult due to the quadratic dependency on uncertainty (ω)**

$$\mathbb{P}((f_{ij}^{p}(\omega))^2 + (f_{ij}^{q}(\omega))^2 \leq (s_{ij}^{max})^2) \geq 1 - \epsilon_I$$

Active flow

Reactive flow

- **Inner approximation of the quadratic dependency (Lubin et al, 2018)**

$$\mathbb{P}((f_{ij}^{p}(\omega))^2 + (f_{ij}^{q}(\omega))^2 \leq (s_{ij}^{max})^2) \geq 1 - \epsilon_I$$

$$\Downarrow$$

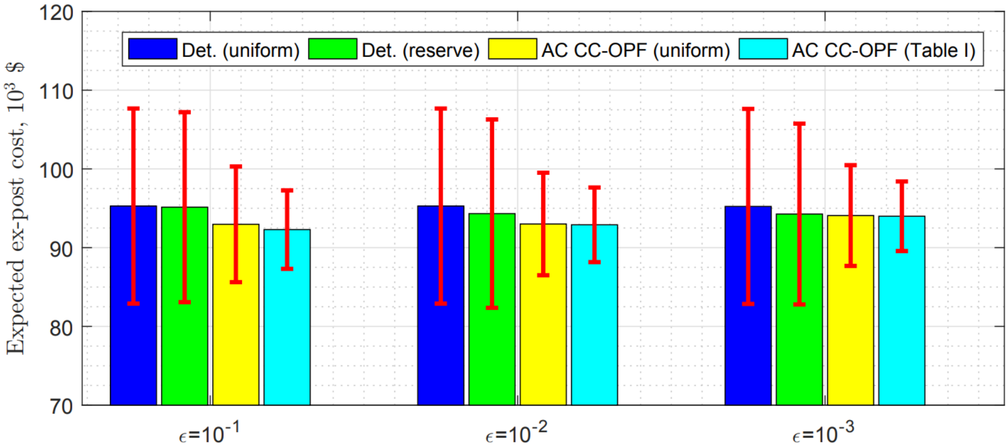
$$\mathbb{P}(|f_{ij}^{p}(\omega)| \leq t_{ij}^{p}) \geq 1 - \frac{\epsilon_I}{2}, \forall ij \in \mathcal{L}$$
$$\mathbb{P}(|f_{ij}^{q}(\omega)| \leq t_{ij}^{q}) \geq 1 - \frac{\epsilon_I}{2}, \forall ij \in \mathcal{L}$$
$$(t_{ij}^{p})^2 + (t_{ij}^{q})^2 \leq (s_{ij}^{max})^2, \forall ij \in \mathcal{L},$$

$$\Downarrow$$

Approximate absolute values with:

$$-t_{ij}^{*} - f_{ij}^{*}(0) \leq \Phi^{-1}(\frac{\epsilon_I}{2.5})\,\mathrm{Stdev}[f_{ij}^{*}(\omega)]$$

- **Inner approximation of the quadratic dependency (Lubin et al, 2018) works quite well**



- **However, the resulting problem is not an LP anymore due to the approximation:**

$$(t_{ij}^{p})^2 + (t_{ij}^{q})^2 \leq (s_{ij}^{max})^2, \forall ij \in \mathcal{L},$$

- **However, the resulting problem is not an LP anymore due to the approximation:**

$$(t_{ij}^{p})^2 + (t_{ij}^{q})^2 \leq (s_{ij}^{max})^2, \forall ij \in \mathcal{L},$$

- **However the program is still convex and the convex duality can be used in this case**
- **The same contract design can be used**
- **New proof is work in progress**

- **Chance constraints offer a great deal of modeling flexibility at an acceptable computational cost**
- **Can be used for pricing under uncertainty**
  - At least, for the single-node case or for the transmission-constrained case with DC assumptions or with deterministic power flow limit
  - Explicit consideration of uncertainty & risk tolerance on the price formation process
- **Can be built on existing practices**
- **More info:**
  - M. Lubin, Y. Dvorkin, and L. Roald, "Chance Constraints for Improving the Security of AC Optimal Power Flow," under review, 2018. Available at: https://arxiv.org/abs/1803.08754
  - X. Kuang, Y. Dvorkin, A. J. Lamadrid, M. Ortega-Vazquez, and L. Zuluaga, "Pricing Chance Constraints in Electricity Markets," IEEE Transactions on Power Systems, early access, 2018.


THANK
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ARIGATO
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JUSPAXAR
KOMAPSUMNIDA
MAAKE

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[2] L. Roald, S. Misra, M. Chertkov, G. Andersson, “Optimal power flow with weighted chance constraints and general policies for generation control”, 2015 IEEE 54th Annual Conference on Decision and Control (CDC), Osaka, Japan, 2015, pp. 6927-6933.

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[8] R. P. O’Neill et al., “Efficient market-clearing prices in markets with nonconvexities,” Eur. J. Oper. Res., vol. 164, no. 1, pp. 269–285, 2005

[9] X. Kuang, Y. Dvorkin, A. J. Lamadrid, M. Ortega-Vazquez, and L. Zuluaga, “Pricing Chance Constraints in Electricity Markets,” IEEE Transactions on Power Systems, early access, 2018.