Federal Energy Regulatory Commission Technical Conference
Washington, DC

Optimizing Sensor Type and Location for Rapid Restoration of Power Grids

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Outline

- Problem Description

- Probability Model for Grid Faults

- Sequential Stochastic Optimization Model for Utility Crew Routing
  - Multistage Lookahead Policy (Monte Carlo Tree Search)

- Stochastic Optimization Model for Sensor and Protective Device Placements
Problem Description – Power System
Problem Description – Power System
Problem Description – Power System
Problem Description – Power System
Problem Description – Power System
Problem Description - Emergency Storm Response

[Diagram of a power grid with buildings, transformers, protective devices, and power lines marked with poles and roadways.]
Problem Description - Emergency Storm Response
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Problem Description - Emergency Storm Response
Problem Description - Emergency Storm Response
Known customers in outage

Unknown outages

Outage calls (known)

Network outages (unknown)

Storm 😊
Problem Description – Research Goals

- How to create a belief about outages?
- How to reconstruct the grid to minimize customer outage-minutes?
- What is the value of a sensor/re-closer in terms of reducing customer outage-minutes?
- What is the best place to put a sensor or re-closer?
- What is the value of upgrades to SCADA systems?
- What is the value of investments in reconfigurable grids?
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Exploiting the information from phone calls

We have to blend the following….

- What we knew before the phone calls came in – this is called the prior belief.
- The outage calls – this is called information.

…to produce the updated estimates of outage – this is called the posterior.

To compute this we have to use Bayes theorem:

$$\text{Prob[segment } l \text{ is out|lights-out calls]} = \frac{\text{Prob[lights-out calls | segment } l \text{ is out]Prob[} l \text{ is out]}}{\text{Prob[lights-out calls]}}$$

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\]

The prior
Exploiting the information from phone calls

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- The prior
- The information
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The posterior

The information

The prior
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**Probability Model for Outage Identification**

- Exploiting the information from phone calls
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    - The outage calls – this is called *information*.
  - …to produce the updated estimates of outage – this is called the *posterior*.
- To compute this we have to use *Bayes theorem*:

$$\text{Prob}[\text{lights-out calls} \mid \text{segment } l \text{ is out}] \times \text{Prob}[l \text{ is out}] = \text{Prob}[\text{lights-out calls}] \times \text{The conditional outage distribution}$$
Probability Model – Prior Probabilities
Posterior Probabilities

- Building
- House
- Transformer
- Protective Device
- Power Line
- Pole
- Roadway

30 Customers

- 0.26
- 0.38
- 0.78
- 0.6
- 0.6

- 0.503
- 0.503
- 0.503
- 0.503
- 0.54
- 0.54

- 0.064
- 0.064
- 0.064

- 0.05
- 0.05
- 0.05
- 0.05
- 0.603
- 0.603

- 0.67
- 0.67

30 Customers

- 0.5
- 0.76
- 0.6
- 0.6
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Grid Restoration Model

- **Objective:** Develop an optimal policy that routes the utility truck in order to minimize the number of customers in outage at any point in time.

\[ R_t: \text{Physical State} \]

\[ I_t: \text{Information State} \]

\[ K_t: \text{Knowledge State} \]

---

Grid Restoration Model

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Information-Collection utility truck routing

Sequential Stochastic Optimization Model

- Five fundamental elements of sequential stochastic optimization:
  - State $S_t$ - information capturing what we know at time $t$;
    $$S_t = (R_t, P_t^L, H_t)$$
    where $R_t$ represents the physical state of the network.
  - Decision $x_t$ - captures the decision made at time $t$;
    Let $X_t^{\pi}(S_t)$ be the policy that determines $x_t \in X_t$ given $S_t$.
  - Exogenous information $W_t$ - new information that arrives between $t - 1$ and $t$;
    Includes new arriving calls, travel time, repair time and outages.
  - Transition function - $S_{t+1} = S^M(S_t, x_t, W_{t+1})$ represents the evolution of the states
    e.g., $H_{t+1} = H_t + \hat{H}_{t+1}, p(L_{t+1,i,j} = 1|x_{t,i,j} = 1) = 0$ in addition to $W_{t+1}$.
  - Objective function - $\min_{\pi} E^{\pi} [\sum_{t=0}^{T} C(S_t, X^{\pi}(S_t))]$;
    minimizes the number of customers in outage at any point in time.
Sequential Stochastic Optimization Model

- Cost function: \( \sum_{t=0}^{T} C(S_t, X_t^\pi(S_t)) \)

Figure 6: Objective function; Customer outage-minute is represented by the shaded area under the curve.
Sequential Stochastic Optimization Model

- Optimization problem

\[
\min_\pi \mathbb{E}^\pi \left[ \sum_{t=0}^{T} C(S_t, X^{\pi}(S_t)) \right]
\]

where

\[
S_{t+1} = S^M(S_t, x_t, W_{t+1})
\]

- Two fundamental strategies for designing policies [5]:
  - Policy Search
  - Lookahead Approximations
    - Value function approximation
    - Direct lookahead

Lookahead Approximations

- Lookahead approximations – Approximate the impact of a decision now on the future:
  - An optimal policy (based on looking ahead):

\[
X_t^*(S_t) = \arg \max_{x_t} \left( C(S_t, x_t) + \mathbb{E} \left\{ \max_{\pi \in \Pi} \left\{ \mathbb{E} \sum_{t'=t+1}^{T} C(S_{t'}, X_{t'}^*(S_{t'})) \mid S_{t+1} \right\} \mid S_t, x_t \right\} \right)
\]
Lookahead Approximations

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\]

2a) Approximating the value of being in a downstream state using machine learning ("value function approximations")

\[
X_t^* (S_t) = \arg \max_{x_t} \left( C(S_t, x_t) + \mathbb{E} \{ V_{t+1}(S_t) \mid S_t, x_t \} \right)
\]

\[
X_t^{VFA} (S_t) = \arg \max_{x_t} \left( C(S_t, x_t) + \mathbb{E} \{ \tilde{V}_{t+1}(S_t) \mid S_t, x_t \} \right)
\]

\[
= \arg \max_{x_t} \left( C(S_t, x_t) + \tilde{V}_t^x (S_t^x) \right)
\]

2b) Approximate lookahead models – Optimize over an approximate model of the future:

\[
X_t^{LA} (S_t) = \arg \max \left( C(S_t, x_t) + \mathbb{E} \left[ \max_{\tilde{\pi} \in \tilde{\Pi}} \left\{ \mathbb{E} \sum_{t'=t+1}^{T} C(\tilde{S}_{t'}, \tilde{X}_{t'}^\tilde{\pi}(\tilde{S}_{t'})) \mid \tilde{S}_{t+1} \right\} \mid S_t, x_t \right] \right)
\]

\[
= \arg \max \left( C(S_t, x_t) + \mathbb{E} \left[ \max_{\tilde{\pi} \in \tilde{\Pi}} \left\{ \tilde{\mathbb{E}} \sum_{t'=t+1}^{T} C(\tilde{S}_{t'}, \tilde{X}_{t'}^{\tilde{\pi}}(\tilde{S}_{t'})) \mid \tilde{S}_{t+1} \right\} \mid S_t, x_t \right] \right)
\]
We can then simulate this lookahead policy over time:

The lookahead model

The base model
Multistage Lookahead Approximations

- We can then simulate this \textit{lookahead policy} over time:
Multistage Lookahead Approximations

- We can then simulate this lookahead policy over time:
Multistage Lookahead Approximations

- We can then simulate this *lookahead policy* over time:
Multistage Lookahead Policy

- The optimal policy is computationally intractable, requiring approximations:

\[
X^*(S_t) = \arg\min_{x_t \in x_t(S_t)} (C(S_t, x_t) + \mathbb{E} \tilde{W}_{t+2} \in \tilde{\Omega}_{t+2} [\min_{\tilde{x}_{t+2} \in \tilde{x}_{t+2}(\tilde{S}_{t+1}, \tilde{x}_{t+1})} \tilde{C}(\tilde{S}_{t+1}, \tilde{x}_{t+1}) + \mathbb{E} \tilde{W}_{t+2} \in \tilde{\Omega}_{t+2} \ldots \mathbb{E} \tilde{W}_{t+H} \in \tilde{\Omega}_{t+H} [\tilde{C}(\tilde{S}_{t+H})|\tilde{S}_{t+H-1}] \ldots |\tilde{S}_{t+1}]|S_t])
\]

where \( \tilde{S}_{t,t'+1} = S^M(\tilde{S}_{tt'}, \tilde{x}_{tt'}, \tilde{W}_{t,t'+1}) \), \( t' = t, \ldots, t + H - 1 \)

- Discretizing the time, states and decision
- Limiting the horizon from \((t, T)\) to \((t, t + H)\)
- Dimensionality reduction e.g., by limiting some variables (e.g., fixing the set of calls, limiting the fault types and travel times)
- Aggregating the outcome or sampling by using Monte Carlo sampling
Monte Carlo Tree Search (MCTS)

- MCTS is a recent research area; the first MCTS algorithm has been developed by Chang et al [2005].

- MCTS biases the growth of the tree towards the most promising moves which decreases the search space [3].

- MCTS mainly applied for deterministic problems but Coutoux et al [2011] extended it to stochastic problems based on *double progressive widening*.


Monte Carlo Tree Search

MCTS Construction Steps

- Decision selection based on UCB (Upper Confidence Bounds for Trees) [7]:

\[
\tilde{x}_{tt'}^* = \arg\max_{\tilde{x}_{tt'} \in \tilde{x}_{tt'}^{e}} \left( -\tilde{C}(\tilde{s}_{tt'}, \tilde{x}_{tt'}) + \tilde{V}_{tt'}(\tilde{s}^{x}_{tt'}) \right) + \alpha \frac{\log N(\tilde{s}_{tt'})}{\sqrt{N(\tilde{s}_{tt'}, \tilde{x}_{tt'})}}
\]

Monte Carlo Tree Search – Convergence Theory

- For deterministic MCTS, the UCB policy samples the actions infinitely often and Kocsis and Szepesvari [2006] exploit this to show that the probability of selecting a suboptimal action converges to zero at the root of the tree.

- Auger et al. [2013] provides convergence results for stochastic MCTS with double progressive widening under an action sampling assumption. The asymptotic convergence of also MCTS relies on some form of "exploring every node infinitely often".

- In Jiang et al. [2017], we design a version of stochastic MCTS that asymptotically does not expand the entire tree, yet is still optimal!

Simulation Policy

- Generate sample path $\tilde{\omega}$ from $\tilde{\Omega}_{t,t'}$ that determines all random events (faults, travel time, repair time).

- Utility truck should visit each fault once to repair it.

- Objective: find the optimal route that minimizes the customer-outage minute.

- In the worst case, computational complexity $O(n!)$. 
Simulation Policy – Optimal Solution

- Define the graph $G(\mathcal{V}, \mathcal{E})$ with connection costs $T_{ij}$.
- $S$: subset of nodes visited by the truck.
- $C(S, i)$: customer-outage minute starting from node 1 and ending at node $i$.
- $f(S)$: function returning the number of customers in outage after visiting the nodes of $S$.
- The cost of moving from node $i$ to node $j$ is
  \[ C(S, j) = C(S - \{j\}, i) + f(S - \{j\}) \times (T_{ij} + R_j) \]

**Dynamic Program**

For all $j \in \mathcal{V}$ do $C(\{1, j\}, j) = \sum_i n_i (T_{1j} + R_j)$

For $s = 3$ to $n$

For all Subsets $S$ of $\mathcal{V}$ of size $s$ do

For all $j \in S, j \neq 1$ do

\[ C(S, j) = \min_{i \in S, i \neq j} C(S - \{j\}, i) + f(S - \{j\}) \times (T_{ij} + R_j) \]

$opt = \min_j C(\mathcal{V}, j)$

Computational Complexity
$O(n^2 2^n)$
Lookahead Policy
Lookahead Policy
Lookahead Policy
Lookahead Policy

[Diagram with various symbols and numbers indicating different locations and values.]

- Building
- House
- Transformer
- Protective Device
- Power Line
- Pole
- Roadway

Numbers and symbols indicate specific values and locations, possibly related to energy distribution or maintenance.
Lookahead Policy
Lookahead Policy
Lookahead Policy
Lookahead Policy
Lookahead Policy
Industrial Heuristics

**Escalation Algorithm**

**Step 1. For** each circuit do

**Step 1a.** Collect all calls and back trace to find the first node that is common to all calls say node $x$.

**Step 1b.** Send the truck to node $x$ and then back trace to the substation to make sure that there is no upstream fault

**Step 1c.** From node $x$, perform down tracing to reach the first segment from which a call was initiated and place it in set $D$.

**Step 2. For** each segment in $D$ do

**Step 2a.** Perform down tracing to cover all nodes that called.
Escalation Algorithm
Escalation Algorithm
Escalation Algorithm
Escalation Algorithm
Escalation Algorithm
Escalation Algorithm
Numerical Results – Customer Outage-Minutes

- Performance metrics
  - Total outage minutes
  - SAIDI, CAIDI, SAIFI, …

![Graph showing performance metrics](image)

- Escalation algorithm
- Lookahead policy
- Total outage minutes: 55110
- Lookahead policy: 36150
Studies

- Possible questions:
  - What is the effect of higher call-in rates?
  - What is the best place to locate a sensor/protective device?
  - What is the value of a sensor vs. protective device?
Figure 10: Average customer outage-hours vs. MCTS budget for ten networks.
Comparison to Industrial Heuristics
Outline

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• Sequential Stochastic Optimization Model for Utility Crew Routing
  • Multistage Lookup Policy (Monte Carlo Tree Search)
  • Monte Carlo Tree Search with Information Relaxation Dual Bounds

• Stochastic Optimization Model for Sensor and Protective Device Placements
Optimal Protective Device and Sensor Placement

- Protective Device vs Sensor Placement to minimize Customer-Outage Minute:
  - A protective device prevents power flow to the downstream segments when a fault is detected.
  - A sensor feeds back information to the utility center whether power is on or off.

- Optimization problem for optimal protective device and sensor placements:

\[
\min_{a} \mathbb{E}^{\pi} \left[ \sum_{t=0}^{T} C(S_t(a), X^{\pi}(S_t(a))) | S_0(a) \right]
\]

where
\[
\pi = MCTS
\]
\[
S_{t+1}(a) = S^M(S_t(a), X^{\pi}(S_t(a)), W_{t+1}(a))
\]
\[
||a||_1 \leq M
\]

\(\boldsymbol{a}\): a binary vector; \(a_i = 1\) if a protective device is placed at node \(i\) and 0 otherwise.

- \(\binom{N}{M}\) combinations of protective devices which results in high computation complexity.
- Solved sequentially by placing one protective device at a time.
Optimal Protective Device and Sensor Placement
Case study A: sensor vs. protective device

Customer Outage-Minute Reduction:
Value of sensor = 0
Value of protective device = 225
Case study B: sensor vs. protective device

Customer Outage-Minute Reduction:
Value of sensor = 3360
Value of protective device = 0

10 Customers

Substation

Building
House
Transformer
Protective Device
Power Line
Pole
Roadway

0.26 0.38 0.78 0.6 0.6
0.01 0.01 0.01 0.18 0.27 0.36 0.36 0.45 0.47 0.47 0.47 0.47
0.501 0.501 0.501 0.501
0.601 0.601
0.58 0.58 0.45
0.71

10 Customers

72
We have developed:

- A detailed storm and grid simulator that simulates storms, outages and lights-out calls
- Models the behavior of protective devices and grid reconfigurations
- An industry-standard escalation policy and a probabilistic lookahead model that uses prior knowledge and anticipates likely outages
- Shown that the lookahead policy produces near-optimal performance when compared against optimal on a deterministic model

Simulations

- We have quantified the value of sensors and protective devices at different locations
- We can find the best locations for each type of device

Conclusion

- We believe that our simulator can be used to develop effective rate cases for the BPU
Summary

- We have developed:
  - A detailed storm and grid simulator that simulates storms, outages and lights-out calls
  - Models the behavior of protective devices and grid reconfigurations
  - An industry-standard escalation policy and a probabilistic lookahead model that uses prior knowledge and anticipates likely outages
  - Shown that the lookahead policy produces near-optimal performance when compared against optimal on a deterministic model

- Simulations
  - We have quantified the value of sensors and protective devices at different locations
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- Conclusion
  - We believe that our simulator can be used to develop effective rate cases for the BPU

Thank you for your attention!
Problem Description – Grid Restoration

- $I$: set of poles, i.e., $I = \{i, i = 1, \ldots, l\}$
- $\mathcal{U}$: set of circuits, i.e., $\mathcal{U} = \{u, u = 1, \ldots, U\}$
- $I_u$: set of power lines on circuit $u$, i.e., $I_u = \{i, i = 1, \ldots, I_u\}$
- $\mathcal{N}^u$: set of nodes on circuit $u$, i.e., $\mathcal{N}^u = \{N_i^u, \forall i \in I_u\}$
- $n_i^u$: number of customers attached to node $N_i^u$
- $n_{ti}^{u,c}$: number of customers that called on node $N_i^u$
- $\Omega_t$: sample space at time $t$; each scenario $\omega$ represents phone calls, faults, travel & repair times
- $H_t$: random vector representing the realizations of received calls at time $t$
- $L_t$: random vector representing the realizations of power line faults at time $t$
- $T_{tij}$: random variable representing the travel time from node $i$ to node $j$ at time $t$
- $R_j^u$: random variable representing the repair time of power line $j$ on circuit $u$
- $x_t$: random vector representing the trajectory of the truck at time $t$; $x_t = \{x_{tij}\}_{i,j \in \mathcal{V}}$

Illustration of distribution grid

- Bayes’ Theorem:

$$P(L_t|H_t) = \frac{p(H_t|L_t)p(L_t)}{\sum_{L_t \in L} p(H_t|L_t)p(L_t)}$$

Problem Description – Simulator

- Features of the simulator:
  - Complete model of the distribution grid:
    - Substations
    - Sensors, protective devices, reconfiguration rules, …
  - Able to simulate:
    - Many storm trajectories with varying sizes
    - Process of outages occurring as storm progresses
    - Model of who loses power (from grid configuration)
    - Process of lights-out calls (with variable call-in rates)
  - Probabilistic model of outages
    - Captures prior knowledge and probabilistic model of lights-out calls
  - Dispatch policies:
    - Industry-standard escalation policy – Uses lights-out calls and grid configuration
    - Stochastic lookahead policy – Exploits uncertainty model of outage probabilities
Monte Carlo Tree Search

Propose a Primal-Dual MCTS, that takes advantage of the information relaxation bound idea that asymptotically converges [8] to the optimal solution while ignoring suboptimal parts of the tree.

Explore a set of actions:

\[ \hat{u}^n(x^n_{\tau e}, a) = c_{\tau e}(s, a, W^n_{\tau e+1}) + \max_a [h_{\tau e+1}(S^n_{\tau e+1}, a, W^n_{\tau e+1}) - \nu_{\tau e+1}(S^n_{\tau e+1}, a, W^n_{\tau e+1})] \]

Smooth with previous estimates:

\[ \hat{u}^n(x^n_{\tau e}, a) = (1 - \alpha^n(x^n_{\tau e}, a)) \hat{u}^{n-1}(x^n_{\tau e}, a) + \alpha^n(x^n_{\tau e}, a) \hat{u}^n(x^n_{\tau e}, a). \]

Expand it if the value is greater than \( \hat{V}^{n-1}(x^n_{\tau e}) \)

Simulation Policy – Integer programming

- The problem can be formulated as a non-linear integer program as follows:

\[
\min_{\tilde{x}_{i'j}} \sum_{t=t'}^{t'+H} \left( N - \sum_{t''=t'}^{t'} \sum_{u \in \mathcal{U}} \sum_{j=1}^{N} \tilde{C}_{i''j} \right)
\]

subject to

\[
\tilde{C}_{i''j} = \sum_{i} \left( \prod_{k \in \mathcal{K}_{i'j}} 1 - \tilde{L}_{i''k} \right) \tilde{L}_{i''j} \left( \sum_{k \in \mathcal{S}_{i'j}} n_k^u + \sum_{s \in \mathcal{W}_j} \left( \prod_{d=\min(\mathcal{W}_j^u)}^{s} \prod_{k \in \mathcal{K}_{i''k}} 1 - \tilde{L}_{i''k} \right) \sum_{k \in \mathcal{S}_{i'j}} n_k^u \right) \tilde{x}_{i''ij}, \quad \forall j \in \mathcal{V}, \forall t''
\]

\[
\tilde{L}_{i''j} = 1 - \sum_{i} \sum_{t=t'}^{t''-1} \tilde{x}_{i''ij}, \quad \text{such that} \quad \tilde{L}_{i''j}(\tilde{\omega}) = 1, \forall j \in \mathcal{I}^u, \forall u \in \mathcal{U}, \forall t''
\]

\[
\tilde{\Delta}_{i''ij} \geq T_{ij}(\tilde{\omega}) \tilde{x}_{i''ij} + \sum_{u} R_{j}^u(\tilde{\omega}) \left( \tilde{x}_{i''ij} - \sum_{t=t'}^{t''-1} \tilde{x}_{i''ji} \right), \forall (i, j) \in \mathcal{A}, \forall t''
\]

\[
\tilde{\xi}_{i''j} \geq \tilde{\xi}_{i''-1i} + \sum_{i} \tilde{\Delta}_{i''ij}, \forall (i, j) \in \mathcal{A}, \forall t''
\]

\[
\tilde{\xi}_{i''j} \leq t'' \sum_{i} \tilde{x}_{i''ij} + \zeta \left( 1 - \sum_{i} \tilde{x}_{i''ij} \right), \forall j \in \mathcal{V}, \forall t''
\]

\[
\sum_{t''=t'}^{t'+H} \tilde{x}_{i''ij} \leq 1, \forall (i, j) \in \mathcal{A}
\]

\[
\sum_{k} \tilde{x}_{(i''+T_{jk}(\tilde{\omega}))jk} + \sum_{k} \tilde{x}_{(i''+T_{jk}(\tilde{\omega})+\sum_{u} R_{j}^u(\tilde{\omega}))jk} \leq \sum_{i} \tilde{x}_{i''ij} \leq 1, \forall j \in \mathcal{V}, \forall t''
\]

\[
\tilde{C}_{i''j} \geq 0, \tilde{\xi}_{i''j} \geq 0, \tilde{\Delta}_{i''ij} \geq 0, \tilde{x}_{i''ij} \in \{0, 1\}
\]

Minimizes the customer-outage minute

Computes number of customers with restored power

Indicates whether power line $j$ is still in outage

Computes required travel and repair time by going from node $i$ to node $j$

Guarantee that the required travel and repair times are met

Guarantees that each arc is visited at most once in one direction

Indicates whether the truck can move to the next node

Domains of the variables