MINLP solved by Outer Approximation (OA)

1. **Initialization**
2. **Approximation MIP**
   - Solve MIP for LB
   - Fix binary
   - Solve SLP ACOPF for UB

3. **Add Constraints to Refine MIP**

   - **Feasible**
     - Yes
     - No
   - **Gap<\(\varepsilon\)**
     - Yes
     - No

   **Local Solution [R2]**

   **Done**
CONTRIBUTIONS
GLOBAL SOLUTION METHOD
ACOPF Second-Order Cone Relaxation \(\S\) (SOCR)

\[
c_{b,b} \equiv (v_{b}^r)^2 + (v_{b}^i)^2 = v_{b}^2
\]

\[
c_{b,k} \equiv v_{b}^r v_{k}^r + v_{b}^i v_{k}^i = |v_{b}| |v_{k}| \cos \theta_{b,k}
\]

\[
s_{b,k} \equiv v_{b}^r v_{k}^i - v_{b}^i v_{k}^r = -|v_{b}| |v_{k}| \sin \theta_{b,k}
\]

\[
c_{b,k}^2 + s_{b,k}^2 = c_{b,b} c_{k,k} \quad \forall \ l = (b, k)
\]

KVL-based constraints (next slide)

\[
c_{b,k}^2 + s_{b,k}^2 \leq c_{b,b} c_{k,k} \quad \forall \ l = (b, k)
\]

\[
c_{b,k} = c_{k,b}, \ s_{b,k} = -s_{k,b} \quad \forall \ l = (b, k)
\]

\[
\begin{align*}
\min & \sum_{g \in \mathcal{G}} [A_g^2(p_g^G)^2 + A_g^1 p_g^G + A_g^0] \\
\text{s.t.} & \sum_{l \in \mathcal{L}_{b}^{in}} p_l^t + \sum_{l \in \mathcal{L}_{b}^{out}} p_l^f + G_b^{sh} c_{b,b} + P_b^D - \sum_{g \in \mathcal{G}_b} p_g^G = 0 \quad \forall \ b \\
& \sum_{l \in \mathcal{L}_{b}^{in}} q_l^t + \sum_{l \in \mathcal{L}_{b}^{out}} q_l^f - B_b^{sh} c_{b,b} + Q_b^D - \sum_{g \in \mathcal{G}_b} q_g^G = 0 \quad \forall \ b \\
& p_l^f = G_l^{ff} c_{b,b} + G_l^{ft} c_{b,k} - B_l^{ft} s_{b,k} \quad \forall \ l \\
& q_l^f = -B_l^{ff} c_{b,b} - B_l^{ft} c_{b,k} - G_l^{ft} s_{b,k} \quad \forall \ l \\
& p_l^t = G_l^{tt} c_{k,k} + G_l^{tf} c_{k,b} - B_l^{tf} s_{k,b} \quad \forall \ l \\
& q_l^t = -B_l^{tt} c_{k,k} - B_l^{tf} c_{k,b} - G_l^{tf} s_{k,b} \quad \forall \ l \\
& (p_l^f)^2 + (q_l^f)^2 \leq (S_{l}^{max})^2, \ (p_l^t)^2 + (q_l^t)^2 \leq (S_{l}^{max})^2 \quad \forall \ l \\
& (V_b^{min})^2 \leq c_{b,b} \leq (V_b^{max})^2 \quad \forall \ b \\
& P_g^{G,min} \leq p_g^G \leq P_g^{G,max}, \ Q_g^{G,min} \leq q_g^G \leq Q_g^{G,max} \quad \forall \ g
\end{align*}
\]

\(\S\) Second-Order Cone Relaxation (Jabr, 2006; Kocuk, 2015)
Improving the Lower Bound of SOCR [R3]

Cycle Constraints:
the sum of angle differences on each cycle equals to zero

\[ \sum_{l \in \mathcal{L}_c} \theta_l = 0 \quad \forall \mathcal{L}_c \]

\[ \theta_l \equiv \theta_{b,k} = -\arctan \left( \frac{s_{b,k}}{c_{b,k}} \right) \quad \forall l = (b, k) \]

Convex Relaxation of $\arctan$:
(PW) Linear Over- and Under-Estimators
Optimality-Based Bound Tightening (OBBT)
Gradually Adding Cycle Constraints
## Global ACOPF Performance

<table>
<thead>
<tr>
<th>Case Name</th>
<th>Optimal Solution</th>
<th>Optimality Gap (%)</th>
<th>CPU Time (s)</th>
<th>Iteration Number</th>
</tr>
</thead>
<tbody>
<tr>
<td>Case6ww</td>
<td>3126.36</td>
<td>0.008</td>
<td>0.26</td>
<td>4</td>
</tr>
<tr>
<td>Case14</td>
<td>8081.52</td>
<td>0.003</td>
<td>0.43</td>
<td>3</td>
</tr>
<tr>
<td>Case30</td>
<td>574.52</td>
<td>0.000</td>
<td>0.95</td>
<td>5</td>
</tr>
<tr>
<td>Case39</td>
<td>41864.18</td>
<td>0.005</td>
<td>1.21</td>
<td>3</td>
</tr>
<tr>
<td>Case57</td>
<td>41737.79</td>
<td>0.006</td>
<td>7.29</td>
<td>12</td>
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<tr>
<td>Case89</td>
<td>5817.60</td>
<td>0.009</td>
<td>46.2</td>
<td>44</td>
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<tr>
<td>Case118</td>
<td>129660.69</td>
<td>0.006</td>
<td>18.5</td>
<td>14</td>
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<tr>
<td>Case300</td>
<td>719725.10</td>
<td>0.009</td>
<td>82.7</td>
<td>49</td>
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<tr>
<td>NESTA Case6ww</td>
<td>3143.97</td>
<td>0.000</td>
<td>0.74</td>
<td>7</td>
</tr>
<tr>
<td>NESTA Case14</td>
<td>244.05</td>
<td>0.003</td>
<td>0.22</td>
<td>3</td>
</tr>
<tr>
<td>NESTA Case30</td>
<td>204.97</td>
<td>0.000</td>
<td>0.57</td>
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<td>NESTA Case39</td>
<td>96505.52</td>
<td>0.009</td>
<td>3.00</td>
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<td>NESTA Case57</td>
<td>1143.27</td>
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<td>9.62</td>
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<td>0.009</td>
<td>55.8</td>
<td>57</td>
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<td>NESTA Case118</td>
<td>3718.64</td>
<td>0.000</td>
<td>93.7</td>
<td>55</td>
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<tr>
<td>NESTA Case300</td>
<td>16891.28</td>
<td>0.000</td>
<td>138.2</td>
<td>26</td>
</tr>
</tbody>
</table>
MINLP solved by Outer Approximation (OA)

- **Initialization**
- **Relaxation MIP**
- **Solve MIP for LB**
- **Solve NLP ACOPF for UB**
- **Add Constraints to Refine MIP**

**Flowchart:**
- **Feasible**
  - Yes: **Global Solution [R4]**
  - No
    - **Gap < ε**
      - Yes: **Done**
      - No
        - **Fix binary**
          - **Yes**
            - **Solving nonlinear, non-convex AC OPF to global optimality?**
          - **No**
            - **Add Constraints to Refine MIP**
CONTRIBUTIONS
UC+ACOPF RESULTS
IEEE-118

118 nodes
54 generators
91 loads
186 network elements/lines
24-hour hourly commitment

<table>
<thead>
<tr>
<th></th>
<th>Cost ($)</th>
<th>AC Feasible?</th>
</tr>
</thead>
<tbody>
<tr>
<td>UC</td>
<td>811,658 (base)</td>
<td>NO</td>
</tr>
<tr>
<td>UC+DCOPF</td>
<td>814,715 (+0.4%)</td>
<td>NO</td>
</tr>
<tr>
<td>Local UC+ACOPF</td>
<td>843,591 (+3.9%)</td>
<td>YES</td>
</tr>
<tr>
<td>UC+DCOPF+RUC</td>
<td>844,922 (+4.1%)</td>
<td>YES</td>
</tr>
<tr>
<td>Global UC+ACOPF</td>
<td>835,926 (+3.0%)</td>
<td>YES</td>
</tr>
</tbody>
</table>

Key Takeaway: Results indicate considerable divergence between the market settlements and stability/reliability requirements

§ Data from Fu et al. (2006)
Computational Results (Local Method)

<table>
<thead>
<tr>
<th></th>
<th>UC MILP</th>
<th>UC+DCOPF MILP</th>
<th>UC+ACOPF MILP</th>
<th>UC+DCOPF+RUC MILP</th>
<th>UC+DCOPF+RUC SLP</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Solution Time (s)</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>6-Bus</td>
<td>0.13</td>
<td>0.21</td>
<td>0.88(3)</td>
<td>0.07(50)</td>
<td>1.02(1, 1)</td>
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<td>RTS-79</td>
<td>1.86</td>
<td>6.76</td>
<td>88.71(3)</td>
<td>0.75(36)</td>
<td>10.37(1, 2)</td>
</tr>
<tr>
<td>IEEE-118</td>
<td>5.04</td>
<td>21.42</td>
<td>110.17(2)</td>
<td>5.06(46)</td>
<td>57.2(1, 1)</td>
</tr>
<tr>
<td><strong>Cost ($)</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>6-Bus</td>
<td>101,270</td>
<td>106,987</td>
<td>101,763</td>
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<td>102,523</td>
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<tr>
<td>RTS-79</td>
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<td>823,894</td>
<td>895,281</td>
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<td>896,169</td>
</tr>
<tr>
<td>IEEE-118</td>
<td>811,658</td>
<td>814,715</td>
<td>843,591</td>
<td></td>
<td>844,922</td>
</tr>
</tbody>
</table>

- Most of the OA algorithm time spent in the MILP (MIP gap tolerance 0.1%)
- UC+ACOPF: 5x-15x slower than the UC+DCOPF
- UC+DCOPF+RUC: 1.5x-5x slower than the UC+DCOPF
## Local v. Global UC+ACOPF Method

<table>
<thead>
<tr>
<th>Case</th>
<th>Problem Formulation</th>
<th>Upper Bound</th>
<th>Lower Bound</th>
<th>Relative Gap (%)</th>
<th>CPU Time (s)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>6-Bus</td>
<td>Global</td>
<td>101,763</td>
<td>101,655</td>
<td>0.11%</td>
<td>3.6</td>
</tr>
<tr>
<td></td>
<td>Local</td>
<td>101,763</td>
<td>-</td>
<td>0.11%</td>
<td>0.95</td>
</tr>
<tr>
<td>RTS-79</td>
<td>Global</td>
<td>895,096</td>
<td>893,967</td>
<td>0.13%</td>
<td>266.4</td>
</tr>
<tr>
<td></td>
<td>Local</td>
<td>895,281</td>
<td>-</td>
<td>0.15%</td>
<td>89.46</td>
</tr>
<tr>
<td>IEEE-118</td>
<td>Global</td>
<td>835,926</td>
<td>833,057</td>
<td>0.34%</td>
<td>8480</td>
</tr>
<tr>
<td></td>
<td>Local</td>
<td>843,591</td>
<td>-</td>
<td>1.25%</td>
<td>115.23</td>
</tr>
</tbody>
</table>

- **Note**: Thermal limits different in global solution method (apparent power thermal limit) and local solution method (current thermal limit) so a direct comparison (above) is *inexact*.

- On the largest test case, the approximation method is over 70x faster, at the cost of 0.91% in relative optimality gap change.
ONGOING WORK
Ongoing Work

- Study of global solution techniques applied to the PSV, RSV and RIV ACOPF formulations

- Implications on market settlements for including AC network constraints in the day-ahead

- Improving the performance of the MIP solution time in the OA algorithm (e.g., hybrid OA + branch-and-bound)

- Comparing the fidelity and computational performance to current market practices on larger scale, more realistic networks (GRIDDATA)
References


