A Toolbox for Exploring AC OPF Formulations, Datasets and Solution Methods

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Today’s electricity market encounters many complex questions, revolving around $\sum \text{Gen MW} = \sum \text{Load MW}$, at all times.

- Operational, planning
- Flexibility in face of new technologies

Many interacting levels, multiple time-scales and agents, increasing levels of stochasticity.

Larger network $\rightarrow$ Larger models
Optimal Power Flow Models

Optimal Power flow (OPF) models are at the heart of it all.

- Well researched, standard models have not changed much.
- Significant progress in decomposition and stochastic methods.
- Lacks cohesiveness in comparative literature due to different models, file formats, software, solvers.
- Core difficulty has always been solving large-scale models.

Consider the following data set:

- **Network**: 13867 buses, 18790 lines
- **Generation capabilities**: 1043 generators
- **Non-zero Loads**: 3753 nodes
- **Time periods**: 24 hours

How far can we take the modeling experience in a dataset of this size?
Objective: Bridge the gap between different software, solvers, data formats, industry and academics.
Toolbox: A general overview

Models and testcases are open source, and written primarily in GAMS. The toolbox consists of:

- Optimization models for different OPF formulations.
- Testcase archive includes IEEE testcases, including Polish testcases (2000-3000+ buses), RTS-96 (6 files containing seasonal 24-hour demand data).
- Data management utilities for format conversion, data generation, and easy viewing output.
- Downloadable at http://www.neos-guide.org/content/optimal-power-flow
- A collection of large-scale datasets is available under Critical Energy Infrastructure Information (CEII) usage and agreement terms (not publicly available).
Why GAMS?

- **One system**
  Designed for modeling multiple types of problems
  e.g. linear, non-linear, mixed integer, stochastic.

- **Write once**
  Integrates multiple high-performance solvers
  e.g. CPLEX, CONOPT, IPOPT, BARON, GUROBI, LINDO, PATH.

- **Flexible**
  Portable between different platforms, models are easily extensible,
  solver integration taken care of on the back-end.

- **Reusability**
  Models and data are easily saved and re-used in future applications.
  Generic GDX data interface.
Toolbox: Standard OPF Models

Core Models:
- Direct current (DC) OPF, with and without shift matrices
- Alternating current (AC) OPF models
  - Polar Power-Voltage Formulation
  - Rectangular Power-Voltage Formulation
  - Rectangular Current-Voltage Formulation
  - Y-bus formulations
- Decoupled OPF
- Unit commitment models, both AC and DC

Stochastic Model extensions:
- Stochastic unit commitment
- Security constrained unit commitment
- Value at Risk
Testcases

Data files include the following information:

- Network, including power and current limits, interfaces, tap transformers
- Generator, including operational, cost, and fuel (where available)
- Active and reactive demand
- Multiple time periods (where available)

Standard data enhancements in testcase archive include:

- Cost function approximations: Linear, quadratic, piecewise linear
- Demand bidding: Bid to shed load
- Generator Capability curves (D-curves)
- Lineflow limit approximation: Alternative approximations
- Generator ramp-rate approximation: Alternative approximations
Data management utilities

- Utilities to facilitate conversions between the following three formats
  - GAMS formatted input files (.gdx)
  - Matpower formatted input files (.m)
  - PSS™ E-31 power flow raw data file (.raw)
- Compute Shift Matrix for a system
- Output data into Excel spreadsheet for easy viewing
Using the OPF toolbox

Examples of model options include:

- **Time**: Select which time periods(s)
- **Objective**: Feasibility, linear, quadratic, piecewise linear functions.
- **Initial conditions**: Starting point methods for AC OPF.
- **D-curve**: Enforce reactive power limits as D-curve circle constraints.
- **Demand bidding**: Incremental elastic demand bidding is considered.
Initial conditions for AC OPF models

Within the toolbox, AC OPF models provide multiple starting point options. Some examples are listed below.

- **ic=0 Midpoint**
  All variables initialized at the midpoint between variable bounds.

- **ic=1 Random**
  All variables initialized using random draws between variable bounds.

- **ic=2 Flat**
  Voltage magnitude = 1, voltage angle = 0. Real, reactive power = 0.

- **ic=3 Random/inferAC**
  Voltage magnitude & voltage angle variables are random draws. Real, reactive power are inferred using AC transmission line equations.

- **ic=4 DC/inferAC**
  Real power and voltage angle values are initialized using a DCOPF model. Voltage magnitudes are initialized at 1 and reactive power is inferred using AC transmission line equations.

**Question:** What expectations would we have?
Comparisons between initial conditions

<table>
<thead>
<tr>
<th>Dataset</th>
<th>ic=0</th>
<th>ic=1</th>
<th>ic=2</th>
<th>ic=3</th>
<th>ic=4</th>
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<td>0.174s</td>
<td>0.1734s</td>
<td>0.1902s</td>
<td>0.2658s</td>
<td>0.3548s</td>
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<tr>
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<td>infeas</td>
<td>1.1414s</td>
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<td>case300</td>
<td>1.0392s</td>
<td>1.0866s</td>
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<td>1.2314s</td>
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<td>case2737sop</td>
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<td>infeas</td>
<td>7.9692s</td>
</tr>
<tr>
<td>case3120sp</td>
<td>8.9356s</td>
<td>41.009s</td>
<td>16.9014s</td>
<td>infeas</td>
<td>11.9338s</td>
</tr>
<tr>
<td>case3375wp</td>
<td>14.2058s</td>
<td>93.9364s</td>
<td>infeas</td>
<td>infeas</td>
<td>16.9038s</td>
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</tbody>
</table>

Table: Comparison of initial conditions
Comparison of solvers

<table>
<thead>
<tr>
<th>Dataset</th>
<th>CONOPT</th>
<th>Knitro</th>
<th>IPOPTTH</th>
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<tbody>
<tr>
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<td>8.736s</td>
<td>7.9692s</td>
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<tr>
<td>case3120sp</td>
<td>56.837s</td>
<td>4m 27s</td>
<td>11.9338s</td>
</tr>
<tr>
<td>case3375wp</td>
<td>1m 58s</td>
<td>12.657s</td>
<td>16.9038s</td>
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<tr>
<td>rts96_winter_wend (UC Polar)</td>
<td>8m 50s</td>
<td>14.806s</td>
<td>15.931s</td>
</tr>
</tbody>
</table>

Table: Comparison of Solvers
Comparison of OPF formulations

- **Polar power-voltage** formulation uses polar form of complex quantities and explicitly uses sines and cosines.
- **Rectangular power-voltage** formulation uses the rectangular form of complex quantities, resulting in quadratic power flow constraints.
- **Rectangular current-voltage** formulation models current flow instead of power on a line. Also uses rectangular form of complex quantities, but has linear current flow equations.

<table>
<thead>
<tr>
<th>Dataset</th>
<th>Polar</th>
<th>Rect-PV</th>
<th>Rect-IV</th>
</tr>
</thead>
<tbody>
<tr>
<td>case118</td>
<td>0.702s</td>
<td>0.757s</td>
<td>0.843s</td>
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<tr>
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<td>1.2314s</td>
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<td>1.369s</td>
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<tr>
<td>rts96_winter_wend (UCAC)</td>
<td>15.931s</td>
<td>33.981s</td>
<td>infeas</td>
</tr>
</tbody>
</table>

**Table:** Comparison of OPF formulations
D-curve constraints

- Generator models primarily use “rectangular constraints” for active and reactive output limits.
- A more detailed model is necessary to accurately characterize generator capability curves, which are also called “D-curves”.

\begin{align}
  P^2 + Q^2 & \leq (R_{\text{max}}^\text{max})^2 \\
  P^2 + (Q - Q_0^\text{field})^2 & \leq (r^\text{field})^2 \\
  P^2 + (Q - Q_0^\text{end})^2 & \leq (r^\text{end})^2
\end{align}
Model result using D-curve constraints

<table>
<thead>
<tr>
<th>Dataset</th>
<th>Time (Standard)</th>
<th>Time (D-curves)</th>
<th>Objective (Standard)</th>
<th>Objective (D-curves)</th>
</tr>
</thead>
<tbody>
<tr>
<td>case14</td>
<td>0.3548s</td>
<td>0.424s</td>
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<tr>
<td>case118</td>
<td>0.734s</td>
<td>0.704s</td>
<td>1.29661e+05</td>
<td>1.29913e+05</td>
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<tr>
<td>case300</td>
<td>1.2314s</td>
<td>0.871s</td>
<td>7.19725e+05</td>
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<tr>
<td>case2737sop</td>
<td>7.9692s</td>
<td>8.689</td>
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<td>2.15042e+06</td>
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<tr>
<td>case3375wp</td>
<td>16.9038s</td>
<td>23.046s</td>
<td>7.41203e+06</td>
<td>7.43363e+06</td>
</tr>
</tbody>
</table>

Table: Rectangular vs. D-curve constraints
Dataset Profile:

- Large scale: 13867/13981 buses and 18790/18626 lines for Winter/Summer datasets respectively.
- Datasets compiled using CEII network information and public information provided on FERC e-Library website.
- Includes information on prime movers, tap transformers, interfaces, fuel.
- **Datasets are non-publicly available and part of Critical Energy Infrastructure Information (CEII).**
 Reactive demand

Question: What is the definition of a “realistic” reactive demand profile?

- Good power factor values at each load bus.
  - \[ PF = \frac{P}{S} = \frac{P}{\sqrt{P^2 + Q^2}} \]
  - Given \( P \) and \( S \) limits, can provide bounds on \( Q \)
  - What about when \( P=0 \)?

- Feasibility of values in the ACOPF model.

- A “reasonable” number of buses with non-zero reactive demand values.
  - L-2 or L-1 norm in objective function

- A larger ratio of withdrawals to injections in the overall system.

Our Solution: Minimize reactive demand with respect to AC OPF constraints.
Solution Process

When considering large scale datasets in the AC models, regular solution practices may be insufficient in finding solutions. Large-scale AC models are much harder, if not impossible to solve without good initial conditions.

Is the toolbox useful for a dataset this size?

Procedure 1: Feasibility methodology

1. $(\tilde{P}, \tilde{\theta}, U) \leftarrow \text{Solve UC\_DC} \quad \text{--lineloss=1.055}$
2. $(P, Q, \theta, V) \leftarrow \text{Solve polar\_acopf}(\tilde{P}, \tilde{\theta}, U) \quad \text{--ic=#}$
3. $(P, Q, \theta, V, U) \leftarrow \text{Solve UC\_AC}(\tilde{P}, \tilde{Q}, \tilde{V}, \tilde{\theta}, \tilde{U})$

<table>
<thead>
<tr>
<th>Dataset</th>
<th>ic=0</th>
<th>ic=4</th>
<th>ic=7</th>
</tr>
</thead>
<tbody>
<tr>
<td>Winter, t=20</td>
<td>14m 28.69s</td>
<td>4m 27.31s</td>
<td>4m 35.52s</td>
</tr>
<tr>
<td>Summer, t=18</td>
<td>infeas</td>
<td>4m 34.74s</td>
<td>3m 50.66s</td>
</tr>
</tbody>
</table>
Handling uncertainty

Stochastic models are becoming increasingly important in today’s electricity delivery landscape.

- Uncertainty stemming from wind forecasts
- Contingency planning
- Simple to model using GAMS EMP, with randvar

**Stochastic unit commitment model**

\[
\begin{align*}
\min \mathbb{E}_s[\text{cost}(P_s, U)] & \quad (4) \\
 g(P_s, \theta_s, U) &= D_s \quad (5) \\
 h(P_s, \theta_s, U) &\leq 0 \quad (6)
\end{align*}
\]
Figure: Validation using 10000 independent samples

Mean = 487 778.3
VaR = 563 848.66
CVaR = 740 190.21
Value at Risk (VaR) and Conditional VaR (CVaR) are risk measures, designed to evaluate effects of uncertainty on the outcomes of interest.

$\text{VaR}_\alpha$ is the Value at Risk at the upper $\alpha$ percentile.

**VaR Model**

$$\min \text{VaR}_\alpha[\text{cost}(P_s, U)]$$

and

$$\text{(5 − 6)}$$
Comparing Stochastic UC with VaR and CVaR

**Expected Value Solution**
Mean = 487,778.3
VaR = 563,848.66
CVaR = 740,190.21

**VaR Solution**
Mean = 489,271.9
VaR = 509,773.61
CVaR = 679,487.18

**CVaR Solution**
Mean = 515,189.3
VaR = 544,568.85
CVaR = 582,425.55

**Figure: Difference: EV-VaR**

**Figure: Difference: EV-CVaR**
Other GAMS extensions

- equilibrium
- vi (agents can solve min/max/vi)
- bilevel (reformulate as MPEC or SOCP)
- dualvar (use multipliers from one agent as variables for another)
- Benders decomposition (available in LINDO)
- Distribution sampling (available in LINDO)
- Conversion techniques to PYOMO, AMPL
Conclusions

- **OPF Toolbox as an analytical and solution tool:**
  - Bridges the gap between work done on different software, solvers, formats.
  - Facilitates structured use and analysis of algorithms for solving large-scale and complex problems.
  - Provides access to powerful, established solvers.
  - Enables us to model complex new devices, test policy.
  - Deal with incomplete data.

- **Domain knowledge/expertise is important,** e.g. good starting points, solvers.

- **Ongoing and future work include:**
  - Exploring structured methods to solve large-scale models.
  - Incorporate/test decomposition methods.
  - Further research into stochastic models and solution methods.