Decentralized Robust Optimization Algorithms for Tie-Line Scheduling of Multi-Area Grid with Variable Wind Energy

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Joint work with
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Outline

- Background and Motivation
- Robust Optimization Formulations and Properties
- Distributed Computing Methods through Alternating Direction Method of Multipliers (ADMM)
- Numerical Study and Demonstrations
- Conclusions
Part I: Background and Motivation

- Multi-area power system
  - Physically separated regions (China)
  - Different electricity market
- Intermittent renewable energy
- Random contingencies
- Aligned LMPs for boundary nodes
Challenges in Multi-Area Power Grids

- Systems are cooperative but operated independently
  - Privacy
  - Commercial information

- Protocols between systems to support cooperation:
  - Coordinated Transaction Scheduling: NYISO and PJM
  - Interchange Optimization: PJM and MISO
  - Inter-Regional Interchange Scheduling: ISONE and NYISO

- Interfaces between multiple systems: tie-lines
  - Power flow decisions (day-ahead and intraday)
  - Unit commitment decisions (day-ahead)
Decision Models for Multi-Area Power Grids

- Lagrangian relaxation
  - Conejo and Aguado (1998), Multi-area coordinated decentralized DC optimal power flow
  - Aguado, Quintana, and Conejo (1999), Optimal power flows of interconnected power systems

- Augmented Lagrangian decomposition
  - Kim and Baldick (1997)
  - Ahmadi-Khatir, Conejo, and Cherkaoui (2014)

- Alternative Direction Method of Multipliers (ADMM)
Uncertainty Consideration in Power Grids

- Scenario based probabilistic models
  - Stochastic programming (SP) and chance constrained formulation
  - Explicit large-scale models but many existing computing methods: e.g., Ahmadi-Khatir, Conejo, and Cherkaoui (2013, 2014):
    - SP+ augmented Lagrangian relaxation
  - Issue: Inaccurate prediction of scenarios and probabilities -> infeasible solutions

- Robust optimization models
  - Uncertainty set based compact formulation
  - Produce highly feasible solutions by considering all possibilities inside uncertainty set
  - Two types of algorithms: Benders decomposition and column-and-constraint generation
  - Our aim: integration of decentralized computing scheme + robust optimization
Part II: Robust Optimization Formulations and Properties
Multi-Area Robust Tie-Line Scheduling

- Two-stage decision making framework

\[
\min_{\xi^f \in \Omega^f} \sum_{a \in A} \left\{ \max_{P^w \in U^w} \left[ \min_{\xi^w_a \in \Omega^w_a} C^{ED}_a \left( \xi^f_a, \tilde{P}_a^w \right) \right] \right\}
\]

- Tie-line interchanges: the first-stage decisions before the availability of wind energy is known
  - Phase angles at boundary buses:

\[
\xi^f_a = \{\xi^f_a, \forall a \in A\}, \quad \xi^w_a = \{\delta_a, i_d, \forall g \in G_a, i \in N^a_{BB} \cup N^{BB}_a, t \in T\}
\]

\[
\Omega^f = \{\xi^f \left| \delta_{\phi(i),i,t} = \delta_{\phi(j),j,t}, \quad \delta_{\phi(i),j,t} = \delta_{\phi(j),j,t}, \quad \left| \delta_{\phi(i),j,t} - \delta_{\phi(i),j,t} \right| \right\} \leq \bar{F}_{i,j}, \quad \forall (i, j) \in E^{tie}, \quad i > j, \quad t \in T
\]

- Economic dispatch: the second-stage decision after wind energy is revealed.
  - Continuous model
Inter-regional constraints

- Region-coupling constraints - perceived phase angles are same at two ends of a tie-line

\[ \delta_{a,i,t} = \delta_{b,i,t}, \quad \delta_{a,j,t} = \delta_{b,j,t}, \quad \forall t \in T \]
Economic Dispatch of Each Area

- Dispatch of conventional generation units, wind farms, and phase angles of internal buses

\[
\begin{align*}
\Omega_s^a (\xi^f_a, \tilde{P}_w^a) &= \{ \xi^f_a \} \\
\sum_{j_c \in \Psi^d_a (i)} (\theta_{i,j} - \theta_{j,i}) / X_{i,j} &= \sum_{g \in \Psi^g_a (i)} p^G_{g,t} + \sum_{k \in \Psi^w_a (i)} p^w_{k,t} - \sum_{d \in \Psi^d_a (i)} p^B_{d,t}, \forall i \in N_a^{\text{IB}}, t, \\
\sum_{j_c \in \Psi^d_a (i)} (\delta_{a,i,j} - \theta_{j,i}) / X_{i,j} &= \sum_{g \in \Psi^g_a (i)} p^G_{g,t} + \sum_{k \in \Psi^w_a (i)} p^w_{k,t} - \sum_{d \in \Psi^d_a (i)} p^B_{d,t}, \forall i \in N_a^{\text{IB}}, t, \\
-F_{i,j} &\leq (\theta_{i,j} - \theta_{j,i}) / X_{i,j} \leq F_{i,j}, \forall i \in N_a^{\text{IB}}, j \in \Psi^d_a (i), j > i, \\
-F_{i,j} &\leq (\delta_{a,i,j} - \theta_{j,i}) / X_{i,j} \leq F_{i,j}, \forall i \in N_a^{\text{IB}}, j \in \Psi^d_a (i), \\
p_{g,t}^G + r_{g,t}^{G^+} &\leq p_{g}^G, \forall g \in G_a, t \in T, \\
r_{g,t}^{G^-} &\geq p_{g,t}^G - p_{a}, \forall g \in G_a, t \in T, \\
\sum_{g \in G_a} r_{g,t}^{G^+} &\geq \sum_{g \in G_a} r_{g,t}^{G^-}, \forall t \in T, \\
0 &\leq r_{g,t}^{G^+} \leq RU_{g}^G, 0 \leq r_{g,t}^{G^-} \leq RD_{g}^G, \forall g \in G_a, t \in T, \\
0 &\leq p_{k,t}^w \leq \tilde{P}_{k,t}^w, \forall k \in W_a, t \in T \}
\end{align*}
\]
Observation:

- If the uncertainty sets $\mathcal{U}_a^w$ are polyhedra, the robust multi-area tie-line schedule problem is a convex optimization (an extremely large-scale linear program).

- Idea: enumerating all extreme points of $\mathcal{U}_a^w$ to construct the equivalent formulation, which is an LP.
Multi-Area Generation Unit and Tie-line Scheduling

- Two-stage decision making framework

\[
\min_{\xi^f \in \Omega} \sum_{a \in A} \left\{ C_{UD}^f (\xi^f_a) + \max_{P^w_a \in U^w_a} \left[ \min_{\xi^f \in \Omega} C_{ED}^a (\xi^f_a, \tilde{P}_a^w) \right] \right\}
\]

- First-stage decisions: unit commitments and tie-line interchanges

\[
\xi^f = \{ \xi^f_a, \forall a \in A \}, \xi_s^f = \{ u^G_{g,t}, x^G_{g,t}, y^G_{g,t}, \delta_{a,i,t}, \forall g \in G_a, i \in N_a^{BB} \cup N_a^{tBB}, t \in T \}
\]

\[
\Omega^f = \{ \xi^f \mid \forall a \in A, g \in G_a, t \in T, \delta_{\phi(j),i,t} = \delta_{\phi(j),j,t}, \delta_{\phi(i),i,t} = \delta_{\phi(i),j,t}, \delta_{\phi(i),i,t} - \delta_{\phi(i),j,t} \}
\]

\[
u^G_{g,t} - \nu^G_{g,t-1} = x^G_{g,t} - y^G_{g,t}, \quad \sum_{t = \max\{1, t - MU^G_{a,t} + 1\}}^{t} x^G_{g,t} \leq u^G_{g,t}, \quad \sum_{t = \max\{1, t - MD^G_{a,t} + 1\}}^{t} y^G_{g,t} \leq 1 - u^G_{g,t}, \forall g, t
\]

\[
u^G_{g,t} \in \{0,1\}, \quad 0 \leq x^G_{g,t} \leq 1, \quad 0 \leq y^G_{g,t} \leq 1\]
Multi-Area Generation Unit and Tie-line Scheduling (Cont’d)

- Second-stage decisions: economic dispatch after the available wind power is revealed and unit status are determined

- Observation:
  - Due to binary variables for unit scheduling, the robust formulation is equivalent to a non-convex and discrete mixed integer program

- Challenge: augmented Lagrangian methods typically do not converge
Part III: Distributed Computing Methods through ADMM
Augmented Lagrangian Relaxation

- Relaxing $\delta_{a,i,t} = \delta_{b,i,t}$, $\delta_{a,j,t} = \delta_{b,j,t}$, $\forall t \in T$

- Averaging $\bar{\delta}_{i,t} = \frac{\sum_{a \in \Phi(i)} \delta_{a,i,t}}{|\Phi(i)|}$

- Augmented model

$$
\min_{\xi_a^f} L_a(\xi_a^f, \lambda_a, \bar{\delta}) = \sum_{i \in \mathbb{N}} \sum_{a \in T} \left[ \lambda_{a,i,t} (\delta_{a,i,t} - \bar{\delta}_{i,t}) + \frac{p}{2} (\delta_{a,i,t} - \bar{\delta}_{i,t})^2 \right]$$

$$+ \max_{P_a^* \in U_a^*} \left[ \min_{\xi_a \in \Omega_a(\xi_a^f, P_a^*)} C_a^{ED}(\xi_a, P_a^*) \right]$$

$$= C_a^f(\xi_a^f, \lambda_a, \bar{\delta}) + \max_{P_a^* \in U_a^*} \left[ \min_{\xi_a \in \Omega_a(\xi_a^f, P_a^*)} C_a^{ED}(\xi_a, P_a^*) \right]
$$
Overall Algorithm Scheme

- Using ADMM, each area can be computed independently
  - distributed computing and privacy protection

- For a single area problem: two-stage robust optimization
  - column and constraint generation method
  - finitely convergent for a polyhedron uncertainty set

- Integrated ADMM+CCG (IAC) Solution Method
  - Multi-area robust tie-line scheduling: ADMM+CCG converges to optimal value
  - Multi-area robust generation unit and tie-line scheduling: convergence is NOT guaranteed
  - Computational enhancements? Speed and convergence
Fast Computing

- **Warm Start (WS):**
  - select initial values of the first-stage variables and dual variables using the deterministic version

- **Scenario Retaining (SR):**
  - CCG is repeatedly called within ADMM framework
  - Keep and re-use existing scenarios generated in previous CCG calls

- **Scenario Discard (SD):**
  - Remove non-critical scenarios to maintain a small pool
  - Dynamically manage a scenario pool through a changing threshold

- SR and SD are key steps in distributed computation of Robust Optimization
Convergence Issue from UC

- Non-convergence due to non-convex structure from binary commitment decisions

- Alternating optimization procedure to ensure convergence (heuristically)
  - Alternatively computing with boundary phase angles or commitment status are fixed
  - A repeated commitment status indicates termination
  - Finitely converged
Part IV: Numerical Study and Demonstrations
Two-Area 12-Bus Interconnected System
Tie-line Flows
IAC Performance for Tie-line Scheduling

(a) Maximum residue and Tolerance

(b) Maximum tie-line flow difference
### Computational Enhancement Strategies

<table>
<thead>
<tr>
<th>Case</th>
<th>WS</th>
<th>SR</th>
<th>SD</th>
<th># iter.</th>
<th>Time (s)</th>
<th># iter.</th>
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*M0: IAC without enhancement*
Coordination Effect with Unit Commitment

Slightly higher than the centralized solution by 0.11%
## Performance in Large Systems

<table>
<thead>
<tr>
<th>System</th>
<th>Areas</th>
<th>Units</th>
<th>Buses</th>
<th>Int. Lines</th>
<th>Tie-lines</th>
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<td>66</td>
<td>48</td>
<td>76</td>
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<tr>
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<td>118</td>
<td>174</td>
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### Simulation Results on Large-Scale Test Systems

<table>
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<tr>
<th>Uncertainty Budget</th>
<th>( \Gamma = 0 )</th>
<th>( \Gamma = 6 )</th>
<th>( \Gamma = 12 )</th>
<th>( \Gamma = 24 )</th>
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<tr>
<td>2A-RTS</td>
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<tr>
<td>IAC</td>
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<tr>
<td>Obj. ($)</td>
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<td>6,182,390</td>
<td>6,203,326</td>
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<td>103</td>
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<td>0.26</td>
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<td>Obj. ($)</td>
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<td>Error (%)</td>
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<td>0.03</td>
<td>0.05</td>
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IAC for Tie-line Scheduling
IAC for Generation Unit and Tie-line Scheduling

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<tr>
<th>System</th>
<th>Uncertainty Budget</th>
<th>$\Gamma = 0$</th>
<th>$\Gamma = 6$</th>
<th>$\Gamma = 12$</th>
<th>$\Gamma = 24$</th>
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<td>Gap (%)</td>
<td>0.85</td>
<td>1.06</td>
<td>1.34</td>
<td>1.54</td>
</tr>
</tbody>
</table>
Part V: Conclusions
Observations and Conclusions

- An integrated ADMM+CCG computing method
  - Supporting information and privacy protection in handling uncertainties
  - Advanced enhancement strategies for fast computation
  - New strategies to ensure convergence in non-convex structures

- Coordination plays a critical role in multi-area grid performance
  - For tie-line scheduling, IAC performs (almost) the same as centralized method
  - For commitment and tie-line scheduling, IAC significantly outperforms non-coordinated control

- Future Improvement
  - Economic implications from IAC computation
  - Novel algorithmic improvement to support fast computing